UDC 528.024:620.174:624.21
Professional paper
Received: 24.09.2007.

Accuracy of trigonometric heighting and monitoring the vertical displacements

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SUMMARY

A lot has been written and said about the measuring of displacements and deformations of structures. This article introduces displacement measurements on the largest viaduct in Slovenia, and an analysis of the results, with particular stress on the accuracy of the calculations. Today there are a lot of sophisticated methods to measure and analyse a bridge during load tests as GPS, photogrammetric measurements, laser scanning, levelling with digital or laser levels, etc. Nevertheless, the use of classical techniques such as trigonometric heighting is still suitable enough for the most demanding field observations and demanded accuracy. In this article the most optimal accuracy for the needs of load tests is presented, the method of trigonometrical heighting is discussed in details as well as the example of its usage on the largest Slovene viaduct Črni Kal.

Key words: *trigonometric heights, vertical displacement, accuracy calculations.*

1. INTRODUCTION

Control measurements can be performed in a variety of ways depending on the structures. In practice, control measurements are performed with the help of geodetic measurements, the basic goal of which is to capture any geometric changes in the measured object. Displacements and deformations are determined. This means defining the position of changes and the object's shape, with respect to the environment and time. In this way, data about the safety of buildings can be obtained to study their behaviour, with the aim of improving the design of similar objects in the future. The main goals of geodetic control measurements are:

– to obtain a certificate for the safe operation and the stability of the measured building,
– to capture geometric changes in the measured object over time,
– to gather data for understanding the causes and creation of changes in geometric attributes,
– to enable predictions for the likely behaviour in the near future and the behaviour under a determined load,
– to control the material characteristics and nature of structures to better model constructional behaviour,
– to gain experience or knowledge in the future, for the planning of similar constructions or their restoration.

The basic tasks of the control measurements performed on buildings define displacements in horizontal and vertical planes and changes in the geometric shape of that object, such as translation, rotation, twisting, shear, bending and torsion.

Every road bridge longer than *15 m* and every rail bridge longer than *10 m* have to be checked (burdened) by legislation in Slovenia [1]. Vertical displacements and different specific deformations have to be measured.

Besides superficial or unskilled defined criteria of demand observation accuracy the extremities appear. One of these is that the displacements should be measured with the maximal possible accuracy. Another extremity is that the level of accuracy of geodetic measurements is identical to the permitted withdrawal of vertical displacements from their calculated (expected) values. More compromising and realistic demand is that the errors of measurements should be negligible small against the size of the displacement.

Today there are a lot of sophisticated methods to measure and analyse a bridge during load tests as GPS [2, 3], photogrammetric measurements [4, 5], laser scanning [6, 7], levelling with digital or laser levels [8], etc.. Nevertheless, the use of classical techniques such as trigonometric heighting is still suitable enough for the most demanding field observations and demanded accuracy.

2. MATERIALS AND METHODS

The issue of the needed accuracy of the measurement of the displacement is very important. The accuracy of the measurements should be set up according to the character of the task which is solved by the analyses of the measured displacements. Extremely high accuracy leads to the choice of unsuitable instrumentation, which is very expensive, and to the methods of measurement, which are more demanding. All in all, this means higher costs and more time for data gathering and processing. On the other side, insufficient accuracy can devaluate the results so far that they are practically unsuitable.

In the last years the problems connected with accuracy of displacement measurements are being solved with the help of probability method.

Probability method: f stands for absolute displacement value. For the necessary needed accuracy (the low border) there is an inevitable condition, that border error (allowed deformation) of its determination Δ_f is smaller, that is:

$$\Delta_f < f \tag{1}$$

If we express the allowed deformation Δ_f with the units of standard deviation of the displacement m_f, we should therefore meet the condition:

$$m_f \cdot \varepsilon < f \tag{2}$$

where ε is the coefficient, which depends on the form of distribution and confidence interval or the probability level.

In surveying, it is known that for the measurement deviations counts the normal distribution law, confidence interval is described in the form of $[+t{\cdot}m_f, -t{\cdot}m_f]$, where t is the coefficient of the normal distribution. Mostly we should count with mutual distribution, that means that $\varepsilon=2\ t$, that is followed by:

$$\frac{m_f}{f} < \frac{1}{2 \cdot t} \tag{3}$$

The choice of confidence interval depends on importance of results, i.e. the needed probability for the conclusions which are used for the interpretation of the measured data. The higher are demands the higher should be the probability level as well as confidence interval.

At measuring data for the lowest probability value $P=0{,}955$ is given, which suits the coefficient $t=2$; from the Eq. (3) of middle relative deformation of displacement determination follows:

$$\frac{m_f}{f} < \frac{1}{4} \tag{4}$$

If the higher accuracy is required, for wider confidence interval the value $P=0{,}997$ is often used because it suits a round value of coefficient $t=3$. In this case the ratio should be:

$$\frac{m_f}{f} < \frac{1}{6} \tag{5}$$

Tolerance for deformation of measured displacements from their standard (approximate or theoretical) value, i. e. critical deformations are mostly lower than the numeric value of one displacement. This allowed dispersion of displacement, according to the sort of object, should be between *50 %* and *25 %* of *f*. In this case it is not hard to conclude that in the Eq. (4) instead of *f*, *f/2* or *f/4* appeared, therefore:

$$\frac{1}{20} < \frac{m_f}{f} < \frac{1}{10} \tag{6}$$

is obtained, that represents optimal accuracy of displacement determination [9]. It can be seen, that middle deformation and displacement are directly proportional, which means that if displacement is smaller the middle deformation can be smaller or the measurement must be more accurate. The greatest problem is represented by objects, on which optimal measurements cannot be applied (because of the form of construction, inaccessibility, etc.) and the expected displacements are relatively small, because the measuring of some centimeters on the big object is no objection.

Having criteria for optimal accuracy, it is essential to find out, what kind of accuracy of displacements measurement we can achieve with the help of trigonometric heighting.

Accuracy of trigonometric heights: With trigonometric heights we can calculate the altitude difference between two points by equation:

$$\Delta H = S_H \cdot \cot Z_A + i_A - l_B + \left(\frac{1-k_a}{2}\right) \cdot \left(\frac{S_H^2}{R}\right) \tag{7}$$

where:

S_H – horizontal measured distance between *A* and *B*;
Z_A – zenith distance;
i_A – height of instrument at point *A*;
l_B – height of prism at point *B*;
k_a – coefficient of refraction (for Slovenia $k_a=0.13$) [10];

R – Earth radius as a sphere ($R = \sqrt{M \cdot N}$; M - radius of curvature along the meridian, N - radius of curvature along the prime vertical (transverse radius of curvature); *R=6370.04 km*).

If we want to calculate the accuracy of this method the height of instrument i_A can be omitted from the Eq. (7), because the measurements are in a relative coordinate system (of a fully local nature) and the measurements are usually made from concrete pillars with steel plates. The height of prisms l_B can also be omitted from Eq. (7), because they usually used reflective tape targets with negligible thickness.

Equation (7) can be simplified:

$$\Delta H_a = S_H \cdot cot\, Z_A + \frac{S_H^2}{2R} - k_a \frac{S_H^2}{2R} \qquad (8)$$

The zenith distance can be replaced by the vertical angle (α) and slope distance (D_p), so Eq. (7) changed into:

$$\Delta H_a = D_p \cdot cos\, z + \frac{{S_H}^2}{2R} - k_a \frac{{S_H}^2}{2R} \qquad (9)$$

The slope distance D_p, the vertical angle α and the coefficient of refraction k_a are considered as the variables. By using a principle of determinining the functions middle errors, function $m(\Delta H)=f(D,\alpha,k_a)$ should be obtained:

$$m_{\Delta H}^2 = \left(\frac{\partial f}{\partial D}\right)^2 \cdot m_D^2 + \left(\frac{\partial f}{\partial \alpha}\right)^2 \cdot m_\alpha^2 + \left(\frac{\partial f}{\partial k}\right)^2 \cdot m_k^2 \qquad (10)$$

where:

$m_{\Delta H}$ – standard deviation of height difference error;
m_D – standard deviation of distance;
m_α – standard deviation of vertical angle;
m_k – standard deviation of coefficient refraction error (by pragmatic experiences $m_k=\pm 0,05$ for Slovenia) [10].

The partial derivatives are:

$$\frac{\partial f}{\partial D_p} = sin\,\alpha + \frac{D_p}{R} - k_a \frac{D_p}{R} \qquad (11)$$

the partial derivative for distance, where the last part can be neglected because it is too small;

$$\frac{\partial f}{\partial \alpha} = D_p\, cos\,\alpha \qquad (12)$$

the partial derivative for vertical angle; and:

$$\frac{\partial f}{\partial k} = -\frac{D_p^2}{2R} \qquad (13)$$

the partial derivative for coefficient of refraction.

The refraction coefficient should be also considered. The closer the line of sight to the earth's surface, the bigger is the influence of refraction. Since, the value of refraction coefficient is relatively stable round noon, most measurements should be made between *9 a.m. – 4 p.m.*.

The final equation is:

$$m_{\Delta H}^2 = \left(sin^2\,\alpha + \frac{D_p^2}{R^2}\right) \cdot m_D^2 + + \left(D_p^2\, cos^2\,\alpha\right) \cdot m_\alpha^2 + m_V^2 + m_k^2 \qquad (14)$$

In Eq. (14) the standard deviations of distance, vertical angle and refraction are calculated. The accuracy of the distance measurements is ±(*a mm+b ppm*), the accuracy of angle measurement is ± *g''* (from instrument manufacturer specifications). In the Eq. (14) the standard deviation of pointing precision m_V is added, where c is the resolution of eye (*2 - 8 mgon*; mean value is *4.5 mgon*) and u is the telescope magnification (in most cases *30x*). The standard deviations m_D, m_α, m_V and m_k from Eq. (14) can be obtained according to the next equations:

$$m_D = \sqrt{a^2 + \left(\frac{D_p[m] \cdot b\, ppm}{1{,}000{,}000}\right)^2} \qquad (15)$$

$$m_\alpha = \frac{g''}{206{,}264.8062''} \qquad (16)$$

$$m_V = \frac{D_p[mm] \cdot \frac{c}{u}}{63{,}662 mgon} \qquad (17)$$

$$m_k = \pm 0.05\, mm \qquad (18)$$

The length and the vertical angle have the biggest influence on the mean error, therefore, it is useful to raise a question, what kind of combination of the length and the vertical angle could be the most ideal (its consequence would be the minimum mean error).

It is shown in Table 1 how the length and the vertical angle should be to reach optimal accuracy $1/20<m_f/f<1/10$ at certain expected displacement.

Let us have a look at the example where the measurements with the instrument for the accuracy of length (*±1mm+1ppm*) and *±0.5''* for angles is performed. Let us calculate expected displacement at *7 mm*. Displacement *7 mm* can be observed by the observation of the point at the length as far as *120 m* under vertical angle as far as *10°* or by the observation of the point at length as far as *30 m* under vertical angle to *40°* where middle error is *0.662 mm* and *0.656 mm* respectively. In Table 1 there are also other examples of expected displacements but just as far as *15 mm*, or theoretically, with the instrument with good accuracy also on bigger distance than *300 m* can be observed, which is not the case in practice. In the practice, the point at shorter length and under bigger vertical angle must be observed mostly (because we are under the object). So we can see, that the displacement of *1 mm* cannot be observed by the most accurate total station (i.e. LEICA TCA2003).

Table 1 Maximal length and vertical angle at expected displacement and suitable mean error

vertical displacement [mm]	accuracy $\frac{1}{20} < \frac{m_f}{f} < \frac{1}{10}$ [mm]		
	achieved at		max. m_f [mm] (by Eq. (14))
	length up to [m]	vertical angle up to [°]	
1	-	-	-
2	10	10	0.182
3	40	10	0.274
4	60 30	10 20	0.364 0.375
5	80 60	10 20	0.460 0.460
6	90 50	10 30	0.510 0.555
7	120 30	10 40	0.662 0.656
8	130 90	10 40	0.714 0.755
9	150 90	10 50	0.817 0.843
10	170 40	20 70	0.937 0.947
15	300	60	1.334

Following example concentrates on the analysis of results, of the biggest Slovenian road viaduct Črni Kal.

Building description*:* The Črni Kal viaduct is the most sophisticated bridge construction in Slovenia regarding its functional demands. These include its constructional and technological components, its complex design and construction regarding the preservation of the environment and the costs of construction and maintenance. It is also the biggest and highest bridge on the Slovene road network (*1065 metres* in length and *95 m* in height).

The lower part of the viaduct features two girders (*7.5 m* high) and 11 columns, five of them are low, double columns, up to *27 m* high, six of them are high single columns, that are, in the surfaces part, y-shaped. The highest column is *87.5 m*. The greatest span between columns is *140 m*. The total width of the two (separated) roads is *26.5 m*.

The columns of the viaduct were built with self-climbing formwork, this technology being used for the first time in Slovenia. The connection of the upper construction was necessary in order to concrete the connecting segments on the left and right sides at the same time. The viaduct contains approximately 50,000 cubic metres of concrete, 8,000 tonnes of reinforcing steel and 1,300 tonnes of prestressed cable.

3. RESULTS AND DISCUSSION

After a precise study of the surface characteristics, climate conditions, configurations in the field and expected strengths of eventual seismic waves, the Engineering Bureau commissioned the viaduct design. This demanding project required a combination of theoretical knowledge of civil engineering, mathematics and mechanics, as well as many years of experience in the field of building technology.

Plans were made for every part of the construction and for every building phase. Detailed static calculations and analysis of the dynamic responses of this construction were performed. The design was also tested in an Austrian wind tunnel in Vienna.

To control the viaduct's behaviour during construction, the temperature of concrete was measured. This required the installation of a considerable number of temperature-measuring devices. The temperature of concrete increased in the early stages of the construction followed by a fall in temperature over 14 days. The temperature was the highest in the middle part of the foundation plate (*130 cm*) while the temperature in the deeper section of the foundation plate (*280 cm*) was almost constant, and it was decreasing very slowly.

Ponting d.o.o., a structural engineering company from Maribor, noticed that the viaduct needed load testing. They prepared an expert's report of the maximum torques for each field (a span between two columns), which also captured the calculated analysis of constructional behaviour. Separate analyses for each field and for each driven construction (both for left and right) were prepared. The calculated maximum vertical displacements are shown in Table 2.

The quantity of load for a load test was determined by using a model of spatial framework (with a computer program). The mathematical model of the construction was identical to the model first used in the project.

Geodetic measurements were done simultaneously on six station points (because of the time available). The measurements were made from concrete pillars that were reinforced with steel reinforcement bars and anchored to the ground. These observation pillars were prepared 14 days before measurements so that the material could consolidate well. Also, a net of geodetic points within a local coordinate system was established (two parts because of field constraints – from Column 1 to Column 5, and from Column 6 to Column 12). These geodetic points were used as a control points, before, during and after each measuring phase.

Prior to each load test, all station point positions were checked for stability. Potential shifts of the columns did not occur. Before every measurement (in the morning / in the afternoon) all instruments were calibrated and the data about temperature and air pressure were entered. First, the zero state was recorded and then one individual phase after another.

Table 2 Calculated maximum vertical displacements by fields in mm

Field	Road direction					
	towards Koper (KP)			towards Ljubljana (LJ)		
	Pylon on KP side	middle - field	Pylon on LJ side	Pylon on KP side	middle - field	Pylon on LJ side
1	-0,01251	-11,20158	-0,15435	-0,01074	-29,77986	-0,15422
2	-0,37819	-35,82911	-0,24398	-0,41834	-38,20297	-0,86234
3	-0,54105	-44,92177	-0,52652	-0,16813	-43,30919	-0,24837
4	-0,73056	-47,85397	-0,75733	-0,33624	-45,33493	-0,19085
5	-0,34358	-46,11753	-0,13221	-0,54641	-44,62204	-0,45344
6	-0,71887	-30,30640	-0,44675	-0,22559	-28,71912	-0,20624
7	0,02588	-13,53882	-0,47841	-0,19849	-13,07834	-0,45314
8	-2,42995	-8,83822	-0,69699	-0,60894	-8,44403	-0,68058
9	-0,64877	-5,44881	-0,65103	-0,63011	-5,19805	-0,63169
10	-0,55073	-5,72538	-0,78389	-0,53315	-5,46094	-0,75938
11	-0,57257	-5,27300	-0,75571	-0,55230	-5,02672	-0,72785
12	-0,41356	-3,05030	-0,04641	-0,39739	-2,90707	-0,04593

The bridge was unloaded after almost each test. Six Nikon series 720 instruments were used with the distance accuracy measurements of (*±3mm+2ppm*) angle accuracy of *±3''* and magnification of *30×* (by instrument manufacturer specifications).

All of the 191 characteristic sight points on the viaduct were observed simultaneously. Leica's reflective tape targets of dimensions *5×5 cm* (for closer targets) and *10×10 cm* (for distant targets) were used on each target point. In Figure 1, the number of targets in the third field and their precise positions in that field are shown (KP – the crossbeam on the Koper side, first 1/4 of the field, 1/3 of the field, the middle of the field and LJ – the crossbeam on the Ljubljana side).

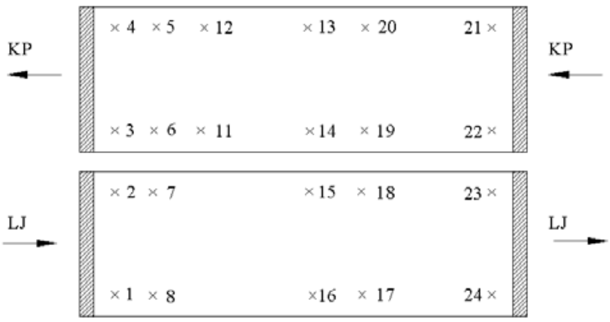

Fig. 1 Points position in field 3

A post-processing of all recorded data was carried out before the analysis (filtering). In this way all errors were eliminated, such as double observation of the same point, wrong order of sightings, etc. The observations were arranged according to individual days and individual station points. Every load test phase was compared (load or relief) with the zero state, which was recorded at the start of each measurement.

A comparison of the calculated and measured maximum vertical displacements was made separately, for both road ways (left: towards Koper (KP), right: towards Ljubljana (LJ)). Every other individual profile was compared; however, in this article only the central and the most interesting profiles will be discussed. Figure 2 shows that the surveyed vertical deformations on all fields, were always smaller then the calculated values (we introduce left road). Figure 3 shows the values of the measured maximal displacement during individual phases of the load tests. The common number of phases was 24, the biggest vertical displacement were in the phases 15-19, when the longest and the highest fields 3, 4 and 5 were loaded.

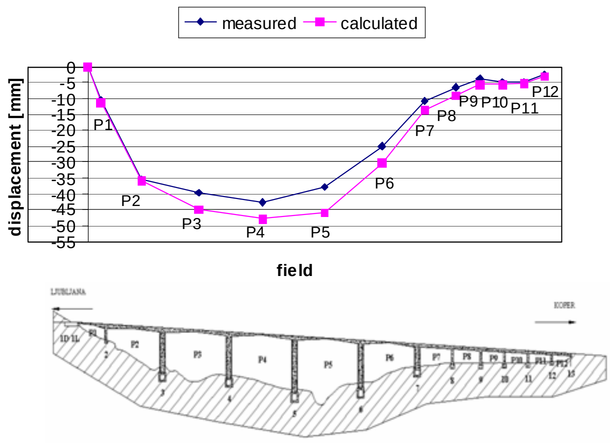

Fig. 2 Comparison of calculated and measured vertical displacement in the middle of the fields - for left road

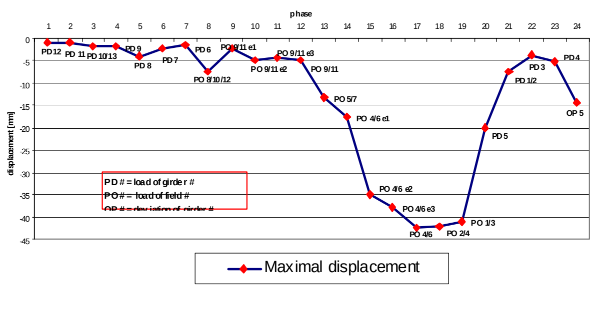

Fig. 3 Measured maximum vertical displacement by phases

The precise processing of the measured data and its analyses were performed after the field measurement.

The standard deviations of height differences for different values of slope distance and angles were calculated. A detailed calculation is given bellow for the longest measured slope distance (*136.5432 m*) and an appropriate vertical angle (*55°13'11''*); the rest can be seen in Table 3.

After inserting the values into Eq. (14) in section Error of altitude difference at trigonometric heights and using $R=6370.04\ km$ a final value for the precision of the height difference was obtained:

$$m_{\Delta H}^2 = \left(sin^2\alpha + \frac{D_p^2}{R^2} \right) \cdot m_D + \left(D_p^2 \cdot cos^2\alpha \right) \cdot m_\alpha + m_V^2 + m_k^2 =$$
$$= \sqrt{6.07\ mm^2 + 1.28\ mm^2 + 0.10\ mm^2 + 2{,}9 \cdot 10^{-7}\ mm^2} =$$
$$= \sqrt{7.45863\ mm^2} \Rightarrow \quad m_{\Delta H} = \pm 2.73\ mm$$

In this case the precision is mainly influenced by the distance measurement, rather than the pointing, and the measurement of the vertical angles. The precision at other slope distances and angles, according to used instrumentation, is shown in Table 3. Column 3 shows measured vertical displacement at a distance in Column 1 and vertical angle in Column 2. In Columns 4 and 6 we can see the precision of height differences depending on instruments we would use. Columns 5 and 7 describe whether ratio m_f/f is in the interval according to Eq. (6) and <0.1 respectively.

We can see that we could improve essentially the accuracy of the measurements using better instruments. The criteria are not met in the *1st*, *2nd* and *3rd* example. In the first case this is due to a very small displacement (*4.5 mm*), while in the second and third case the combination of the length and vertical angle is very inconvenient (short length but high vertical angle).

For every target the standard deviation of distance measurement was also calculated. As an example, only three targets on the longest (and highest) field 4 of the left road are considered, where the biggest vertical displacements were expected.

Ten readings were made for each target (distance and vertical angle) in the precise measurement mode (PMRS) of the instruments. The arithmetic mean values of the distance readings were calculated. For each target, the standard deviation (see Column 2 in Table 3) and the variance were calculated (see Column 3 in Table 3).

The variance was calculated according to:

$$\sigma^2 = \frac{\sum v^2}{(n-1)} \qquad (19)$$

and the standard deviation by equation:

$$s = \sqrt{\frac{\sum v^2}{(n-1)}} \qquad (20)$$

where:

s – standard deviation,
v^2 – squared deviation from arithmetic mean,
n – number of observations.

Table 4 shows the calculation of the standard deviation for field 4 (the crossbeam on Ljubljana side (LJ), the centre of the field, the crossbeam on Koper side (KP)).

The measurement precision of *±0.2 mm* was achieved (average value of the standard deviation of the distance measurement to targets near pylons) and *±0.3 mm* (average value of the standard deviation of distance measurements to the middle of the field).

In Table 5 the calculated and measured values of the deformation of the middle points of both roads are shown. The measured vertical displacements for the road direction to Koper, is between *68.8 %* and *98.8 %* (average *83.5 %*). For the road direction to Ljubljana, the measured values are between 7*3.2 %* and *97.5 %* (average *86.9 %*) of the calculated value. In practice, there is an unwritten rule that the ratio should be about *75 %*. If we compare only those ratios for the smallest vertical displacement (because they are the most questionable) for the two parallel road ways, that is *68.8 %* and *82.6 %*, a ratio that is almost exactly as expected ((*68.8 %+82.6 %*)/*2=75.7 %*). We conclude that a proper measurement method was selected. It is essential, what size of displacement is measured at a certain distance and under certain angle.

4 CONCLUSION

Today, the method of trigonometric heighting is still the most commonly used in determining the vertical displacement with total stations. For large load tests, as in the case of the Črni Kal viaduct, GPS receivers could be also used but just for the biggest displacements on the field 3-5. Levelling was not suitable at all.

The method used for the precision testing of the Črni Kal viaduct was suitable, since the measurement precision in comparison to the magnitude of the measured vertical displacement was sufficient. Six instruments (total stations) of the same accuracy were used. Every measurement was repeated 10 times which turned out to be appropriate. The reflective tape targets were big enough. The measured values of the vertical displacements of the fields increased with the field length, as it was expected. The accuracy of the measurements could be improved with more accurate total stations.

Table 3 Precision of the height difference for a number of distance-vertical angles pairs

				instrumentation 1[1]		instrumentation 2[2]	
No.	D_p [m]	vertical angle α	measured vertical displacement f [mm]	$m_{\Delta H}$ [mm] [1]	$m_{\Delta H} / f < 0.1$	$m_{\Delta H}$ [mm] [2]	$m_{\Delta H} / f < 0.1$
	1	2	3	4	5	6	7
1	5.8474	20º 40' 23"	4.5	1.06	NO [3]	0.35	YES
2	21.5514	75º 32' 53"	10.9	2.91	NO [4]	0.97	YES
3	32.8236	55º 18' 17"	18.7	2.48	NO [4]	0.83	YES
4	50.3400	22º 37' 55"	20.2	1.34	YES	0.42	YES
5	87.0210	59º 40' 20"	37.9	2.68	YES	0.89	YES
6	91.7401	28º 54' 53"	24.9	1.87	YES	0.56	YES
7	114.6949	45º 48' 57"	35.4	2.46	YES	0.79	YES
8	136,5432	55º 13' 11"	40.5	2,73	YES	0.90	YES

[1] instrument with distance accuracy of (± 3mm+2 ppm), angle accuracy of ± 3" and magnification of 30x
[2] instrument with distance accuracy (± 1mm + 1 ppm), angle accuracy of ± 0.5" and magnification of 30x
[3] because of a very small vertical displacement
[4] because of inconvenient combination of distance and vertical angle

Table 4 Calculation of standard deviation of slope distances

	point on girder of left construction on LJ side			**point on the middle of the field of the left construction**			**point on girder of left construction on KP side**		
	reading [m]	**deviation v [m]**	**squared deviation v^2 [m]^2**	**reading [m]**	**deviation v [m]**	**squared deviation v^2 [m]^2**	**reading [m]**	**deviation v [m]**	**squared deviation v^2 [m]^2**
	1	**2**	**3**	**1**	**2**	**3**	**1**	**2**	**3**
	0.0020	0.0000	0	0,0412	-0,0004	$1.6{\cdot}10^{-8}$	0,0080	0,0000	0
	0.0022	-0.0002	$4.0{\cdot}10^{-8}$	0,0412	-0,0004	$1.6{\cdot}10^{-8}$	0,0082	-0,0002	$4.0{\cdot}10^{-8}$
	0.0022	-0.0002	$4.0{\cdot}10^{-8}$	0,0409	-0,0001	$1.0{\cdot}10^{-8}$	0,0083	-0,0003	$9.0{\cdot}10^{-8}$
	0.0020	0.0000	0	0,0411	-0,0003	$9.0{\cdot}10^{-8}$	0,0081	-0,0001	$1.0{\cdot}10^{-8}$
	0.0020	0.0000	0	0,0407	0,0001	$1.0{\cdot}10^{-8}$	0,0077	0,0003	$9.0{\cdot}10^{-8}$
	0.0018	0.0002	$4.0{\cdot}10^{-8}$	0,0405	0,0003	$9.0{\cdot}10^{-8}$	0,0080	0,0000	0
	0.0020	0.0000	0	0,0410	-0,0002	$4.0{\cdot}10^{-8}$	0,0084	-0,0004	$1.6{\cdot}10^{-8}$
	0.0018	0.0002	$4.0{\cdot}10^{-8}$	0,0404	0,0004	$1.6{\cdot}10^{-8}$	0,0078	0,0002	$4.0{\cdot}10^{-8}$
	0.0020	0.0000	0	0,0404	0,0004	$1.6{\cdot}10^{-8}$	0,0079	0,0001	$1.0{\cdot}10^{-8}$
	0.0020	0.0000	0	0,0406	0,0002	$4.0{\cdot}10^{-8}$	0,0076	0,0004	$1.6{\cdot}10^{-8}$
sum	0.0200	0.0000	$16.0{\cdot}10^{-8}$	0,4080	0,0000	$9.2{\cdot}10^{-7}$	0,0800	0,0000	$6.0{\cdot}10^{-7}$
mean	0.0020			0,0408			0,0080		
Variance σ		$1.7{\cdot}10^{-8}$			$10.2{\cdot}10^{-8}$			$6.6{\cdot}10^{-8}$	
Standard deviation s [mm]		**± 0.13**			**± 0,32**			**± 0,26**	

Table 5 Comparison of calculated and measured values

Field No.	road direction to Koper (KP)				road direction to Ljubljana (LJ)			
	calculated value [mm]	measured value [mm]	meas/calc	%	calculated value [mm]	measured value [mm]	meas/calc	%
1	- 11.2	- 10.5	0.94	93.7	- 29.8	- 19.5	0.65	65.5
2	- 35.8	- 35.4	0.99	98.8	- 38.2	- 33.8	0.88	88.5
3	- 44.9	- 39.5	0.88	87.9	- 43.3	- 40.9	0.94	94.4
4	- 47.9	- 40.5	0.85	84.6	- 45.3	- 42.5	0.94	93.7
5	- 46.1	- 37.9	0.82	82.2	- 44.6	- 39.7	0.89	89.0
6	- 30.3	- 24.9	0.82	82.2	- 28.7	- 27.4	0.95	95.4
7	- 13.5	- 11.0	0.81	81.2	- 13.1	- 11.5	0.88	87.9
8	- 8.8	- 6.5	0.74	73.5	- 8.4	- 7.5	0.89	88.8
9	- 5.4	- 3.9	0.72	71.6	- 5.2	- 4.5	0.87	86.6
10	- 5.7	- 4.7	0.82	82.1	- 5.5	- 4.0	0.73	73.2
11	- 5.3	- 5.0	0.95	94.8	- 5.0	- 4.9	0.97	97.5
12	- 3.1	- 2.1	0.69	68.8	- 2.9	- 2.4	0.83	82.6
			average	83.5			average	86.9

In this paper the optimal accuracy for geodetic measurements is proposed. Some tenth of the millimeter accuracy is obtained on a very big construction using the trigonometric heights. The advantage of this method is that it can be used at longer sighting distances than in levelling. In addition to vertical displacements, horizontal displacement can be also obtained. The measurements are quicker and, most important, partial results can be already obtained in the field. Therefore the main remaining problem relates to the operator, such as the errors that occur due to inaccurate pointings.

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TOČNOST TRIGONOMETRIJSKOG VISINOMJERSTVA I PROMATRANJE VERTIKALNIH POMAKA

SAŽETAK

Puno toga se napisalo o mjerenju pomaka i o deformacijama građevina. Ovaj rad govori o mjerenju pomaka na najvećem vijaduktu u Sloveniji i o analizi rezultata, s posebnim naglaskom na točnosti proračuna. Danas postoje mnoge sofisticirane metode za mjerenje i analizu nekog mosta tijekom testova opterećenja kao što su GPS, fotogrametrijska mjerenja, lasersko skeniranje, niveliranje digitalnim laserskim libelama itd. Usprkos tome, korištenja klasičnih tehnika poput trigonometrijskog visinomjerstva još je uvijek dovoljno dobro za najzahtjevnija promatranja i veliku točnost. Ovaj članak govori o najvećoj mogućoj točnosti za potrebe testova opterećenja, pri čemu se o metodi trigonometrijskog visinomjerstva raspravljalo detaljno kao i o njezinom korištenju na najvećem slovenskom vijaduktu Črni kal.

Ključne riječi: *trigonometrijsko visinomjerstvo, vertikalni pomak, proračun točnosti.*