Binary-coded and real-coded genetic algorithm in pipeline flow optimization*

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Abstract. *The mathematical model for the liquid-gas mixture flow in pipelines is an initial-boundary value problem for a nonlinear hyperbolic conservation law system. This hyperbolic conservation law system together with boundary conditions is numerically solved using the essentially non-oscillatory (ENO) schemes.*

The optimization problem is a boundary control problem, i.e. boundary conditions that cause pressure values in the pipeline as close as possible to the desired ones are to be found, considering given constraints. The applied optimization method is the genetic algorithm (GA) with two different variable-to-chromosome coding strategies: binary coding and real coding.

The results of both GA strategies applied to two pipeline flow optimization problems are presented and compared.

Key words: *pipeline, liquid-gas mixture, water-hammer, cavitation, optimization, genetic algorithm, binary-coded and real-coded GA*

AMS subject classifications: 65C20, 35L65, 49M99, 76 - 04

1. Introduction

We are concerned with pipelines consisting of pipes, reservoirs, valves, pumps, turbines, surge tanks, air chambers etc. filled with a liquid-gas mixture. Perturbations at the boundaries of the pipes induce nonstationary fluid flow characterized by waterhammer and cavitation phenomena. Our goal is to control the operations with valves, turbines, pumps and other pipeline elements in order to minimize pressure oscillations.

In this paper we not only propose a solution of this problem using essentially non-oscillatory schemes for the computation of the flow and the genetic algorithm as the optimization method (see [8]), but also further improve of the optimization results obtained by applying of real coding instead of binary coding.

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2. ENO-schemes in computation of liquid-gas mixture flow in pipelines

The model for nonstationary liquid-gas mixture flow in a pipe is the conservation law system:

$$\frac{\partial \varrho}{\partial t} + \frac{\partial}{\partial x}(\varrho\nu) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\varrho\nu) + \frac{\partial}{\partial x}(\varrho\nu^2 + p) = -\varrho\left(g\frac{dz}{dx} + \frac{\lambda}{2D}\nu|\nu|\right) \tag{2}$$

$$\frac{\partial}{\partial t}(\kappa) = c_1\frac{p_s - p}{\sqrt{p - p_\nu}}\sqrt{\kappa} \tag{3}$$

for $x \in (0, L)$, $t > 0$ together with relations:

$$\varrho = \left(1 - \frac{c_2\kappa}{p - p_\nu}\right)\varrho_t \ , \quad a_t = \sqrt{\frac{dp}{d\varrho_t}}. \tag{4}$$

Here: x - position along the pipe axis, t - time, ϱ - density, ν - flow velocity, p - pressure, g - gravity acceleration, z - height measured from a given level, λ - friction factor, D - circular cross section diameter of the pipe, κ - quantity of gas in a bubble, c_1, c_2 - constants of proportionality, p_s - liquid-gas saturation pressure, p_ν - vaporization pressure, ϱ_t - liquid density, a_t - sound velocity in the liquid in the pipe, L - pipe length.

The equations are the mass conservation law (1), the momentum conservation law (2) (see [3]) and the ideal gas law combined with the Boussinesque relation (3) (see [6]). Initial and boundary conditions are needed in order to solve the system (1)-(4). Boundary conditions are derived from the influence of other pipeline elements: reservoirs, pumps, valves, turbines, air chambers, surge tanks etc. on the fluid flow and are usually non-linear (see [2]). The physical phenomena in that way modeled are waterhammer and cavitation.

The system (1)-(4) is a hyperbolic conservation law system (see [4]):

$$\frac{\partial \vec{u}}{\partial t} + \frac{\partial \vec{f}(\vec{u})}{\partial x} = \vec{g}(\vec{u}, x, t) \tag{5}$$

where:

$$\vec{u} = \begin{pmatrix} \varrho \\ \varrho\nu \end{pmatrix}, \ \vec{f}(\vec{u}) = \begin{pmatrix} u_2 \\ \frac{u_2^2}{u_1} + p(u_1) \end{pmatrix}, \ \vec{g}(\vec{u}, x, t) = \begin{pmatrix} 0 \\ -u_1\left(g\frac{dz}{dx}\cdot x + \frac{\lambda}{2D}\frac{u_2|u_2|}{u_1^2}\right) \end{pmatrix} \tag{6}$$

For this system Hugoniot loci in 1- and 2-characteristic fields are 1-parametric (ξ-parameter) curves:

$$\nu_s = \frac{\hat{u}_2}{\hat{u}_1} \mp \sqrt{\frac{p(\hat{u}_1(1+\xi)) - p(\hat{u}_1)}{\hat{u}_1\xi}}\sqrt{1+\xi}, \ \tilde{\vec{u}} = \hat{\vec{u}} + \xi\begin{pmatrix} \hat{u}_1 \\ \sqrt{\frac{p(\hat{u}_1(1+\xi)) - p(\hat{u}_1)}{\hat{u}_1\xi}}\sqrt{1+\xi} \end{pmatrix} \tag{7}$$

differential equations for 1- and 2- rarefaction integral curves:

$$\vec{u}'(\xi) = \frac{1}{\frac{a(u_1)}{u_1} \pm a'(u_1)} \begin{pmatrix} 1 \\ \frac{u_2}{u_1} \pm a(u_1) \end{pmatrix} \tag{8}$$

and 1- and 2- Riemann invariants:

$$R^{(1)}(\vec{u}) = \frac{u_2}{u_1} - \int \frac{a(u_1)}{u_1} du_1, \quad R^{(2)}(\vec{u}) = \frac{u_2}{u_1} + \int \frac{a(u_1)}{u_1} du_1\,, \tag{9}$$

where: $a = \sqrt{\frac{dp}{du_1}}$ - sound speed.

In order to numerically solve the initial-boundary value problem for (1)-(4), we applied essentially non-oscillatory (ENO) schemes (see [7]) and with particular care we numerically modeled boundary conditions (see [8]).

3. Binary-coded and real-coded genetic algorithm in optimization of pipeline flows

Valve closures, pump failures, turbine operations etc. are boundary perturbations that generate pressure oscillations propagating through the pipeline. The optimization goal is to minimize these oscillations, i.e. the function:

$$\varphi = \max_{t \in [t_1, t_2]} |p(x_0, t) - p_0(t)|\,, \tag{10}$$

where: x_0 - chosen position in the pipeline, $[t_1, t_2]$ - time interval, p_0 - desired pressure values.

The optimization variables are the cross section area of the valve opening as a function of time, the rotational speed of a centrifugal pump or of a turbine as a function of time etc.

We chose the genetic algorithm (GA) (see [5]) as the optimization method. For the coding of the goal function real variables into the chromosome of an individual we used two different approaches. In the binary-coded GA the chromosome is:

$$g_1 \cdots g_l \cdots g_{(i-1)l+1} \cdots g_{il} \cdots g_{(n-1)l+1} \cdots g_{nl}\,, \tag{11}$$

$$g_j \in \{0, 1\}\,, \quad j = 1, \dots, nl, \tag{12}$$

$$\eta_i = g_{(i-1)l+1}2^0 + g_{(i-1)l+2}2^1 + \cdots + g_{il}2^{l-1} \in \left\{0, 1, \dots, 2^{l-1}\right\}\,, \tag{13}$$

$$\xi_i = \frac{\xi_{i,max} - \xi_{i,min}}{2^l - 1}\eta_i + \xi_{i,min} \in [\xi_{i,min}, \xi_{i,max}], \quad i = 1, \dots, n \tag{14}$$

and in the equivalent real-coded GA:

$$g_1 \cdots g_n, \tag{15}$$

$$g_i \in \left\{0, 1, \dots, 2^{l-1}\right\}, \quad i = 1, \dots, n, \tag{16}$$

$$\xi_i = \frac{\xi_{i,max} - \xi_{i,min}}{2^l - 1}g_i + \xi_{i,min} \in [\xi_{i,min}, \xi_{i,max}], \quad i = 1, \dots, n \tag{17}$$

where: g_j - gene, ξ_i - goal function variable.

Since the crossover operator makes cuts between genes in the binary-coded GA, it changes the value of the goal function variables, while in the real-coded GA it just interchanges the variables between individuals. The mutation operator is the only one that creates new variable values in the real-coded GA, so we must take a small mutation probability ($p_m = 0.05$) in the binary-coded GA, and a much greater one ($p_m = 0.3$) in the real-coded GA.

4. Results

We implemented the chosen numerical and optimization methods in the C++ program. The results were obtained using the ENO-Localy-Lax-Friedrichs scheme of 3^{rd} order, and the GA with linear scaling of raw fitness, stochastic universal sampling for the selection operator, two-point crossover operator and multi-bit mutation operator. In both of the following two computations the real-coded GA gave better results than the binary-coded GA. We can conclude that for the solution of optimization problems where the goal function has real variables the real coding should be preferred to the binary coding.

4.1. Valve-closure optimization

In the pipeline with the structure: reservoir - pipe - junction - pipe - valve, the valve has to be closed in the time interval $[0s, 6s]$. The binary-coded GA gave us the valve-closure strategy in *Figure 2* that reduces pressure oscillations in respect to the initial value down to the value of $\varphi_{min} = 784015$ Pa, while with the real-coded GA we obtained an improved valve-closure strategy in *Figure 4* that gave us $\varphi_{min} = 736218$ Pa. Pressure and velocities at the valve in both cases can be seen in *Figure 1* and *Figure 3*, respectively.

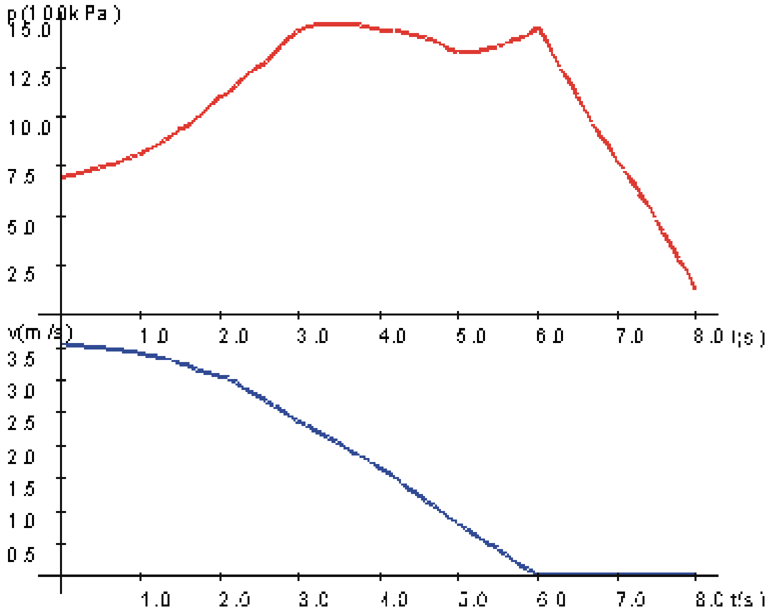

Figure 1. *Binary-coded GA result: pressure and velocity at the valve*

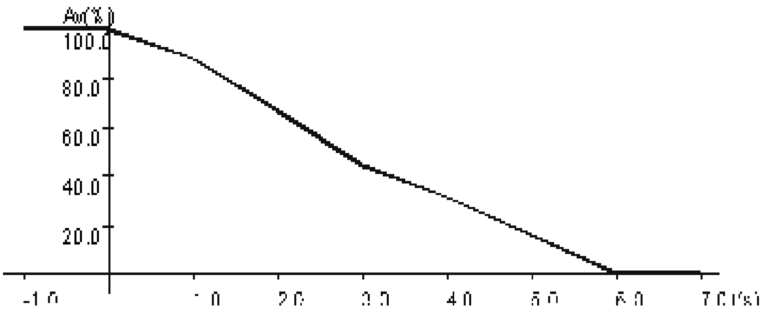

Figure 2. *Binary-coded GA result: optimal value-closure strategy*

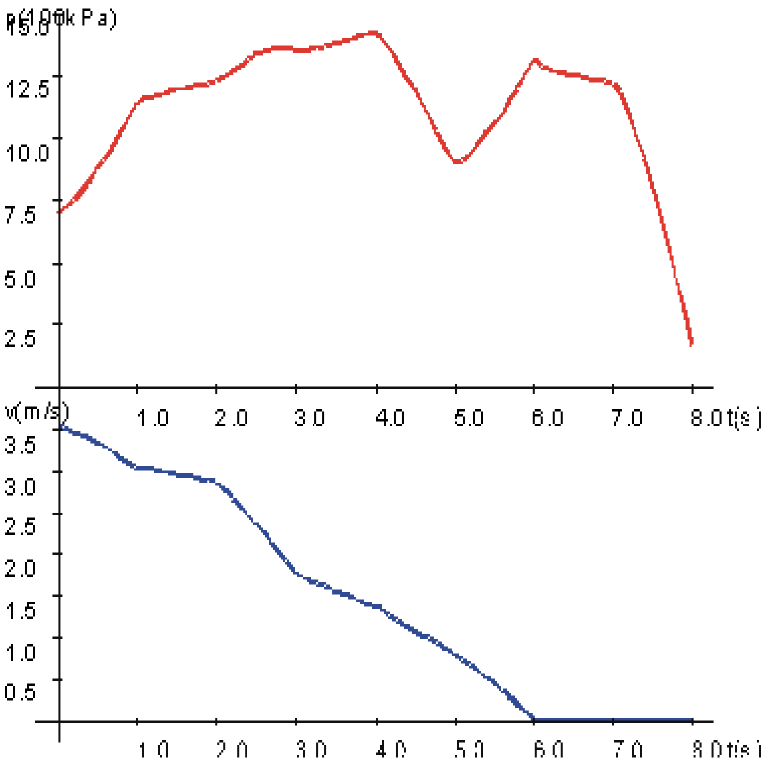

Figure 3. *Real-coded GA result: pressure and velocity at the valve*

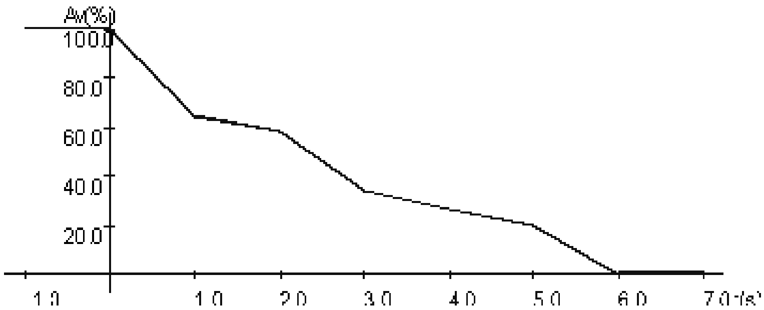

Figure 4. *Real-coded GA result: optimal value-closure strategy*

4.2. Pump-stoppage optimization

In the pipeline with the structure: reservoir - pipe - pump - pipe - junction - pipe - reservoir, the pump has to be stopped in the time interval $[0s, 4s]$. The binary-coded GA gave us the pump-stoppage strategy in *Figure 6* that reduces pressure oscillations in respect to the initial value down to the value of $\varphi_{min} = 310760$ Pa, while with the real-coded GA we obtained an improved pump-stoppage strategy in *Figure 8* that gave us $\varphi_{min} = 308562$ Pa. Pressure and velocities at the valve in both cases can be seen in *Figure 5* and *Figure 7*, respectively.

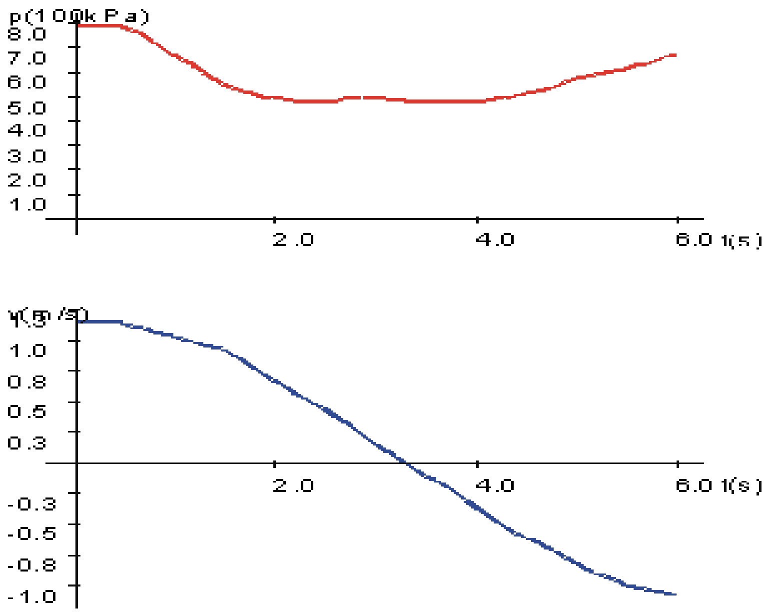

Figure 5. *Binary-coded GA result: pressure and velocity at the pipe junction*

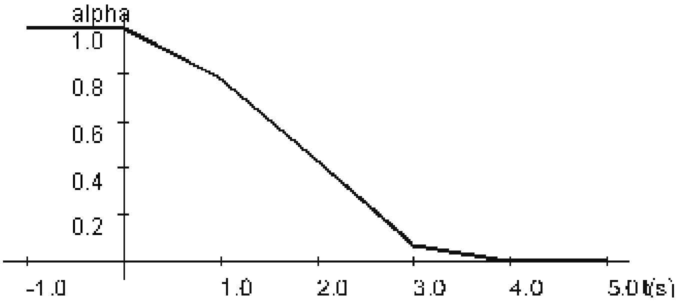

Figure 6. *Binary-coded GA result: optimal pump-stoppage strategy*

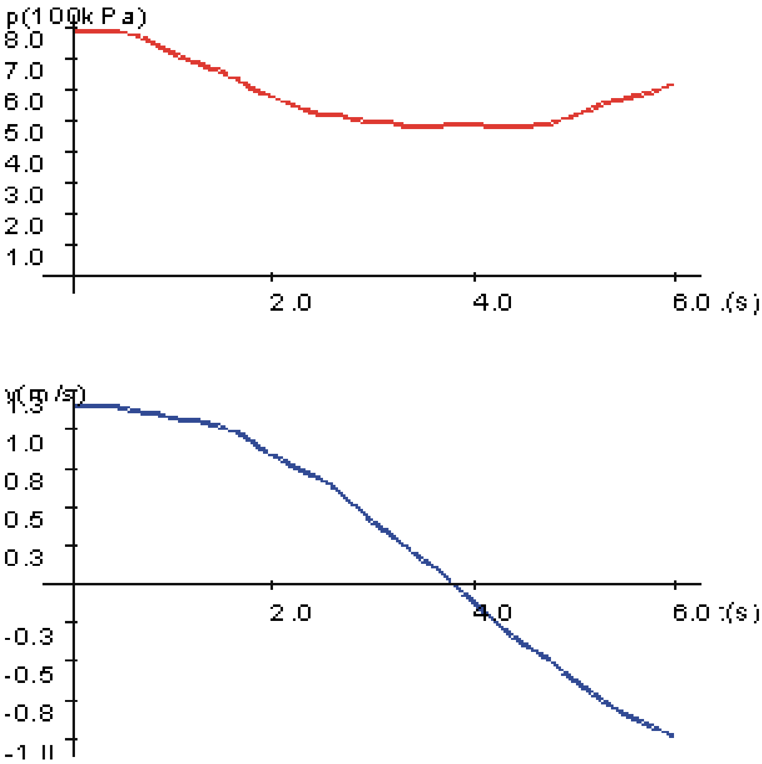

Figure 7. *Real-coded GA result: pressure and velocity at the pipe junction*

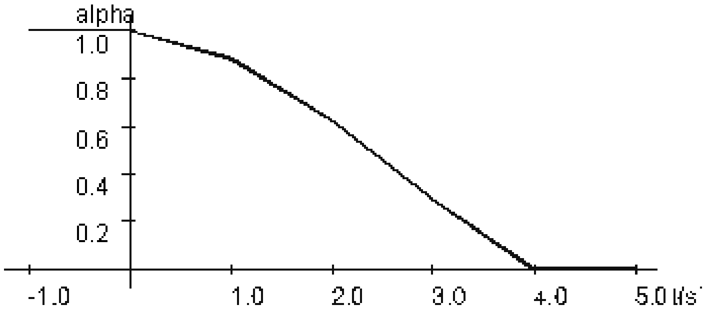

Figure 8. *Real-coded GA result: optimal pump-stoppage strategy*

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