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Incorporating machine reliability issue and backlogging into the EMQ model - Part I: Random breakdown occurring in backorder filling time

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SUMMARY

This study is concerned with determination of the optimal replenishment policy for economic manufacturing quantity (EMQ) model with backlogging and machine reliability issue. Classic EMQ model does not consider nonconforming items generated during a production cycle, nor does it deal with the machine breakdown situation. It is noted that in manufacturing system when back-ordering is permitted, a random machine failure can take place in either backorder filling time or in on-hand inventory piling period. The first phase of this study examines the aforementioned practical issues by incorporating rework process of defective items, scrap and random machine failure taking place specifically in backorder satisfying time into the EMQ model. The objective is to determine the optimal replenishment lot-size that minimizes the overall production-inventory costs. Mathematical modelling and analysis is used and the renewal reward theorem is employed to cope with the variable cycle length. Theorem on conditional convexity of total cost function is proposed and proved. The optimal lot size for such a real-life imperfect manufacturing system is derived. A numerical example is given to demonstrate its practical usage.

Key words: *production, machine breakdown, EMQ model, rework, lot sizing, scrap, random defective rate.*

1. INTRODUCTION

The classic economic manufacturing quantity (EMQ) model with backlogging [1, 2] assumes that all items produced are of perfect quality and the production equipments are always in good condition. However, in real world due to different reasons, generation of random defective items and occasional breakdowns of machine are inevitable. Hence, studies have been carried out to address the issues of defective items produced in EMQ model [3-8]. The imperfect quality items fall into two groups, the scrap and the repairable. By reworking defective items, for example, manufacturing processes in printed circuit board assembly or in plastic injection molding, etc., the overall production-inventory costs can be significantly reduced [9-14].

Another common and inevitable reliability factors that trouble the production planners and practitioners is mostly the breakdown of production equipments. To effectively manage and control the disruption and to minimize total production costs are critical tasks to most manufacturing firms. It is not surprising that determination of optimal lot-size for systems with machine failures has received attention from researchers in recent decades [15-22]. Groenevelt et al. [15] proposed two inventory control policies to deal with machine breakdown. They are (1) no resumption (NR) policy and (2) abort/resume (AR) policy. Under the NR policy, the production of the interrupted lot is not resumed after a breakdown. While the AR policy considers that after the machine is fixed and restored, the production of the interrupted lot will be immediately resumed if the current on-hand inventory falls below a certain threshold level. The effects of breakdown under both policies and their corrective maintenance on the economic lot sizing decisions have been investigated respectively. Abboud [17] considered an EMQ model with Poisson machine failures and random machine repair time. A simple approximation model was developed to describe the behavior of such systems, and specific formulations were derived for the cases where the repair times are exponential and constant. Giri and Dohi [20] presented the exact formulation of stochastic EMQ model for an unreliable production system. Their EMQ model is formulated based on the net present value (NPV) approach and by taking limitation on the discount rate the traditional long-run average cost model is obtained. They also provided the criteria for the existence and uniqueness of the optimal production time and computational results showing that the optimal decision based on the NPV approach is superior to that based on the long-run average cost approach. Lin and Kroll [21] examined an EMQ model for an imperfect production process that is subject to random machine breakdowns. The objective of their paper was to find an optimal production lot size that minimizes the expected (long-run) total cost per unit time. Several models are investigated and a numerical approach is developed to obtain an optimal production lot size. Chiu [22] investigated the optimal run time for EPQ model with random breakdown, rework, and no shortages permitted. Theorems on conditional convexity of the integrated cost function and on bounds of the production run time is proposed and proved. An optimal run time was located by the use of the bisection method based on the intermediate value theorem. This paper reexamines the imperfect quality EMQ model studied by Chiu [13] and takes an additional reliability factor – machine breakdown into consideration. For the reason that little attention has been paid to the aforementioned area, this study intends to bridge the gap.

2. MODEL DESCRIPTIOM AND NOTATION

Reconsider the imperfect production system studied by Chiu [13], a manufactured item can be produced at a rate of P per year, and its demand rate is λ units per year. The production rate P is much larger than the demand rate d. Shortages are allowed and backordered, they will be satisfied when the next replenishment production cycle starts. This system may randomly generate x portion of defective items at a rate d, where d=Px. Assume that the production rate of perfect quality items must always be greater than or equal to the sum of the demand rate λ and the defective rate d. Hence, the following condition: $(P\text{-}d\text{-}\lambda)\geq 0$ or $(1\text{-}x\text{-}\lambda/P)\geq 0$ must hold. All items produced are screened and the inspection cost per item is included in the unit production cost C. The imperfect quality items fall into two groups, a θ portion of them is the scrap and the other $(1\text{-}\theta)$ portion of the defective items is considered to be reworkable. When regular production ends, the rework process starts immediately at a rate of P_1, in each cycle. A θ_1 portion of the reworked items fails the repairing and becomes scrap.

Further, according to the mean time between failures (MTBF) data, a machine breakdown may take place randomly in the backorder filling stage (see Figure 1), and an abort/resume production control policy is adopted. Under such a policy, when a random breakdown occurs, the machine is under corrective maintenance immediately, a constant repair time is assumed and the interrupted lot will be resumed right after the restoration of machine. It is also assumed that during the setup time, prior to the production uptime, the working function of machine is fully checked and confirmed. Hence, the chance of breakdown in a very short period of time when production begins is small. Also, due to tight preventive maintenance schedule, the probability of more than one machine breakdown occurrences in a production cycle is assumed to be very small. However, if it does happen, safety stock will be used to satisfy the demand during machine repairing time. Therefore, multiple machine failures are assumed to have insignificant effect on the proposed model.

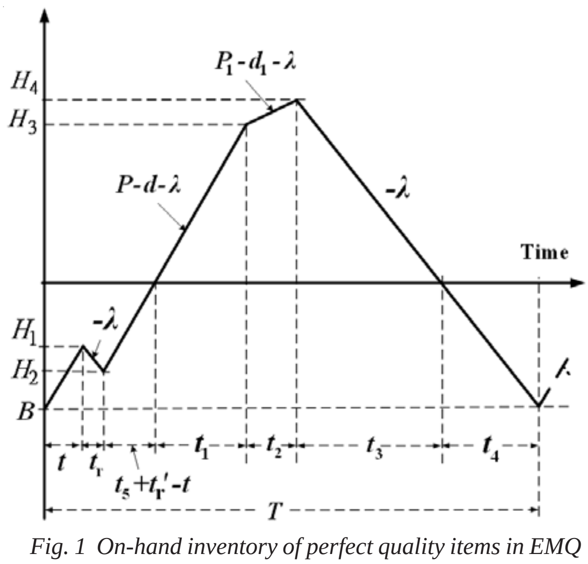

Fig. 1 On-hand inventory of perfect quality items in EMQ model with imperfect rework and breakdown taking place in backorder filling time

Figure 1 depicts the level of on-hand inventory of perfect quality items in the proposed EMQ. Cost parameters considered in the study include: unit manufacturing cost C, unit shortage/backordered cost b, disposal cost for each scrap item C_S, setup cost K, unit holding cost h, unit repair cost for each defective item reworked C_R, unit holding cost per reworked item h_1, and the cost for repairing and restoring machine M. Other notations used are listed as follows:

T – the production cycle length,

T_1 – optimal production time (i.e. production uptime), the decision variable to be solved for the proposed EPQ model,
t – production time before a random breakdown occurs,
Q – production lot size for each cycle,
B – the maximum backorder level allowed for each cycle,
H_1 – the level of backorder quantity when machine breakdown occurs,
H_2 – the level of backorder quantity when machine is repaired and restored,
H_3 – the level of perfect quality inventory when regular production process ends,
H_4 – the maximum level of perfect quality inventory when rework finishes,
t_r – time required for repairing and restoring the machine,
t_r' – time required for producing sufficient stocks to satisfy the demand during machine repair time t_r,
t_5 – time required for filling the backorder quantity B (excluding t_r and t_r'),
t_1 – time for piling up stocks during the production uptime in each cycle,
t_2 – time needed to rework ($1-\theta$) of the repairable defective items,
t_3 – time required for depleting all available perfect quality on-hand items,
t_4 – shortage permitted time,
$TC(T_1,B)$ – total production-inventory costs per cycle,
$TCU(T_1,B)$ – total production-inventory costs per unit time (e.g. annual),
$E[TCU(T_1,B)]$ – the expected total production-inventory costs per unit time.

3. FORMULATION

In Figure 1, let t denote the production time before a breakdown occurring in the backorder filling time t_5, that is $t<t_5$. Let the maximum machine repair time be a constant and $t_r=g$. In this study, it is conservatively assumed that if a failure of a machine cannot be fixed within a certain allowable amount of time, then a spare machine will be in place to avoid further delay of production. The following derivation procedure is similar to what was used by past studies [10, 13]. From Figure 1, one can obtain the following relationships for: production lot size Q; the levels of backorder H_1 and H_2; the levels of on-hand perfect quality inventory H_3 and H_4; the cycle length T; production uptime T_1; t_r'; time required for satisfying B (maximum backorder quantity) t_5; time for piling up stocks t_1; time for reworking repairable items t_2; and time required for depleting all available on-hand items t_3 and t_4:

$$Q = P \cdot T_1 \tag{1}$$

$$H_1 = B - \left(P - d - \lambda\right)t \tag{2}$$

$$H_2 = H_1 + t_r \lambda \tag{3}$$

$$H_3 = \left(P - d - \lambda\right)t_1 \tag{4}$$

$$H_4 = H_3 + \left(P_1 - d_1 - \lambda\right)t_2 \tag{5}$$

$$T = \sum_{i=1}^{5} t_i + \left(t_r + t_r'\right) \tag{6}$$

$$T_1 = t_5 + t_r' + t_1 \tag{7}$$

$$t_r' = \frac{g\lambda}{P - d - \lambda} \tag{8}$$

$$t_5 = \frac{B}{P - d - \lambda} \tag{9}$$

$$t_1 = \frac{H_3}{P - d - \lambda} \tag{10}$$

$$t_2 = \frac{\left(d \cdot T_1\right)}{P_1} \tag{11}$$

$$t_3 = \frac{H_4}{\lambda} \tag{12}$$

$$t_4 = \frac{B}{\lambda} \tag{13}$$

where $d=Px$.

Figure 2 illustrates the level of on-hand inventory of defective items for the proposed EMQ model, where total defective items produced during the production uptime T_1 can be computed as shown in Eq. (14):

$$d \cdot T_1 = P \cdot x \cdot \left[t_5 + t_r' + t_1\right] = x \cdot Q \tag{14}$$

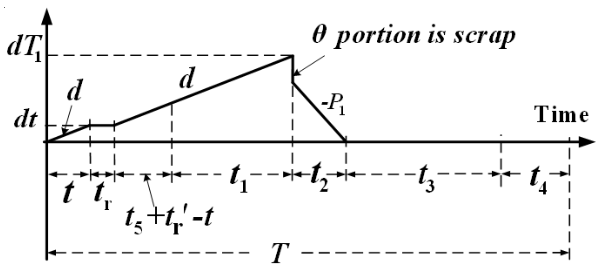

Fig. 2 On-hand inventory of defective items in the proposed EMQ model

Figure 3 displays the on-hand inventory level of scrap items for the proposed model. During the rework process, the production rate of scrap items can be rewritten as in Eq. (15) and the total scrap items produced can be calculated by Eq. (16):

$$d_1 = P_1 \cdot \theta_1 \;;\; where \;\; 0 \le \theta_1 \le 1 \tag{15}$$

$$\begin{aligned} & d \cdot \theta \cdot T_1 + P_1 \cdot \theta_1 \cdot t_2 = \\ & = Q \cdot x \cdot \theta + Q \cdot x \cdot \left(1-\theta\right) \cdot \theta_1 = \\ & = Q \cdot x \cdot \left[\theta + \left(1-\theta\right) \cdot \theta_1\right] \end{aligned} \tag{16}$$

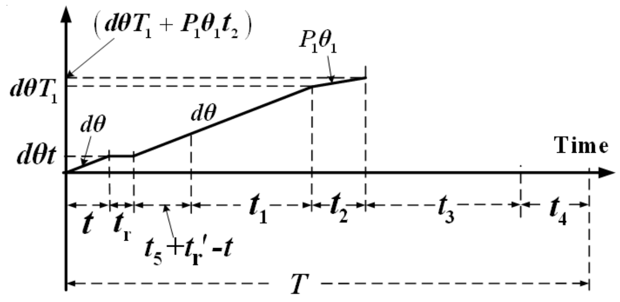

Fig. 3 On-hand inventory level of scrap items in the proposed EMQ model

Let $\varphi = \left[\theta + (1-\theta)\cdot\theta_1\right]$, then φ denotes the total scrap rate among the defective items. From Figures 1 to 3, one obtains the total production-inventory cost per cycle $TC(T_1,B)$ as follows:

$$\begin{aligned} TC(T_1,B) = {} & C\cdot(P\cdot T_1) + K + M + C_R\cdot(P\cdot T_1)\cdot x\cdot(1-\theta) + C_S\cdot(P\cdot T_1)\cdot x\cdot\varphi + \\ & + h\cdot\left[\frac{H_3}{2}\cdot(t_1) + \frac{H_3+H_4}{2}\cdot(t_2) + \frac{H_4}{2}\cdot(t_3)\right] + h_1\cdot\left[\frac{d\cdot T_1}{2}\right]\cdot(t_2) + \\ & + b\cdot\left[\frac{(B+H_1)}{2}\cdot(t) + \frac{(H_1+H_2)}{2}\cdot(t_r) + \frac{(H_2)}{2}\cdot(t_5+t_r'-t) + \frac{B}{2}\cdot(t_4)\right] + \\ & + h\cdot\left[\frac{(d\cdot t)}{2}\cdot(t) + (d\cdot t)\cdot t_r + \frac{(d\cdot t + d\cdot T_1)}{2}\cdot(t_5+t_r'-t+t_1)\right] \end{aligned} \tag{17}$$

By substituting all related parameters from Eqs. (1) to (16) into Eq. (17), one has $TC(T_1,B)$ as follows:

$$\begin{aligned} TC(T_1,B) = {} & K + P\cdot T_1\cdot\left[C + C_R\cdot x\cdot(1-\theta) + C_S\cdot x\cdot\varphi\right] + M + gPt\cdot\left[hx - b[1-x]\right] + \\ & + \frac{(b+h)}{2}\left\{g(2B+g\lambda) + \frac{B^2}{\lambda}\right\}\cdot\left(\frac{1-x}{1-x-\frac{\lambda}{P}}\right) + \frac{T_1^2P^2x^2(1-\theta)^2}{2P_1}\left[h_1 - h(1-\theta_1)\right] - \\ & - hT_1P(1-x\varphi)\cdot\left[g + \frac{B}{\lambda}\right] + \frac{hT_1^2P}{2}\left[\frac{P\left(1-2x\varphi+x^2\varphi^2\right)}{\lambda} + 2x\varphi - 1\right] \end{aligned} \tag{18}$$

Due to random defective/scrap rates, the production cycle length is not constant. Therefore, to take the randomness of scrap rate into account, one can employ the renewal reward theorem in production-inventory cost analysis to cope with the variable cycle length. Also, because a Poisson machine breakdown (with mean equals to β per unit time) is assumed to occur in backorder satisfying time, one can use the integration of $TC(T_1,B)$ to deal with the random breakdown happening in time t_5. The long-run expected costs per unit time $E[TCU(T_1,B)]$ can be calculated as follows:

$$E\left[TCU(T_1,B)\right] = \frac{E\left[\int_0^{t_5} TC(T_1,B)\cdot f(t)\,dt\right]}{E\left[\int_0^{t_5} T\cdot f(t)\,dt\right]} = \frac{E\left[\int_0^{t_5} TC(T_1,B)\cdot\left(\beta e^{-\beta t}\right)dt\right]}{\left[T_1\cdot P\cdot\left(1-E[x]\cdot\varphi\right)/\lambda\right]\cdot\left(1-e^{-\beta t_5}\right)} \tag{19}$$

Substituting $TC(T_1,B)$ (Eq. (18)) in Eq. (19) and with further arrangement one can obtain $E[TCU(T_1,B)]$ as follows (see Appendix *A* for detailed derivations):

$$E\left[TCU\left(T_1,B\right)\right]=\frac{\lambda\left(K+M\right)}{T_1P\left(1-E\left[x\right]\varphi\right)}+\frac{\lambda\left[C+C_RE\left[x\right](1-\theta)+C_SE\left[x\right]\varphi\right]}{\left(1-E\left[x\right]\varphi\right)}+\frac{\lambda g\left[hE\left[x\right]-b\left(1-E\left[x\right]\right)\right]}{T_1\beta\left(1-E\left[x\right]\varphi\right)}+$$
$$+\frac{\lambda}{T_1P\left(1-E\left[x\right]\varphi\right)}\cdot\frac{\left(b+h\right)}{2}\cdot E\left[\frac{1-x}{1-x-\lambda/P}\right]\cdot\left[g^2\lambda+\frac{B^2}{\lambda}\right]+\frac{\lambda T_1P\left(E\left[x\right]\right)^2\left(1-\theta\right)^2}{2P_1\left(1-E\left[x\right]\varphi\right)}\left[h_1-h\left(1-\theta_1\right)\right]+$$
$$+\frac{\lambda hT_1}{2\left(1-E\left[x\right]\varphi\right)}\left[\frac{P\left[1-2\varphi E\left[x\right]+\varphi^2\left(E\left[x\right]\right)^2\right]}{\lambda}+2\varphi E\left[x\right]-1\right]+\frac{\lambda hgB}{T_1P\left(1-E\left[x\right]\varphi\right)}\cdot E\left[\frac{1}{1-x-\lambda/P}\right]-$$
$$-hB-hg\lambda+\frac{\lambda Bg}{T_1P\left(1-E\left[x\right]\varphi\right)(1-e^{-\beta t_5})}\cdot\left\{b\cdot E\left[\frac{1-x}{1-x-\lambda/P}\right]-h\cdot E\left[\frac{x}{1-x-\lambda/P}\right]\right\} \tag{20}$$

Let:

$$E_0=\frac{1}{1-\varphi E\left[x\right]};\quad E_1=\frac{E\left[x\right]}{1-\varphi E\left[x\right]};\quad E_2=\frac{\left(E\left[x\right]\right)^2}{1-\varphi E\left[x\right]};\quad E_3=\frac{1}{1-\varphi E\left[x\right]}E\left[\frac{1-x}{1-x-\lambda/P}\right];$$
$$E_4=\frac{1}{1-\varphi E\left[x\right]}E\left[\frac{1}{1-x-\lambda/P}\right];\quad E_5=\frac{1}{1-\varphi E\left[x\right]}E\left[\frac{x}{1-x-\lambda/P}\right] \tag{21}$$

then Eq. (20) becomes:

$$E\left[TCU\left(T_1,B\right)\right]=\frac{\lambda\left(K+M\right)}{T_1P}E_0+\lambda\left\{CE_0+\left[C_R(1-\theta)+C_S\varphi\right]E_1\right\}+\frac{\lambda g}{T_1\beta}\left[hE_1-b\left(E_0-E_1\right)\right]+$$
$$+\frac{\lambda\left(b+h\right)}{2T_1P}\cdot\left[g^2\lambda+\frac{B^2}{\lambda}\right]\cdot E_3+\frac{\lambda hT_1}{2}\left[\frac{P\left[E_0-2\varphi E_1+\varphi^2E_2\right]}{\lambda}+2\varphi E_1-E_0\right]+$$
$$+\frac{\lambda T_1P\left(1-\theta\right)^2}{2P_1}\left[h_1-h\left(1-\theta_1\right)\right]\cdot E_2+\frac{\lambda hgB}{T_1P}\cdot E_4-hB-hg\lambda+\frac{\lambda Bg}{T_1P(1-e^{-\beta t_5})}\cdot\left[b\cdot E_3-h\cdot E_5\right] \tag{22}$$

4. CONVEXITY OF COST FUNCTION AND OPTIMAL LOT SIZE

In order to find the optimal production run time T_1*, a theorem is proposed in this study. Let δ denote the following:

$$\delta=\beta\left[2\left(K+M\right)+\left(b+h\right)\cdot\lambda g^2\cdot E\left(\frac{1-x}{1-x-\lambda/P}\right)\right]+2gP\left(b+h\right)E\left[x\right] \tag{23}$$

Theorem 1: $E[TCU(T_1,B)]$ is convex if $\delta > 2gPb$.

Applying the Hessian matrix equations [23] to Eq. (22), one obtains the following (see Appendix *B* for detailed computation):

$$\left[T_1\quad B\right]\cdot\begin{pmatrix}\frac{\partial^2E\left[TCU\left(T_1,B\right)\right]}{\partial T_1^2} & \frac{\partial^2E\left[TCU\left(T_1,B\right)\right]}{\partial T_1\partial B}\\ \frac{\partial^2E\left[TCU\left(T_1,B\right)\right]}{\partial T_1\partial B} & \frac{\partial^2E\left[TCU\left(T_1,B\right)\right]}{\partial B^2}\end{pmatrix}\cdot\begin{bmatrix}T_1\\ B\end{bmatrix}=$$
$$=\frac{1}{1-E\left[x\right]\varphi}\cdot\frac{\lambda}{T_1P\beta}\cdot\left\{\beta\left[2\left(K+M\right)+\left(b+h\right)\cdot\lambda g^2\cdot E\left(\frac{1-x}{1-x-\lambda/P}\right)\right]+2gP\left[\left(b+h\right)E\left[x\right]-b\right]\right\} \tag{24}$$

Because the first part of the right-hand-size (RHS) of Eq. (24) is greater than zero:

$$\frac{1}{1-E[x]\varphi}\cdot\frac{\lambda}{T_1P\beta}>0$$

it implies that if the following (the second part of RHS of Eq. (24)) is greater than zero then the Hessian matrix equations for $E[TCU(T_1,B)]$ is greater than 0:

$$\begin{aligned}&\text{if}: \quad \left\{\beta\left[2(K+M)+(b+h)\cdot\lambda g^2\cdot E\left(\frac{1-x}{1-x-\lambda/P}\right)\right]+2gP\left[(b+h)E[x]-b\right]\right\}>0\\&\text{then}: \quad E\left[TCU(T_1,B)\right]>0\end{aligned} \tag{25}$$

or:

$$\begin{aligned}&\text{if}: \quad \left(\beta\left[2(K+M)+(b+h)\lambda g^2\cdot E\left(\frac{1-x}{1-x-\lambda/P}\right)\right]+2gP(b+h)E[x]>2gPb\right)\\&\text{then}: \quad E\left[TCU(T_1,B)\right]>0\end{aligned} \tag{26}$$

From Eqs. (23) and (26), the proof of Theorem 1 is completed.

In order to minimize the expected overall costs $E[TCU(T_1,B)]$, Eq. (26) must be satisfied. Now, to search for the optimal uptime T_1 and optimal backorder level B, one can differentiate $E[TCU(T_1,B)]$ with respect to T_1 and with respect to B separately, then solve linear systems of Eqs. (27) and (28) by setting these partial derivatives equal to zero:

$$\begin{aligned}\frac{\partial E\left[TCU\left(T_1,B\right)\right]}{\partial T_1}=&-\frac{\lambda(K+M)\cdot E_0}{T_1^{\,2}P}+\frac{\lambda P(1-\theta)^2}{2P_1}\left[h_1-h\left(1-\theta_1\right)\right]\cdot E_2-\frac{(b+h)}{2T_1^{\,2}P}(g^2\lambda^2+B^2)\cdot E_3-\\&-\frac{\lambda g}{T_1^{\,2}\beta}\left[h\cdot E_1+b(E_1-E_0)\right]-\frac{\lambda hgB}{T_1^{\,2}P}\cdot E_4-\frac{\lambda Bg}{T_1^{\,2}P(1-e^{-\beta t_5})}\cdot(b\cdot E_3-h\cdot E_5)+\\&+\frac{h}{2}\left(PE_0-2P\varphi E_1+P\varphi^2E_2+2\lambda\varphi E_1-\lambda E_0\right)\end{aligned} \tag{27}$$

$$\frac{\partial E\left[TCU\left(T_1,B\right)\right]}{\partial B}=\frac{B}{T_1P}(b+h)E_3-h+\frac{h\lambda g}{T_1P}E_4+\frac{g\lambda}{T_1P(1-e^{-\beta t_5})}(bE_3-hE_5) \tag{28}$$

With extra derivations (see Appendix C), the resulting optimal uptime T_1*, optimal production lot size Q*, and backorder level B* can be obtained as follows:

$$T_1^*=\frac{1}{P}\sqrt{\frac{\left[2\lambda(K+M)+(g^2\lambda^2)\omega_2-\frac{h^2g^2\lambda^2\omega_3^{\,2}}{\omega_2}-\frac{g^2\lambda^2\omega_1^{\,2}}{\omega_2(1-e^{-\beta t_5})^2}-\frac{2hg^2\lambda^2\omega_3\omega_1}{(1-e^{-\beta t_5})\omega_2}+\frac{2Pg\lambda}{\beta}\left[hE[x]-b(1-E[x]\right]\right]}{\frac{\lambda(1-\theta)^2}{P_1}\left[h_1-h(1-\theta_1)\right]\left(E[x]\right)^2+h\left[\left(1-\frac{\lambda}{P}\right)\left(1-2\varphi E[x]\right)+\varphi^2\left(E[x]\right)^2\right]-\frac{h^2\left(1-\varphi E[x]\right)^2}{\omega_2}}} \tag{29}$$

$$Q^*=\sqrt{\frac{\left[2\lambda(K+M)+(g^2\lambda^2)\omega_2-\frac{h^2g^2\lambda^2\omega_3^{\,2}}{\omega_2}-\frac{g^2\lambda^2\omega_1^{\,2}}{\omega_2(1-e^{-\beta t_5})^2}-\frac{2hg^2\lambda^2\omega_3\omega_1}{(1-e^{-\beta t_5})\omega_2}+\frac{2Pg\lambda}{\beta}\left[hE[x]-b(1-E[x]\right]\right]}{\frac{\lambda(1-\theta)^2}{P_1}\left[h_1-h(1-\theta_1)\right]\cdot\left(E[x]\right)^2+h\cdot\left[\left(1-\frac{\lambda}{P}\right)\left(1-2\varphi E[x]\right)+\varphi^2\left(E[x]\right)^2\right]-\frac{h^2\left(1-\varphi E[x]\right)^2}{\omega_2}}} \tag{30}$$

$$B^*=\left(\frac{h}{b+h}\right)\left(\frac{1}{E_3}\right)Q^*-\frac{hg\lambda E_4}{(b+h)E_3}-\frac{g\lambda(bE_3-hE_5)}{(b+h)E_3(1-e^{-\beta t_5})} \tag{31}$$

where ω_1, ω_2 and ω_3 denote the following:

$$\omega_1 = b \cdot E\left[\frac{1-x}{1-x-\lambda/P}\right] - h \cdot E\left[\frac{x}{1-x-\lambda/P}\right]; \quad \omega_2 = (b+h)E\left[\frac{1-x}{1-x-\lambda/P}\right]; \quad \omega_3 = E\left[\frac{1}{1-x-\lambda/P}\right].$$

4.1 Verification of the results

Suppose factor of machine breakdown is not considered, then machine repair cost and time for failure correction are both zero, i.e. $M=0$ and $g=0$; Eqs. (30) and (31) become the same equations as were given by Chiu [13]:

$$\therefore Q^* = \sqrt{\frac{2K\lambda}{h\left(1-\frac{\lambda}{P}\right)+\left\{\frac{\lambda(1-\theta)^2}{P_1}\left[h_1-h(1-\theta_1)\right]+h\varphi^2\right\}E\left[x^2\right]-2h\varphi\left(1-\frac{\lambda}{P}\right)\cdot E[x]-\frac{h^2\left[1-E[x]\varphi\right]^2}{(b+h)\cdot E\left[\frac{1-x}{1-x-\lambda/P}\right]}}} \tag{32}$$

$$B^* = \left(\frac{h}{b+h}\right)\left(\frac{1-E[x]\varphi}{E\left[\frac{1-x}{1-x-\lambda/P}\right]}\right)Q^* \tag{33}$$

Further, suppose that the regular production process produces no defective items, i.e. $x=0$, then Eqs. (32) and (33) become the same equations as were presented by the classic EMQ model with shortages permitted and backordered [2]:

$$Q^* = \sqrt{\frac{2K\lambda}{h\left(1-\frac{\lambda}{P}\right)}}\cdot\sqrt{\frac{b+h}{b}} \tag{34}$$

$$B^* = \left[\frac{h}{(b+h)}\left(1-\frac{\lambda}{P}\right)\right]\cdot Q^* \tag{35}$$

5. NUMERICAL EXAMPLE AND DISCUSSION

Consider a manufactured product has an annual demand rate 4600 units and its production rate is 11500 units per year. Machine in the production system is subject to a random breakdown that follows a Poisson distribution with mean $\beta=2$ times per year and according to the MTBF data from the maintenance department, a breakdown is expected to occur in backorder filling time. Abort/resume (AR) policy is used when a random breakdown takes place. Under such policy, the interrupted lot will be resumed right after restoration of the machine. The percentage x of defective items produced follows a uniform distribution over the interval $[0, 0.2]$. Among all imperfect quality items, a $\theta=0.15$ portion is the scrap and the other $(1-\theta)$ portion of the defective items are reworked at a rate of $P_1= 600$ units per year. During the reworking time, a $\theta_1=0.15$ portion of the reworked items fails and becomes scrap. Therefore, after the rework process, overall scrap rate $\varphi = [\theta + (1-\theta)\ \theta_1] = 0.2775$. Other related parameters are summarized as follows:

C = $2 per item,
K = $450 for each production run,
h = $0.6 per item per unit time,
b = $0.2 per item backordered per unit time,
C_R = $0.5 for each item reworked,
h_1 = $0.8 per item per unit time,
C_S = $0.3 disposal cost for each scrap item,
g = 0.018 years, time needed to repair and restore the machine,
M = $500 repair cost for each breakdown.

To test for convexity of the cost function, from Eq. (23) and theorem 1 one obtains $\delta=3{,}837.2$ and $2gPb=82.8$. Because $d>2gPb$, therefore $E[TCU(T_1,B)]$ is convex. Then by applying Eqs. (29) to (31) and (22), one obtains the optimal production lot-size $Q^* = 8{,}096$ (or the optimal run time $T_1^* = 0.7040$ years), the backorder level $B^*=3{,}216$, and $E[TCU(T1^*,B^*)]=\$10{,}757.88$. Variation of defective rate x and scrap rate φ effects on the optimal Q^* is depicted in Figure 4, where each x-value represents a uniform distributed random variable over the interval $[0, x]$. In Figure 4, it is noted that as the overall scrap rate φ increases, the value of Q^* increases; and for different x values, as x increases, Q^* decreases significantly.

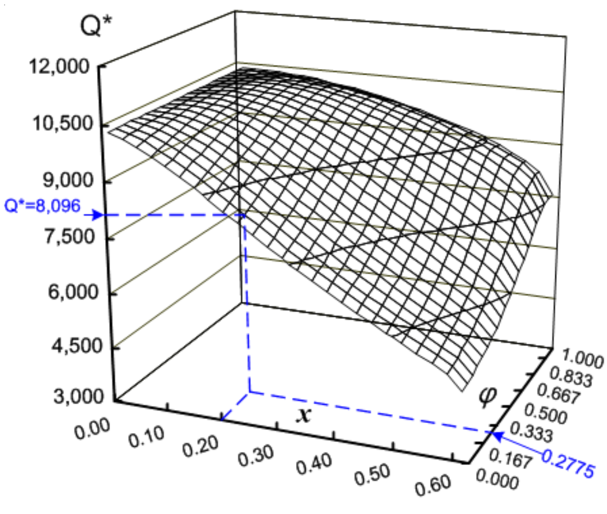

*Fig. 4 Variation of defective rate and scrap rate effects on optimal lot size Q**

Figure 5 shows the behaviour of the long-run average cost function $E[TCU(Q^*,B^*)]$ with respect to defective rate x and overall scrap rate φ. It may be noted that as x increases, the value of $E[TCU(Q^*,B^*)]$ increases; and for different φ values, as φ increases, the $E[TCU(Q^*,B^*)]$ increases significantly. Figure 6 demonstrates convexity of the long-run average cost function $E[TCU(Q^*,B^*)]$.

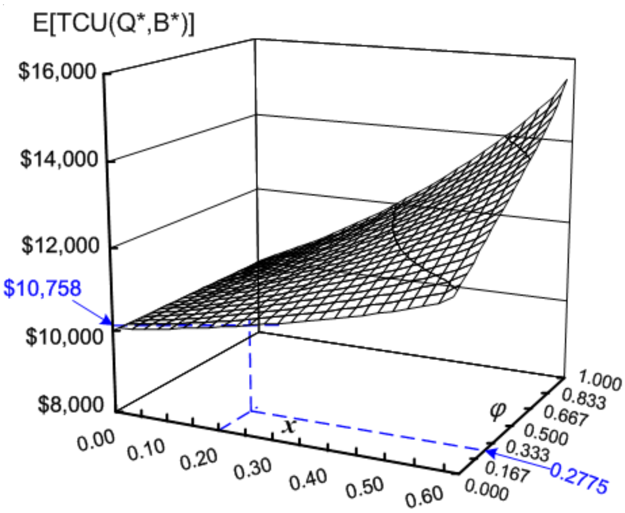

Fig. 5 Variation of defective rate and scrap rate effects on E[TCU(Q,B)]

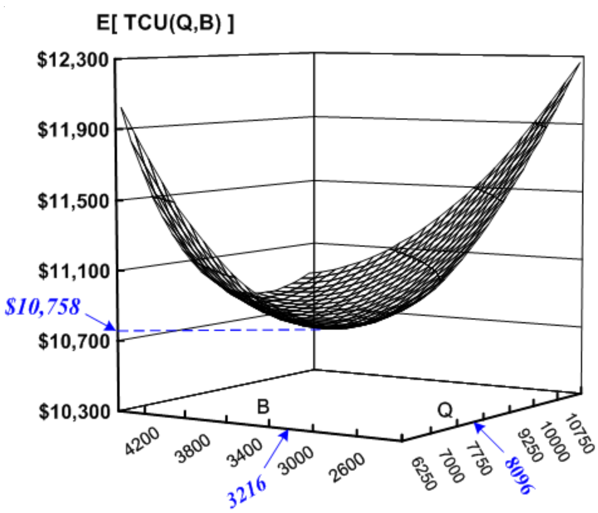

Fig. 6 Convexity of the long-run cost function E[TCU(Q,B)]

It may be noted that if a random breakdown is expected to take place and the result of the present study is not available, one can probably use the most related lot size solutions given by Chiu [13] to solve such an unreliable EMQ model and obtain $Q=5,606$ (or $T_1=0.4875$) and $B=2,257$. Then by plugging T_1 and B in Eq. (20) one obtains $E[TCU(Q,B)]=\$10,841.68$.

Excluding *$9703.60* (i.e. total manufacturing, rework, and scrap costs) and the expected annual machine repair cost *$421.97* from $E[TCU(Q,B)]$, our optimal cost $E[TCU(Q^*,B^*)]$ is *13.3%* less (on total setup and holding costs) in comparison with the corresponding costs computed without using the exact model as presented in this paper.

6. CONCLUSION

In real life manufacturing environments, random machine breakdown and defective items produced are inevitable. One cannot count on classic EMQ model to deal with the reliability and imperfect quality issues. Effect of these practical situations on optimal replenishment policy for EMQ model must be specifically studied, so that the overall production-inventory costs can be minimized. This research incorporates the reliability issues such as random machine breakdown, the reworking of random defective items, and scrap into the EMQ model with backlogging. The objective is to determine the optimal replenishment lot size that minimizes the overall production-inventory costs. Mathematical modeling is used and the renewal reward theorem is employed to cope with the variable cycle length. Theorem on conditional convexity of total cost function is proposed and proved. The optimal lot size for such a real-life imperfect manufacturing system is derived. A numerical example is provided to demonstrate its practical usage. For future study, considering multiple machine failures during uptime and random repair time for breakdown (these may be applied to certain production systems) into the similar imperfect EMQ model may be one of the interesting studies.

ACKNOWLEDGEMENTS

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APPENDIX A

Computational procedures for Eq. (20):

Because: $f(t)=\beta e^{-\beta t}$, so: $\int_0^{t_5} f(t)\,dt=\int_0^{t_5}(\beta e^{-\beta t})dt=1-e^{-\beta t_5}$, and:

$$\int_0^{t_5} t\cdot f(t)=\int_0^{t_5} t(\beta e^{-\beta t})dt=(1-e^{-\beta t_5})\left[\frac{B}{P(1-x-\lambda/P)}+\frac{1}{\beta}\right]-\left[\frac{B}{P(1-x-\lambda/P)}\right] \tag{A.1}$$

Recall Eq. (19) from Section 3:

$$E\left[TCU(T_1,B)\right]=\frac{E\left[\int_0^{t_5} TC(T_1,B)\cdot f(t)\,dt\right]}{E\left[\int_0^{t_5} T\cdot f(t)\,dt\right]}=\frac{E\left[\int_0^{t_5} TC(T_1,B)\cdot\left(\beta e^{-\beta t}\right)dt\right]}{\left[T_1\cdot P\cdot\left(1-E[x]\cdot\varphi\right)/\lambda\right]\cdot\left(1-e^{-\beta t_5}\right)} \tag{19}$$

Substituting equations (A.1) and (18) into Eq. (19), one has:

$$E\left[TCU(T_1,B)\right]=\frac{\lambda}{T_1P\left(1-E[x]\varphi\right)(1-e^{-\beta t_5})}\cdot$$
$$\left\{\begin{array}{l}(1-e^{-\beta t_5})\cdot\left[\begin{array}{l}K+M+P\cdot T_1\left[C+C_R\cdot E[x]\cdot(1-\theta)+C_S\cdot E[x]\cdot\varphi\right]-hT_1P\left(1-\varphi E[x]\right)\left[B/\lambda+g\right]+\\ +\dfrac{(b+h)}{2}E\left[\dfrac{1-x}{1-x-\lambda/P}\right]\cdot\left[g^2\lambda+\dfrac{B^2}{\lambda}\right]+\dfrac{T_1^2P^2\left(E[x]\right)^2(1-\theta)^2}{2P_1}\left[h_1-h\left(1-\theta_1\right)\right]+\\ +\dfrac{hT_1^2P}{2}\left[\dfrac{P\left[1-2\varphi E[x]+\varphi^2\left(E[x]\right)^2\right]}{\lambda}+2\varphi E[x]-1\right]+hgB\cdot E\left[\dfrac{1}{1-x-\lambda/P}\right]+\\ +gP\left[hE[x]-b\left(1-E[x]\right)\right]/\beta\end{array}\right]+\\ +Bg\left[b\cdot E\left[\dfrac{1-x}{1-x-\lambda/P}\right]-h\cdot E\left[\dfrac{x}{1-x-\lambda/P}\right]\right]\end{array}\right\} \tag{A.2}$$

Rearranging Eq. (A.2) one obtain:

$$E\left[TCU(T_1,B)\right]=$$
$$=\frac{\lambda}{T_1P\left(1-E[x]\varphi\right)}\cdot\left[\begin{array}{l}K+M+P\cdot T_1\left[C+C_R\cdot E[x]\cdot(1-\theta)+C_S\cdot E[x]\cdot\varphi\right]-hT_1P\left(1-\varphi E[x]\right)\left[B/\lambda+g\right]+\\ +\dfrac{(b+h)}{2}E\left[\dfrac{1-x}{1-x-\lambda/P}\right]\cdot\left[g^2\lambda+\dfrac{B^2}{\lambda}\right]+\dfrac{T_1^2P^2\left(E[x]\right)^2(1-\theta)^2}{2P_1}\left[h_1-h\left(1-\theta_1\right)\right]+\\ +\dfrac{hT_1^2P}{2}\left[\dfrac{P\left[1-2\varphi E[x]+\varphi^2\left(E[x]\right)^2\right]}{\lambda}+2\varphi E[x]-1\right]+hgB\cdot E\left[\dfrac{1}{1-x-\lambda/P}\right]+\\ +gP\left[hE[x]-b\left(1-E[x]\right)\right]/\beta\end{array}\right]+$$
$$+\frac{\lambda}{T_1P\left(1-E[x]\varphi\right)(1-e^{-\beta t_5})}\cdot\left\{Bg\left[b\cdot E\left[\frac{1-x}{1-x-\lambda/P}\right]-h\cdot E\left[\frac{x}{1-x-\lambda/P}\right]\right]\right\} \tag{A.3}$$

With further derivations, Eq. (A.3) becomes Eq. (20) as follows:

$$E\left[TCU\left(T_1,B\right)\right]=\frac{\lambda\left(K+M\right)}{T_1P\left(1-E\left[x\right]\varphi\right)}+\frac{\lambda\left[C+C_RE\left[x\right](1-\theta)+C_SE\left[x\right]\varphi\right]}{\left(1-E\left[x\right]\varphi\right)}+\frac{\lambda g\left[hE\left[x\right]-b\left(1-E\left[x\right]\right)\right]}{T_1\beta\left(1-E\left[x\right]\varphi\right)}+$$
$$+\frac{\lambda}{T_1P\left(1-E\left[x\right]\varphi\right)}\cdot\frac{\left(b+h\right)}{2}\cdot E\left[\frac{1-x}{1-x-\lambda/P}\right]\cdot\left[g^2\lambda+\frac{B^2}{\lambda}\right]+\frac{\lambda T_1P\left(E\left[x\right]\right)^2(1-\theta)^2}{2P_1\left(1-E\left[x\right]\varphi\right)}\left[h_1-h\left(1-\theta_1\right)\right]+$$
$$+\frac{\lambda hT_1}{2\left(1-E\left[x\right]\varphi\right)}\left[\frac{P\left[1-2\varphi E\left[x\right]+\varphi^2\left(E\left[x\right]\right)^2\right]}{\lambda}+2\varphi E\left[x\right]-1\right]+\frac{\lambda hgB}{T_1P\left(1-E\left[x\right]\varphi\right)}\cdot E\left[\frac{1}{1-x-\lambda/P}\right]-$$
$$-hB-hg\lambda+\frac{\lambda Bg}{T_1P\left(1-E\left[x\right]\varphi\right)(1-e^{-\beta t_5})}\cdot\left\{b\cdot E\left[\frac{1-x}{1-x-\lambda/P}\right]-h\cdot E\left[\frac{x}{1-x-\lambda/P}\right]\right\} \tag{20}$$

APPENDIX B

Computational procedures for Eq. (24):
From Eq. (22) one has the following:

$$\frac{\partial^2E\left[TCU\left(T_1,B\right)\right]}{\partial B^2}=\frac{1}{T_1P}\left(b+h\right)\cdot E_3 \tag{B.1}$$

$$\frac{\partial^2E\left[TCU\left(T_1,B\right)\right]}{\partial T_1\partial B}=-\frac{1}{T_1^2P}\left(b+h\right)B\cdot E_3-\frac{hg\lambda}{T_1^2P}\cdot E_4-\frac{g\lambda}{T_1^2P(1-e^{-\beta t_5})}(b\cdot E_3-h\cdot E_5) \tag{B.2}$$

$$\frac{\partial^2E\left[TCU\left(T_1,B\right)\right]}{\partial T_1^2}=\frac{2\lambda(K+M)}{T_1^3P}E_0+\frac{1}{T_1^3P}\left(b+h\right)(g^2\lambda^2+B^2)\cdot E_3+\frac{2hgB\lambda}{T_1^3P}\cdot E_4$$
$$+\frac{2hg\lambda}{\beta T_1^3}\cdot E_1+\frac{2bg\lambda}{\beta T_1^3}\cdot(E_1-E_0)+\frac{2Bg\lambda}{T_1^3P(1-e^{-\beta t_5})}(b\cdot E_3-h\cdot E_5) \tag{B.3}$$

then:

$$\left[T_1\quad B\right]\cdot\begin{pmatrix}\dfrac{\partial^2E\left[TCU\left(T_1,B\right)\right]}{\partial T_1^2} & \dfrac{\partial^2E\left[TCU\left(T_1,B\right)\right]}{\partial T_1\partial B}\\ \dfrac{\partial^2E\left[TCU\left(T_1,B\right)\right]}{\partial T_1\partial B} & \dfrac{\partial^2E\left[TCU\left(T_1,B\right)\right]}{\partial B^2}\end{pmatrix}\cdot\begin{bmatrix}T_1\\ B\end{bmatrix}=$$

$$=\left\{\begin{aligned}&\frac{2\left(K+M\right)\lambda}{T_1P}E_0+\frac{1}{T_1P}\left(b+h\right)g^2\lambda^2E_3+\frac{1}{T_1P}\left(b+h\right)B^2E_3+2hB\frac{\lambda g}{T_1P}E_4+\\&+\frac{2gh\lambda}{\beta T_1}E_1+\frac{2bg\lambda}{\beta T_1}E_1-\frac{2bg\lambda}{\beta T_1}E_0+\frac{2Bg\lambda}{T_1P(1-e^{-\beta t_5})}\cdot(b\cdot E_3)-\frac{2Bg\lambda}{T_1P(1-e^{-\beta t_5})}\cdot(h\cdot E_5)-\\&-\frac{2B^2}{T_1P}\left(b+h\right)E_3-\frac{2Bh\lambda g}{T_1P}E_4-\frac{2Bg\lambda}{T_1P(1-e^{-\beta t_5})}\cdot(b\cdot E_3)+\frac{2Bg\lambda}{T_1P(1-e^{-\beta t_5})}\cdot(h\cdot E_5)+\frac{B^2}{T_1P}\left(b+h\right)E_3\end{aligned}\right\}=$$
$$=\frac{2\left(K+M\right)\lambda}{T_1P}E_0+\frac{1}{T_1P}\left(b+h\right)g^2\lambda^2E_3+\frac{2g\lambda}{\beta T_1}\left(b+h\right)E_1-\frac{2bg\lambda}{\beta T_1}E_0 \tag{B.4}$$

Substituting Eq. (21) in the RHS of Eq. (B.4), one obtain Eq. (24) as follows:

$$\begin{bmatrix} T_1 & B \end{bmatrix} \cdot \begin{pmatrix} \dfrac{\partial^2 E\left[TCU\left(T_1, B\right)\right]}{\partial T_1^2} & \dfrac{\partial^2 E\left[TCU\left(T_1, B\right)\right]}{\partial T_1 \partial B} \\ \dfrac{\partial^2 E\left[TCU\left(T_1, B\right)\right]}{\partial T_1 \partial B} & \dfrac{\partial^2 E\left[TCU\left(T_1, B\right)\right]}{\partial B^2} \end{pmatrix} \cdot \begin{bmatrix} T_1 \\ B \end{bmatrix} =$$

$$= \frac{1}{1 - E[x]\varphi} \cdot \frac{\lambda}{T_1 P \beta} \cdot \left\{ \beta \left[2(K+M) + (b+h) \cdot \lambda g^2 \cdot E\left(\frac{1-x}{1-x-\lambda/P} \right) \right] + 2gP\left[(b+h)E[x] - b \right] \right\} \quad (24)$$

APPENDIX C

Computational procedures for Eqs. (29)-(31).
By setting Eqs. (27) and (28) equal to zero, one obtain:

$$\frac{1}{T_1^2} \left\{ \begin{array}{l} \dfrac{1}{2P}\left[2\lambda(K+M)E_0 + (b+h)(g^2\lambda^2 + B^2)\cdot E_3 + 2hgB\lambda E_4 + \dfrac{2Bg\lambda}{(1-e^{-\beta t_5})}\cdot(b\cdot E_3 - h\cdot E_5) \right] + \\ + \dfrac{1}{\beta}\left[bg\lambda(E_1 - E_0) + hg\lambda E_1 \right] \end{array} \right\} =$$

$$= \frac{\lambda P(1-\theta)^2}{2P_1}\left[h_1 - h(1-\theta_1) \right]\cdot E_2 + \frac{h}{2}\left(PE_0 - 2P\varphi E_1 + P\varphi^2 E_2 + 2\lambda\varphi E_1 - \lambda E_0 \right) \quad \text{(C.1)}$$

$$B = \frac{hP}{(b+h)E_3}T_1 - \frac{hg\lambda E_4}{(b+h)E_3} - \frac{g\lambda(bE_3 - hE_5)}{(b+h)E_3(1-e^{-\beta t_5})} \quad \text{(C.2)}$$

Substituting Eq. (C.2) in Eq. (C.1), one obtain equation (C.3):

$$\frac{1}{T_1^2} \left\{ \begin{array}{l} \dfrac{1}{2P} \left[\begin{array}{l} 2\lambda(K+M)E_0 + (b+h)(g^2\lambda^2)E_3 + \\ +(b+h)\left[\dfrac{hP}{(b+h)E_3}T_1 - \dfrac{hg\lambda E_4}{(b+h)E_3} - \dfrac{g\lambda(bE_3 - hE_5)}{(b+h)E_3(1-e^{-\beta t_5})} \right]^2 \cdot E_3 + \\ +2hg\left[\dfrac{hP}{(b+h)E_3}T_1 - \dfrac{hg\lambda E_4}{(b+h)E_3} - \dfrac{g\lambda(bE_3 - hE_5)}{(b+h)E_3(1-e^{-\beta t_5})} \right]\lambda E_4 + \\ + \dfrac{2g\lambda}{(1-e^{-\beta t_5})}\cdot(b\cdot E_3 - h\cdot E_5)\left[\dfrac{hP}{(b+h)E_3}T_1 - \dfrac{hg\lambda E_4}{(b+h)E_3} - \dfrac{g\lambda(bE_3 - hE_5)}{(b+h)E_3(1-e^{-\beta t_5})} \right] \end{array} \right] + \\ + g\lambda\left[b(E_1 - E_0) + hE_1 \right]/\beta \end{array} \right\} =$$

$$= \frac{\lambda P(1-\theta)^2}{2P_1}\left[h_1 - h(1-\theta_1) \right]\cdot E_2 + \frac{h}{2}\left(PE_0 - 2P\varphi E_1 + P\varphi^2 E_2 + 2\lambda\varphi E_1 - \lambda E_0 \right) \quad \text{(C.3)}$$

Rearranging Eq. (C.3) one has:

$$T_1^2 = \frac{ \dfrac{1}{P}\left[\begin{array}{l} 2\lambda(K+M)E_0 + (b+h)(g^2\lambda^2)E_3 - \\ -\dfrac{h^2g^2\lambda^2E_4^2}{(b+h)E_3} - \dfrac{g^2\lambda^2(bE_3 - hE_5)^2}{(b+h)E_3(1-e^{-\beta t_5})^2} - \dfrac{2hg^2\lambda^2E_4(bE_3 - hE_5)}{(b+h)E_3(1-e^{-\beta t_5})} \end{array} \right] + \dfrac{2g\lambda}{\beta}\left[b(E_1 - E_0) + hE_1 \right] }{ \dfrac{\lambda P(1-\theta)^2}{P_1}\left[h_1 - h(1-\theta_1) \right]\cdot E_2 + h\left(PE_0 - 2P\varphi E_1 + P\varphi^2 E_2 + 2\lambda\varphi E_1 - \lambda E_0 \right) - \dfrac{h^2P}{(b+h)E_3} } \quad \text{(C.4)}$$

By substituting Eq. (21) in the RHS of Eq. (C.4) and let ω_1, ω_2 and ω_3 denote the following, one obtains equations (29) to (31):

$$\omega_1 = b \cdot E\left[\frac{1-x}{1-x-\lambda/P}\right] - h \cdot E\left[\frac{x}{1-x-\lambda/P}\right]; \quad \omega_2 = (b+h)E\left[\frac{1-x}{1-x-\lambda/P}\right]; \quad \omega_3 = E\left[\frac{1}{1-x-\lambda/P}\right].$$

$$T_1^* = \frac{1}{P}\sqrt{\frac{\left[2\lambda(K+M)+(g^2\lambda^2)\omega_2 - \frac{h^2g^2\lambda^2\omega_3^2}{\omega_2} - \frac{g^2\lambda^2\omega_1^2}{\omega_2(1-e^{-\beta t_5})^2} - \frac{2hg^2\lambda^2\omega_3\omega_1}{(1-e^{-\beta t_5})\omega_2} + \frac{2Pg\lambda}{\beta}\left[hE[x]-b(1-E[x])\right]\right]}{\frac{\lambda(1-\theta)^2}{P_1}\left[h_1 - h(1-\theta_1)\right]\left(E[x]\right)^2 + h\left[\left(1-\frac{\lambda}{P}\right)\left(1-2\varphi E[x]\right)+\varphi^2\left(E[x]\right)^2\right] - \frac{h^2\left(1-\varphi E[x]\right)^2}{\omega_2}}} \tag{29}$$

$$Q^* = \sqrt{\frac{\left[2\lambda(K+M)+(g^2\lambda^2)\omega_2 - \frac{h^2g^2\lambda^2\omega_3^2}{\omega_2} - \frac{g^2\lambda^2\omega_1^2}{\omega_2(1-e^{-\beta t_5})^2} - \frac{2hg^2\lambda^2\omega_3\omega_1}{(1-e^{-\beta t_5})\omega_2} + \frac{2Pg\lambda}{\beta}\left[hE[x]-b(1-E[x])\right]\right]}{\frac{\lambda(1-\theta)^2}{P_1}\left[h_1 - h(1-\theta_1)\right]\cdot\left(E[x]\right)^2 + h\cdot\left[\left(1-\frac{\lambda}{P}\right)\left(1-2\varphi E[x]\right)+\varphi^2\left(E[x]\right)^2\right] - \frac{h^2\left(1-\varphi E[x]\right)^2}{\omega_2}}} \tag{30}$$

$$B^* = \left(\frac{h}{b+h}\right)\left(\frac{1}{E_3}\right)Q^* - \frac{hg\lambda E_4}{(b+h)E_3} - \frac{g\lambda(bE_3 - hE_5)}{(b+h)E_3(1-e^{-\beta t_5})} \tag{31}$$

UKLJUČIVANJE PROBLEMA POUZDANOSTI OPREME I IZOSTALIH POTRAŽIVANJA U EMQ MODEL - I. DIO: SLUČAJNI PREKID U VREMENSKOM INTERVALU POPUNJAVANJA ZAOSTALIH NARUDŽBI

SAŽETAK

Ovaj rad bavi se određivanjem optimalne metode nadopunjavanja u modelu ekonomične količine proizvodnje (EMQ) uključujući problem pouzdanosti opreme i izostalih potraživanja. Klasični EMQ model ne uzima u obzir neispravne predmete, koji nastaju tijekom proizvodnog ciklusa, kao ni kvar stroja. Uočeno je da se u proizvodnom sustavu, kada je dozvoljeno popunjavanje zaostalih narudžbi, slučajni kvar opreme može pojaviti u tom vremenskom intervalu, ali i tijekom gomilanja inventara u skladištu. Prva faza studije ispituje ranije spomenute praktične probleme u EMQ modelu uvodeći proces obnavljanja oštećenih predmeta, otpadaka te slučajni kvar opreme koji se posebno pojavljuje u situaciji popunjavanja zaostalih narudžbi. Svrha istraživanja je odrediti optimalno popunjavanje zaostalih narudžbi što će smanjiti troškove proizvodnje i skladištenja. Da bi se savladala promjenjiva duljina ciklusa korišteni su matematički model i analiza te "renewal reward" teorem. Predložen je i dokazan teorem o uvjetnoj konveksnosti sveukupne funkcije troška. Dobiveno je optimalno određivanje količine robe za stvarni nesavršeni proizvodni sustav. Praktična primjena je pokazana na jednom numeričkom primjeru.

Ključne riječi: *proizvodnja, kvar opreme, EMQ model, ponovni rad, određivanje količine robe, otpadak, prirast slučajnih oštećenja.*