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Mathematical formulation of the space curvature of the tendon in the PC structures

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SUMMARY

This paper presents the mathematical formulation of the space curvature of the prestressing tendon in the nonlinear analysis of prestressed concrete (PC) structures. The nonlinear behaviour of prestressed tendons is described by the one dimensional elasto-viscoplastic model. The tendon element geometry is described by the second order space function which is determined by its projections. These elements make it possible to model arbitrarily curved prestressing tendons in space, thus they can be determined independently of the three-dimensional (3D) finite element mesh. This is very important in the case when the prestressing tendon can not be located in a plane. The transfer of prestressing force on the concrete was modelled numerically. The developed model makes it possible to compute prestressing structures in phases: before prestressing, during prestressing and after prestressing. The described models are implemented in the computer programme for a 3D analysis of the prestressed concrete structures where the structures are discretized by three-dimensional finite elements with an embedded one-dimensional element of prestressed tendons.

***Key words**: space curvature, prestressing tendon, prestressing, nonlinear analysis, prestressed concrete structures.*

1. INTRODUCTION

This paper presents a numerical treatment of prestressing tendons in three-dimensional (3D) numerical modelling of prestressed concrete structures according to the work performed in Ref. [1].

The proposed model for the numerical treatment of prestressed concrete structures consists of 3D concrete elements with embedded 1D element of prestressing tendons. Geometry of the tendon is described by the space function of second order. In this way any position of the tendon can be described, curved into one or more planes. This model offers different possibilities for cable description but it requires more input data necessary for defining its position. In this model the tendon position is defined by coordinates of two nodes and the location of the two tangents at each boundary node. The tendon position is determined by nodes whose coordinates are defined in a global coordinate system. In order to ensure the continuity between tendon elements, the boundary nodes have to be placed at the intersection points of the tendon and boundary plane of 3D concrete element. The attention is mainly focused on the description of the tendon element and its compatibility with the parent element.

Equations of the finite element method have been obtained from the incremental form of the principle of virtual work. The influence of the prestressing tendon on the concrete is modelled by distributed normal and tangential load along the tendon and the two forces concentrated on the anchors. Tendon can be prestressed at one end or at both ends. The prestressed equivalent nodal forces are computed to get the contribution of the prestressing to the global stiffness.

2. MATHEMATICAL FORMULATION OF PRESTRESSING

According to balancing loading method, the tendon element and the concrete element are assumed to be separate and free bodies [2]. Loads which are consequences of prestressing are defined in local coordinate system t-n-b, where axis t is located tangentially along the tendon, axis n is normal on the space curvature and axis b is binormal on the tendon, Figure 1.

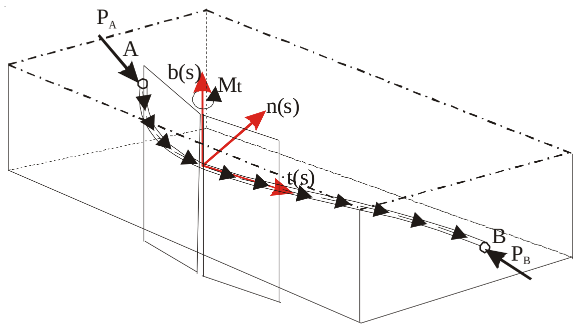

Fig. 1 The influences of the tendon forces

Mathematical formulation of all these influences can be expressed by the virtual work principle:

$$\int_{V_0} (\delta \boldsymbol{\varepsilon})\, \boldsymbol{\sigma} dV_0 + \int_{V_0} (\delta \boldsymbol{u})\, \boldsymbol{v} dV_0 + \int_{S_0} (\delta \boldsymbol{u})\, \boldsymbol{s} dS_0 + \int_{l_0} (\delta \boldsymbol{u}(s))\, \boldsymbol{p}^z(s) ds + (\delta \boldsymbol{u}_A) \boldsymbol{F}_A^{\,z} + (\delta \boldsymbol{u}_B) \boldsymbol{F}_B^{\,z} = 0 \tag{1}$$

where:

$\int_{V_0} (\delta \boldsymbol{\varepsilon}) \boldsymbol{\sigma}\, dV_0$ is the virtual work of the internal forces of concrete and reinforcing bars,

$\int_{V_0} (\delta \boldsymbol{u}) \boldsymbol{v}\, dV_0$ and $\int_{S\sigma} (\delta \boldsymbol{u}) \boldsymbol{s}\, dS_\sigma$ are the virtual work of volume and area forces, and

$\int_{l_0} (\delta \boldsymbol{u}(s))\, \boldsymbol{p}^z(s) ds + (\delta \boldsymbol{u}_A) \boldsymbol{F}_A^{\,z} + (\delta \boldsymbol{u}_B) \boldsymbol{F}_B^{\,z}$ is the virtual work of prestressing.

The balancing condition of the tendon is defined by:

$$-A_s \int_{l_0} \delta \varepsilon(s) \boldsymbol{\sigma}(s) ds - \left[\int_{l_0} \delta \boldsymbol{u}(s)\, \boldsymbol{p}^z(s) ds + (\delta \boldsymbol{u}_A) \boldsymbol{F}_A^{\,z} + (\delta u_B) \boldsymbol{F}_B^{\,z} \right] = 0 \tag{2}$$

where A_s is the cross-section area of the tendon, s is the arch length of the tendon from the beginning to the section in which the stresses are calculated, l_0 is the length of the tendon, $\boldsymbol{p}^z(s)$ is the vector of the line load along the tendon, and $\boldsymbol{F}_A^z$ and $\boldsymbol{F}_B^z$ are the vectors of the anchorage forces. If we superpose Eqs. (1) and (2) the virtual work of the line load $\boldsymbol{p}^z(s)$ will disappear. The virtual work of the internal and external forces in the prestressed structures can be defined as:

$$-\int_{V_0} (\delta \varepsilon) \boldsymbol{\sigma} dV - A^z \int_{l_0} \delta \boldsymbol{\varepsilon}(s) \boldsymbol{\sigma}(s) ds + \int_{V_0} (\delta \boldsymbol{u}) \boldsymbol{v}\, dV_0 + \int_{S_0} (\delta \boldsymbol{u}) \boldsymbol{s}\, dS_0 = 0 \tag{3}$$

3. TENDON FORCES

The prestressing forces are exceptionally high so that even small deviations in position provoke great changes in the time of prestressing. As they are external forces and as they are mutually in equilibrium they do not induce any bearing forces at statically determined systems. The prestressed tendons are usually curved whereby they could take bigger bending moment with as possibly smaller force. The curvilinear tendons decrease the transverse forces too. The deviation forces occur at every disruption or detour of the tendon. They take effect from tendon onto concrete as line forces when the tendon is bended upon uninterrupted curve, Figure 2.

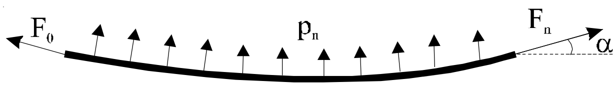

Fig. 2 Deviation forces caused by the tendon curvature

Special threats are horizontally improperly sited tendons because deviation forces can generate unfavourable lateral and torsional moments.

The mechanical behaviour of the prestressed concrete structure is under the great influence of the prestressing forces since the stress changes they directly influence the bearing capacity of the structure. Due to the stretching of the tendon the prestressing force occur which changes along the tendon during the time causing the variations of stresses in the concrete and the tendon. So, the introduction of the prestressing force into tendon and afterwards the transfer of that influence along the tendon has to be modelled. After anchoring, the tendon try to return to the aboriginal position that causes the occurrence of the concentrated compression force on the side of the concrete element. Consequently, the starting prestressing force is modelled as a concentrated force in the node of the finite element of the tendon and the influence along the tendon as continuously distributed loading. The size of the prestressing force depends on the intensity of the given forces at anchorages (F_A and F_B) and on the losses and droops which will occur at the same time.

3.1 Mathematical formulation of the tendon forces transfer

The prestressing force obtained by tendon steel extension changes during time along the tendon element consequently changing stresses and strains in concrete and tendons. The changes of prestressing forces have great influence on the mechanical behaviour of the prestressed concrete structures and its bearing capacity. The prestressing forces depend on the intensity of the prescribed forces on the anchorages and instantaneous as well as time-dependent prestress losses. In the actual model, implemented in the computer programme PRECON3D [3], only the losses due to the friction between tendon and concrete are taken into account. The prestressing force values on the ends are known, so, the influence of the prestressing in the internal points of the tendon should be determined. Figure 3 shows the differentially small arc element *ds* of the tendon and acting forces.

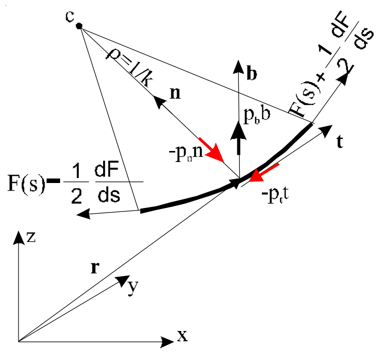

Fig. 3 Forces at differential element ds of the tendon

The vectors $\boldsymbol{t}(s)$, $\boldsymbol{n}(s)$ and $\boldsymbol{b}(s)$ are computed from the analytical representation of the tendon curve as:

$$\begin{aligned} \boldsymbol{t}(s) &= \frac{d\boldsymbol{r}}{ds}, \\ \boldsymbol{n}(s) &= \frac{1}{k(s)}\frac{d^2\boldsymbol{r}}{ds^2}, \\ \boldsymbol{b}(s) &= \boldsymbol{t}(s)\times\boldsymbol{n}(s) \end{aligned} \tag{4}$$

The vectors $\boldsymbol{t}(s)$, $\boldsymbol{n}(s)$ and $\boldsymbol{b}(s)$ are unit vectors of the tangent vector $\boldsymbol{t}$, the principal normal $\boldsymbol{n}$ and the binormal vector $\boldsymbol{b}$ respectively, vector $\boldsymbol{r}$ is position vector of the analyzing point and $k(s)$ denotes the curvature of the tendon. According to the balancing method applied on the differentially small element of the arch it can be written in the form:

$$\begin{aligned} &-\left(F(s)-\frac{1}{2}\frac{dF}{ds}ds\right)\left(\boldsymbol{t}-\frac{1}{2}\frac{dt}{ds}ds\right)+ \\ &+\left(F(s)+\frac{1}{2}\frac{dF}{ds}ds\right)\left(\boldsymbol{t}+\frac{1}{2}\frac{dt}{ds}ds\right)+ \\ &+\left(-p_t\boldsymbol{t}-p_n\boldsymbol{n}+p_b\boldsymbol{b}\right)ds=0 \end{aligned} \tag{5}$$

After assortment the Eq. (5) can be written as:

$$F\frac{d\boldsymbol{t}}{ds}+\frac{dF}{ds}t=p_t\boldsymbol{t}+p_n\boldsymbol{n}-p_b\boldsymbol{b} \tag{6}$$

Using geometrical relationship of the form:

$$\frac{d\boldsymbol{t}}{ds}=k(s)\boldsymbol{n} \tag{7}$$

the expression (6) can be written as:

$$\boldsymbol{F}(s)k(s)\boldsymbol{n}+\frac{d\boldsymbol{F}}{ds}\boldsymbol{t}=p_t(s)\boldsymbol{t}+p_n(s)\boldsymbol{n}-p_b(s)\boldsymbol{b} \tag{8}$$

or as scalars:

$$\begin{aligned} p_n(s) &= k(s)F(s), \\ p_t(s) &= \frac{dF(s)}{ds}, \\ p_b(s) &= 0 \end{aligned} \tag{9}$$

The tangential load component $p_t(s)$ can be expressed by function of the normal load component $p_n(s)$ and the friction coefficient μ (coefficient between tendon and concrete):

$$p_t(s)=\pm\mu p_n(s) \tag{10}$$

The equilibrium equation on the element (Figure 3) can be expressed as:

$$\begin{aligned} F(s)k(s)ds-p_n(s)ds&=0, \\ dF(s)-p_t(s)ds&=0 \end{aligned} \tag{11}$$

Substituting Eq. (10) into Eq. (11) and integrating over the interval $[s_1,s_2]$ (where $s_2>s_1$), it can be obtained:

$$F(s_2)=F(s_1)\,e^{\pm\mu\int_{s_1}^{s_2}k(s)\,ds} \tag{12}$$

where s is smooth curve on that interval.

If the tendon is prestressed from one end, and if the forces at both tendon ends are known, then the friction coefficient can be computed according to the expression:

$$\mu=\frac{1}{\int_A^B k(s)ds}\,ln\frac{\boldsymbol{F}_A}{\boldsymbol{F}_B} \tag{13}$$

If the tendon is prestressed at both ends, the force decreases due to the friction between the tendon and concrete, if the distance from the end is increased. In symmetrical prestressing, the problem can be simply solved since the decrease in force is the greatest at the middle of the beam. If the tendon is asymmetric or if the prestressing forces at the tendon ends are not equal, the procedure is more complex because we can not accurately predict where the minimum of the prestressing force is. The force F_{min} in some arbitrary choose cross section l_x can be calculate by the forces on the beginning (F_A) and on the end (F_B) of the prestressing tendon:

$$F^L_{min} = F_A e^{-\mu\int_B^{l_x} k(s)ds}, \qquad F^R_{min} = F_A e^{-\mu\int_B^{l-l_x} k(s)ds} \tag{14}$$

The forces should be equal regardless of the ends at which they were computed. By equating terms in Eq. (14), i.e. $F^L_{min}=F^R_{min}$, we shall obtain an equation where the integration limit is an unknown value. This equation can be solved numerically.

If we are going to determine prestressing force in the arbitrary section of the tendon it is necessary to define the tendon curvature in that section, Figure 4.

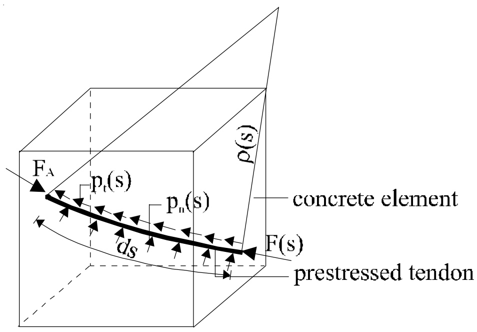

Fig. 4 Concrete finite element with embedded prestressed tendon finite element

Spatial curvature of the curve can be expressed as:

$$k(s)=\frac{d^2\mathbf{r}}{ds^2}=\sqrt{\left(\frac{d^2x}{ds^2}\right)^2+\left(\frac{d^2y}{ds^2}\right)^2+\left(\frac{d^2z}{ds^2}\right)^2}=\frac{1}{\rho} \tag{15}$$

where ρ is radius of the curve and ds is differential element of the arc.

So, with Eq. (15), using differential element of the arc, the spatial curvature is defined in the line coordinate system χ that will request double mapping. The calculation of the components $\frac{d^2x}{ds^2}$, $\frac{d^2y}{ds^2}$ and $\frac{d^2z}{ds^2}$ which are imperative for curvature definition require the mapping from the line coordinate system of the tendon into local coordinate system of the basic concrete finite element ξ–η–ζ and afterwards into global coordinate system of the structure x-y-z. The procedure of the calculation of these components will be shown in detail in continuation. The differential element of the arc of the space curve in the line coordinate system can be expressed in the form:

$$ds=\sqrt{\left(\frac{dx}{d\chi}\right)^2+\left(\frac{dy}{d\chi}\right)^2+\left(\frac{dz}{d\chi}\right)^2}\,d\chi \tag{16}$$

Respective derivations on the element of the arc can be expressed with the following terms:

$$\frac{dx}{ds}=\frac{dx}{d\chi}\frac{d\chi}{ds}; \quad \frac{d^2x}{ds^2}=\frac{d}{ds}\left(\frac{dx}{d\chi}\frac{d\chi}{ds}\right) \tag{17}$$

Developing Eq. (17) one can obtain:

$$\frac{d^2x}{ds^2}=\frac{d}{d\chi}\left(\frac{dx}{d\chi}\frac{d\chi}{ds}\right)\frac{d\chi}{ds}=\frac{d^2x}{d\chi^2}\frac{d\chi}{ds}\frac{d\chi}{ds}+\frac{dx}{d\chi}\frac{d^2\chi}{ds^2} \tag{18}$$

where:

$$\frac{dx}{d\chi}=\sum_{i=1}^{3}\frac{dN_i}{d\chi}x_i, \quad \frac{d^2x}{d\chi^2}=\sum_{i=1}^{3}\frac{d^2N_i}{d\chi^2}x_i \tag{19}$$

and:

$$\frac{d\chi}{ds}=\frac{1}{\sqrt{\left(\frac{dx}{d\chi}\right)^2+\left(\frac{dy}{d\chi}\right)^2+\left(\frac{dz}{d\chi}\right)^2}} \tag{20}$$

$$\frac{d^2\chi}{ds^2}=\frac{d}{d\chi}\left(\frac{1}{\sqrt{\left(\frac{dx}{d\chi}\right)^2+\left(\frac{dy}{d\chi}\right)^2+\left(\frac{dz}{d\chi}\right)^2}}\right)\frac{d\chi}{ds}$$

Developing Eq. (20) follows:

$$\frac{d^2\chi}{ds^2}=\left(\left(\frac{dx}{d\chi}\right)^2+\left(\frac{dy}{d\chi}\right)^2+\left(\frac{dz}{d\chi}\right)^2\right)^{-\frac{3}{2}}\cdot \cdot\left(\frac{dx}{d\chi}\frac{d^2x}{d\chi^2}+\frac{dy}{d\chi}\frac{d^2y}{d\chi^2}+\frac{dz}{d\chi}\frac{d^2z}{d\chi^2}\right) \tag{21}$$

Analogously, according to Eqs. (18) and (19), the remaining components can be defined as:

$$\frac{dy}{d\chi}=\sum_{i=1}^{3}\frac{dN_i}{d\chi}y_i; \quad \frac{dz}{d\chi}=\sum_{i=1}^{3}\frac{dN_i}{d\chi}z_i;$$
$$\frac{d^2y}{d\chi^2}=\sum_{i=1}^{3}\frac{d^2N_i}{d\chi^2}y_i; \quad \frac{d^2z}{d\chi^2}=\sum_{i=1}^{3}\frac{d^2N_i}{d\chi^2}z_i$$

Finally, component $\frac{d^2x}{ds^2}$ can be expressed in the form:

$$\frac{d^2x}{ds^2}=\left(\frac{d^2x}{d\chi^2}\right)\left(\frac{d\chi}{ds}\right)^2+\left(\frac{dx}{d\chi}\right)\frac{d^2\chi}{ds^2}, \tag{22}$$

and, analogously, components $\frac{d^2y}{ds^2}$ and $\frac{d^2z}{ds^2}$ can be carried out in a form:

$$\frac{d^2 y}{ds^2}=\left(\frac{d^2 y}{d\chi^2}\right)\left(\frac{d\chi}{ds}\right)^2+\left(\frac{dy}{d\chi}\right)\frac{d^2\chi}{ds^2}, \qquad \frac{d^2 z}{ds^2}=\left(\frac{d^2 z}{d\chi^2}\right)\left(\frac{d\chi}{ds}\right)^2+\left(\frac{dz}{d\chi}\right)\frac{d^2\chi}{ds^2} \tag{23}$$

So, the space curvature $k(s)$ of the tendon after double mapping (firstly into the local coordinate system ξ–η–ζ, and secondly into the global coordinate system x-y-z of the 3D concrete element) can be expressed as [3]:

$$\mathrm{k(s)}=\sqrt{\left(\frac{d^2x}{d\chi^2}\left(\frac{d\chi}{ds}\right)^2+\frac{dx}{d\chi}\frac{d^2\chi}{ds^2}\right)^2+\left(\frac{d^2y}{d\chi^2}\left(\frac{d\chi}{ds}\right)^2+\frac{dy}{d\chi}\frac{d^2\chi}{ds^2}\right)^2+\left(\frac{d^2z}{d\chi^2}\left(\frac{d\chi}{ds}\right)^2+\frac{dz}{d\chi}\frac{d^2\chi}{ds^2}\right)^2} \tag{24}$$

After defining the space curvature $k(s)$ of the tendon, the Eq. (12) can be finally used to define the prestressing force in the tendon in arbitrary section.

With Eq. (12) we can also define prestressing forces at the ends of the 1D tendon finite element i.e. forces F_1 and F_3 in the first and the third node. The tendon geometry is approximated with three basic functions of the form:

$$\begin{aligned} N_1(\chi)&=-\frac{1}{2}\chi\left(1-\chi\right)\\ N_2(\chi)&=\left(1+\chi\right)\left(1-\chi\right)\\ N_3(\chi)&=\frac{1}{2}\chi\left(1+\chi\right)\end{aligned} \tag{25}$$

Alongside the tendon the prestressing force $F(\chi)$ is approximated linearly suppressing izoparametric mapping. To avoid introduction of the new basic functions the force F_2 in the middle node can be calculated according to the expression:

$$F_2(s)=\frac{F_1(s)+F_3(s)}{2} \tag{26}$$

while the force in any section of the 1D tendon element can be expressed in the form:

$$F(\chi)=\sum_{k=1}^{3}N_k\left(\chi\right)F_k \tag{27}$$

where F_k is the force in the node of 1D element while N_k are basic functions of the 1D element defined with Eq. (25).

3.2 Torsion of the tendon represented by space curvature

In the prestressed structures discretized with three-dimensional model the prestressing tendon sometimes can not be placed into one plane with all its length. The influences along the tendon appearing as the effect of the tendon forces depend on the curvature of the tendon $k(s)$ which is characterized by the deviation of the axis of the tendon from the tangent on the tendon and by the binormal position changes. Binormals are not mutually parallel but form some angle ψ. The consequence of these changes is the torsion of the tendon φ, what can be mathematically expressed as:

$$\varphi=\frac{d\psi}{ds}=\left|\frac{d\boldsymbol{b_0}}{ds}\right| \tag{28}$$

where ψ is the angle between unit vectors $\boldsymbol{b}$ and $\boldsymbol{b}_0$ of the binormals drawn in the two considered neighbouring cross-sections.

The binormal can be expressed as:

$$\mathbf{b}_0=\mathbf{t}_0\times\mathbf{n}_0 \tag{29}$$

Deriving the vector product given by Eq. (29) over the length s of the arch the following equation can be obtained:

$$\frac{d\boldsymbol{b}_0}{ds}=\boldsymbol{t}_0\times\frac{d\boldsymbol{n}_0}{ds}-\boldsymbol{n}_0\times\frac{d\boldsymbol{t}_0}{ds} \tag{30}$$

where $\frac{d\boldsymbol{t_0}}{ds}$ can be expressed as:

$$\frac{d\boldsymbol{t_0}}{ds}=\frac{d^2\boldsymbol{r}}{ds^2}=k(s)\boldsymbol{n_0}=\frac{\boldsymbol{n_0}}{\rho} \tag{31}$$

Substituting Eq. (31) into Eq. (30) one can obtain:

$$\frac{d\boldsymbol{b_0}}{ds}=\boldsymbol{t_0}\times\frac{d\boldsymbol{n_0}}{ds} \tag{32}$$

Considering the fact that the vector which represents the derivation of the unit vector $\frac{d\boldsymbol{b}_0}{ds}$ is perpendicular on that vector, one can conclude that the vector is perpendicular on $\boldsymbol{b}_0$, i.e. on the binormal. The vector product is perpendicular on both vectors in vector product, i.e. on $\boldsymbol{t}_0$ and $\frac{d\boldsymbol{n}_0}{ds}$, (see Eq. (32)). So, one can conclude that the vector $\frac{d\boldsymbol{b}_0}{ds}$ is perpendicular on tangent too. As the vector is perpendicular both on binormal $\boldsymbol{b}_0$ and on tangent $\boldsymbol{t}_0$, it coincides with principal normal vector $\boldsymbol{n}_0$ of the curve in the considered point and one can write:

$$\frac{d\boldsymbol{b}_0}{ds} = \left|\frac{d\boldsymbol{b}_0}{ds}\right| \boldsymbol{n}_0 \tag{33}$$

or according to Eq. (28):

$$\frac{d\boldsymbol{b}_0}{ds} = \pm \varphi\, \boldsymbol{n}_0 \tag{34}$$

The double sign in Eq. (34) appears because the vector $\frac{d\boldsymbol{b}_0}{ds}$ can have identical or opposite direction of the vector $\boldsymbol{n}_0$. According to the sign convention the torsion is positive if the rotation of the binormal is to the right regarding unit vector of the tangent $\boldsymbol{t}_0$ while moving along the curve. Using the Frenet equation which shows the connection between the changes of the principal directions of the space curve, curvature $k(s)$ and torsion, the torsion φ of the tendon in the considered cross-section can be expressed with scalar components of the vectors $\frac{dr}{ds}$, $\frac{d^2r}{ds^2}$ and $\frac{d^3r}{ds^3}$ as:

$$\varphi = \frac{-\begin{vmatrix} \frac{dx}{ds} & \frac{dy}{ds} & \frac{dz}{ds} \\ \frac{d^2x}{ds^2} & \frac{d^2y}{ds^2} & \frac{d^2z}{ds^2} \\ \frac{d^3x}{ds^3} & \frac{d^3y}{ds^3} & \frac{d^3z}{ds^3} \end{vmatrix}}{\left(\frac{d^2x}{ds^2}\right)^2 + \left(\frac{d^2y}{ds^2}\right)^2 + \left(\frac{d^2z}{ds^2}\right)^2} \tag{35}$$

The previous equations represent the mathematical formulation of the torsion of the tendon represented with the space curve. Considering the equilibrium equation (5) on differential element of the arch it has been shown that the component of the load in the direction of the binormal on the tendon is equal to the zero even in the case of spatially sited tendon.

4. NUMERICAL EXAMPLE

The described modelling of the prestressed tendons is implemented in the computer programme PRECON3D [3]. The prestressed beams and/or girders used in everyday engineering structures generally have *I*, *T*, *Π* or similar cross-sections. The beams and/or girders with those cross-sections due to apparent three-dimensional stress state can not be analyzed exactly with two-dimensional model [4-7] especially in the situations when tendon laying is performed in space, not in one plane.

In this Section we will firstly perform a verification of presented model on one I-beam analysed numerically with three-dimensional model and experimentally tested [8] while in the second part we will replace the tendon with two tendons and lay the tendons in space not in one plane.

4.1 Model verification

The prestressed I-beam taken from Ref. [8] is analyzed to verify our developed model. The I-beam geometry and loading are shown in Figure 5. The material characteristics of the I-beam are [5]: the modulus of elasticity of the concrete is $Ec=35000\ N/mm^2$, Poisson's ratio of the concrete is $\nu=0.25$, the uniaxial compressive strength of the concrete is $\sigma=30\ N/mm^2$, the uniaxial tensile strength of the concrete is $\sigma_t=3\ N/mm^2$, the compressive strain of the concrete is $\varepsilon_c=0.0035$, the tensile strain of the concrete is $\varepsilon_t=0.002$, the uniaxial tensile strength of the steel is $\sigma_y=1500\ N/mm^2$, the modulus of elasticity of the steel is $E_s=210000\ N/mm^2$ and the cross-section area of the prestressed tendon is $As=1962.5\ mm^2$.

The I-beam concrete structure is discretized with 550 three-dimensional isoparametric 20-node finite elements and with 55 one-dimensional isoparametric 3-node elements for tendon.

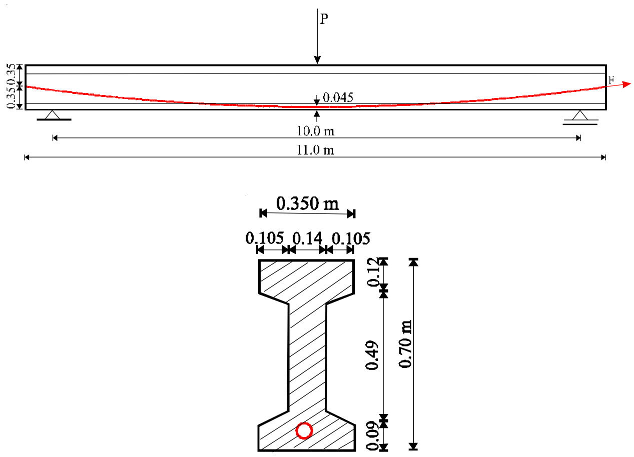

Fig. 5 Geometry of the analyzed I-beam

The load-deflection diagrams for three different analyses: (1) three-dimensional numerical analysis according to Nguyen, Ref. [8]; (2) experimental investigations according to Nguyen, Ref. [8]; and (3) three-dimensional numerical analysis according to the presented model and the computer programme PRECON3D, Ref. [3], are shown in Figure 6. These lines present mid-span deflection under the presented loading case, i.e. under monotonically increasing mid-span force *P* up to the beam failure. A good agreement of the obtained results for all three analyses is evident. Furthermore, results obtained with PRECON3D are almost identical with Nguyen's numerical results and the failure is closer to the experiment.

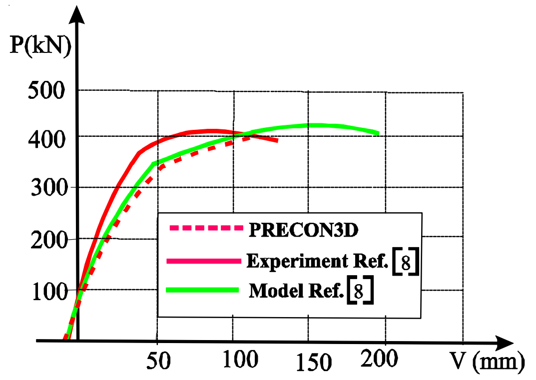

Fig. 6 Load-deflection diagrams

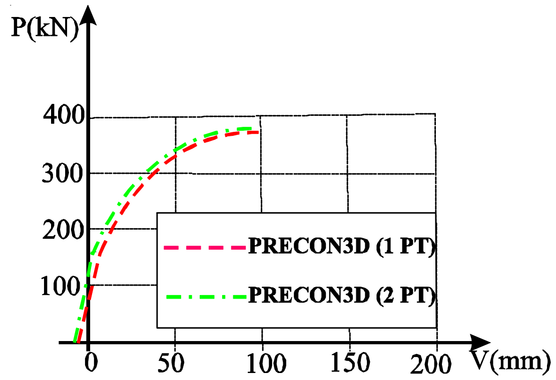

Fig. 8 Load-deflection diagrams

4.2 Tendon laying in space

The same I-beam is analysed again but with two prestressed tendons, instead with one prestressed tendon, retaining the same prestressed force and the tendon cross-section area, Figure 7. The only difference is in tendon laying, i.e. tendon in one plane is replaced with two tendons laying in space not in one plane as shown in Figure 7. The material characteristics are the same except the cross-section area of the prestressed tendon which is now twice smaller, i.e. the cross-section of each tendon is A_s=*981.25 mm*^{2}.

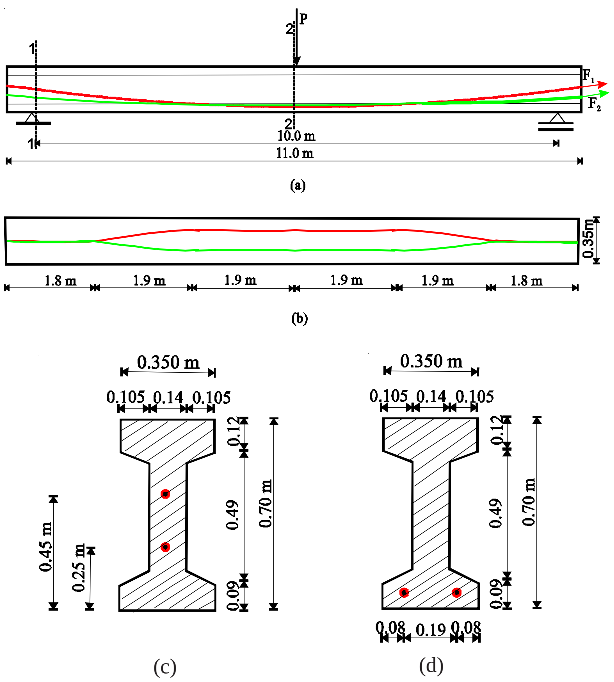

Fig. 7 Geometry of the analyzed I-beam with two prestressed tendons: (a) longitudinal section; (b) plan view; (c) cross-section 1-1; (d) cross-section 2-2

The load-deflection diagrams for two analyses performed with computer programme PRECON3D: (1) three-dimensional numerical analysis of the I-beam with one prestressed tendon laying in one plane; (2) three-dimensional analysis of the I-beam with two prestressed tendons laying in space, are shown in Figure 8.

The obtained results are almost the same with the notice that the displacements of the I-beam prestressed with two tendons laying in space are little smaller then the displacements of the I-beam prestressed with one tendon laying in one plane.

5. CONCLUSIONS

This paper presents the influence of the space curvature of the prestressing tendon in the nonlinear analysis of three-dimensional numerical modelling of prestressed concrete structures. The advantage of the proposed modelling is complete freedom in prescribing the location and geometry of prestressing tendons.

The described modelling of the prestressed tendons in 3D is implemented in the computer programme PRECON3D [3]. The full advantage of the proposed 3D modelling is evident when the width of the cross-section over the height is not constant, e.g. when we have *I*, *T*, *Π* or similar cross-sections, and when we have to describe the tendon which can not be placed into one plane with all its length.

The programme is tested on a few real life structures and compared with the known numerical and experimental results. The obtained results show good agreement with the published ones. Furthermore, this programme allows:

* Precise geometry description of prestressed concrete structures because the complex geometry of the concrete is discretized by 3D finite elements while the arbitrarily curved spatial prestressed tendons are discretized by embedded 1D finite elements;

* Analysis and monitoring of the structure behaviour in phases (before prestressing, during prestressing and after prestressing) and during the exploitation of the structure;

* Analysis of stresses and strains in characteristic sections;

* Calculation of the influence of the phase prestress on the stresses and strains in concrete and prestressed tendons;

* Calculation of the losses caused by the friction and the concrete deformation.

The analyses performed with numerical programme PRECON3D can be used as the numerical tests of the loaded structures until their collapse. So far, the obtained results show a very good agreement with experimental data and accuracy is in the interval of *5-8%* (model is always on the side of safety). These numerical tests can be used for the computation of the bearing capacity of new and existing structures. In this way the expensive experimental test can be reduced.

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MATEMATIČKA FORMULACIJA PROSTORNE ZAKRIVLJENOSTI KABELA U PREDNAPETIM BETONSKIM KONSTRUKCIJAMA

SAŽETAK

U ovom radu prikazana je matematička formulacija prostorne zakrivljenosti prednapetog kabela pri nelinearnoj analizi prednapetih betonskih konstrukcija. Nelinearno ponašanje prednapetog kabela opisano je jednodimenzonalnim elasto-viskoplastičnim modelom. Geometrija kabela je opisana prostornom krivuljom drugog reda koja je definirana svojim projekcijama. Takav način omogućava dobar opis geometrije prostorno zakrivljenog kabela te se može definirati neovisno o mreži 3D konačnih elemenata. To je posebno važno pri opisu kabela koji se ne mogu cijelom svojom dužinom smjestiti u jednu ravninu. Prijenos sile prednapinjanja je modeliran numerički. Razvijeni model omogućava računanje prednapetih konstrukcija u fazama: prije, za vrijeme i nakon prednapinjanja. Opisani model je implementiran u računalni program za 3D analizu prednapetih betonskih konstrukcija gdje je konstrukcija diskretizarana 3D konačnim elementima s uključenim 1D elementom za opis prednapetog kabela.

Ključne riječi: *prostorna zakrivljenost, prednapeti kabel, prednaprezanje, nelinearna analiza, prednapete betonske konstrukcije.*