UDC 336.645:339.1:519.86
Original scientific paper
Received: 22.10.2013.

Revisiting "integrating a cost-reduction shipment plan into a single-producer multi-retailer system with rework" using an alternative approach

Yuan-Shyi Peter Chiu(1)**, Kuo-Wei Chiang**(1)**, Mei-Fang Wu**(2) and **Singa Wang Chiu**(3)

(1)Department of Industrial Engineering & Management, Chaoyang University of Technology, Gifong East Road, Wufong, Taichung 413, TAIWAN, e-mail: ypchiu@cyut.edu.tw
(2)Department of Industrial Engineering & Systems Management, Feng Chia University, Taichung, TAIWAN
(3)Department of Business Administration, Chaoyang University of Technology, Gifong East Road, Wufong, Taichung 413, TAIWAN, e-mail: swang@cyut.edu.tw

SUMMARY

This study revisits a single-producer multi-retailer system with a cost-reduction shipment plan and rework [1] using an alternative approach. Unlike the conventional method that uses differential calculus on system cost function to prove its convexity and derive the optimal production-shipment policy, we proposed an algebraic solution procedure to the problem. Such a straightforward approach may enable the practitioners with little knowledge of calculus to understand real supply chain systems more easily.

Key words: *optimization, supply chain systems, production-shipment decision, rework, multi-shipment, algebraic approach.*

1. INTRODUCTION

The most economical production lot problem, also known as economic production quantity (EPQ) model, was first studied by Taft [2] with a help of mathematical modelling. The EPQ model assumed a continuous issuing policy to satisfy product demand. However, in real vendor-buyer integrated systems, the multi-delivery policy is commonly adopted. Therefore, the decision on determining an optimal replenishment lot-size and number of deliveries that minimizes the system's production-inventory-delivery cost becomes crucial to the management in the field.

Schwarz [3] has been the first to examine a one-warehouse, N-retailer inventory system with the objective of defining the stock refilling policy that minimizes the system cost. Goyal [4] has studied a single-supplier single-customer integrated inventory problem, wherein a product made by a single-supplier is acquired by a single-customer. Kim and Hwang [5] have developed the formulation of a quantity discount pricing schedule for a supplier. They have considered a single incremental discount system and proposed an algorithm for deriving an optimal discount schedule. They have studied cases in which both the discount rate and the break point are unknown and either one is prescribed, and used numerical example to illustrate their algorithm. Many studies have since been carried out to address different aspects of the supply chain optimization issues [6-16].

Vendor's product quality assurance is another critical issue management faced in supply chains' environments. Due to various unpredictable factors in any given production run, it is inevitable to produce random nonconforming items. Many studies in the past decades have attempted to address the issues related to defective products and quality assurance in production systems [17-23]. Chiu et al. [1] have integrated a cost-reduction shipment plan into a single-producer, multi-retailer system with rework with the purpose of cutting down system's stock-holding cost for both producer and retailers. Their study has considered a practical, multi-delivery policy and production quality assurance issues. All random defective items produced have assumed to be repairable through a rework process, and a multi-shipment policy has been adopted to synchronously deliver finished items to multiple retailers in order to satisfy customers' demands. An optimal production lot-size and shipment policy that minimized the expected system costs has been derived with the help of a mathematical model.

Grubbström and Erdem [24] recently proposed an algebraic derivation to solve the economic order quantity (EOQ) model with backlogging. The approach does not reference to the first- or second-order derivatives. Similar approaches were applied to deal with various different aspects of production and supply chain optimization issues [25-28]. Such an alternative approach is adopted in this paper to revisit a single-producer multi-retailer system with a cost reduction shipment plan and rework introduced in Ref. [1].

2. MODELLING AND ALGEBRAIC APPROACH

A single-producer, multi-retailer system with a cost-reduction shipment plan and rework introduced in Ref. [1] is revisited in this paper using an algebraic approach. The description and modelling of such a specific model is given as follows. It has been assumed that a producer can manufacture a product at an annual production rate P, and the process may randomly generate an x portion of nonconforming items at a rate d. All items produced are screened and the inspection cost is included in the unit production cost C. All nonconforming items are assumed to be repairable at a rate of rework P_1, and this rework process starts at the end of the regular production in each cycle (see Figure 1). To disallow shortages, a constant production rate P must satisfy $(P-d-\lambda)>0$, where λ is the sum of the demands of all m retailers (i.e., sum of λ_i where i=$1, 2, \ldots, m$), and d can be expressed as $d=Px$.

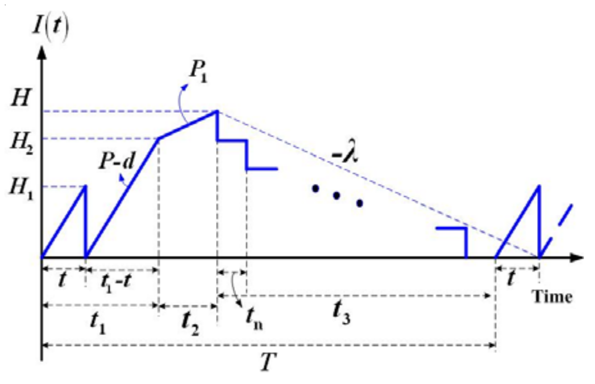

Fig. 1 On-hand inventory of perfect quality items in producer's side [1]

An $n+1$ multi-shipment policy is used in this study with the purpose of reducing inventory holding cost for both producer and retailers. In accordance with the proposed $n+1$ delivery policy, an initial shipment of finished goods is delivered to multiple retailers to meet their demands during the producer's uptime and reworking time. Upon the completion of the rework process, i.e. when the remaining production lot's quality has been assured, n fixed quantity installments of the finished products are transported to different retailers, at a fixed time interval t_n (see Figures 1 and 2).

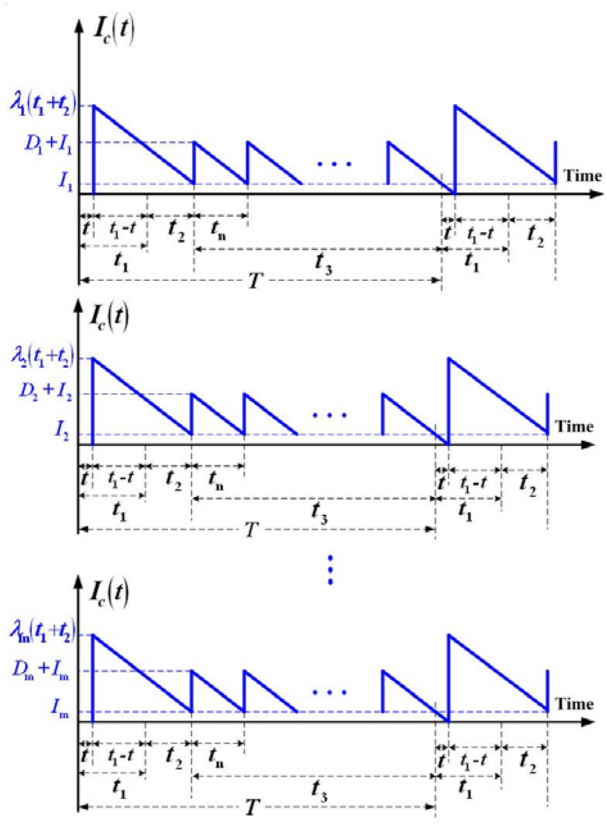

Fig. 2 On-hand inventory levels in m retailer sides under our proposed n+1 delivery policy [1]

Cost parameters used in this study include: (1) production setup cost K, (2) unit holding cost h, (3) unit reworking cost C_R, (4) holding cost h_1 for each reworked item, (5) fixed delivery cost K_{1i} per shipment delivered to retailer i, (6) holding cost h_{2i} for each item stocked by retailer i, and (7) unit shipping cost C_{Ti} for an item shipped to retailer i. Additional notation is listed as follows:

Q = production lot-size per cycle, a decision variable,
T = production cycle length,
n = number of fixed quantity installments of the finished batch to be delivered to retailers in each cycle, another decision variable,
t = time required for producing items to meet retailers' demands during producer's uptime t_1 and reworking time t_2,
t_1 = production uptime,
t_2 = time required for reworking nonconforming

items in each cycle,

t_3 = time required to deliver all remaining quality assured products in a lot to retailers,

H_1 = level of on-hand inventory in units for meeting retailers' product demands during t_1 and t_2,

H_2 = level of on-hand inventory in units when regular production process ends,

H = maximum level of on-hand inventory in units when rework process ends,

t_n = a fixed interval of time between each installment of finished products delivered during t_3,

m = number of regional sales offices,

D_i = number of fixed quantity finished items distributed to retailer i per delivery,

I_i = number of left over items per delivery after the depletion during t_{ni} for retailer i,

$I(t)$ = on-hand inventory of perfect quality items at time t,

$Ic(t)$ = on-hand inventory at the retailers at time t,

$TC(Q, n+1)$ = total production-inventory-delivery costs per cycle for the proposed system,

$E[TCU(Q, n+1)]$ = total expected production-inventory-delivery costs per unit time for the proposed system.

From Figures 1 and 2, it can be seen that total production-inventory-delivery cost per cycle $TC(Q,n+1)$ consists of the following variables: the production cost, production setup cost, reworking cost, fixed and variable delivery cost, producer's inventory holding cost during t_1, t_2 and t_3, and holding cost for finished goods stocked by m retailers (Figure 2):

$$TC(Q,n) = CQ + K + C_R xQ + (n+1)\sum_{i=1}^{m} K_{1i} + \sum_{i=1}^{m} C_{Ti}Q + \\ + h\left[\frac{H_1}{2}(t) + \frac{H_2}{2}(t - t_1) + \frac{H_2 + H}{2}(t_2) + \frac{dt_1}{2}(t_1) + \left(\frac{n-1}{2n}\right)Ht_3\right] + \\ + h_1 \cdot \frac{dt_1}{2} \cdot (t_2) + \sum_{i=1}^{m} h_{2i}\left[\frac{\lambda_i (t_1 + t_2)^2}{2} + n\left(\frac{D_i + 2I_i}{2}\right)t_n\right] \tag{1}$$

Since the defective rate x is assumed to be a random variable, we use the expected values of x in the consequence cost analysis to cope with the randomness of x. The following expected system cost function $E[TCU(Q, n+1)]$ can be obtained with further derivations [1]:

$$E\left[TCU(Q,n+1)\right] = C\lambda + \frac{K\lambda}{Q} + C_R\lambda E[x] + \sum_{i=1}^{m} C_{Ti}\lambda_i + \frac{(n+1)\lambda}{Q}\sum_{i=1}^{m} K_{1i} + \\ + \frac{hQ\lambda}{2}\left\{\frac{1}{\lambda} - \frac{1}{P} - \frac{1}{P_1}\left[1 + \frac{\lambda}{P_1}\right]E[x]^2 - \left(\frac{1}{n}\right)E_3 + E_4\right\} + \frac{h_1 Q\lambda E[x]^2}{2P_1} + \\ + \frac{\sum_{i=1}^{m} h_{2i}Q\lambda_i}{2}\left\{\frac{\lambda E[x]^2}{P_1^2} + \frac{2\lambda E_0}{P^2} + \frac{2\lambda E_1}{PP_1} + \left(\frac{1}{n}\right)E_3 - E_4\right\} \tag{2}$$

where E_i denotes the following:

$$E_0 = E\left(\frac{1}{1-x}\right);\ E_1 = E\left(\frac{x}{1-x}\right);\ E_2 = E\left(\frac{x^2}{1-x}\right); \\ E_3 = \left[\frac{1}{\lambda} - \frac{2}{P} - \frac{2E[x]}{P_1} + \frac{\lambda}{P^2} + \frac{2\lambda E[x]}{PP_1} + \frac{\lambda E[x]^2}{P_1^2}\right]; \\ E_4 = \left[\frac{2\lambda^2 E_0}{P^3} + \frac{4\lambda^2 E_1}{P^2 P_1} + \frac{2\lambda^2}{PP_1^2}E_2 - \frac{\lambda}{P^2} - \frac{2\lambda E[x]}{PP_1}\right]. \tag{3}$$

2.1 The algebraic approach

It can be seen that Eq. (2) contains two decision variables (i.e., Q and n), and there are different forms of decision variables in the right-hand side of Eq. (2), such as Q, Q^{-1}, nQ^{-1}, and Qn^{-1}. Suppose we let Ω_1, Ω_2, Ω_3, Ω_4, and Ω_5 denote the following coefficients of different forms of decision variables in the right hand side of Eq. (2):

$$\Omega_1 = C\lambda + C_R \lambda E[x] + \sum_{i=1}^{m} C_{Ti}\lambda_i \tag{4}$$

$$\Omega_2 = \lambda\left[K + \sum_{i=1}^{m} K_{1i}\right] \tag{5}$$

$$\Omega_3 = \lambda \cdot \left(\sum_{i=1}^{m} K_{1i}\right) \tag{6}$$

$$\begin{aligned}\Omega_4 = {} & \frac{h\lambda}{2}\left\{\frac{1}{\lambda} - \frac{1}{P} - \frac{1}{P_1}\left[1 + \frac{\lambda}{P_1}\right]E[x]^2 + E_4\right\} + \\ & + \frac{h_1 \lambda E[x]^2}{2P_1} + \frac{\sum_{i=1}^{m} h_{2i}\lambda_i}{2}\left\{\frac{\lambda E[x]^2}{P_1^2} + \frac{2\lambda E_0}{P^2} + \frac{2\lambda E_1}{PP_1} - E_4\right\}\end{aligned} \tag{7}$$

$$\Omega_5 = \frac{E_3}{2}\left[\left(\sum_{i=1}^{m} h_{2i}\lambda_i\right) - h\lambda\right] \tag{8}$$

Thus, Eq. (2) becomes:

$$E\left[TCU(Q,n+1)\right] = \Omega_1 + \Omega_2\left(\frac{1}{Q}\right) + \Omega_3\left(\frac{n}{Q}\right) + \Omega_4 \cdot Q + \Omega_5\left(\frac{Q}{n}\right) \tag{9}$$

or:

$$E\left[TCU(Q,n+1)\right] = \Omega_1 + \Omega_2 Q^{-1} + \Omega_3\left(nQ^{-1}\right) + \Omega_4 \cdot Q + \Omega_5\left(Qn^{-1}\right) \tag{10}$$

Equation (10) can also be rearranged as:

$$E\left[TCU(Q,n+1)\right] = \Omega_1 + \left(Q^{-1}\right)\left[\left(\sqrt{\Omega_2}\right)^2 + \left(\sqrt{\Omega_4}\right)^2 Q^2\right] + \left(nQ^{-1}\right)\left[\left(\sqrt{\Omega_3}\right)^2 + \left(\sqrt{\Omega_5}\right)^2\left(Qn^{-1}\right)^2\right] \tag{11}$$

or:

$$\begin{aligned}E\left[TCU(Q,n+1)\right] = {} & \Omega_1 + \left(Q^{-1}\right)\left[\sqrt{\Omega_2} - Q\sqrt{\Omega_4}\right]^2 + \left(nQ^{-1}\right)\left[\sqrt{\Omega_3} - \left(Qn^{-1}\right)\sqrt{\Omega_5}\right]^2 + \\ & + 2\sqrt{\Omega_2}\cdot\sqrt{\Omega_4} + 2\sqrt{\Omega_3}\cdot\sqrt{\Omega_5}\end{aligned} \tag{12}$$

It can be seen that Eq. (12) is minimized if both its second and third square terms equal zero. That is:

$$\left[\sqrt{\Omega_2} - Q\sqrt{\Omega_4}\right] = 0 \quad \text{and} \quad \left[\sqrt{\Omega_3} - \left(Qn^{-1}\right)\sqrt{\Omega_5}\right] = 0 \tag{13}$$

or:

$$Q = \sqrt{\frac{\Omega_2}{\Omega_4}} \tag{14}$$

and:

$$n = Q\frac{\sqrt{Q_5}}{\sqrt{Q_3}} = \sqrt{\frac{\Omega_2}{\Omega_4}} \cdot \frac{\sqrt{Q_5}}{\sqrt{Q_3}} \tag{15}$$

Substituting Eqs. (5) to (8) in Eq. (15), one obtains the optimal number of shipments n^* as:

$$n^{*}=\sqrt{\frac{\lambda\left(K+\sum_{i=1}^{m}K_{1i}\right)\cdot\left[\left(\sum_{i=1}^{m}h_{2i}\lambda_i\right)-h\lambda\right]\cdot\frac{E_3}{2}}{\lambda\cdot\left(\sum_{i=1}^{m}K_{1i}\right)\cdot\left\{\begin{array}{l}\frac{h\lambda}{2}\left\{\frac{1}{\lambda}-\frac{1}{P}-\frac{1}{P_1}\left[1+\frac{\lambda}{P_1}\right]E\left[x\right]^2+E_4\right\}+\frac{h_1\lambda E\left[x\right]^2}{2P_1}+\\+\frac{\sum_{i=1}^{m}h_{2i}\lambda_i}{2}\left\{\frac{\lambda E\left[x\right]^2}{P_1^2}+\frac{2\lambda E_0}{P^2}+\frac{2\lambda E_1}{PP_1}-E_4\right\}\end{array}\right\}}}\tag{16}$$

or:

$$n^{*}=\sqrt{\frac{\left(\sum_{i=1}^{m}K_{1i}+K\right)\left(\sum_{i=1}^{m}h_{2i}\lambda_i-h\lambda\right)E_3}{\left(\sum_{i=1}^{m}K_i\right)\left\{\begin{array}{l}h\lambda\left\{\frac{1}{\lambda}-\frac{1}{P}-\frac{1}{P_1}\left[1+\frac{\lambda}{P_1}\right]E\left[x\right]^2+E_4\right\}+\frac{h_1\lambda E\left[x\right]^2}{P_1}+\\+\sum_{i=1}^{m}h_{2i}\lambda_i\left\{\frac{\lambda E\left[x\right]^2}{P_1^2}+\frac{2\lambda E_0}{P^2}+\frac{2\lambda E_1}{PP_1}-E_4\right\}\end{array}\right\}}}\tag{17}$$

It can be seen that Eq. (17) is identical to that obtained by using the conventional differential calculus method in Ref. [1].

Furthermore, in order to find the integer value of n^* that minimizes the expected system cost two integers adjacent to n must be examined to test and select the one that minimizes $E[TCU(Q, n+1)]$ [29]. Upon obtaining the integer value of n, we now treat $E[TCU(Q, n+1)]$ (i.e. Eq.(2)) as an expected cost function with single decision variable Q.

Thus, from Eq. (9) $E[TCU(Q, n+1)]$ can be rewritten as:

$$E\left[TCU\left(Q,n+1\right)\right]=\Omega_1+\Omega_6\cdot\left(\frac{1}{Q}\right)+\Omega_7\cdot Q\tag{18}$$

where Ω_6 and Ω_7 denote the following:

$$\Omega_6=\left(\Omega_2+n\cdot\Omega_3\right);\quad \Omega_7=\left(\Omega_4+n^{-1}\cdot\Omega_5\right).\tag{19}$$

Equation (18) can also be rearranged as:

$$E\left[TCU\left(Q,n+1\right)\right]=\Omega_1+\left(Q^{-1}\right)\left[\sqrt{\Omega_6}-Q\sqrt{\Omega_7}\right]^2+2\sqrt{\Omega_6}\cdot\sqrt{\Omega_7}\tag{20}$$

It can be seen that Eq. (20) is minimized by the second square terms in the right hand side equals zero, that is:

$$Q=\sqrt{\frac{\Omega_6}{\Omega_7}}\tag{21}$$

Substituting Eqs. (5) to (8) in Eq. (19) and then in Eq. (21), one obtains the optimal number of shipments Q^* as:

$$Q^{*}=\sqrt{\frac{\lambda\left[K+\sum_{i=1}^{m}K_{1i}\right]+n\cdot\left[\lambda\cdot\left(\sum_{i=1}^{m}K_{1i}\right)\right]}{\begin{array}{l}\frac{h\lambda}{2}\left\{\frac{1}{\lambda}-\frac{1}{P}-\frac{1}{P_1}\left[1+\frac{\lambda}{P_1}\right]E\left[x\right]^2+E_4\right\}+\frac{h_1\lambda E\left[x\right]^2}{2P_1}+\\+\frac{\sum_{i=1}^{m}h_{2i}\lambda_i}{2}\left\{\frac{\lambda E\left[x\right]^2}{P_1^2}+\frac{2\lambda E_0}{P^2}+\frac{2\lambda E_1}{PP_1}-E_4\right\}+\frac{1}{n}\cdot\left[\frac{E_3}{2}\left[\left(\sum_{i=1}^{m}h_{2i}\lambda_i\right)-h\lambda\right]\right]\end{array}}}\tag{22}$$

or:

$$Q^* = \sqrt{\frac{2\left[K+(n+1)\sum_{i=1}^{m}K_{1i}\right]\lambda}{h\lambda\left\{\frac{1}{\lambda}-\frac{1}{P}-\frac{1}{P_1}\left[1+\frac{\lambda}{P_1}\right]E[x]^2-\left(\frac{1}{n}\right)E_3+E_4\right\}+\frac{h_1\lambda E[x]^2}{P_1}+ \\ +\sum_{i=1}^{m}h_{2i}\lambda_i\left\{\frac{\lambda E[x]^2}{P_1^2}+\frac{2\lambda E_0}{P^2}+\frac{2\lambda E_1}{PP_1}+\left(\frac{1}{n}\right)E_3-E_4\right\}}} \tag{23}$$

It can be seen that the optimal replenishment lot size Q^* in Eq. (23) is identical to that obtained by using conventional differential calculus method in Ref. [1]. To facilitate the comparison for readers, the details of the conventional method are provided in Appendix.

Finally, one can obtain the optimal system cost by applying the optimal production-shipment (Q^*, n^*+1) policy in the expected system cost function $E[TCU(Q^*, n^*+1)]$.

3. EXAMPLE

The aforementioned results are verified using the same numerical example as in Ref. [1] to facilitate the comparison. It has been assumed that a product in a single-producer, multi-retailer integrated system can be manufactured at an annual production rate $P=60,000$ units. Its annual demands λ_i from *5* different retailers are *650*, *350*, *450*, *800*, and *750* (the total demand λ is *3,000* per year). During the production process, there is a random defective rate during production uptime which follows a uniform distribution over the interval [*0*, *0.3*]. All defective items are repairable at the end of the regular production for each cycle and its reworking rate $P_1=3,600$ per year.

An $n+1$ multi-shipment policy is adopted in this study. Under the proposed $n+1$ delivery policy, an initial shipment of finished goods is delivered to multiple retailers to meet their demands during the producer's uptime and reworking time (Figure 1). Upon the completion of the rework process, namely, when the remaining production lot's quality has been assured, n fixed quantity installments of the finished products are transported to different retailers, at a fixed time interval t_n.

Values of other parameters include: (1) $C=\$100$ per item; (2) $K=\$35,000$ per production run; (3) $h=\$25$ per item per year; (4) $h_1=\$60$ per item per year; (5) $C_R=\$60$ for each items reworked; (6) $K_{1i}=\$400$, *\$100*, *\$300*, *\$450*, and *\$250* for retailer i, respectively; (7) $h_{2i}=\$70$, *\$80*, *\$75*, *\$60*, and *\$65* per item; and (8) $C_{Ti}=\$0.5$, *\$0.4*, *\$0.3*, *\$0.2*, and *\$0.1* for retailer i, respectively.

To demonstrate how to derive the aforementioned optimal production-shipment policy, we first apply Eq. (17) and find that $n=5.136$. In order to determine the optimal integer number of n^* that minimizes the expected cost function, two integers adjacent to n are examined [29]. Applying Eq. (23) we obtain (Q, n^+)=(*2310, 6*) and (Q, n^-)=(*2228*, *5*), respectively. Substituting these (Q, n^+) and (Q, n^-) in Eq. (2), respectively, and by selecting the one that gives the minimum system cost, we obtain the optimal number of deliveries $n^*=5$, optimal lot-size $Q^*=2,835$, and optimal expected cost $E[TCU(2835, 6)]=\$420,967$.

It is observable that these results prove to be identical to those obtained by using the conventional differential calculus method [1].

4. CONCLUSIONS

In this study, an alternative approach to solving a single-producer, multi-retailer system with rework and a specific, cost-reduction shipment plan. Unlike conventional methods that have to use differential calculus on the system cost function to find the optimal policy presented in Ref. [1], our approach uses simple algebraic derivations. Such a straightforward method may enable the practitioners possessing little knowledge of calculus to easily understand real supply chain systems.

ACKNOWLEDGEMENTS

The authors greatly appreciate National Science Council of Taiwan for supporting this research under grant number: NSC 99-2410-H-324-007-MY3.

APPENDIX

The conventional differential calculus approach to the proposed problem is as follows [1]:

Upon obtaining the expected system cost function (Eq. (2)), first the convexity of $E[TCU(Q, n+1)]$ needs to be proved, by the use of Hessian matrix equations [30] to test if the following Eq. (A-1) holds:

$$[Q \quad n]\cdot\begin{pmatrix} \frac{\partial^2 E\left[TCU\left(Q,n+1\right)\right]}{\partial Q^2} & \frac{\partial^2 E\left[TCU\left(Q,n+1\right)\right]}{\partial Q\partial n} \\ \frac{\partial^2 E\left[TCU\left(Q,n+1\right)\right]}{\partial Q\partial n} & \frac{\partial^2 E\left[TCU\left(Q,n+1\right)\right]}{\partial n^2} \end{pmatrix}\cdot\begin{bmatrix} Q \\ n \end{bmatrix} > 0 \tag{A-1}$$

Applying the differentiation calculus, we obtain the following:

$$\begin{aligned} \frac{\partial E\left[TCU\left(Q,n+1\right)\right]}{\partial Q} = &-\frac{1}{Q^2}\lambda\left[\left(n+1\right)\sum_{i=1}^{m}K_{1i}+K\right]+\frac{h_1\lambda}{2P_1}\left(E\left[x\right]\right)^2+ \\ &+\frac{h\lambda}{2}\left\{\frac{1}{\lambda}-\frac{1}{P}-\frac{1}{P_1}\left[1+\frac{\lambda}{P_1}\right]\left(E\left[x\right]\right)^2-\left(\frac{1}{n}\right)E_3-E_4\right\}+ \\ &+\frac{\sum_{i=1}^{m}h_{2i}\lambda_i}{2}\left\{\frac{\lambda E\left[x\right]^2}{P_1^2}+\frac{2\lambda E_0}{P^2}+\frac{2\lambda E_1}{PP_1}+\left(\frac{1}{n}\right)E_3-E_4\right\} \end{aligned} \tag{A-2}$$

$$\frac{\partial^2 E\left[TCU\left(Q,n+1\right)\right]}{\partial Q^2} = \frac{2\left[\left(n+1\right)\sum_{i=1}^{m}K_{1i}+K\right]\lambda}{Q^3} \tag{A-3}$$

$$\frac{\partial E\left[TCU\left(Q,n+1\right)\right]}{\partial n} = \lambda\sum_{i=1}^{m}K_{1i}\frac{1}{Q}+\left(\frac{1}{n^2}\right)\frac{1}{2}\left[hQ\lambda-\sum_{i=1}^{m}h_{2i}Q\lambda_i\right]\cdot E_3 \tag{A-4}$$

$$\frac{\partial^2 E\left[TCU\left(Q,n+1\right)\right]}{\partial n^2} = \frac{-1}{n^3}\left[hQ\lambda-\sum_{i=1}^{m}h_{2i}Q\lambda_i\right]E_3 \tag{A-5}$$

$$\frac{\partial E\left[TCU\left(Q,n+1\right)\right]}{\partial Q\partial n} = -\frac{\lambda\sum_{i=1}^{m}K_{1i}\cdot\sum_{i=1}^{m}\lambda_i}{Q^2}+\left(\frac{1}{n^2}\right)\frac{1}{2}\left[h\lambda-\sum_{i=1}^{m}h_{2i}\lambda_i\right]E_3 \tag{A-6}$$

Substituting Eqs. (A-3), (A-5) and (A-6) in Eq. (A-1) we have:

$$[Q \quad n]\cdot\begin{pmatrix} \frac{\partial^2 E\left[TCU\left(Q,n+1\right)\right]}{\partial Q^2} & \frac{\partial^2 E\left[TCU\left(Q,n+1\right)\right]}{\partial Q\partial n} \\ \frac{\partial^2 E\left[TCU\left(Q,n+1\right)\right]}{\partial Q\partial n} & \frac{\partial^2 E\left[TCU\left(Q,n+1\right)\right]}{\partial n^2} \end{pmatrix}\cdot\begin{bmatrix} Q \\ n \end{bmatrix} = \frac{2\lambda}{Q}\left(K+\sum_{i=1}^{m}K_{1i}\right) > 0 \tag{A-7}$$

Eq. (A-7) becomes positive, because λ, Q, K, and K_{1i} are all positive. Hence, the convexity of $E[TCU(Q, n+1)]$ is proved, and there is a minimum of $E[TCU(Q, n+1)]$.

To simultaneously determine the production-shipment policy for the proposed model, one can solve the linear system of Eqs. (A-2) and (A-4) by setting these partial derivatives equal to zero. With further derivations one obtains:

$$Q^{*}=\sqrt{\frac{2\left[K+(n+1)\sum_{i=1}^{m}K_{1i}\right]\lambda}{h\lambda\left\{\frac{1}{\lambda}-\frac{1}{P}-\frac{1}{P_1}\left[1+\frac{\lambda}{P_1}\right]E[x]^2-\left(\frac{1}{n}\right)E_3+E_4\right\}+\frac{h_1\lambda E[x]^2}{P_1}+\sum_{i=1}^{m}h_{2i}\lambda_i\left\{\frac{\lambda E[x]^2}{P_1^2}+\frac{2\lambda E_0}{P^2}+\frac{2\lambda E_1}{PP_1}+\left(\frac{1}{n}\right)E_3-E_4\right\}}} \tag{A-8}$$

and:

$$n^{*}=\sqrt{\frac{\left(\sum_{i=1}^{m}K_{1i}+K\right)\left(\sum_{i=1}^{m}h_{2i}\lambda_i-h\lambda\right)E_3}{\left(\sum_{i=1}^{m}K_i\right)\left\{h\lambda\left\{\frac{1}{\lambda}-\frac{1}{P}-\frac{1}{P_1}\left[1+\frac{\lambda}{P_1}\right]E[x]^2+E_4\right\}+\frac{h_1\lambda E[x]^2}{P_1}+\sum_{i=1}^{m}h_{2i}\lambda_i\left\{\frac{\lambda E[x]^2}{P_1^2}+\frac{2\lambda E_0}{P^2}+\frac{2\lambda E_1}{PP_1}-E_4\right\}\right\}}} \tag{A-9}$$

5. REFERENCES

[1] S.W. Chiu, N. Pan, K.-W. Chiang and Y.-Sh.P. Chiu, Integrating a cost-reduction shipment plan into a single-producer multi-retailer system with rework, *Int. Journal for Engineering Modelling*, Vol. 27, No. 1-2, pp. 33-41, 2014.

[2] E.W. Taft, The most economical production lot, *Iron Age*, Vol. 101, pp. 1410-1412, 1918.

[3] L.B. Schwarz, A simple continuous review deterministic one-warehouse N-retailer inventory problem, *Management Science*, Vol. 19, pp. 555-566, 1973.

[4] S.K. Goyal, Integrated inventory model for a single-supplier single-customer problem, *Int. Journal of Production Research*, Vol. 15, pp. 107-111, 1977.

[5] K.H. Kim and H. Hwang, An incremental discount pricing schedule with multiple customers and single price break, *European Journal of Operational Research*, Vol. 35, No. 1, pp. 71-79, 1988.

[6] R.W. Hall, On the integration of production and distribution: economic order and production quantity implications, *Transportation Research Part B: Methodological*, Vol. 30, No. 5, pp. 387-403, 1996.

[7] R.M. Hill, The single-vendor single-buyer integrated production-inventory model with a generalised policy, *European Journal of Operational Research*, Vol. 97, No. 3, pp. 493-499, 1997.

[8] G.R. Parija and B.R. Sarker, Operations planning in a supply chain system with fixed-interval deliveries of finished goods to multiple customers, IIE Transactions, Vol. 31. No. 11, pp. 1075-1082, 1999.

[9] M.-J. Yao and C.-C. Chiou, On a replenishment coordination model in an integrated supply chain with one vendor and multiple buyers, *European Journal of Operational Research*, Vol. 159, pp. 406-419, 2004.

[10] M.A. Hoque, Synchronization in the single-manufacturer multi-buyer integrated inventory supply chain, *European Journal of Operational Research*, Vol. 188, No. 3, pp. 811-825, 2008.

[11] B.R. Sarker and A. Diponegoro, Optimal production plans and shipment schedules in a supply-chain system with multiple suppliers and multiple buyers, *European Journal of Operational Research*, Vol. 194, No. 3, pp. 753-773, 2009.

[12] Y.-Sh.P. Chiu, S.-C. Liu, C.-L. Chiu and H.-H. Chang, Mathematical modelling for determining the replenishment policy for EMQ model with rework and multiple shipments, *Mathematical and Computer Modelling*, Vol. 54, pp. 2165-2174, 2011.

[13] Y.-Sh.P. Chiu, C.-Y. Chang, C.-K. Ting and S.W. Chiu, Effect of variable shipping frequency on production-distribution policy in a vendor-buyer integrated system, *Int. Journal for Engineering Modelling*, Vol. 24, No. 1-4, pp. 11-20, 2011.

[14] B. Pal, S.S. Sana and K. Chaudhuri, A three layer multi-item production-inventory model for multiple suppliers and retailers, *Economic Modelling*, Vol. 29, pp. 2704-2710, 2012.

[15] S.W. Chiu, Y.-Sh.P. Chiu and J.-C. Yang, Combining an alternative multi-delivery policy into economic production lot size problem with partial rework, *Expert Systems with Applications*, Vol. 39, pp. 2578-2583, 2012.

[16] S.W. Chiu, C.-H. Lee, F.-T. Cheng and C.-K. Ting, Production-shipment policy for EPQ model with quality assurance and an improved delivery schedule, *Mathematical and Computer Modelling of Dynamical Systems*, Vol. 19, No. 4, pp. 344-352, 2013.

[17] K.L. Mak, Inventory control of defective products when the demand is partially captive, *Int. Journal of Production Research*, Vol. 23, No. 3, pp. 533-542, 1985.

[18] T. Bielecki and P.R. Kumar, Optimality of zero-inventory policies for unreliable production facility, *Operations Research*, Vol. 36, pp. 532-541, 1988.

[19] H.-M. Wee, Economic production lot size model for deteriorating items with partial back-ordering, *Computers & Industrial Engineering*, Vol. 24, No. 3, pp. 449-458, 1993.

[20] R.H. Teunter and S.D.P. Flapper, Lot-sizing for a single-stage single-product production system with rework of perishable production defectives, *OR Spectrum*, Vol. 25, No. 1, pp. 85-96, 2003.

[21] Y.-Sh.P. Chiu, Y.-L. Lien and C.-A.K. Lin, Incorporating machine reliability issue and backlogging into the EMQ model - I: random breakdown occurring in backorder filling time, *Int. Journal for Engineering Modelling*, Vol. 22, No. 1-4, pp. 1-13, 2009.

[22] Y.-Sh.P. Chiu, K.-K. Chen and C.-K. Ting, Replenishment run time problem with machine breakdown and failure in rework, *Expert Systems with Applications*, Vol. 39, pp. 1291-1297, 2012.

[23] S.W. Chiu, C.-L. Chou and W.-K. Wu, Optimizing replenishment policy in an EPQ-based inventory model with nonconforming items and breakdown, *Economic Modelling*, Vol. 35, pp. 330-337, 2013.

[24] R.W. Grubbström and A. Erdem, The EOQ with backlogging derived without derivatives, *Int. Journal of Production Economics*, Vol. 59, pp. 529-530, 1999.

[25] S.W. Chiu, Production lot size problem with failure in repair and backlogging derived without derivatives, *European Journal of Operational Research*, Vol. 188, pp. 610-615, 2008.

[26] H.-D. Lin, Y.-Sh.P. Chiu and C.-K. Ting, A note on optimal replenishment policy for imperfect quality EMQ model with rework and backlogging, *Computers and Mathematics with Applications*, Vol. 56, No. 11, pp. 2819-2824, 2008.

[27] K.-K. Chen, M.-F. Wu, S.W. Chiu and C.-H. Lee, Alternative approach for solving replenishment lot size problem with discontinuous issuing policy and rework, *Expert Systems with Applications*, Vol. 39, No. 2, pp. 2232-2235, 2012.

[28] S.W. Chiu, H.-D. Lin, L.-W. Lin and Y.-Sh.P. Chiu, Reexamining a single-producer multi-retailer integrated inventory model with rework using algebraic method, *Int. Journal for Engineering Modelling*, Vol. 25, No. 1-4, pp. 37-43, 2012.

[29] Y.-Sh.P. Chiu, C.-C. Huang, M.-F. Wu and H.-H. Chang, Joint determination of rotation cycle time and number of shipments for a multi-item EPQ model with random defective rate, *Economic Modelling*, Vol. 35, pp. 112-117, 2013.

[30] R.L. Rardin, *Optimization in Operations Research*, Int. Ed., Prentice-Hall, New Jersey, 1998.

REVIZIJA STUDIJE "INTEGRIRANJE PLANA SMANJENJA TROŠKOVA ISPORUKA ROBE U SUSTAV S JEDNIM PROIZVOĐAČEM, VIŠE MALOPRODAJNIH TRGOVACA TE PROCESOM OBRADE PROIZVODA S GREŠKOM" ALTERNATIVNIM PRISTUPOM

SAŽETAK

U ovome se radu usvajanjem alternativnog pristupa revidira plan smanjenja troškova isporuke u sustavu s jednim proizvođačem i više maloprodajnih trgovaca te procesom obrade proizvoda s greškom [1]. Za razliku od konvencionalne metode koja se temelji na primjeni diferencijalnog računa radi određivanja funkcije troška sustava, kako bi se dokazala njegova konveksnost i iznašao optimalan plan proizvodnje i isporuke dobara, ovdje je predloženo algebarsko rješenje problema. Takvo jednostavno rješenje omogućuje korisnicima, koji nemaju potrebna matematička znanja, da lakše shvate opskrbni lanac u stvarnom vremenu.

Ključne riječi: *optimizacija, sustav opskrbnih lanaca, odluka o proizvodnji i isporuci, proces obrade proizvoda s greškom, višestruka pošiljka, algebarski pristup.*