UDC 519.8:339.1:336.6:519.61
Original scientific paper
Received: 29.06.2010.

Production lot sizing with rework and fixed quantity deliveries

Singa Wang Chiu(1), **Yuan-Shyi Peter Chiu**(2), **Li-Wen Lin**(2) and **Yi-Chun Lin**(2)

(1)Department of Business Administration, Chaoyang University of Technology,
Gifong East Road, Wufong, Taichung 413, TAIWAN, e-mail: swang@cyut.edu.tw
(2)Department of Industrial Engineering, Chaoyang University of Technology,
Gifong East Road, Wufong, Taichung 413, TAIWAN, e-mail: ypchiu@cyut.edu.tw

SUMMARY

This paper is concerned with determination of the optimal lot size for an economic production quantity (EPQ) model with the reworking of random defective items and fixed quantity multiple deliveries. Classic EPQ model assumes continuous issuing policy for satisfying product demand and perfect quality production for all items produced. However, in real life vendor-buyer integrated production-inventory system, multi-delivery policy is used practically in lieu of the continuous issuing policy and generation of defective items during production run is inevitable. In this study, all nonconforming items produced are considered to be repairable and are reworked in each cycle when regular production ends. The finished items can only be delivered to customers if the whole lot is quality assured at the end of the rework. Fixed quantity multiple installments of the finished batch are delivered to customers at a fixed interval of time. The long-run average integrated cost function per unit time is derived. A closed-form optimal batch size solution to the problem is obtained. A numerical example demonstrates its practical usage.

Key words: *production lot sizing, rework, multiple deliveries, EPQ model, inventory.*

1. INTRODUCTION

"*When should an order be placed*" and "*how much should be ordered*" are two fundamental questions that must be answered by production-inventory practitioners for the items they stock [1]. Using mathematical model to assist corporations in minimizing overall inventory costs dates back almost a century, when the economic order quantity (EOQ) model was first introduced [2]. EOQ model employs mathematical techniques to balance setup and inventory holding costs, and derives the optimal order quantity (as well as time between orders) which minimizes the long-run average costs. In manufacturing sector, when products are produced in-house instead of being acquired from outside suppliers, the economic production quantity (EPQ) model (also known as the economic manufacturing quantity (EMQ) model [3]) is often utilized to cope with the finite production-inventory replenishment rate in order to minimize total costs per unit time [3-4].

Classic EPQ model assumes that all items produced are of perfect quality. However, in real life production systems, due to process deterioration or other factors, generation of imperfect quality items is inevitable. Studies have been carried out to enhance the classic EPQ model by addressing the issue of defective items produced [5-12]. The nonconforming items sometimes can be reworked and repaired hence overall production costs can be significantly reduced [13-19]. For instance, manufacturing processes in printed circuit board assembly, or in plastic injection molding, etc., sometimes employs rework as an acceptable process in terms of level of quality. Jamal et al. [14] studied the optimal manufacturing batch size with rework process at a single-stage production system. Cases of rework being completed within the same production cycle as well as rework being done after *N* cycles are examined. They developed mathematical models for each case, and derived total system costs and the optimal batch sizes accordingly. Chiu and Chiu [15] proposed mathematical modeling for determining optimal batch size and backordering level for an imperfect finite production rate model with backlogging and failure in repair. They derived the long-run average function cost and also proved its optimality. Closed-form optimal solutions for the lot size as well as optimal backordering quantity are obtained.

The continuous inventory issuing policy for satisfying product demand is another unrealistic assumption of classic EPQ model. In real life vendor-buyer integrated production-inventory system, multiple or periodic deliveries of finished products are commonly adapted at customer's request. Goyal [20] first considered integrated inventory model for a single supplier-single customer problem. He proposed a method that is typically applicable to those inventory problems where a product is procured by a single customer from a single supplier. Example was given to illustrate the method proposed. Studies have since been carried out to address various aspects of supply chain optimization [21-32]. Golhar and Sarker [22] developed a simple algorithm to compute the optimal batch size for a system where a just-in-time (JIT) buyer demands frequent deliveries of small lots of certain products. They found that the generalized total cost function for the proposed model is a piecewise convex function and when production uptime and cycle time are each equal to an integer multiple of the shipment interval, a perfect matching of shipment size occurs, and for such a situation, the generalized model specializes to more traditional inventory models. Economic impact of ordering and setup costs reduction was also investigated. Hill [24] examined a model in which a manufacturing company purchases a raw material, manufactures a product (at a finite rate) and ships a fixed quantity of the product to a single customer at fixed and regular intervals of time, as specified by the customers. The objective is to determine a purchasing and production schedule which minimizes total cost of purchasing, manufacturing and stockholding. Sarker and Khan [26] considered a manufacturing system that procures raw materials from suppliers in a lot and processes them into finished products which are then delivered to outside buyers at fixed points in time. A general cost model was formulated considering both raw materials and finished products. Using this model, a simple procedure was developed to determine optimal ordering policy for procurement of raw materials as well as manufacturing batch size, to minimize the total cost of meeting customer demands in time. Ouyang et al. [29] investigated the single-vendor single-buyer integrated production inventory models with stochastic demand in controllable lead time. They assumed that shortage during lead time is permitted and lead time can be reduced at an added cost. Two models were considered in their paper. The first model assumed that demand of lead time follows a normal distribution and the second model, the demand of lead time is distribution-free. They developed effective iterative procedures for finding the optimal policy for both cases, respectively.

For the reason that little attention was paid to the area of investigating the joint effect of multiple deliveries and reworking of defective items on the optimal replenishment lot size of EPQ model, this paper intends to bridge the gap.

2. THE PROPOSED MODEL AND DERIVATIONS

The aim of this study is to determine the optimal lot size for the EPQ model with the reworking of random defective items and fixed quantity multiple deliveries. Consider that during regular production time, there is x portion of defective items produced randomly, at a production rate d. All defective items are assumed to be repairable and are reworked and repaired at a rate P_1 in each cycle when regular production ends.

In order to avoid shortage from occurring, the constant production rate P has to be larger than the sum of demand rate λ and production rate of defective items d. That is: $(P-d-\lambda) > 0$ or $(1-x-\lambda/P) > 0$; where $d=Px$. Unlike the classic EPQ model assuming a continuous issuing policy for satisfying demand λ, this paper considers a multi-delivery policy. It is assumed that the finished items can only be delivered to customers if the whole lot is quality assured at the end of rework. Fixed quantity n installments of the finished batch are delivered to customers, at a fixed interval of time during the production downtime t_3 (see Figure 1).

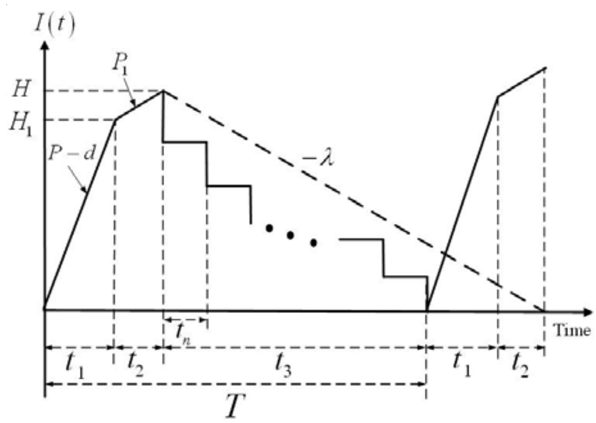

Fig. 1 On-hand inventory of perfect quality items in EPQ model with multi-delivery policy and rework

Cost parameters considered in the proposed model include: unit production cost C, unit holding cost h, setup cost K, unit rework cost C_R, holding cost h_1 for each reworked item, fixed delivery cost K_1 per shipment, and delivery cost C_T per item shipped to customers. Additional notation is listed under nomenclature. From Figure 1, the following variables can be obtained:

$$T = t_1 + t_2 + t_3 \tag{1}$$

$$t_1 = \frac{Q}{P} = \frac{H_1}{P-d} \tag{2}$$

$$t_2 = \frac{xQ}{P_1} \tag{3}$$

$$t_3 = nt_n = T - (t_1 + t_2) = Q\left(\frac{1}{\lambda} - \frac{1}{P} - \frac{x}{P_1}\right) \tag{4}$$

$$H_1 = (P-d)t_1 = (P-d)\frac{Q}{P} = (1-x)Q \tag{5}$$

$$H = H_1 + P_1 t_2 = Q \tag{6}$$

The on-hand inventory of defective items during production uptime t_1 and rework time t_2 is illustrated in Figure 2. It is noted that maximum level of on-hand defective items is dt_1, and:

$$dt_1 = Pxt_1 = xQ \tag{7}$$

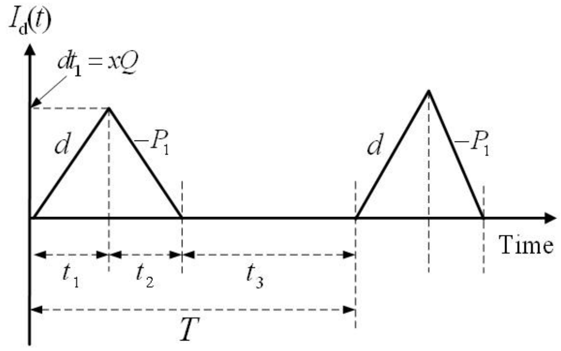

Fig. 2 On-hand inventory of defective items in EPQ model with multi-delivery policy and rework

Total costs per cycle $TC(Q)$ consists of setup cost, variable production cost, variable rework cost, fixed and variable delivery cost, holding cost during production uptime t_1 and reworking time t_2, variable holding cost for items reworked, and holding cost for finished goods during the delivery time t_3 where n fixed-quantity installments of the finished batch are delivered to customers at a fixed interval of time. Cost for each delivery is:

$$K_1 + C_T\left(\frac{H}{n}\right) \tag{8}$$

Total delivery costs for n shipments in a cycle are:

$$n\left[K_1 + C_T\left(\frac{H}{n}\right)\right] = nK_1 + C_T H = nK_1 + C_T Q \tag{9}$$

Therefore, $TC(Q)$ is (see Appendix A for computation of the very last term of holding cost during delivery time t_3 in Eq. (10)):

$$TC(Q) = CQ + K + C_R[xQ] + C_T Q + nK_1 + h_1 \cdot \frac{P_1 \cdot t_2}{2} \cdot (t_2) + h\left[\frac{H_1 + dt_1}{2}(t_1) + \frac{H_1 + H}{2}(t_2)\right] + h\left(\frac{n-1}{2n}\right)Ht_3 \tag{10}$$

Since the proportion x of defective items is assumed to be a random variable with a known probability density function, in order to take the randomness of defective rate into account, one can use the expected values of x in the inventory cost analysis. Substituting all related parameters from Eqs. (1) to (9) in $TC(Q)$, one obtains the expected production-inventory-delivery cost per unit time, $E[TCU(Q)]$ as follows (see Appendix B for detailed computation):

$$\begin{aligned} E[TCU(Q)] = \frac{E[TC(Q)]}{E[T]} &= C\lambda + \frac{(K + nK_1)\lambda}{Q} + C_R E[x]\lambda + C_T\lambda + \frac{hQ\lambda}{2P} + \\ &+ \frac{hQ\lambda}{2P_1}\left[2E[x] - (E[x])^2\right] + \left(\frac{n-1}{n}\right)\left[\frac{hQ}{2} - \frac{hQ\lambda}{2P} - \frac{hQE[x]\lambda}{2P_1}\right] + \frac{h_1 (E[x])^2 Q\lambda}{2P_1} \end{aligned} \tag{11}$$

3. CONVEXITY OF COST FUNCTION AND OPTIMAL LOT SIZE

The optimal production lot size can be obtained by minimizing the expected cost function $E[TCU(Q)]$. Differentiating $E[TCU(\mathrm{Q})]$ with respect to Q, the first and the second derivatives of $E[TCU(Q)]$ are shown in Eqs. (12) and (13):

$$\frac{dE[TCU(Q)]}{dQ} = -\frac{K\lambda}{Q^2} - \frac{nK_1\lambda}{Q^2} + \frac{h\lambda}{2P} + \frac{h\lambda}{2P_1}\left[2E[x] - (E[x])^2\right] + \frac{h_1(E[x])^2\lambda}{2P_1} + \left(\frac{n-1}{n}\right)\left[\frac{h}{2} - \frac{h\lambda}{2P} - \frac{hE[x]\lambda}{2P_1}\right] \tag{12}$$

$$\frac{d^2E\left[TCU\left(Q\right)\right]}{dQ^2}=\frac{2\left(K+nK_1\right)\lambda}{Q^3} \tag{13}$$

Equation (13) is resulting positive, because K, n, K_1, λ, and Q are all positive. The second derivative of $E[TCU(Q)]$ with respect to Q is greater than zero, hence, $E[TCU(Q)]$ is a convex function for all Q different from zero. The optimal production lot size Q^* can be obtained by setting the first derivative of $E[TCU(Q)]$ equal to zero (referring to Eq. (12)):

$$\frac{dE\left[TCU\left(Q\right)\right]}{dQ}=0=$$

$$=-\frac{K\lambda}{Q^2}-\frac{nK_1\lambda}{Q^2}+\frac{h\lambda}{2P}+\frac{h\lambda}{2P_1}\left[2E\left[x\right]-\left(E\left[x\right]\right)^2\right]+\frac{h_1\left(E\left[x\right]\right)^2\lambda}{2P_1}+\left(\frac{n-1}{n}\right)\left[\frac{h}{2}-\frac{h\lambda}{2P}-\frac{hE\left[x\right]\lambda}{2P_1}\right] \tag{14}$$

One obtains the following after further rearrangement:

$$\frac{\left(K+nK_1\right)\lambda}{Q^2}=\frac{h\lambda}{2P}+\frac{h\lambda}{2P_1}\left[2E\left[x\right]-\left(E\left[x\right]\right)^2\right]+\left(\frac{n-1}{2n}\right)\left[h-h\left(\frac{\lambda}{P}+\frac{E\left[x\right]\lambda}{P_1}\right)\right]+\frac{h_1\left(E\left[x\right]\right)^2\lambda}{2P_1} \tag{15}$$

and:

$$Q^*=\sqrt{\frac{2\left(K+nK_1\right)\lambda}{\frac{h\lambda}{P}+\frac{h\lambda}{P_1}\left[2E\left[x\right]-\left(E\left[x\right]\right)^2\right]+\frac{h_1\left(E\left[x\right]\right)^2\lambda}{P_1}+\left(\frac{n-1}{n}\right)\left[h-h\left(\frac{\lambda}{P}+\frac{E\left[x\right]\lambda}{P_1}\right)\right]}} \tag{16}$$

4. SPECIAL CASE

Suppose that all items produced are of perfect quality. The proposed model becomes the same as the classic EPQ model with multi-delivery policy. The total cost per cycle is:

$$TC_1\left(Q\right)=CQ+K+C_TQ+nK_1+h\frac{H}{2}\left(t_1\right)+h\left(\frac{n-1}{2n}\right)Ht_2 \tag{17}$$

The expected production-inventory-delivery cost per unit time for this special model is:

$$E\left[TCU_1\left(Q\right)\right]=\frac{TC_1\left(Q\right)}{T}=C\lambda+\frac{\left(K+nK_1\right)\lambda}{Q}+C_T\lambda+\frac{hQ\lambda}{2P}+\left(\frac{n-1}{n}\right)\left(\frac{hQ}{2}-\frac{hQ\lambda}{2P}\right) \tag{18}$$

The first and the second derivatives of $E[TCU_1(Q)]$ are shown in Eqs. (19) and (20):

$$\frac{dE\left[TCU_1\left(Q\right)\right]}{dQ}=-\frac{K\lambda}{Q^2}-\frac{nK_1\lambda}{Q^2}+\frac{h\lambda}{2P}+\left(\frac{n-1}{n}\right)\frac{h}{2}\left(1-\frac{\lambda}{P}\right) \tag{19}$$

$$\frac{d^2E\left[TCU_1\left(Q\right)\right]}{dQ^2}=\frac{2\left(K+nK_1\right)\lambda}{Q^3} \tag{20}$$

From Eq. (20), one can identify that $E[TCU_1(Q)]$ is a convex function and the optimal batch size can be derived as follows:

$$Q^*=\sqrt{\frac{2\left(K+nK_1\right)\lambda}{\left\{\frac{h\lambda}{P}+\left(\frac{n-1}{n}\right)h\left(1-\frac{\lambda}{P}\right)\right\}}} \tag{21}$$

5. RESULTS VERIFICATION USING NUMERICAL EXAMPLE

Suppose that a product can be manufactured at a rate of *60,000* units per year and this item has experienced a flat demand rate of *3,400* units per year. During the production uptime, the random defective rate is assumed to be uniformly distributed over the interval [*0*, *0.3*]. All defective items can be reworked and repaired at a rate $P_1=2{,}200$ units per year. Additional parameters considered by this example are given as follows:

C_R = *$60*, repaired cost for each item reworked,
C = *$100* per item,
K = *$20,000* per production run,
h = *$20* per item per year,
h_1 = *$40* per item reworked per unit time (year),
n = *4* installments of the finished batch are delivered per cycle,
K_1 = *$4,400* per shipment, a fixed cost,
C_T = *$0.1* per item delivered.

The optimal batch size $Q^*=3{,}427$ can be obtained from Eq. (16) and the long-run average production-inventory-delivery costs $E[TCU(Q^*)]=\$445{,}554$ from Eq. (11).

Figure 3 depicts the convexity of the long-run cost function $E[TCU(Q)]$. Additional analysis indicates that as random defective rate x increases, the value of the long-run cost function $E[TCU(Q^*)]$ increases significantly. Furthermore, for the special case (i.e. situation when all items produced are of perfect quality), the optimal batch size $Q^*=4{,}090$ and the long-run average cost $E[TCU_1(Q^*)] = \$402{,}853$ can also be calculated by using Eqs. (21) and (18), respectively.

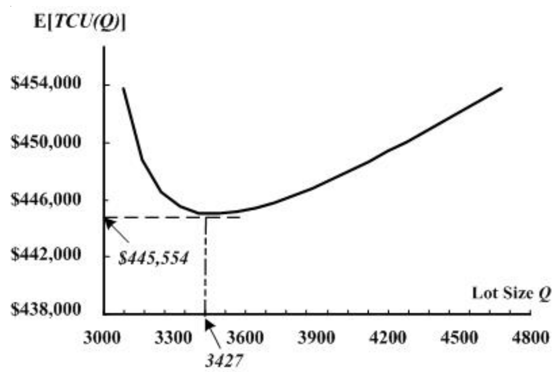

Fig. 3 Convexity of the long-run cost function E[TCU(Q)]

6. CONCLUSION

Classic EPQ model assumes continuous issuing policy for satisfying demand and all items produced are of perfect quality. In real life situation, multi-delivery policy is used practically and generation of defective items during a production run is inevitable. This study incorporates fixed-quantity multiple deliveries and quality assurance (the reworking of defective items) into an imperfect EPQ model. By using the mathematical modeling and analyses, the long-run average production-inventory-delivery cost per unit time is derived. Convexity of this integrated cost function is proved. A closed-form optimal lot-size solution to the problem is obtained.

7. ACKNOWLEDGEMENTS

Authors would like to express their appreciation to National Science Council of Taiwan for supporting this research under Grant number NSC-98-2410-H-324-005.

APPENDIX A

Computation of holding cost of finished products during delivery time t_3 (i.e. the very last term in Eq. (10)):

(1) When $n=1$, total holding cost in delivery time $t_3=0$.
(2) When $n=2$, total holding costs in delivery time t_3 become (see Figure 4):

$$h\left(\frac{H}{2}\times\frac{t_3}{2}\right)=h\left(\frac{1}{2^2}\right)Ht_3 \qquad \text{(A.1)}$$

(3) When $n=3$, total holding costs in delivery time t_3 become (see Figure 5):

$$h\left(\frac{2H}{3}\times\frac{t_3}{3}+\frac{1H}{3}\times\frac{t_3}{3}\right)=h\left(\frac{2+1}{3^2}\right)Ht_3 \qquad \text{(A.2)}$$

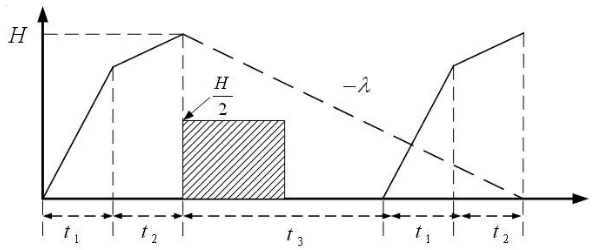

Fig. 4 Average on-hand inventory of finished products during delivery time t_3 when n=2

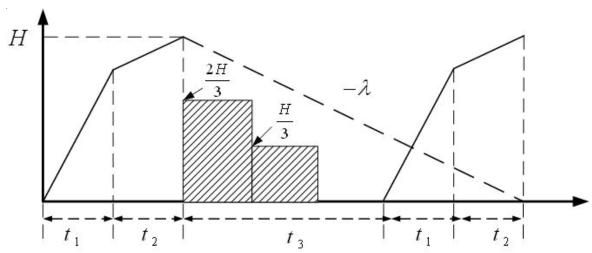

Fig. 5 Average on-hand inventory of finished products during delivery time t_3 when n=3

(4) When $n=4$, total holding costs in delivery time t_3 become:

$$h\left(\frac{3H}{4}\times\frac{t_3}{4}+\frac{2H}{4}\times\frac{t_3}{4}+\frac{1H}{4}\times\frac{t_3}{4}\right)=h\left(\frac{3+2+1}{4^2}\right)Ht_3 \tag{A.3}$$

It follows that the general term for total holding costs during delivery time t_3 is:

$$h\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right)Ht_3=h\left(\frac{1}{n^2}\right)\left[\frac{n(n-1)}{2}\right]Ht_3=h\left(\frac{n-1}{2n}\right)Ht_3 \tag{A.4}$$

APPENDIX B

Computation of Eq. (11).

Recall Eq. (10) as follows:

$$TC(Q)=CQ+K+C_R[xQ]+C_TQ+nK_1+h_1\cdot\frac{P_1\cdot t_2}{2}\cdot(t_2)+ \\ +h\left[\frac{H_1+dt_1}{2}(t_1)+\frac{H_1+H}{2}(t_2)\right]+h\left(\frac{n-1}{2n}\right)Ht_3 \tag{10}$$

then:

$$TC(Q)=CQ+K+nK_1+C_R[xQ]+C_TQ+\frac{hQ^2}{2P}+\frac{hQ^2}{2P_1}\left[\left(2x-x^2\right)\right]+ \\ +\left(\frac{n-1}{n}\right)\left[\frac{hQ^2}{2\lambda}-\frac{hQ^2}{2P}-\frac{hQ^2x}{2P_1}\right]+\frac{h_1x^2Q^2}{2P_1} \tag{B.1}$$

and:

$$T=\frac{Q}{\lambda} \tag{B.2}$$

Since:

$$E[TCU(Q)]=\frac{E[TC(Q)]}{E[T]} \tag{B.3}$$

then:

$$E[TCU(Q)]=\frac{E[TC(Q)]}{E[T]}=C\lambda+\frac{(K+nK_1)\lambda}{Q}+C_RE[x]\lambda+C_T\lambda+\frac{hQ\lambda}{2P}+ \\ +\frac{hQ\lambda}{2P_1}\left[2E[x]-(E[x])^2\right]+\left(\frac{n-1}{n}\right)\left[\frac{hQ}{2}-\frac{hQ\lambda}{2P}-\frac{hQE[x]\lambda}{2P_1}\right]+\frac{h_1(E[x])^2Q\lambda}{2P_1} \tag{11}$$

NOMENCLATURE

n – number of fixed quantity installments of the finished batch to be delivered by request to customers,
H_1 – maximum level of on-hand inventory in units when regular production process ends,
H – the maximum level of on-hand inventory in units when rework process finishes,
Q – production lot size to be determined for each cycle,
t_1 – the production uptime for the proposed EPQ model,
t_2 – time required for reworking of defective items,
t_3 – time required for delivering all quality assured finished products,
t_n – a fixed interval of time between each installment of finished products delivered during production downtime t_3,
T – cycle length,
$I(t)$ – on-hand inventory of perfect quality items at time t,
$I_d(t)$ – on-hand inventory of defective items at time t,
$TC(Q)$ – total production-inventory-delivery costs per cycle for the proposed model,
$TC_1(Q)$ – total production-inventory-delivery per

cycle when no defective items produced (i.e. special case - the classic EPQ model with multi-delivery policy),

$E[TCU(Q)]$ = the long-run average costs per unit time for the proposed model,

$E[TCU_1(Q)]$ = the long-run average costs per unit time for model in the special case.

8. REFERENCES

[1] S. Nahmias, *Production & Operations Analysis*, 5th edition, McGraw-Hill, New York, pp. 195-223, 2005.

[2] F.W. Harris, How many parts to make at once, *Factory - The Magazine of Management*, Vol.10, pp. 135-136, 1913.; Reprinted in: *Operations Research*, Vol. 38, pp. 947-950, 1990.

[3] E.A. Silver, D.F. Pyke and R. Peterson, *Inventory Management and Production Planning and Scheduling*, John Wiley & Sons, New York, pp. 151-172, 1998.

[4] P.H. Zipkin, *Foundations of Inventory Management*, McGraw-Hill, New York, pp. 30-60, 2000.

[5] M.J. Rosenblatt and H.L. Lee, Economic production cycles with imperfect production processes, *IIE Transactions*, Vol. 18, pp. 48-55, 1986.

[6] T.C.E. Cheng, An economic order quantity model with demand-dependent unit production cost and imperfect production processes, *IIE Transactions*, Vol. 23, pp. 23-28, 1991.

[7] K. Moinzadeh and P. Aggarwal, Analysis of a production/inventory system subject to random disruptions, *Management Science*, Vol. 43, pp. 1577-1588, 1997.

[8] C.H. Kim, Y. Hong and S.-Y. Kim, An extended optimal lot sizing model with an unreliable machine, *Production Planning and Control*, Vol. 8, pp. 577-585, 1997.

[9] S.W. Chiu, Y.-Sh.P. Chiu and C.-C. Shih, Determining expedited time and cost of the end product with defective component parts using critical path method (CPM) and time-costing method, *Journal of Scientific & Industrial Research*, Vol. 65, pp. 695-701, 2006.

[10] M.A. Rahim and M. Ben-Daya, Joint determination of production quantity, inspection schedule and quality control for imperfect process with deteriorating products, *Journal of the Operational Research Society*, Vol. 52, pp. 1370-1378, 2001.

[11] Y.-Sh.P. Chiu and S.W. Chiu, Determining the materials procurement policy based on the economic order/production models with backlogging permitted, *International Journal of Advanced Manufacturing Technology*, Vol. 30, pp. 156-165, 2006.

[12] S.W. Chiu, C. Shih and Y.-Sh.P. Chiu, A revised cost-benefit algorithm for solving ECTEP problem with defective materials in product structure diagram, *P I Mech Eng Part B: Journal of Engineering Manufacture*, Vol. 221, pp. 489-497, 2007.

[13] T. Dohi, N. Kaio and S. Osaki, Minimal repair policies for an economic manufacturing process, *Journal of Quality in Maintenance Engineering*, Vol. 4, pp. 248-262, 1998.

[14] A.M.M. Jamal, B.R. Sarker and S. Mondal, Optimal manufacturing batch size with rework process at a single-stage production system, *Computers & Industrial Engineering*, Vol. 47, pp. 77-89, 2004.

[15] S.W. Chiu and Y.-Sh.P. Chiu, Mathematical modeling for production system with backlogging and failure in repair, *Journal of Scientific & Industrial Research*, Vol. 65, pp. 499-506, 2006.

[16] S.W. Chiu, C.-K. Ting and Y.-Sh.P. Chiu, Optimal production lot sizing with rework, scrap rate, and service level constraint, *Mathematical and Computer Modelling*, Vol. 46, pp. 535-549, 2007.

[17] Y.-Sh.P. Chiu, Y.-L. Lien, Ch.-A.K. Lin, Incorporating machine reliability issue and backlogging into the EMQ model – Part I: Random breakdown occurring in backorder filling time, *Int. J. for Engineering Modelling*, Vol. 22, No. 1-4, pp. 1-13, 2009.

[18] S.W. Chiu, Y.-Sh.P. Chiu, H.-J. Chuang, Ch.-H. Lee, Incorporating machine reliability issue and backlogging into the EMQ model – Part II: Random breakdown occurring in inventory piling time, *Int. J. for Engineering Modelling*, Vol. 22, No. 1-4, pp. 15-24, 2009.

[19] S.W. Chiu, H.-D. Lin, Ch.-B. Cheng, Ch.-L. Chung, Optimal production- shipment decisions for the finite production rate model with scrap, *Int. J. for Engineering Modelling*, Vol. 22, No. 1-4, pp. 25-34, 2009.

[20] S.K. Goyal, Integrated inventory model for a single supplier-single customer problem, *Int. J. of Production Research*, Vol. 15, pp. 107-111, 1977.

[21] A. Banerjee, A joint economic lot size model for purchaser and vendor, *Decision Sciences*, Vol. 17, No. 3, pp. 292-311, 1986.

[22] D. Golhar and B. Sarker, Economic manufacturing quantity in a just-in-time delivery system, *Int. J. of Production Research*, Vol. 30, pp. 961-972, 1992.

[23] L. Lu, A one-vendor multi-buyer integrated inventory model, *European Journal of Operational Research*, Vol. 81, No. 2, pp. 312-323, 1995.

[24] R.M. Hill, Optimizing a production system with a fixed delivery schedule, *Journal of the Operational Research Society*, Vol. 47, pp. 954-960, 1996.

[25] S. Viswanathan, Optimal strategy for the integrated vendor-buyer inventory model, *European Journal of Operational Research*, Vol. 105, pp. 38-42, 1998.

[26] R.A. Sarker and L.R. Khan, Optimal batch size for a production system operating under periodic delivery policy, *Computers and Industrial Engineering*, Vol. 37, pp. 711-730, 1999.

[27] P.C. Yang and H.M. Wee, A single-vendor and multiple-buyers production-inventory policy for a deteriorating item, *European J. of Operational Research*, Vol. 143, pp. 570-581, 2002.

[28] M. Khouja, Optimizing inventory decisions in a multi-stage multi-customer supply chain, *Transportation Research Part E: Logistics and Transp. Review*, Vol. 39, No. 3, pp. 193-208, 2003.

[29] L.-Y. Ouyang, K.-S. Wu and C.-H. Ho, Integrated vendor-buyer cooperative models with stochastic demand in controllable lead time, *Int. J. of Production Economics*, Vol. 92, pp. 255-266, 2004.

[30] Y.-Sh.P. Chiu, S.W. Chiu, C.-Y. Li and C.-K. Ting, Incorporating multi- delivery policy and quality assurance into economic production lot size problem, *Journal of Scientific & Industrial Research*, Vol. 68, pp. 505-512, 2009.

[31] Y.-Sh.P. Chiu, K.-K. Chen and H.-H. Chang, Solving an economic production lot size problem with multi-delivery policy and quality assurance using an algebraic approach, *Journal of Scientific & Industrial Research*, Vol. 69, No. 12, pp. 926-929, 2010.

[32] Y.-Sh.P. Chiu, Ch.-A.K. Lin, H.-H. Chang and V. Chiu, Mathematical modeling for determining economic batch size and optimal number of deliveries for EPQ model with quality assurance, *Mathematical and Computer Modelling of Dynamical Systems*, Vol. 16, No. 4, pp. 373-388, 2010.

PROIZVODNJA UKUPNE KOLIČINE S POPRAVCIMA I FIKSNIM ISPORUKAMA KOLIČINA

SAŽETAK

Ovaj rad bavi se određivanjem optimalnih ukupnih količina za model proizvodnje ekonomične količine (EPQ) uz popravke ponekih oštećenih predmeta i fiksnim količinama višestrukih isporuka. Klasičan EPQ model pretpostavlja politiku kontinuiranog izdavanja da bi se zadovoljila potražnja kao i besprijekorna proizvodnja kvalitete svih proizvedenih predmeta. Međutim, u stvarnom prodavač-kupac integriranom proizvodno-skladišnom sustavu koristi se politika višekratne isporuke pa je neizbježna pojava oštećenih proizvoda tijekom proizvodnje. U ovom radu se svi neodgovarajuće proizvedeni predmeti smatraju popravljivima te se ponovo obrade u svakom ciklusu kada završi redovna proizvodnja. Dovršeni predmeti mogu se isporučiti kupcima samo ako je ukupna proizvedena količina dokazano kvalitetna na kraju proizvodnje. Fiksne količine isporuka dovršene robe isporučuju se kupcima u fiksnim vremenskim intervalima. Izvedena je funkcija dugoročnoga prosječnog integriranoga troška u jedinici vremena. Na kraju se dobiva rješenje problema optimalne količine robe u zatvorenom obliku. Numerički primjer prikazuje njegovu praktičnu upotrebu.

Ključne riječi: *Proizvodnja sveukupne količine, popravak, višekratne isporuke, EPQ model, inventar.*