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Integrating a cost-reduction shipment plan into a single-producer multi-retailer system with rework

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SUMMARY

This study integrates a cost-reduction shipment plan into a single-producer, multi-retailer system with rework process. In a recent article, Chiu et al. [1] have examined a single-producer, multi-retailer integrated inventory model with a rework process. For the purpose of reducing the inventory holding cost, this study combines an alternative n+1 product distribution policy into their model. Under the proposed shipment plan, an extra (initial) delivery of finished items takes place during the production uptime to meet the retailers' product demands for the periods of the producer's uptime and reworking time. Upon the completion of rework, multiple shipments will be delivered synchronously to m different retailers. The objectives are to find an optimal production-shipment policy that minimizes the expected system cost for such a supply chain system, and to demonstrate that the result of this study gives significant holding cost savings in comparison with Chiu et al.'s model [1]. With the help of mathematical modelling and Hessian matrix equations, the optimal operating policy for the proposed model is derived. Through a numerical example, we demonstrate our model gives significant savings in stock holding cost for both the producer and retailers.

Key words: *optimization, supply chains, production lot-size, multiple retailers, rework, multi-delivery.*

1. INTRODUCTION

Taft [2] has been first to study the most economical production lot (the so-called economic production quantity (EPQ) model). He has considered a production planning for single product with perfect production process and a continuous-inventory issuing policy for finished goods. However, in a real world production environment, due to many uncontrollable factors, a generation of defective items is inevitable.

Many studies have since been carried out to address various aspects of imperfect production systems [3-12]. Shih [4] has studied two inventory models where the proportion of defective units in an accepted lot is a random variable with known probability distributions. Optimal solutions for the proposed systems have been developed, and through numerical examples, comparisons to the traditional models have been presented. Schwaller [5] has examined the EOQ model by incorporating both fixed and variable inspection costs, for finding and removing a known proportion of defective items in incoming lots. Chelbi and Rezg [8] have investigated a production-inventory system with a randomly failing production unit subjected to a minimum required availability level. They have assumed that a production unit is submitted to a maintenance action as soon as it reaches a certain age T or at failure, whichever occurs first. A buffer stock h is built up in order to guarantee a continuous supply of the assembly line during repair or preventive maintenance actions. Under the constraint of a minimum required availability level A, they have derived an optimal operating policy (in terms of h^* and T^*) that minimizes overall cost per unit time. Taleizadeh et al. [10] have studied a multiproduct, single-machine production system employing economic production quantity (EPQ) model, in which one machine produces all products using a common cycle length, the production defective rates are random variables, shortages are allowed and take a combination of backorder and lost sale, and there is a service rate constraint for the company. A mathematical model of the problem was derived to determine the optimal production quantity, allowable shortage level, and period length of each product such that the total expected cost is minimized.

In real-life supply chain environments, a multi-delivery policy for shipping the end products is commonly used in lieu of the continuous issuing policy as assumed by a classic EPQ model.

Furthermore, there is often a producer that supplies a product to several clients/retailers. In such an internal type of vendor-buyer integrated system, the management would like to attain an optimal production-shipment policy in order to minimize the total expected system costs. Schwarz [13] has considered a one-warehouse, *N*-retailer, deterministic inventory system with the objective of deriving a stocking policy that minimizes the long-run average system cost per time unit. The optimal solutions along with a few necessary properties are derived. Heuristic solutions for the general problem have been also suggested in his study.

Goyal [14] has examined an integrated inventory model for a single supplier-single customer problem. He has proposed a method to solve those inventory problems, wherein a product made by a single supplier is acquired by a single customer. A numerical example was provided as a verification of his solution process.

Schwarz et al. [15] have studied the system fill-rate of a one-warehouse, *N*-identical retailer distribution system as a function of warehouse and retailer safety stock. They have used an approximation model from a prior study to maximize the system fill-rate subject to a constraint on system safety stock. As a result, the properties of fill-rate policy lines have been suggested.

Banerjee [16] has examined a joint economic lot-size model for a purchaser and vendor, with the focus on the minimization of the joint total relevant costs. He has concluded that a jointly optimal ordering policy, together with an appropriate price adjustment, could be economically beneficial for both parties, but definitely not disadvantageous to either of the parties.

Hall [17] has studied how attributes of the distribution system affect inventory accounting and EOQ/EPQ decisions. A range of “characteristic inventory curves” is developed to represent situations encountered in integrated production/distribution systems, and it is shown how system attributes define the inventory curve and the resulting EOQ/EPQ equations. It has been concluded that (1) accounting for inventory at both the origin and the destination can yield significantly different EOQ/EPQ results, but relatively modest regret; and (2) failure to account for consolidation effects among multiple products sent to a common destination can lead to substantial errors.

Parija and Sarker [18] have considered an ordering policy for raw materials as well as an economic batch size for finished products that are delivered to multiple customers, with a fixed-quantity at a fixed time-interval to each of the customers. In their model, an optimal multi-ordering policy for procurement of raw materials for a single manufacturing system is developed to minimize the total cost incurred due to raw materials and finished goods inventories. A closed-form solution to the problem has been obtained for the minimal total cost and the algorithm for multiple customer systems demonstrated.

Sucky [19] has studied a supply chain management from the perspective of inventory management. The coordination of order and production policies between buyers and suppliers in supply chains was of particular interest. This study has provided several bargaining models depending on alternative production policies of the supplier. With these bargaining models, the offered cooperative policy and the offered side payment can be derived.

Chiu et al. [1] have studied a single-producer, multi-retailer, integrated inventory system with rework. All random defective items produced have been assumed repairable through a rework process, and a multi-shipment policy has been adopted to synchronously deliver finished items to multiple retailers in order to satisfy customer's demands. An optimal production lot-size and shipment policy that minimizes the expected system costs has been derived with the help of a mathematical model.

Additional studies related to various aspects of supply-chain issues can also be found among other articles [20-27].

For the purpose of reducing the inventory-holding cost, this study integrates a cost-reduction shipment plan into Chiu et al.’s model [1]. The objectives are to find an optimal production-shipment policy that minimizes the expected system cost for such a supply chains system, and to demonstrate that the result of this study offers significant savings in the stock-holding cost for both the producer and the retailers.

2. PROBLEM DESCRIPTION AND MODELLING

Operating in highly-competitive business environments, the management of contemporary firms constantly seeks to cut down various operating costs, such as the inventory-holding costs, in their production units and affiliated retailers. This study integrates a cost-reduction delivery plan into a specific, single-producer, multi-retailer system [1] and aims at reducing the inventory-holding costs for both the producer and the retailers. To facilitate comparison, the present study adopts the same notations as those employed in Ref. [1]. Our proposed model is described as follows. A single-producer, multi-retailer, integrated system with rework and a cost reduction shipment plan is considered. It is supposed that an end product is produced at an annual rate P by a producer, and the manufacturing process may randomly generate an x portion of nonconforming items at a production rate d. All products made are screened and unit inspection cost is included into the unit production cost C. All of the nonconforming items are assumed to be repairable at a reworking rate P_1. The rework process takes place at the end of the regular production in each cycle. To prevent possible shortages, it is assumed that the constant production rate P must satisfy $(P–d–\lambda) > 0$ or $(1–x–\lambda/P) > 0$, where λ is the sum of the demands of all m retailers (i.e., sum of λ_i where $i=1,2,\ldots,m$), and d can be expressed as $d=Px$.

Aiming at the inventory-holding cost reduction, we adopt an $n+1$ multi-shipment policy. Under the proposed $n+1$ delivery policy, an initial shipment of finished goods is delivered to multiple retailers to meet the demand during the producer's uptime and reworking time. Upon the completion of the rework process, that is, when the remaining production lot's quality has been assured, n fixed quantity installments of the finished products are transported to different retailers, at a fixed time interval t_n (see Figure 1).

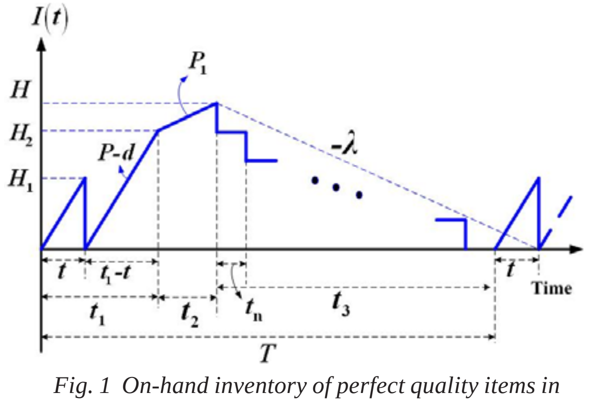

Fig. 1 On-hand inventory of perfect quality items in producer's side

Figure 2 depicts the expected reduction in the producer's inventory-holding costs (in light-blue shaded area) of the proposed model (in blue - lower lines), in comparison to that of Chiu et al.'s model [1] (in black - upper lines).

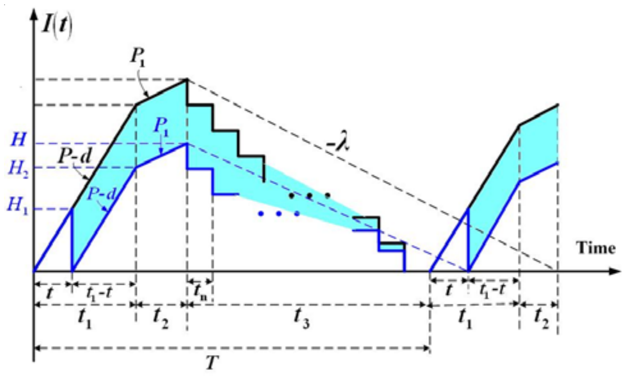

Fig. 2 Expected reduction in producer's inventory-holding costs (in the light-blue shaded area) in comparison to that of Chiu et al.'s model [1]

Figure 3 illustrates the on-hand inventory levels in m retailer sides under our proposed $n+1$ delivery policy. One notes that each retailer has its own annual demand rate λ_i. Upon the completion of the rework process, a fixed quantity of n installments of the finished batch are delivered to m retailers synchronously, at a fixed interval of time during the downtime t_3 (refer to Figure 1 and 3).

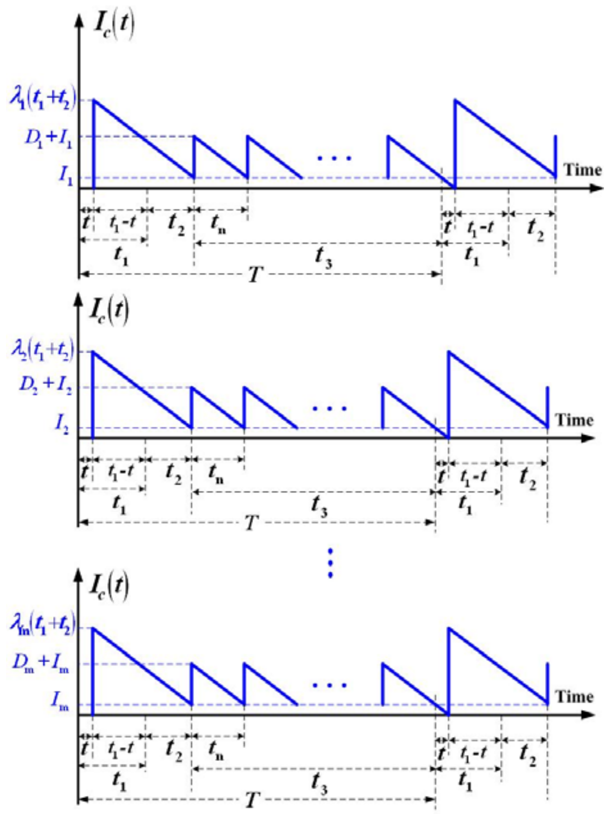

Fig. 3 On-hand inventory levels in m retailer sides under our proposed n+1 delivery policy

Figure 4 shows the expected reduction in retailers' stock-holding costs (in light-blue shaded area), as opposed to that of Chiu et al.'s model [1].

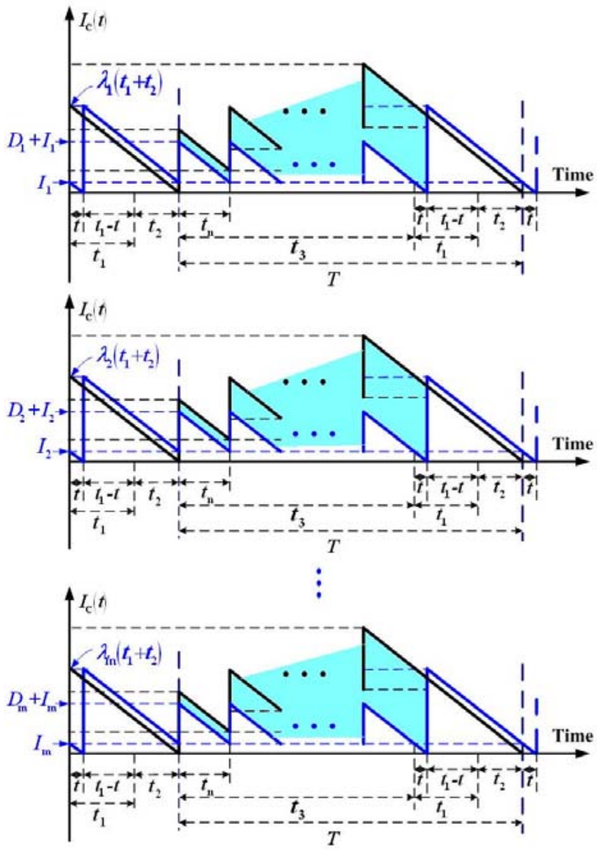

Fig. 4 Expected reduction in retailers' stock-holding costs (in the light-blue shaded area) in comparison to that of Chiu et al.'s model [1]

Cost variables used in this study include: the production setup cost K, unit holding cost h, unit reworking cost C_R and unit holding cost h_1 for each reworked item, the fixed delivery cost K_{1i} per shipment delivered to retailer i, unit holding cost h_{2i} for item stocked by retailer i, and unit shipping cost C_{Ti} for item shipped to retailer i. Additional notation is listed below:

t = time required for producing items to meet retailers' demands during producer's uptime t_1 and reworking time t_2,

t_1 = the production uptime,

t_2 = time required for reworking nonconforming items in each cycle,

t_3 = time required to deliver all remaining quality assured products in a lot to retailers,

Q = production lot-size per cycle, a decision variable,

n = number of fixed quantity installments of the finished batch to be delivered to retailers in each cycle, another decision variable,

T = production cycle length,

H_1 = level of on-hand inventory in units for meeting retailers' product demands during t_1 and t_2,

H_2 = level of on-hand inventory in units when regular production process ends,

H = maximum level of on-hand inventory in units when the rework process ends,

t_n = a fixed interval of time between each installment of finished products delivered during t_3,

m = number of regional sales offices,

D_i = number of fixed quantity finished items distributed to retailer i per delivery,

I_i = number of left over items per delivery after the depletion during t_{ni} for retailer i,

$I(t)$= on-hand inventory of perfect quality items at time t,

$I_c(t)$ = on-hand inventory at the retailers at time t,

$TC(Q, n+1)$ = total production-inventory-delivery costs per cycle for the proposed system,

$E[TCU(Q, n+1)]$ = total expected production-inventory-delivery costs per unit time for the proposed system.

The following equations can be directly obtained from Figures 1 and 3:

$$T = t_1 + t_2 + t_3 = \frac{Q}{\lambda} \tag{1}$$

$$t = \frac{\lambda(t_1 + t_2)}{P - d} \tag{2}$$

$$t_1 = \frac{Q}{P} = \frac{H_1 + H_2}{P - d} \tag{3}$$

The on-hand inventory of defective items during production uptime t_1 is:

$$dt_1 = Pxt_1 = xQ \tag{4}$$

$$t_2 = \frac{xQ}{P_1} \tag{5}$$

$$t_3 = T - (t_1 + t_2) = Q\left(\frac{1}{\lambda} - \frac{1}{P} - \frac{x}{P_1}\right) = nt_n \tag{6}$$

$$H_1 = \lambda(t_1 + t_2) = \lambda\left(\frac{Q}{P} + \frac{xQ}{P_1}\right) \tag{7}$$

$$H_2 = (t_1 - t)\cdot(P - d) \tag{8}$$

$$H = H_1 + P_1 t_2 \tag{9}$$

$$\lambda = \sum_{i=1}^{m} \lambda_i \tag{10}$$

Total delivery costs of $n+1$ shipments to m retailers in a production cycle are:

$$(n+1)\sum_{i=1}^{m} K_{1i} + \sum_{i=1}^{m} C_{Ti} Q \tag{11}$$

The variable holding costs at producer's side during the delivery time t_3, where n fixed-quantity installments of the finished batch are delivered to retailers at a fixed interval of time are [22]:

$$h\left(\frac{n-1}{2n}\right)Ht_3 \tag{12}$$

Total inventory holding costs for items stocked by retailers during the cycle are (see Figure 3):

$$\sum_{i=1}^{m} h_{2i}\left[\frac{\lambda_i(t_1+t_2)^2}{2}+n\left(\frac{D_i+2I_i}{2}\right)t_n\right] \tag{13}$$

Total production-inventory-delivery cost per cycle $TC(Q, n+1)$ consists of the following variables: the production cost, setup cost, reworking cost, fixed and variable delivery cost, producer's inventory holding cost during t_1, t_2, and t_3, and holding cost for finished goods stocked by m retailers:

$$\begin{aligned} TC(Q,n) &= CQ+K+C_R xQ+(n+1)\sum_{i=1}^{m}K_{1i}+\sum_{i=1}^{m}C_{Ti}Q+ \\ &+h\left[\frac{H_1}{2}(t)+\frac{H_2}{2}(t-t_1)+\frac{H_2+H}{2}(t_2)+\frac{dt_1}{2}(t_1)+\left(\frac{n-1}{2n}\right)Ht_3\right]+ \\ &+h_1\cdot\frac{dt_1}{2}\cdot(t_2)+\sum_{i=1}^{m}h_{2i}\left[\frac{\lambda_i(t_1+t_2)^2}{2}+n\left(\frac{D_i+2I_i}{2}\right)t_n\right] \end{aligned} \tag{14}$$

Since the defective rate x is assumed to be a random variable, taking the randomness of x into account we use the expected values of x in the consequence cost analysis. Substituting all parameters from Eqs. (1) to (13) in $TC(Q, n+1)$, and with further derivations, we obtain the expected cost $E[TCU(Q, n+1)]$ as:

$$\begin{aligned} E\left[TCU(Q,n+1)\right] &= C\lambda+\frac{K\lambda}{Q}+C_R\lambda E[x]+\sum_{i=1}^{m}C_{Ti}\lambda_i+\frac{(n+1)\lambda}{Q}\sum_{i=1}^{m}K_{1i}+ \\ &+\frac{hQ\lambda}{2}\left\{\frac{1}{\lambda}-\frac{1}{P}-\frac{1}{P_1}\left[1+\frac{\lambda}{P_1}\right]E[x]^2-\left(\frac{1}{n}\right)E_3+E_4\right\}+\frac{h_1Q\lambda E[x]^2}{2P_1}+ \\ &+\frac{\sum_{i=1}^{m}h_{2i}Q\lambda_i}{2}\left\{\frac{\lambda E[x]^2}{P_1^2}+\frac{2\lambda E_0}{P^2}+\frac{2\lambda E_1}{PP_1}+\left(\frac{1}{n}\right)E_3-E_4\right\} \end{aligned} \tag{15}$$

where E_i denotes the following:

$$\begin{aligned} E_0 &= E\left(\frac{1}{1-x}\right);\ E_1=E\left(\frac{x}{1-x}\right);\ E_2=E\left(\frac{x^2}{1-x}\right); \\ E_3 &= \left[\frac{1}{\lambda}-\frac{2}{P}-\frac{2E[x]}{P_1}+\frac{\lambda}{P^2}+\frac{2\lambda E[x]}{PP_1}+\frac{\lambda E[x]^2}{P_1^2}\right]; \\ E_4 &= \left[\frac{2\lambda^2E_0}{P^3}+\frac{4\lambda^2E_1}{P^2P_1}+\frac{2\lambda^2}{PP_1^2}E_2-\frac{\lambda}{P^2}-\frac{2\lambda E[x]}{PP_1}\right]. \end{aligned} \tag{16}$$

3. DETERMINING THE OPTIMAL POLICY

The Hessian matrix equations [28, 29] is employed here to prove the convexity of $E[TCU(Q, n)]$, that is, to verify whether Eq. (17) holds:

$$\begin{bmatrix} Q & n \end{bmatrix}\cdot\begin{pmatrix} \dfrac{\partial^2E\left[TCU(Q,n+1)\right]}{\partial Q^2} & \dfrac{\partial^2E\left[TCU(Q,n+1)\right]}{\partial Q\partial n} \\ \dfrac{\partial^2E\left[TCU(Q,n+1)\right]}{\partial Q\partial n} & \dfrac{\partial^2E\left[TCU(Q,n+1)\right]}{\partial n^2} \end{pmatrix}\cdot\begin{bmatrix} Q \\ n \end{bmatrix}>0 \tag{17}$$

From Eq. (15) we have:

$$\frac{\partial E\left[TCU\left(Q,n+1\right)\right]}{\partial Q}=-\frac{1}{Q^2}\lambda\left[\left(n+1\right)\sum_{i=1}^{m}K_{1i}+K\right]+\frac{h_1\lambda}{2P_1}\left(E\left[x\right]\right)^2+ +\frac{h\lambda}{2}\left\{\frac{1}{\lambda}-\frac{1}{P}-\frac{1}{P_1}\left[1+\frac{\lambda}{P_1}\right]\left(E\left[x\right]\right)^2-\left(\frac{1}{n}\right)E_3-E_4\right\}+ +\frac{\sum_{i=1}^{m}h_{2i}\lambda_i}{2}\left\{\frac{\lambda E\left[x\right]^2}{P_1^2}+\frac{2\lambda E_0}{P^2}+\frac{2\lambda E_1}{PP_1}+\left(\frac{1}{n}\right)E_3-E_4\right\} \tag{18}$$

$$\frac{\partial^2 E\left[TCU\left(Q,n+1\right)\right]}{\partial Q^2}=\frac{2\left[\left(n+1\right)\sum_{i=1}^{m}K_{1i}+K\right]\lambda}{Q^3} \tag{19}$$

$$\frac{\partial E\left[TCU\left(Q,n+1\right)\right]}{\partial n}=\lambda\sum_{i=1}^{m}K_{1i}\frac{1}{Q}+\left(\frac{1}{n^2}\right)\frac{1}{2}\left[hQ\lambda-\sum_{i=1}^{m}h_{2i}Q\lambda_i\right]\cdot E_3 \tag{20}$$

$$\frac{\partial^2 E\left[TCU\left(Q,n+1\right)\right]}{\partial n^2}=\frac{-1}{n^3}\left[hQ\lambda-\sum_{i=1}^{m}h_{2i}Q\lambda_i\right]E_3 \tag{21}$$

$$\frac{\partial E\left[TCU\left(Q,n+1\right)\right]}{\partial Q\partial n}=-\frac{\lambda\sum_{i=1}^{m}K_{1i}\cdot\sum_{i=1}^{m}\lambda_i}{Q^2}+\left(\frac{1}{n^2}\right)\frac{1}{2}\left[h\lambda-\sum_{i=1}^{m}h_{2i}\lambda_i\right]E_3 \tag{22}$$

Substituting Eqs. (19), (21) and (22) in Eq. (17) we obtain:

$$\left[Q\quad n\right]\cdot\begin{pmatrix}\frac{\partial^2 E\left[TCU\left(Q,n+1\right)\right]}{\partial Q^2} & \frac{\partial^2 E\left[TCU\left(Q,n+1\right)\right]}{\partial Q\partial n}\\ \frac{\partial^2 E\left[TCU\left(Q,n+1\right)\right]}{\partial Q\partial n} & \frac{\partial^2 E\left[TCU\left(Q,n+1\right)\right]}{\partial n^2}\end{pmatrix}\cdot\begin{bmatrix}Q\\ n\end{bmatrix}=\frac{2\lambda}{Q}\left(K+\sum_{i=1}^{m}K_{1i}\right)>0 \tag{23}$$

The solution to Eq. (23) is positive, because λ, Q, K, and K_{1i} are all positive. Hence, $E[TCU(Q, n+1)]$ is a strictly convex function for all Q and n different from zero. Therefore, the convexity of $E[TCU(Q, n+1)]$ is proved, and there is a minimum of $E[TCU(Q, n+1)]$.

To simultaneously determine the production-shipment policy for the proposed model, one can solve the linear system of Eqs. (18) and (20) by setting these partial derivatives equal to zero. With further derivations one obtains:

$$Q^*=\sqrt{\frac{2\left[K+\left(n+1\right)\sum_{i=1}^{m}K_{1i}\right]\lambda}{h\lambda\left\{\frac{1}{\lambda}-\frac{1}{P}-\frac{1}{P_1}\left[1+\frac{\lambda}{P_1}\right]E\left[x\right]^2-\left(\frac{1}{n}\right)E_3+E_4\right\}+\frac{h_1\lambda E\left[x\right]^2}{P_1}+ +\sum_{i=1}^{m}h_{2i}\lambda_i\left\{\frac{\lambda E\left[x\right]^2}{P_1^2}+\frac{2\lambda E_0}{P^2}+\frac{2\lambda E_1}{PP_1}+\left(\frac{1}{n}\right)E_3-E_4\right\}}} \tag{24}$$

and:

$$n^* = \sqrt{\frac{\left(\sum_{i=1}^{m} K_{1i} + K\right)\left(\sum_{i=1}^{m} h_{2i}\lambda_i - h\lambda\right)E_3}{\left(\sum_{i=1}^{m} K_i\right)\left\{\begin{array}{l} h\lambda\left\{\frac{1}{\lambda} - \frac{1}{P} - \frac{1}{P_1}\left[1 + \frac{\lambda}{P_1}\right]E[x]^2 + E_4\right\} + \frac{h_1\lambda E[x]^2}{P_1} + \\ +\sum_{i=1}^{m} h_{2i}\lambda_i\left\{\frac{\lambda E[x]^2}{P_1^2} + \frac{2\lambda E_0}{P^2} + \frac{2\lambda E_1}{PP_1} - E_4\right\}\end{array}\right\}}} \qquad (25)$$

It can be noted that the computational result of Eq. (25) does not necessarily have to be an integer number. However, in a real-world situation, the number of deliveries can only be an integer number. Therefore, in order to obtain the integer value of n^* that minimizes the expected system cost, two adjacent integers to n must be examined [30]. Let n^+ denote the smallest integer greater than or equal to n (derived from Eq. (23)) and n^- denote the largest integer less than or equal to n. Substitute n^+ and n^- in Eq. (22), then apply the resulting (Q, n^+) and (Q, n^-) in Eq. (14). By selecting the one that gives the minimal long-run average cost as the optimal replenishment-distribution policy (Q^*, n^*). An example is provided in the next section to demonstrate the practical usage of the obtained results.

4. NUMERICAL EXAMPLE

In order to facilitate the comparison of the results, this section adopts the same numerical example as the one used in Ref. [1]. We have assured that a product's annual demands λ_i from *5* different retailers are *650*, *350*, *450*, *800*, and *750* respectively (the total demand λ is *3000* per year) and this product can be made by a vendor at an annual production rate $P=60,000$ units. There is a random defective rate during production uptime which follows a uniform distribution over the interval [*0*, *0.3*]. All nonconforming items can be repaired during the rework process at a rate $P_1=3600$ per year.

Values for other variables are as follows: $C=\$100$ per item; $K=\$35,000$ per production run; $h = \$25$ per item per year; $h_1=\$60$ per item per year; $C_R=\$60$ for each item reworked; $K_{1i}=\$400, \$100, \$300, \$450$, and $\$250$ for retailer i, respectively; $h_{2i}=\$70, \$80, \$75, \$60$, and $\$65$ per item; and $C_{Ti}=\$0.5, \$0.4, \$0.3, \$0.2$, and $\$0.1$ for retailer i, respectively.

To demonstrate the inventory-holding cost savings, we first take the production-shipment policy (i.e. $(Q, n)=(2310, 5)$) obtained by Chiu et al. [1] and integrate it in our cost function $E[TCU(Q, n+1)]$ (i.e. Eq. (15)). We obtain $E[TCU(2310, 6)]=\$422,667$ versus *\$438,211* in Ref. [1], the savings amounts to *\$15,544*. Figure 5 shows the comparison of different cost components in the proposed model and in Chiu et al.'s model [1] using policy $(Q, n)=(2310, 5)$. One notes that the main contribution of the cost savings comes from the reduction of inventory holding costs for both producer and retailers.

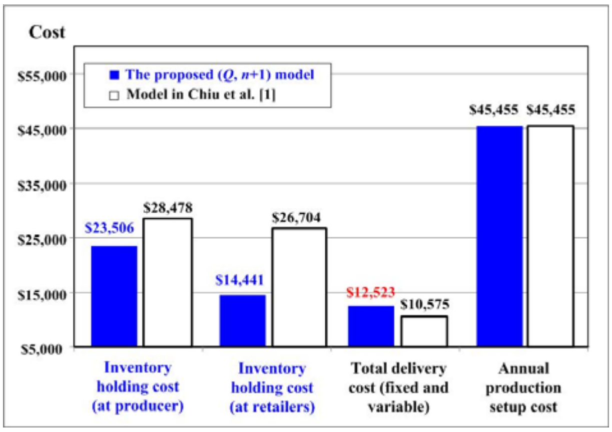

Fig. 5 Comparisons of different cost components in the proposed model and in Chiu et al.'s model [1] using (Q, n)=(2310, 5)

In addition, to derive the optimal production-shipment policy of our proposed model, we first apply Eq. (25) and find that $n=5.136$. In order to determine the optimal integer number of n^* that minimizes the expected cost function $E[TCU(Q, n+1)]$, two adjacent integers to n are examined [30]. Applying Eq. (24), we obtain $(Q, n^+)=(2310, 6)$ and $(Q, n^-)=(2228, 5)$. Substituting these (Q, n^+) and (Q, n^-) into equation $E[TCU(Q, n+1)]$ (Eq. (15)), and by selecting the one that gives the minimum system cost, we obtain the optimal number of deliveries $n^*=5$, the optimal lot-size $Q^*=2835$, and the optimal expected cost $E[TCU(2835, 6)]=\$420,967$.

The optimal solution gives us savings of *\$17,244* (i.e., *\$438,211–\$420,967*) or equivalent to *12.48%* of the total setup-holding-delivery costs, as opposed to that in Chiu et al.'s model [1].

Finally, the variation of Q and n effects on the optimal $E[TCU(Q, n+1)]$ are illustrated in Figure 6. It is noted that as the replenishment lot-size Q changes, i.e. moves away from the optimal Q^* in either direction, the expected system cost $E[TCU(Q, n+1)]$ slightly increases; but as the number of deliveries n changes, i.e. moves away from the optimal n^* in either direction, the expected system cost increases significantly.

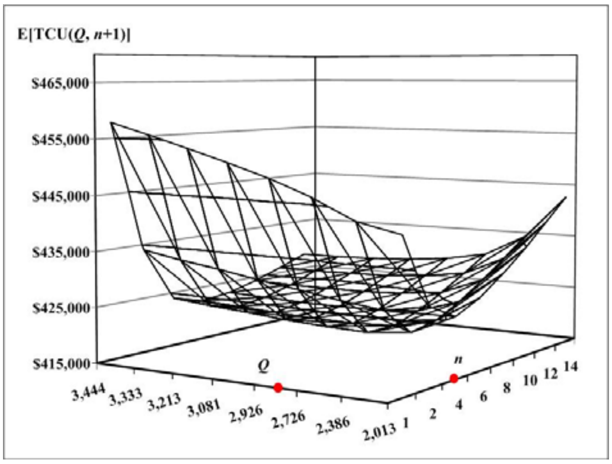

Fig. 6 Variation of Q and n effects on the cost function E[TCU(Q, n+1)]

5. CONCLUSIONS

Operating in highly competitive business environments, the management of contemporary firms constantly seeks to cut down on various operating costs, such as the inventory-holding costs in their production units and affiliated retailers. This study integrates a cost-reduction, *n+1* delivery plan into a specific, single-producer, multi-retailer system [1] and aims at reducing the inventory-holding costs for both the producer and the retailers. With the help of mathematical modelling, the closed-form, optimal production-shipment policy that minimizes the long-run average system cost is derived. Through a numerical example, we demonstrate that our research result (Figure 5) in comparison to that in Ref. [1] gives significant savings in inventory-holding costs for both producer and retailers. The effects of decision variables on the expected system cost have also been analyzed (Figure 6), to provide the management in the field with insights into the proposed single-producer, multi-retailer, integrated inventory model.

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INTEGRIRANJE PLANA SMANJENJA TROŠKOVA ISPORUKE ROBE U SUSTAV S JEDNIM PROIZVOĐAČEM, VIŠE MALOPRODAJNIH TRGOVACA TE PROCESOM OBRADE PROIZVODA S GREŠKOM

SAŽETAK

U ovome se radu razmatra uključivanje plana smanjenja troškova isporuke robe u opskrbni sustav koji se sastoji od jednog proizvođača, više maloprodajnih trgovaca te procesa obrade proizvoda s greškom. U nedavno objavljenom članku, Chiu et al. [1] razmatraju model s integriranim nadzorom robe koji obuhvaća jednog proizvođača, više maloprodajnih trgovaca te proces obrade robe s greškom. Kako bi se smanjio trošak zadržavanja zaliha robe, u ovome se radu razmatra kombiniranje alternativne metode distribucije dobara i već spomenutog razrađenog modela. Predloženi model isporuke robe uključuje dodatnu pošiljku gotovog proizvoda koja se odvija tijekom neprekidne proizvodnje kako bi se zadovoljili trgovčevi zahtjevi za robom u vrijeme kada proizvođač prerađuje robu s greškom. Po završetku obrade, mnoštvo pošiljki se istovremeno isporučuje različitim trgovcima na malo. Cilj ovoga rada je iznaći optimalan plan odnosa proizvodnje i isporuke robe koji bi sveo očekivane troškove sustava na najmanju moguću mjeru, a isto tako i pokazati da predloženi model ima značajne prednosti glede uštede u odnosu na model koji su predložili Chiu et al. [1]. Pomoću matematičkog modeliranja i jednadžbi Hessianove matrice iznjedren je optimalan operativni plan za predloženi model. Numeričkim primjerom je pokazano da novopredloženi model pruža značajne uštede glede troškova skladištenja zaliha robe, kako proizvođačima tako i trgovcima na malo.

Ključne riječi: *optimizacija, opskrbni lanac, serijska proizvodnja, sustav s više maloprodajnih trgovaca, obrada proizvoda s greškom, mnogostruka isporuka.*