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Modelling a wormhole structure

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SUMMARY

This paper proposes a model of a hypothetical wormhole done by using non-linear equations of a high degree of complexity to obtain exact solutions. Modelling of a wormhole implies creating a particular gravitational field with a hyperbolically symmetric and static geometric body produced for a hyper-cylindrical and symmetric rest mass that can occur in its short tunnel, and its subsequent thermodynamic study. Starting out from the metric in General Relativity, in order that a short wormhole tunnel might be modelled, it is essential to define the coordinates and covariant metric of a 4-dimensional space with three spatial coordinates and one temporal by applying the summation convention on repeated indices in the system of four generalized coordinates. The limiting form of the line element for flat spacetime at large distances from the origin must be Lorentzian. The line element corresponding to the solution proposed here has to remain invariant under inversion of time interval. In this paper, the simplest quadratic scalar invariant of the Riemann tensor, the Kretschmann invariant, has been taken. Furtehmore, curvilinear coordinates in which the metric tensor components that are outside the main diagonal and that have to have zero value for not having a determined direction in space have been used. The metric can be reduced to a flat metric for an observer at rest located in the "infinite" so that the effects caused by the mass in the gravitational field could be deemed negligible. The results obtained a quasi-exact solution to the Einstein's equations that describes the gravitational field induced for a hyper-cylindrical geometry similar to a wormhole tunnel. The so-called "exotic matter" that is hypothesized to reside inside wormhole tunnels and is involved in the energy transport phenomenon has subsequently been studied.

Key words: *wormhole, Einstein field equations, metric, quasi-exact solutions, hyperbolic geometry.*

1. INTRODUCTION

A body with a hyperbolically symmetric geometry resembles a hypothetical wormhole as well as one of the the ways in which the universe is conceived to a local and global geometry. This paper is a contribution to the state-of-the-art modelling and simulation of a geometry that envelops the matter in space-time continuum conceived in a hyperbolic form as a wormhole [1, 2], a problem extensively studied in the area of quantum gravity [3, 4].

This study allows us to set the foundations for investigations into the transport phenomena possible to occur in the short tunnel of the hole for which it has been assumed to contain a kind of "exotic matter" [5, 6] that acquires considerable energy from elementary particles of near zero or zero mass, photons being one of these [7-9].

The modelling process makes use of the Einstein field equations in a 4-dimensional space with three spatial coordinates and one time coordinate. A two-component hyperbolic metric is proposed for a symmetrically hyperbolic and static body produced by a hyperbolic symmetrical body at rest. The other two components are based on the boundary conditions of the Schwarzschild metric [10] for an observer at rest, located at an infinitely large distance from a body producing gravitational effects in space-time continuum.

This paper aims to present a model of the structure of a hypothetical wormhole having quantum gravity with thermodynamic implications [8, 9, 11] containing matter inside its structure (for further study). To this end, "quasi-exact" solutions to the Einstein field equations [12] will be employed to simulate the possible geometric behaviour that the matter adopts in space-time continuum.

The Einstein's general relativity theory suited to the modelling of equations describing the gravitational interaction between matter and space-time, offers non-linear equations with which it is difficult to obtain exact solutions [12] to define a given geometry [2, 13, 14]. For the reasons mentioned, this paper presents "quasi-exact" solutions [12] to the Einstein equations based on the solutions obtained for the Ricci tensor, for a wormhole structure that could contain so-called "exotic matter". The "exotic matter" is thought to may have properties which allow establishing communication with parallel universes, namely through its input and output modelled as a hyperboloid sheets [5, 6] and connected by a short tunnel.

The results obtained suggest that the proposed metric reduces to the Lorentzian metric [15-17] for the line element used to calculate the Kretschmann invariant [18].

2. MODELLING

The Schwarzschild solution [10] to the Einstein field equations is considered to be the most important achievement of general relativity in the field of Celestial Mechanics because it presents an exact solution to the field equations [2, 12-14] that historically corresponds with the Newtonian result of the square inverse of the universal attractive force of classical gravitational theory [19].

The Einstein field equations for free space are non-linear and therefore very complicated, making it difficult to obtain exact solutions [12]. There is, however, a special case that can be solved without considerable difficulties, the case of a symmetrically spherical and static field produced by a spherically symmetrical body at rest [10].

This paper presents a model of the special case of a symmetrically hyperbolic and static field produced by a hyperbolically symmetric body at rest.

The common starting point for the theoretical considerations of this type is the general metric used in the 4-dimensional space of general relativity [15], applied commonly as the summation convention on the repeated indices:

$$ds^2 = g_{\delta y} dx^{\delta} dx^{y} \qquad (1)$$

in the system of four generalized coordinates (x^1, x^2, x^3, x^4). The limiting form of the line element ds^2 for a flat space-time at large distances from the origin ($r \to \infty$) must be Lorentzian:

$$ds^2 = C_1^2 dt^2 - dx^2 - dy^2 - dz^2 \qquad (2)$$

This is so because the Special Theory of Relativity is deemed a special case of General Relativity when the curvature of space-time is reduced to zero at large distances [20]. In order to construct a valid line element ds^2 at distances close to the origin of a static field, we assume that this line element remains invariant under the inversion of the time interval dt; that is, ds^2 must remain equal to the change of dt for $-dt$, as the static condition allows us to use a static coordinate system where the metric components g_{ij} are independent of the time coordinate ($t=x^1$). This determines the use of curvilinear coordinates in which the elements g_{ij} "outside the main diagonal" are zero and that the line element has the following form:

$$g_{11}(dt)^2 + g_{ik} dx^i dx^k \qquad (3)$$

g_{ik} being independent of time coordinate ($t=x^1$). This is reffered to as a static metric and it has to be distinguished from a metric that is merely independent of time or stationary (as the metric for cylindrical coordinates in a space of three dimensions is).

Moreover, if there is no preferred angular direction in space, the line element ds^2 should not depend on the change from $d\theta$ to $-d\theta$ and change from $d\phi$ to $-d\phi$. This situation requires that no crusaders terms such as $dr{\cdot}d\theta$, $d\theta{\cdot}d\phi$ and $dr{\cdot}d\phi$ are present in the metric since the absence of such terms ensures the tensor to be completely diagonal for the type of solution being sought for. The limiting form of the hyper-cylindrical line element (hyperbolic cylinder) is expressed as:

$$ds^2 = AC_1^2 dt^2 - \\ - \left(Bdr^2 + C\frac{1}{2}\frac{d\theta^2}{\sqrt{r^2+\theta^2}} + D\frac{1}{2}\frac{d\phi^2}{\sqrt{r^2+\theta^2}} \right) \qquad (4)$$

Under the assumptions that our hypothetical body is radially symmetrical, the functions A, B, C and D ought to be functions only of the radial coordinate r:

$$A=A(r),\ B=B(r),\ C=C(r),\ D=D(r) \qquad (5)$$

Since the displacement of arc $\varepsilon=rd\theta$ from the North Pole corresponds to an interval in the line element $ds^2=-C\varepsilon^2$ and an arc displacement $\varepsilon=rd\phi$ along the Equator corresponds to an interval in the line element $ds^2=-D\varepsilon^2$, it is assumed that $C(r)=D(r)$. If θ and ϕ represent angular coordinates, one would expect that these amounts were identical for isotropy, from which ensues that C equals D, a condition of symmetry leading to the simplification of our line element ds^2 as:

$$ds^2 = AC_1^2 dt^2 - \\ - Bdr^2 - C\left(\frac{1}{2}\frac{d\theta^2}{\sqrt{r^2+\theta^2}} + \frac{1}{2}\frac{d\phi^2}{\sqrt{r^2+\theta^2}} \right) \qquad (6)$$

The presented line element is the simplest solution governed by the elements of symmetry. A further simplification can be obtained by a judicious choice of the radial coordinate. It is assumed that A and B are some sort of function so the proposed metric may be reduced to a flat metric for a stationary observer located at a distance large enough so that the effects caused by mass on the gravitational field are numerically insignificant. Otherwise, the gravitational effect caused by a body would extend to infinity. This metric would correspond to Eq. (2).

Following the mentioned changes, $A(r)$ and $B(r)$ shoul be reduced to a unit at an enormous distance from the body that causes curvature in space-time:

$$A(r)\to 1 \quad \text{for} \quad r\to\infty \tag{7}$$
$$B(r)\to 1 \quad \text{for} \quad r\to\infty \tag{8}$$

A tentative mathematical function reduced to the unit for large negative values of x is the exponential mathematical function $f(x)=e^x$. The derivative of this function is the same function, except for the sign, which can be translated into possible simplifications.

Since it is required that both $A(r)$ and $B(r)$ reduce to the unit for large values of r, an exponential character with a dependent exponents of r is assigned to both of them:

$$A(r) = e^{u(r)} \tag{9}$$
$$B(r) = -e^{v(r)} \tag{10}$$

To be able to fulfill the requirement according to which coefficients $A(r)$ and $B(r)$ reduce to a unit at large distances, exponents $u(r)$ and $v(r)$ should be reduced to zero:

$$u(r) \to 0 \text{ for } r \to \infty \tag{11}$$
$$v(r) \to 0 \text{ for } r \to \infty \tag{12}$$

As the gravitational field around a symmetrically hyper-cylindrical mass should also be symmetrically hyper-cylindrical, the field must be independent of the angular coordinates θ and ϕ, so that the possibilities:

$$u = u(t,r,\theta,\phi) \tag{13}$$
$$v = v(t,r,\theta,\phi) \tag{14}$$

are necessarily reduced to:

$$u = u(t,r) \tag{15}$$
$$v = v(t,r) \tag{16}$$

In the present claculation, it is assumed that, besides being symmetric and hyper-cylindrical, the gravitational field should also be static and time-invariant. Since static and time-invariant, the field is independent of the temporal coordinate t, consequently:

$$u = u(r) \tag{17}$$
$$v = v(r) \tag{18}$$

which is precisely what has been put forth in the paper.

At this point of the research, functions $u(r)$ and $v(r)$ are unknown and not possible to be determined. Without loss of generality, we can multiply the functions by a numerical constant without changing the nature of the behaviour of the functions over longer distances. An appropriate numerical constant would be number 2 because they are all squared with respect to the preliminary proposed metric, thus in order to simplify the metric even further, the following equations are offered:

$$A(r) = e^{2u(r)} \tag{19}$$
$$B(r) = -e^{2v(r)} \tag{20}$$

Following the given proposal, a tentative metric for a symmetrically hyper-cylindrical and static field produced by a hyper-cylindrical symmetrical body at rest is:

$$ds^2 = e^{2u(r)}\left(C_1 dt\right) - \\ - e^{2v(r)}dr^2 - \frac{1}{2}\frac{d\theta^2}{\sqrt{r^2+\theta^2}} - \frac{1}{2}\frac{d\phi^2}{\sqrt{r^2+\theta^2}} \tag{21}$$

Having defined the proposed tentative metric, we can obtain the Christoffel symbols different from zero, corresponding to the following metric:

$$\begin{bmatrix} e^{2u(r)} & 0 & 0 & 0 \\ 0 & -e^{2v(r)} & 0 & 0 \\ 0 & 0 & -\frac{1}{2}\frac{1}{\sqrt{r^2+\theta^2}} & 0 \\ 0 & 0 & 0 & -\frac{1}{2}\frac{1}{\sqrt{r^2+\theta^2}} \end{bmatrix} \tag{22}$$

The coordinates of this 4-dimensional space are t, r, θ and ϕ, and the only components $g_{\delta y}$ belonging to the metric tensor $\boldsymbol{g}$ that are not zero are:

$$g_{11}=g_{tt}=e^{2u(r)}, \quad g_{22}=g_{rr}=-e^{2v(r)} \tag{23}$$

$$g_{33} = g_{\theta\theta} = -\frac{1}{2}\frac{1}{\sqrt{r^2+\theta^2}}, \quad g_{44} = g_{\phi\phi} = -\frac{1}{2}\frac{1}{\sqrt{r^2+\theta^2}} \tag{24}$$

Given that matrix $[g_{\delta y}]$ which lumps together the components of the metric tensor is a diagonal matrix, matrix $[g^{\delta y}]$ that lumps together the components of the conjugate metric tensor will also be a diagonal matrix and their respective elements would simply be reciprocals of the elements of the matrix $[g_{\delta y}]$. For that reason, the only components of the conjugate metric tensor $\boldsymbol{g}^{-1}$ that are not equal to zero are:

$$g^{11} = g^{tt} = e^{-2u(r)}, \quad g^{22} = g^{rr} = -e^{-2v(r)} \tag{25}$$

$$g^{33} = g^{\theta\theta} = -2\sqrt{r^2+\theta^2}, \quad g^{44} = g^{\phi\phi} = -2\sqrt{r^2+\theta^2} \tag{26}$$

At this point, all the information needed to obtain the Christoffel symbols for the tentative metric under construction are known.

Christoffel symbols of the second class, the cornerstone of the Riemann-Christoffel tensor or Einstein contracted tensor and the Ricci tensor are:

$$\begin{gathered} \{1,12\} = u', \quad \{2,11\} = u'e^{2u}/e^{2v} \quad \{2,22\} = v', \\ \{2,33\} = r\Big/\left(4e^{2v}\left(r^2+\theta^2\right)^{3/2}\right), \quad \{2,44\} = r\Big/\left(4e^{2v}\left(r^2+\theta^2\right)^{3/2}\right), \quad \{3,23\} = -r\Big/2\left(r^2+\theta^2\right), \\ \{3,33\} = -\theta\Big/2\left(r^2+\theta^2\right), \quad \{3,44\} = \theta\Big/2\left(r^2+\theta^2\right), \quad \{4,24\} = -r\Big/2\left(r^2+\theta^2\right), \\ \{4,34\} = -\theta\Big/2\left(r^2+\theta^2\right). \end{gathered} \tag{27}$$

With the Christoffel symbols, we proceed to evaluate the components of the Ricci tensor by tensorial contraction on the Riemann tensor by repeated indices which make use of the summation convention:

$$R_{ij} = R^k_{ijk} = R^1_{ij1} + R^2_{ij2} + R^3_{ij3} + R^4_{ij4} \tag{28}$$

Furthermore, we need to obtain the components of the Riemann tensor. The components of the Ricci tensor, which is a diagonal tensor, in generalized coordinates and specific coordinates to the metric, respectively, are:

$$\begin{array}{l} R_{11} = R^1_{111} + R^2_{112} + R^3_{113} + R^4_{114} \\ R_{22} = R^1_{221} + R^2_{222} + R^3_{223} + R^4_{224} \\ R_{33} = R^1_{331} + R^2_{332} + R^3_{333} + R^4_{334} \\ R_{44} = R^1_{441} + R^2_{442} + R^3_{443} + R^4_{444} \end{array} ; \quad \begin{array}{l} R_{tt} = R^t_{ttt} + R^r_{ttr} + R^\theta_{tt\theta} + R^\phi_{tt\phi} \\ R_{rr} = R^t_{rrt} + R^r_{rrr} + R^\theta_{rr\theta} + R^\phi_{rr\phi} \\ R_{\theta\theta} = R^t_{\theta\theta t} + R^r_{\theta\theta r} + R^\theta_{\theta\theta\theta} + R^\phi_{\theta\theta\phi} \\ R_{\phi\phi} = R^t_{\phi\phi t} + R^r_{\phi\phi r} + R^\theta_{\phi\phi\theta} + R^\phi_{\phi\phi\phi} \end{array} \tag{29}$$

requiring for it only 16 components of the Riemann tensor. In making these evaluations, only non-zero components are the diagonal components of the Ricci tensor:

$$\begin{aligned} R_{11} &= -e^{2u-2v}\left(u''r^2 + u''\theta^2 + (u')^2 r^2 + (u')^2\theta^2 - u'v'r^2 - u'v'\theta^2 - u'r\right) \Big/ \left(r^2+\theta^2\right) \\ R_{22} &= \left(\begin{array}{l} 2u''r^4 + 4u''r^2\theta^2 + 2u''\theta^4 + 2(u')^2 r^4 + 4(u')^2 r^2\theta^2 + 2(u')^2\theta^4 - \\ -2u'v'r^4 - 4u'v'r^2\theta^2 - 2u'v'\theta^4 + 2v'r^3 + 2v'r\theta^2 + 3r^2 - 2\theta^2 \end{array}\right) \Big/ 2\left(r^2+\theta^2\right) \\ R_{23} &= r\theta \Big/ \left(r^2+\theta^2\right)^2 \\ R_{33} = R_{44} &= -\frac{1}{4}\frac{1}{e^{2v}\left(r^2+\theta^2\right)^{15/2}} \left(\begin{array}{l} -15v'r^9\theta^4 - 20v'r^7\theta^6 - 15v'r^5\theta^8 - 6v'r^3\theta^{10} - v'r\theta^{12} - \\ -2\left(r^2+\theta^2\right)^{5/2} e^{2v}\theta^8 + 2\left(r^2+\theta^2\right)^{5/2} e^{2v} r^8 + 6u'r^{11}\theta^2 + \\ +15u'r^9\theta^4 + 20u'r^7\theta^6 + 15u'r^5\theta^8 + 6u'r^3\theta^{10} + u'r\theta^{12} - \\ -9r^{10}\theta^2 - 15r^8\theta^4 - 10r^6\theta^6 + 3r^2\theta^{10} - v'r^{13} + u'r^{13} - \\ -4\left(r^2+\theta^2\right)^{5/2} e^{2v}\theta^6 r^2 - 2r^{12} + \theta^{12} - 6v'r^{11}\theta^2 \end{array}\right) \end{aligned} \tag{30}$$

In vacuum, the field equations of General Relativity require that these expressions vanish for a sufficiently small test particle (with mass in the case of a material particle and without rest mass in the case of a photon of light) so that this does not produce a curvature in space-time:

$$R_{11} = R_{22} = R_{33} = R_{44} = 0 \tag{31}$$

The solution of the Ricci tensor equations shows us that R_{33}–$R_{44}=0$ and $R_{23}=0$ as well. This nullification (Eq. (31)) imposes the following condition:

$$\frac{dv(r)}{dr} = -\frac{1}{2}\frac{3r^2 - 2\theta^2}{2r\left(r^2+\theta^2\right)} \tag{32}$$

The Ricci tensor equations were solved as a system and the following three solutions were obtained:

$$\frac{1}{\sqrt{r^2+\theta^2 r^2}} = \left(\begin{array}{l} 2v''r^6\sqrt{r^2+\theta^2} - 16e^{2v}r^6 - 12v'e^{2v}r^7 - 12v'e^{2v}r^5\theta^2 + 6v'r^5\sqrt{r^2+\theta^2} + 4v''r^4\sqrt{r^2+\theta^2}\,\theta^2 + \\ +7r^4\sqrt{r^2+\theta^2} - 4e^{2v}r^4\theta^2 + 8r^4\left(r^2+\theta^2\right)^{3/2}\left(e^{2v}\right)^2 + 4v'r^3\sqrt{r^2+\theta^2}\,\theta^2 + 2v''r^2\sqrt{r^2+\theta^2}\,\theta^4 - \\ -16r^2\left(r^2+\theta^2\right)^{3/2}\left(e^{2v}\right)^2\theta^2 - 2rv'\sqrt{r^2+\theta^2}\,\theta^4 + 12r^3v'e^{2v}\theta^4 + 12rv'e^{2v}\theta^6 - 12e^{2v}\theta^6 + \\ +8r^2\left(r^2+\theta^2\right)^{3/2}\left(e^{2v}\right)^2\theta^4 + 4\sqrt{r^2+\theta^2}\,\theta^4 \end{array}\right) \tag{33}$$

$$-\frac{1}{r^2\left(r^2+\theta^2\right)^{3/2}}=$$
$$=\left(\left(\begin{array}{l} v''r^6\sqrt{r^2+\theta^2}-6e^{2v}r^6-6v'e^{2v}r^7-6v'e^{2v}r^5\theta^2+ \\ +v'r^5\sqrt{r^2+\theta^2}+2v''r^4\sqrt{r^2+\theta^2}\theta^2+4r^4\left(r^2+\theta^2\right)^{3/2}\left(e^{2v}\right)^2- \\ -2e^{2v}\theta^2r^4-2e^{2v}\theta^4r^2+2r^2\sqrt{r^2+\theta^2}\theta^2+v''r^2\sqrt{r^2+\theta^2}\theta^4- \\ -8r^2\left(r^2+\theta^2\right)^{3/2}\left(e^{2v}\right)^2\theta^2-rv'\sqrt{r^2+\theta^2}\theta^4+6r^3v'e^{2v}\theta^4+ \\ +6rv'e^{2v}\theta^6+2\sqrt{r^2+\theta^2}\theta^4+4\left(r^2+\theta^2\right)^{3/2}\left(e^{2v}\right)^2\theta^4-6\theta^6e^{2v} \end{array}\right)e^{2\left(\int\frac{vr^3-2e^{2v}\sqrt{r^2+\theta^2}r^2+2r^2+v'r\theta^2-\theta^2+\sqrt{r^2+\theta^2}e^{2v}\theta^2}{r\left(r^2+\theta^2\right)}dr\right)}+2\_C1\right)=0 \tag{34}$$

$$u=\int\frac{v'r^3-2e^{2v}\sqrt{r^2+\theta^2}r^2+2r^2+v'r\theta^2-\theta^2+\sqrt{r^2+\theta^2}e^{2v}\theta^2}{r\left(r^2+\theta^2\right)}dr+2\_C1 \tag{35}$$

Finally, the solutions to the Eq. (33) are:

$$particular\_sol=v(r)=\frac{-r-e^{-C1}r+\_C2}{e^{-C1}} \tag{36}$$

$$sol\_red\_2=\_f1(\_c)=\frac{e^{e^{-C}}}{2Ei\left(1,-e^{-C}\right)+\_C1} \tag{37}$$

The general solution to the ordinary differential equation (ODE), follows:

$$general\_sol:=y(x)=x+RootOf\left(-x+\sqrt{2}\left(\int^{\_Z}\frac{e^{-C1}}{\sqrt{2Ei\left(1,-e^{-b}\right)+\_C3}}d\_b\right)+\_C2\right) \tag{38}$$

Comparing Eq. (32) to Eq. (36), it is obtained that:

$$e^{-C1}=\frac{2r^2\left(r^2+\theta^2\right)}{2r^3+2r\theta^2+3r^2-2\theta^2} \tag{39}$$

$$\_C2=-e^{-C1} \tag{40}$$

Introducing these constants, that is, Eqs. (39) and (40), into the Eq. (36), the following statement is obtained:

$$v(r)=\frac{1}{2}\frac{1}{r\left(r^2+\theta^2\right)}\left(\left(-r+\frac{2r^2\left(r^2+\theta^2\right)+2r\left(r^2+\theta^2\right)}{2r^3+2r\theta^2+3r^2-2\theta^2}\right)\left(2r^3+2r\theta^2+3r^2-2\theta^2\right)\right) \tag{41}$$

Introducing Eq. (41) in the component of the metric $g_{22}=e^{2v(r)}$, we get:

$$g_{22}=e^{2v(r)}=e^{\frac{1}{r\left(r^2+\theta^2\right)}\left(\left(-r+\frac{2r^2\left(r^2+\theta^2\right)+2r\left(r^2+\theta^2\right)}{2r^3+2r\theta^2+3r^2-2\theta^2}\right)\left(2r^3+2r\theta^2+3r^2-2\theta^2\right)\right)} \tag{42}$$

In the limit, when $r\to\infty$, the Eq. (42) tends to 1 (just as in the boundary conditions imposed by Schwarzschild [10] for spherical symmetry).

Therefore, $\underset{r\to\infty}{limit}\left(e^{2v(r)}\right)\to 1$.

This is an exact solution to Einstein vacuum equations since the obtained metrics gives Lorentzian metrics [15, 16] for the component $g_{22}=e^{2v(r)}$, so:

$$ds^2=e^{2u(r)}dt^2-1\cdot dr^2-C\left(\frac{1}{2}\frac{d\theta^2}{\sqrt{r^2+\theta^2}}+\frac{1}{2}\frac{d\phi^2}{\sqrt{r^2+\theta^2}}\right) \tag{43}$$

Alternative solutions of the Ricci tensor, taking into account the nullification of Eq. (31) and comparing R_{11} to R_{22} we find out that:

$$r=\sqrt{5}/5; \qquad \theta=\sqrt{30}/10 \tag{44}$$

In addition, introducing Eq. (44) in the single equation of the Ricci tensor resulting from vanishing R_{33} expressed by Eq. (31), the following expression is obtained:

$$R_{33}=-\left(32\left((-1/320)v'\sqrt{5}-(17/40000)\sqrt{2e}^{2v}+(1/320)u'\sqrt{5}-1/320-(27/2000)\sqrt{2e}^{2v}\left(\sqrt{5}/5\right)^{2}\right)\sqrt{2}\right)\Big/e^{2v} \tag{45}$$

This differential equation is classified as quadrature to $u(r)$ and of "1^{st} order, with symmetry [$F(r)$, $G(r)$]" for $v(r)$. Resolving Eq. (45) with symmetries *ábaco1* and *ábaco2* using the Maple software, it is possible to find the following solutions for both symmetries.

Computing symmetries using: *way=abaco1*, starting from: *symgen* (Eq. (45), *way=abaco1, v(r)*):

$$v(r)=\frac{-125}{34}\sqrt{10}e^{-2u(r)+\frac{2}{5}\sqrt{5}r+\frac{216}{125}e^{v}\left(\frac{\sqrt{5}}{5}\right)^{4}r\sqrt{10}} \tag{46}$$

$$v(r)=\frac{-5}{34}e^{-2u+\frac{2}{5}\sqrt{5}r+\frac{216}{125}e^{v}\left(\frac{\sqrt{5}}{5}\right)^{4}r\sqrt{10}}\left(25u'\sqrt{10}-25\sqrt{2}-216e^{v}\left(\frac{\sqrt{5}}{5}\right)^{4}\right) \tag{47}$$

The computation of symmetries has been successful.

Making comparisons between Eq. (46) and Eq. (47), we solve e^{v} obtaining the following equation:

$$e^{v(r)}=-\frac{125}{432}\left(-10u'(r)+\sqrt{10}\sqrt{2}+10\right)\sqrt{10} \tag{48}$$

Eq. (48) is solved for $u(r)$ as:

$$u(r)=e^{v(r)}-\frac{625}{216}\left(e^{-\_C1\sqrt{10}}-\sqrt{2}\right) \tag{49}$$

Then the limit, when $r\to\infty$:

$$\lim_{r\to\infty} it\left(e^{2u(r)}\right)\to e^{2+\frac{625}{216}\sqrt{2}} \tag{50}$$

which is a small finite number negligible for an observer located at a distance large enough to consider it infinitely large.

This is yet another proof that it fulfill the Lorentzian metric [15] for the component $g_{11}=e^{2u(r)}$ of the proposed metric.

Introducing Eqs. (36) and (39) into Eq. (49), the mathematical expression for $u(r)$ is obtained:

$$u(r)=e^{\frac{1}{2}\frac{1}{r\left(r^{2}+\frac{1}{4}\pi^{2}\right)}\left(\left(-r+\frac{2r^{2}\left(r^{2}+\frac{1}{4}\pi^{2}\right)+2r\left(r^{2}+\frac{1}{4}\pi^{2}\right)}{2r^{3}+\frac{1}{2}r\pi^{2}+3r^{2}-\frac{1}{2}\pi^{2}}\right)\left(2r^{3}+\frac{1}{2}r\pi^{2}+3r^{2}-\frac{1}{2}\pi^{2}\right)-\right.}$$
$$\left.-\frac{625}{216}\left(e^{-\_C1\sqrt{10}}-\sqrt{2}\right)\right) \tag{51}$$

$$g_{11}=e^{2u(r)}=e^{2e^{\frac{1}{2}\frac{\left(-r+\frac{2r^{2}\left(r^{2}+\theta^{2}\right)}{2r^{3}+2r\theta^{2}+3r^{2}-2\theta^{2}}+\frac{2r^{2}\left(r^{2}+\theta^{2}\right)}{2r^{3}+2r\theta^{2}+3r^{2}-2\theta^{2}}\right)\left(2r^{3}+2r\theta^{2}+3r^{2}-2\theta^{2}\right)}{r\left(r^{2}+\theta^{2}\right)}}+\frac{625}{216}\frac{\left(2r^{3}+2r\theta^{2}+3r^{2}-2\theta^{2}\right)\sqrt{10}}{r\left(r^{2}+\theta^{2}\right)}+\frac{625}{216}\sqrt{2}} \tag{52}$$

Further on, $g_{11}=e^{2u(r)}$, when included in the limit where $r\to\infty$, has a value corresponding to the value obtained from Eq. (50). Having done this, the Lorentzian metric for g^{11} is proved and the arc element becomes:

$$ds^{2}=e^{2+\frac{625}{108}\sqrt{2}}dt^{2}-1\cdot dr^{2}-C\left(\frac{1}{2}\frac{d\theta^{2}}{\sqrt{r^{2}+\theta^{2}}}+\frac{1}{2}\frac{d\phi^{2}}{\sqrt{r^{2}+\theta^{2}}}\right) \tag{53}$$

In the same way, the boundary conditions imposed by Schwarzschild for a static and symmetric spherical body reduce to Lorentz metric [10, 15-17]. Furthermore, it has been shown that the metrics proposed here for a symmetrically hyperbolic and static mass, Eq. (6), reduced to the Lorentz metric expressed in Eq. (2) as well. These solutions are valid for sufficiently large distances from the body or bodies that generate gravitational fields causing the curvature. Therefore, the Special Theory of Relativity is actually a special case of the General Theory of Relativity.

Turning the hyperbolic coordinates to Cartesian coordinates, we have the following situation:

if Cartesian:

$$x = r \tag{54}$$

$$y = \theta \tag{55}$$

then hyperbolic:

$$x = \sqrt{\sqrt{r^2+\theta^2}+r} \tag{56}$$

$$y = \sqrt{\sqrt{r^2+\theta^2}-r} \tag{57}$$

Eq. (6) turned into Eq. (53) in hyperbolic coordinates becomes the Lorentzian metric [15] according the following procedure:

Deriving Eq. (56) for x with respect to r and at θ, we have:

$$dx = \frac{1}{2}\frac{\frac{r}{\sqrt{r^2+\theta^2}}+1}{\sqrt{\sqrt{r^2+\theta^2}+r}}dr + \frac{1}{2}\frac{\theta}{\sqrt{\sqrt{r^2+\theta^2}+r}\sqrt{r^2+\theta^2}}d\theta \tag{58}$$

The next step is squaring Eq. (58):

$$(dx)^2 = \left(\frac{1}{2}\frac{\frac{r}{\sqrt{r^2+\theta^2}}+1}{\sqrt{\sqrt{r^2+\theta^2}+r}}dr + \frac{1}{2}\frac{\theta}{\sqrt{\sqrt{r^2+\theta^2}+r}\sqrt{r^2+\theta^2}}d\theta\right)^2 \tag{59}$$

Likewise, for Eq. (57):

$$dy = \frac{1}{2}\frac{\frac{r}{\sqrt{r^2+\theta^2}}-1}{\sqrt{\sqrt{r^2+\theta^2}-r}}dr + \frac{1}{2}\frac{\theta}{\sqrt{\sqrt{r^2+\theta^2}-r}\sqrt{r^2+\theta^2}}d\theta \tag{60}$$

$$(dy)^2 = \left(\frac{1}{2}\frac{\frac{r}{\sqrt{r^2+\theta^2}}-1}{\sqrt{\sqrt{r^2+\theta^2}-r}}dr + \frac{1}{2}\frac{\theta}{\sqrt{\sqrt{r^2+\theta^2}-r}\sqrt{r^2+\theta^2}}d\theta\right)^2 \tag{61}$$

Then, simplifying the sum of Eqs. (59) and (61), it is obtained:

$$dx^2 + dy^2 = \frac{1}{2}\frac{dr^2+d\theta^2}{\sqrt{r^2+\theta^2}} \tag{62}$$

Having done this, we have proved that the proposed metric corresponds to the Lorentzian metric [15]. Introducing and implementing these expressions in the line element on which we started off, i.e. Eq. (6), we are a step from the final answer for the metric that has been sought for, that is, a metric for a symmetrically hyperbolic and static field produced by a hyperbolically symmetric body at rest. This is achieved by inserting Eqs. (42) and (52) into the Eq. (21). The process gives the dimensionally correct metric as follows:

$$ds^2 = e^{2e^{\frac{1}{2}\frac{\left(-r+\frac{2r^2(r^2+\theta^2)}{2r^3+2r\theta^2+3r^2-2\theta^2}+\frac{2r^2(r^2+\theta^2)}{2r^3+2r\theta^2+3r^2-2\theta^2}\right)(2r^3+2r\theta^2+3r^2-2\theta^2)}{r(r^2+\theta^2)}}+\frac{625}{216}\frac{(2r^3+2r\theta^2+3r^2-2\theta^2)\sqrt{10}}{r(r^2+\theta^2)}+\frac{625}{216}\sqrt{2}(C_1dt)^2} -$$

$$-e^{\frac{1}{r(r^2+\theta^2)}\left(\left(-r+\frac{2r^2(r^2+\theta^2)2r(r^2+\theta^2)}{2r^3+2r\theta^2+3r^2-2\theta^2}\right)(2r^3+2r\theta^2+3r^2-2\theta^2)\right)}dr^2 - \frac{1}{2}\frac{d^2\theta}{\sqrt{r^2+\theta^2}} - \frac{1}{2}\frac{d^2\phi}{\sqrt{r^2+\theta^2}} \tag{63}$$

We can compare this element of arc with the Schwarzschild counterpart [19]:

$$ds^2 = (1+2m/r)c^2dt^2 - (1+2m/r)^{-1}dr^2 - r^2\left(d\theta^2 + sin^2(\phi)d\phi^2\right); \quad m = GM/c^2$$

where m is known as the geometric mass of the central body and has units of distance, c is the speed of light, and G is the gravitational constant.

The offered solution presents a mathematically quasi-exact solution to the Einstein field equations and which, in turn, allows us to model a wormhole structure having a hyperbollicaly symmetrical and static body as the starting point.

The solutions to the Eq. (35) for $v(r)$ are the following:

$$v(r) = \frac{1}{2}\frac{\left(r^2+\theta^2\right)^{7/2}}{r(r-\theta)(r+\theta)} \tag{64}$$

$$v(r) = -\frac{1}{2}\frac{\left(r^2+\theta^2\right)^{5/2}\left(2r^2-\theta^2\right)}{r^2(r-\theta)(r+\theta)} \tag{65}$$

$$v(r) = \left(r^4+3r^2\theta^2+3\theta^4+\theta^6/r^2\right)e^{2v(r)} \tag{66}$$

$$v(r)=0 \tag{67}$$

$$v(r) = \frac{1}{2}ln\left(\frac{15}{2}r^2 \middle/ \left(\begin{array}{l}-15\_C1r^6 - 45\_C1r^4\theta^2 - 45\_C1r^2\theta^4 - \\ -15\_C1\theta^6 - 2\theta^2\sqrt{r^2+\theta^2} + 10\sqrt{r^2+\theta^2}\,r^2\end{array}\right)\right) \tag{68}$$

Integrating Eqs. (64) and (65) to obtain $u(r)$, we have:

$$z = \frac{1}{2}\int\frac{\left(r^2+\theta^2\right)^{7/2}}{r(r-\theta)(r+\theta)} \tag{69}$$

$$k = -\frac{1}{2}\int\frac{\left(r^2+\theta^2\right)^{5/2}\left(2r^2-\theta^2\right)}{r^2(r-\theta)(r+\theta)}dr \tag{70}$$

The result of the integration of z and k is unwieldy and, therefore, not presented in the paper. Deriving the results of Eqs. (69) and (70), two solutions are obtained for $u(r)$:

$$u(r)_1 = \frac{dz}{dr} \tag{71}$$

$$u(r)_2 = \frac{dk}{dr} \tag{72}$$

Afterwards, two results for g_{11} are:

$$g_{11} = e^{2u(r)_1} \tag{73}$$

$$g_{11} = e^{2u(r)_2} \tag{74}$$

Deriving the Eq. (36) twice, and introducing these derivatives and $v(r)$ of the mentioned equation in the Eq. (33), the constants obtained are:

$$\_C2 = \frac{2r}{2r+2} \tag{75}$$

$$e^{-C1} = -\_C2 \tag{76}$$

Introducing the constants, Eqs. (75) and (76) in Eq.(36), it is obtained:

$$v(r) = \frac{1}{2}\frac{\left(-r+\frac{2r^2}{2r+2}+\frac{2r}{2r+2}\right)(2r+2)}{r} \tag{77}$$

Another solution to Eq. (33) would be:

$$v(r)=\frac{1}{2}ln\left(-\frac{1}{8}\frac{1}{r^6-r^4\theta^2-r^2\theta^4+\theta^6}\left(\left(\begin{array}{l}\sqrt{r^2+\theta^2}\,r^2-9\sqrt{r^2+\theta^2}\,\theta^2-\\-\sqrt{-7r^6-9r^4\theta^2+71r^2\theta^4+73\theta^6}\end{array}\right)r^2\right)\right) \tag{78}$$

The above-presented solution, i.e. Eq. (78), is obtained by deriving the condition imposed on the vanishings, Eq. (32).

Incorporating derivatives and $v(r)$ of Eq. (36) in Eq. (32), the following constants are obtained:

$$\_C2=r+e^{-C1}r-\frac{1}{2}ln\left(-\frac{1}{8}\frac{1}{r^6-r^4\theta^2-r^2\theta^4+\theta^6}\left(\left(\begin{array}{l}\sqrt{r^2+\theta^2}\,r^2-9\sqrt{r^2+\theta^2}\,\theta^2-\\-\sqrt{-7r^6-9r^4\theta^2+71r^2\theta^4+73\theta^6}\end{array}\right)r^2\right)\right)e^{-C1} \tag{79}$$

$$e^{-C1}=-\_C2 \tag{80}$$

Equations (79) and (80) are introduced into Eq. (36) to obtain Eq. (78).

Introducing derivatives and $v(r)$ of Eq. (36) into Eq. (33), two expressions are obtained for e^{-C1}, one of them being (the other expression is omitted since unwieldy):

$$e^{-C1}=\frac{-r+\_C2}{r} \tag{81}$$

3. RESULTS AND DISCUSSION

The results obtained by modelling a hyperbolically symmetric and static geometric body simulating a possible hypothetical wormhole with the Einstein field equations and with the boundary conditions imposed by Schwarzschild for a spherically symmetric and static body, are displayed in the graphs presented in Figures 1 to 4. The graphs present some of the possible illustrations, that is, models of the hypothetical wormhole in hyperbolic coordinates.

Figure 1(a) exemplifies the graph pertaining to Eq. (41) and obtained as a possible solution of the Ricci tensor of the proposed metric. In Figure 1(b), the contour of the hyperbolic behaviour can be seen, while in Figure 1(c) the contour plot of two-sheet hyperboloids is observable. In addition, Figure 1(d) displays a hyperboloid having two sheets.

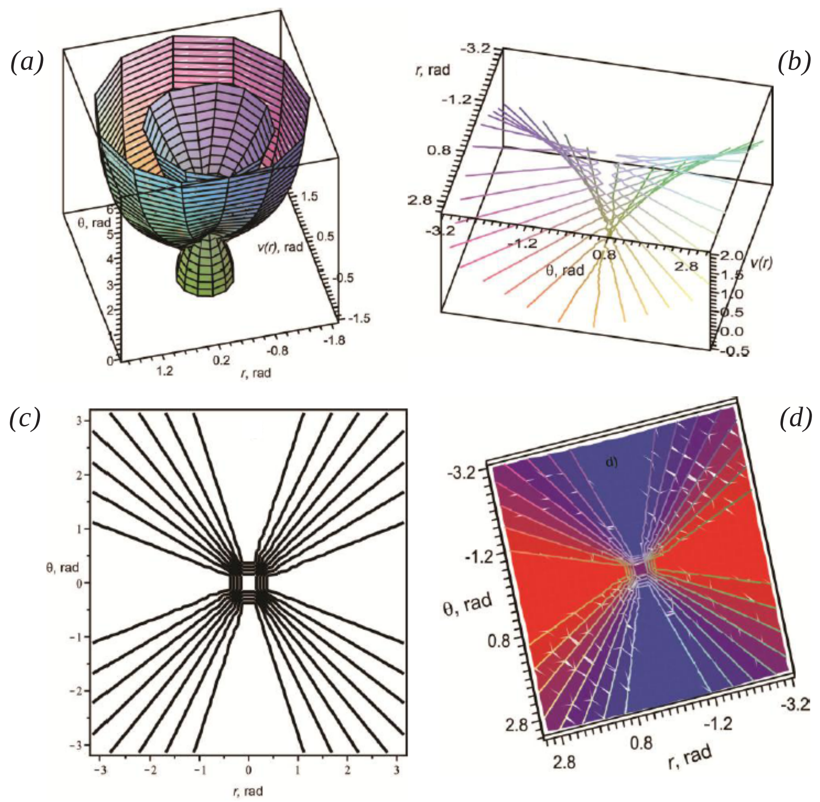
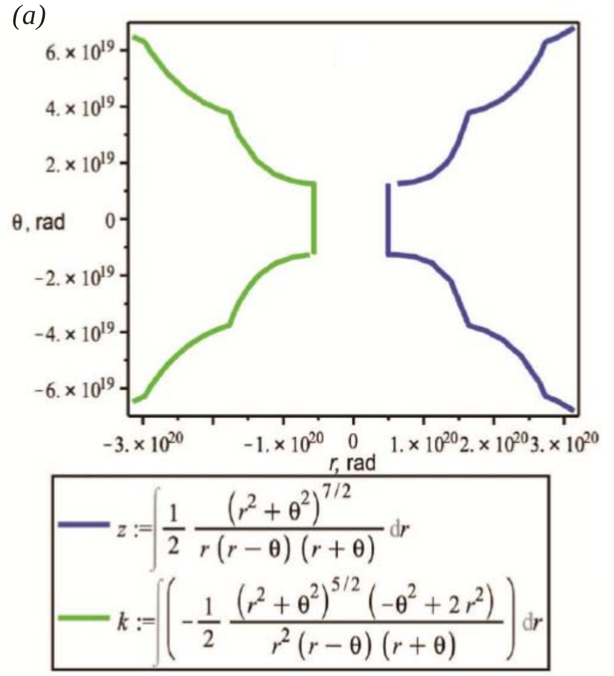


Fig. 1 Views of wormholes obtained on the basis of one of the solutions to Einstein field equations: (a) 3D graph, cylindrical coordinates and patch style; (b) 3D graph, contour style, boxed axes style; (c) a contour plot of two-sheet hyperboloids; and (d) a two-sheet hyperboloid with filled regions, boxed axes style

Other views (models) of hypothetical wormholes are obtained solving Eq. (35), but which do not fulfill the Schwarzschild boundary conditions at large distances from the body producing gravitational effects in space-time (see Figures 2(a) and 2(b)). Graph representing Eqs. (69) and (70) are presented in Figure 2(a), while the Figure 2(b) contains graphs of Eqs. (73) and (74).

Figures 2(a) and 2(b) are graphical views of one-sheet hyperboloids resembling a wormhole with a tunnel and an exit to two parallel universes.

(a)



(b)

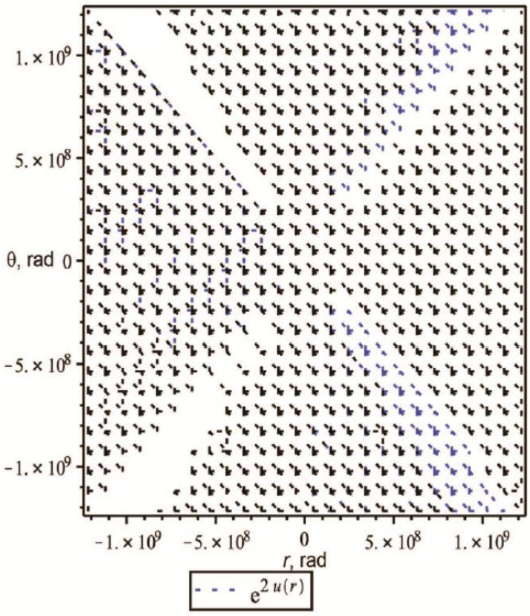


Fig. 2 Other views of hypothetical wormholes. Implicit graphs obtained from: (a) two solutions to Einstein's equations; and (b) one solution to Einstein's equations with the proposed metric. Graphical views of one-sheet hyperboloids resembling inter-womholes.

Figure 3 is the graph of Eq. (77) and, as observable from the Figure, for a large r, $v(r)$ is very small and this condition is true for one of the Schwarzschild boundary conditions, namely, when $v(\infty)=0$.

Thus, Figure 3 shows a horizontal wormhole across the hyperboloid of a horizontal sheet.

Figure 4 shows the graph of Eq. (78) presenting it as a view of a two-sheet hyperboloid: (a) with filled regions; and (b) in line-style resembling a wormhole.

Introducing Eq. (81) in Eq. (36), $v(r)=0$ is obtained; consequently, $r = \frac{\_C2}{e^{-\_C1}+1}$ from Eq. (81) for $v(r)=0$.

It is concluded that, while in the Schwarzschild metric is found that $v(r)=0$ when $r \to \infty$, we found for our metric that $v(r)=0$ when $r = \frac{\_C2}{e^{-\_C1}+1}$. With both metrics $e^{2v(r)}=0$, as expected according to the assumptions.

The gallery of graphs in Figures from 1 to 4 present the views of hypothetical wormholes fulfilling thus the main objective of this research consequently representing an introduction to and a cornerstone of the further inquiries into the possible contents of the "exotic matter" situated in wormholes [5].

The results have given us the possibility to reach some conslusions.

Namely, at large distances from the body generating a hyperbolically, symmetric and static field ($r \to \infty$), two terms tending to a unit and a constant of finite value were obtained, allowing us to confirm that the proposed metric leads to the Lorentz metric [15-17] for a flat space-time, which is typical of the Special Theory of Relativity [20].

Therefore, the mathematical model for the study of curved space-time (structure of a wormhole) proposed in General Relativity reduces to the Lorentzian metric at sufficiently large distances from the body or bodies generating gravitational fields that cause curvature. The conclusion also gives a rather acceptable justification of our study.

Finally, it is arguable that the Special Theory of Relativity is actually a special case of the General Theory of Relativity.

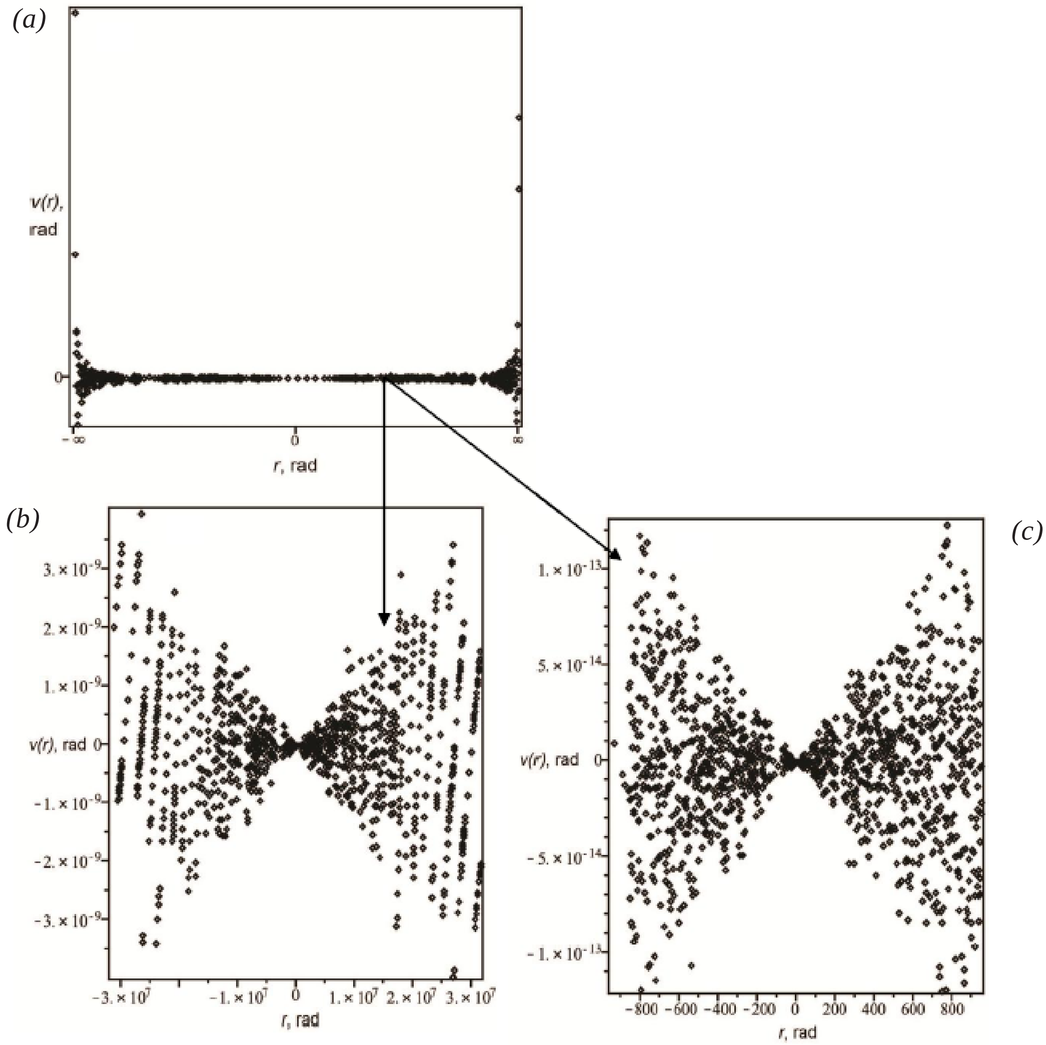


Fig. 3 (a) Invested, one-sheet hyperboloid; (b) Enlargement of (a); (c) Another enlargement of (a) that coincidentally contains the value of the slow rate of rotation of the semi-major axis of the planet Mercury (6 x 10^{-14} radians/second)

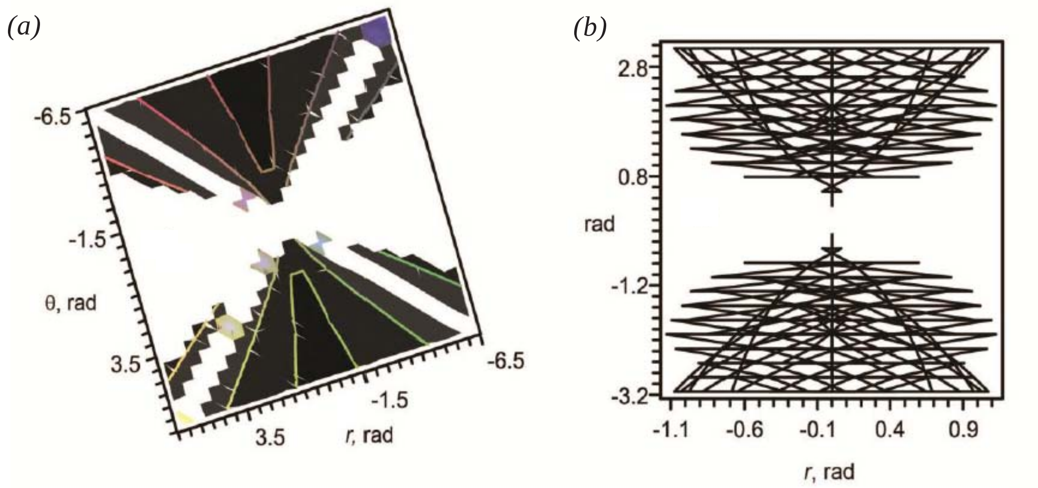


Fig. 4 Graphic of one quasi-exact solution of the Einstein field equations, in which the Schwarzschild boundary conditions are satisfied: (a) 3D contour plot, filled regions; and (b) 3D graphics, cylindrical coordinates, line-style, boxed axes style

4. CONCLUSIONS

The modelling of the structure of a hypothetical wormhole, from solutions to the Ricci tensor of General Relativity, taking into account the Schwarzschild solution and the calculation of the Kretschmann invariant, resulted in "quasy-exact" solutions to the Einstein equations. Additionally, this presents the basis for further thermodynamic study, heat transfer and fluids analysis of the possible “exotic matter” that might exist in the field of quantum gravity, what is in accordance with researches conducted by other authors.

The study of static body with hyper-cylindrical coordinates allowed the simulation of the hyperbolic cylindrical geometry similar to a wormhole using a hyperboloid of one or two sheets.

From the results obtained from the graphs of the structure of possible forms of wormholes, we can conclude that the stated objective was achieved given the fact that the results satisfy the Schwarzschild's boundary conditions in the general solution, the most often repeated solution among all of the possible solutions to the Einstein field equations.

The metric proposed in the General Theory of Relativity for the study of curved space-time reduces to the Lorentzian metric at sufficiently large distances from the body generating the gravitational field and causing the curvature. We can conclude that the Special Theory of Relativity is effectively a special case of the General Theory of Relativity.

5. REFERENCES

[1] A. Borel and L. Ji, *Compactifications of Symmetric and Locally Symmetric Spaces*, Birkhäuser, Boston, 2006.

[2] S. Chandrasekhar, *An Introduction to the Study of Stellar Structure*, Dover, New York, 2005.

[3] C. Kiefer, T. Luck and P. Moniz, The semiclassical approximation to supersymmetric quantum gravity, *Phys. Rev. D.*, Vol. 72, No. 4, 19p, 2005. (doi: 10.1103/PhysRevD.72.045006)

[4] A. DeBenedictis and A. Das, On a general class of wormhole geometries, *Classical and Quantum Gravity*, Vol. 18, No. 7, pp. 1187-1204, 2001.

[5] M.S. Morris, K.S. Thorne and U. Yurtsever, Wormholes, time machines, and the weak energy condition, *Phys. Rev. Lett.*, Vol. 61, No. 13, pp. 1446-1449, 1988. (doi:10.1103/PhysRevLett. 61.1446)

[6] M.S. Morris and K.S. Thorne, Wormholes in spacetime and their use for interstellar travel: A tool for teaching general relativity, *Am. J. Phys.*, Vol. 56, No. 5, pp. 395-412, 1988. (doi:10.1119/ 1.15620)

[7] K. Nakamura et al. (Particle Data Group), *The Review of Particle Physics*, *J. Phys. G: Nucl. Part. Phys.*, Vol. 37, No. 7A, 1422p, IOP Publ., 2010. (doi:10.1088/0954-3899/37/7A/075021)

[8] R.L. Corral-Bustamante, A.R. Rodríguez-Corral, T.J. Amador-Parra, E. Martínez-Loera and G. Irigoyen-Chávez, Modeling of virtual particles of the Big Bang, *Comput. Therm. Sci.*, Vol. 4, No. 2, pp. 1-13, 2012.

[9] R.L. Corral-Bustamante and A.R. Rodríguez-Corral, Transport phenomena in an evaporated Black Hole, In: A. Öchsner, L.F.M. Da Silva, H. Altenbach (eds.), *Materials with Complex Behaviour II*, pp. 469-482, Springer, Heidelberg, 2012.

[10] R.M. Wald, *General Relativity*, University of Chicago, Chicago, 1984.

[11] R. Brustein and M. Hadad, Einstein equations for generalized theories of gravity and the thermodynamic relation $\delta Q = T\delta S$ are equivalent, *Phys. Rev. Lett.*, Vol. 103, No. 10, 4p, 2009. (doi:10.1103/PhysRevLett.103.101301)

[12] H. Stephani, D. Kramer, M. MacCallum, C. Hoenselaers and E. Herlt, *Exact Solutions of Einstein's Field Equations*, Cambridge University Press, Cambridge, 2009.

[13] S.M. Carroll, *Spacetime and Geometry: An Introduction to General Relativity*, Addison-Wesley, San Francisco, 2004.

[14] J.B. Hartle, *Gravity: An Introduction to Einstein's General Relativity*, Addison-Wesley, San Francisco, 2003.

[15] G.S. Hall and D.P. Lonie, On the compatibility of Lorentz metrics with linear connections on four-dimensional manifolds, *J. Phys. A: Math. Gen.*, Vol. A39, No. 12, pp. 2995-3010, 2006. (doi:10.1088/0305-4470/39/12/009)

[16] H.A. Lorentz, Considerations on gravitation, *Proc. Royal Neth. Acad. Arts & Sciences*, Vol. 2, pp. 559–574, 1900.

[17] H.A. Lorentz, *Lectures on Theoretical Physics*, McMillan & Co., New York, 1927-1931.

[18] M.D. Roberts, Imploding scalar fields, *J. Math. Phys.*, Vol. 37, No. 9, pp. 4557-4573, 1996. (doi.org/10.1063/1.531642)

[19] J.H. Heinbockel, *Introduction to Tensor Calculus and Continuum Mechanics*, Trafford, Norfolk, 2001.

[20] W. Rindler, *Relativity: Special, General, and Cosmological*, Oxford University Press, New York, 2001.

MODELIRANJE STRUKTURE SVEMIRSKE CRVOTOČINE

SAŽETAK

U ovome se radu nudi model hipotetske svemirske crvotočine napravljen uz pomoć nelinearnih jednadžbi visokog stupnja složenosti. Modeliranje svemirske crvotočine podrazumijeva kreiranje posebnog gravitacijskog polja s hiperboličko-simetričnim i statičnim geometrijskim tijelom stvorenim za hipercilindričnu i simetričnu masu u mirovanju te njenu termodinamičku analizu. Prema metrici opće teorije relativnosti, kako bi se kratki tunel svemirske crvotočine mogao izmodelirati, nužno je definirati koordinate i kovarijantnu metriku četverodimenzionalnog prostora koji se sastoji od tri prostorne i jedne vremenske koordinate uz primjenu konvencije zbrajanja za ponavljajuće indekse u sustavu s četiri generalizirane koordinate. Limitirajuća forma linijskog elementa za plošno prostor-vrijeme na velike udaljenosti od izvora mora biti Lorentzova. Linijski element koji odgovara rješenju ponuđenom u ovome radu treba ostati konstantan s obzirom na inverziju vremenskog intervala. Za potrebe modeliranja svemirske crvotočine, uzeta je najjednostavnija kvadratična skalarna invarijanta Riemannova tenzora - Kretschmannova invarijanta. Nadalje, korištene su krivolinijske koordinate u kojima se metričke, tenzorne komponente nalaze van glavne dijagonale te koje imaju nultu vrijednost zbog neodređenosti u prostoru. Metrika opće teorije relativnosti se može reducirati na plošnu metriku za promatrača u mirovanju koji je lociran u "beskonačnosti" kako bi se zanemarili efekti uzrokovani gibanjem mase u gravitacijskom polju. Analizom rezultata dobiveno je kvazi-egzaktno rješenje Einsteinovih jednadžbi koje opisuju gravitacijsko polje u geometriji oblika hiper-cilindra koje sliči tunelu svemirske crvotočine. Naposlijetku, promatrana je i takozvana egzotična materija za koju se smatra da se nalazi u tunelu svemirske crvotočine te da ima ulogu u fenomenu prijenosa energije.

Ključne riječi: *svemirska crvotočina, Einsteinove jednadžbe polja, metrika, kvazi-egzaktno rješenje, hiperbolička geometrija.*