Two-stage optimisation of the multiphase production*

JANEZ ARTENJAK† AND POLONA TOMINC‡

Abstract. *Two approaches to the optimisation of the multiphase production process are presented in this paper. The first model - model I is a static one, using enterprise input/output modelling and linear programming simultaneously. The second approach - model II is a dynamic one and represents the combination between discrete dynamic deterministic programming in the first stage and the model I in the second stage. An application from building industry is also presented in the paper.*

Key words: *multiphase production models, enterprise modelling, production optimisation*

AMS subject classifications: 90A11, 90B30, 90C05, 90C11,90C39

1. Introduction

The paper deals with the determination of the optimal production quantities in the processing industries for a fixed number of periods assuming that the capacities are fixed and the demand for each period is known, but varies from one to another. Two linear optimisation models are presented. Optimisation Model I is a static approach to production problem using enterprise input/output modelling (EIOM) and linear programming (LP) simultaneously. Optimisation Model II is a two-stage model representing a partly dynamic approach by using dynamic programming in the first stage of optimisation and model I in the second stage.

The presented models have already been applied in practice in Slovenia and in this paper an application from the building industry is presented. This approach represents the continuation and extension of the work, which has started some years ago.

*This paper was presented at the 7^{th} *International Conference on OR, Rovinj, 1998.*

†Faculty of Economics and Business, Razlagova 14, 2000 Maribor, Slovenia, e-mail: janez.artenjak@uni-mb.si

‡Faculty of Economics and Business, Razlagova 14, 2000 Maribor, Slovenia, e-mail: polona.tominc@uni-mb.si

2. Optimisation model I - static approach to the production problem. Enterprise input/output models (EIOM)

A general EIOM starts with a structural analysis of the multiphase production process, resulting in defining n production centres, each producing a single product which can be used as intermediate output in reproduction process and/or sold on the market (or markets) [1]. These relations are expressed in the matrix form as

$$x = A\,x + q \tag{1}$$

where vector x stands for production quantities of production centres, vector q denotes selling quantities of products. Matrix A represents the matrix of input coefficients of production.

For the production quantities of production centres, denoted by vector x, it is necessary to use appropriate input quantities of raw materials and labour hours, that is

$$x^m = A^m x \tag{2}$$

where vector x^m stands for input quantities and matrix A^m for input coefficients of raw materials and labour hours.

In each production centre of the enterprise it is therefore possible, using model (1), to decompose the quantity produced, to the part, which is consumed in production process and to the part, which is to be sold. If in (1), instead of vector x, we put vector c for capacities of production centres, the relation

$$q_{max} = c - Ac \tag{3}$$

denotes the maximum possible selling quantities of products.

For analysing the impact of different selling quantities on produced quantities, it is possible to use model

$$x = (E - A)^{-1}\,q \tag{4}$$

which represents the solution of (1) for x.

It is possible to extend this EIOM for analysing also the effects of changing production levels, of changing input prices or technological and institutional conditions, on the production prices.

Optimisation model (linear programming)

Considering relations (1), (2) and (4), the optimal selling quantities of products in period t can be obtained by solving the following optimisation model

$$\max(z_t^T q_t) \tag{5}$$

subject to

$$x_t = (E - A)^{-1} q_t \le c_t \tag{6}$$

$$x_t^m = A^m (E - A)^{-1} q_t \le b_t \tag{7}$$

$$p^{mT} A^m (E - A)^{-1} q_t \le C_t \tag{8}$$

$$m_{lt} \le q_t \le m_{ut} \tag{9}$$

where

(5)- maximisation of the contribution to cover fixed costs, where the elements of vector z_t are defined as the difference between the selling price and cost price, calculated by using only variable costs, in period t

(6)- capacity limitations, where c_t is the vector of available capacities in period t,

(7)- limited quantities of materials and working hours, where b_t is the vector of available quantities of materials and working hours in period t,

(8)- financial limitations, where C_t is the vector of available amount of financial sources in period t,

(9)- represents market limitations, where m_{lt} is the vector of minimum selling quantities of products and m_{ut} is the vector of maximum possible selling quantities of products.

After solving the model (5)-(9), the optimal production quantities of production centres can be obtained by using equation (4).

3. Optimisation model II - discrete dynamic deterministic approach to production problem

Let us assume to have an enterprise with n production centres, each producing a single product, which can be used as an intermediate output in the reproduction process and/or sold on the market (or markets).

This second approach starts with the selection of one or more products, which are of the greatest importance for business. Their optimal production quantities for the fixed number of periods T are obtained by using dynamic programming and assuming that the production and storage capacities are fixed and that the demand for each period t, $t = 1, 2, \ldots, T$ is known, but may vary.

Let us assume that only one, the most important product, product A, is selected. By using dynamic programming, the following problem, presented in the form of recursion equations for $t = 1, 2, \ldots, T$, is to be solved

$$S_t(y_t) = \min_{x_t}[s(y_{t-1}) + s(x_t) + s_{t-1}(y_{t-1})] \tag{10}$$

and $s_0 = 0$
subject to

$$y_0 = a \tag{11}$$

$$y_t = y_{t-1} + x_t - d_t \text{ for } t = 1, 2, ..., T \tag{12}$$

$$y_T = b \tag{13}$$

$$y_t \in E \text{ for } t = 1, 2, ..., T \tag{14}$$

$$x_t \in F \text{ for } t = 1, 2, ..., T \tag{15}$$

where
y_{t-1} - stock at the beginning of period t,
y_t - stock at the end of period t,

x_t - production quantity in period t,
d_t - demand in period t,
$s(x_t)$ - production and set-up costs for period t,
$s(y_{t-1})$ - storage costs at period t for the stock at the beginning of period t,
$s_t(y_t)$ - minimum production, set-up and storage costs till the end of period t, meeting cumulative demands and stock at the end of period t,
E - set of possible states, which are all possible quantities of the stock for product A,
F - set of possible states, which are all possible production quantities of the product A.

By solving the dynamic model (10)-(15), the optimal production quantities of the selected product A for each period t are obtained.

At the second stage of the model II the optimal selling and production quantities of other products, produced in other production centres, are to be obtained, by using the model (5)-(9), which must be modified and is expressed in the form

$$\max(\underline{z}_t^T \underline{q}_t) \tag{16}$$

subject to

$$x_t = (E - A)^{-1}\underline{q}_t \leq \underline{c}_t \tag{17}$$

$$x_t^m = A^m(E - A)^{-1}\underline{q}_t \leq \underline{b}_t \tag{18}$$

$$p^{mT} A^m(E - A)^{-1}\underline{q}_t \leq \underline{A}_t \tag{19}$$

$$\underline{m}_{lt} \leq \underline{q}_t \leq \underline{m}_{ut} \tag{20}$$

where vectors $\underline{z}_t$, $\underline{q}_t$, $\underline{m}_{lt}$ and $\underline{m}_{ut}$ are reduced for one component. Vectors $\underline{c}_t$, $\underline{b}_t$ and constant $\underline{A}_t$ are obtained by using relations

$$\underline{c}_t = c_t - (E - A)^{-1} q_{ti}^{opt} \tag{21}$$

$$\underline{b}_t = A_t - A^m(E - A)^{-1} q_{ti}^{opt} \tag{22}$$

$$\underline{A}_t = A_t - p^{mT} A^m(E - A)^{-1} q_{ti}^{opt} \tag{23}$$

where the value of the component i of the vector q_{ti}^{opt} in period t represents the optimal selling quantity of the discussed product (if the optimal selling quantity equals optimal producing quantity), but all other components are equal to zero.

4. An application from the building industry

The described procedure can be used in practice in many different ways. We get a simplified case if only one product is selected as the most important for the enterprise. But the same procedure may be used if two or more products are recognised as very important. We also get a special case, if the most important product (or products) is produced only from the raw materials, without any element, which is to be produced in the previous phase of the multiphase business process. This is also the case in the application presented in this section.

The multiphase business process of the enterprise TAME, which is the part of the joint-stock company *Konstruktor d.d. Maribor*, is presented in this paper. The ferro-concrete prefabricated elements are produced in this enterprise.

The multiphase business process is decomposed to the production, selling and purchasing activities. All inputs and outputs of the multiphase business process as well as the intermediate outputs, which are used in the further production process, are considered as elements. They are described in *Table 1* as well as the corresponding purchasing or selling activities, purchasing or selling prices (in 1000 sit) and also market limitation (maximal possible purchasing or selling quantities). Production activities, which can also be considered as the production centres, and costs (in 1000 sit), that are not connected with the use of the elements in *Table 1*(for example: water, electricity, ...) are given in *Table 2*.

Elem.	Unit	Description	Purch. (y)/ Sell. (z) act.	Purchasing/ Selling price	Maxim. quant.
E1	m3	Concrete μB30	y1	8,780	It is possible
E2	m3	Concrete μB35	y2	9,300	to purchase max.
E3	m3	Concrete μB40	y3	10,88	18000 m3 of all four
E4	m3	Concrete μB45	y4	14,21	concr. in a month
E5	kg	Reinforcement 6-12	y5	0,01	
E6	kg	Reinforcement 14	y6	0,011	
E7	kg	Steel	y7	0,012	
E8	kg	Planking	y8	0,420	
E10	hour	Planning	y10	1,600	360
E11	hour	Crane	y11	3	360
E12	hour	Fork-lift	y12	1,8	360
E13	hour	Concrete-pipes machine	y13	2,7	360
E14	hour	Hollow-plate machine	y14	3,2	180
E15	hour	Hollow-plate cut. machine	y15	3,5	180
E16	1200 m2	Hollow-plate 200			
E17	1200 m2	Hollow-plate 265			
E18	1200 m2	Hollow-plate400			
E19	piece	Fe-conc. Pillar 9m	z19	69	100
E20	piece	Fe-conc. Pillar 2m	z20	25	100
E21	piece	Fe-conc. Pillar 7m	z21	53	100
E22	piece	Fe-co. Roof Support 18m	z22	171,45	10
E23	piece	Fe-co. Roof Support 16m	z23	192,9	30
E24	piece	Fe-co. Roof T-Support 12m			
E25	piece	Fe-co.Roof T-Support 8m			
E27	piece	Concrete Pipes Φ20	z27	0,813	360
E28	piece	Concrete Pipes Φ50	z28	2,437	350
E29	piece	Concrete Pipes Φ80	z29	5,425	400
E30	piece	Concrete Pipes Φ100	z30	7,712	420
E31	piece	Concrete Pipes Φ120	z31	9,890	550
E32	piece	Conc. Paving plates	z32	0,520	190000
E33	piece	Conc. Borders 15cm	z33	0,945	500
E34	piece	Conc. Borders 5cm	z34	0,454	500
E35	piece	Conc. Cover Φ50	z35	0,990	5
E36	piece	Conc. Cover Φ80	z36	2,661	5
E37	piece	Conc. Cover Φ100	z37	6,190	5
E38	piece	Conc. Cover Φ120	z38	7,691	7
E39	m2	Tin-plates	y39	0,350	
E40	m	Roof manger	y40	0,150	
E41	m2	Cutted Hollow-plate 200	y41	5,040	
E42	m2	Cutted Hollow-plate 265	y42	6,240	
E43	m2	Cutted Hollow-plate 400	y43	8,400	
E44	piece	Prefabricated Hall 6m /12/7m	z44	3763,2	1
E45	piece	Prefabricated Hall 16m /12/9m	z45	4800,0	1
E46	piece	Prefabricated Hall 16m /8/7m	z46	5145,6	1
E47	piece	Prefabricated Hall 16m /8/9m	z47	5683,2	1
E48	piece	Prefabricated Hall 18m /12/9m	z48	7300,8	1
E49	piece	Prefabricated Hall 18m /8/9m	z49	8337,6	1

Table 1. *Elements of the multiphase business process*

Production of the ferro-concrete prefabricated elements is therefore the multiphase business process with described production activities, which are considered as the production centres, each producing a single product, which can be used as an intermediate output in the production process and /or sold on the market. The element E23 -Ferro-concrete roof support, of 16 meters length, is an intermediate output, which causes high storage costs because of its special shape and other characteristics. But this ferro-concrete prefabricated element is at the same time at a very important place in the production program of the enterprise, since the demand for it on the market is almost permanent and very stable.

Prod. Activity	Description	Costs
X1	Production of the hollow-plates 200	218
X2	Production of the hollow-plates 265	220
X3	Production of the hollow-plates 400	340
X4	Production of the Fe-concrete pillar 9m	6,9
X5	Production of the Fe-concrete pillar 2m	2,5
X6	Production of the Fe-concrete pillar 7m	5,36
X7	Production of the Fe-concrete Roof support 18m	19,290
X8	Production of the Fe-concrete Roof support 16m	17,145
X9	Production of the Fe-concrete T-Roof support 12m	1,295
X10	Production of the Fe-concrete T-Roof support 8m	1,010
X12	Production of the paving plates	0,023
X13	Production of the concrete borders 5 cm	0,034
X14	Production of the concrete borders 15 cm	0,045
X15	Production of the concrete pipes Φ20	0,155
X16	Production of the concrete pipes Φ50	0,233
X17	Production of the concrete pipes Φ80	0,295
X18	Production of the concrete pipes Φ100	0,420
X19	Production of the concrete pipes Φ120	0,545
X20	Production of covers Φ50	0,110
X21	Production of covers Φ80	0,140
X22	Production of covers Φ100	0,180
X23	Production of covers Φ120	0,205
X24	Cutting of the hollow-plates 200	19,100
X25	Cutting of the hollow-plates 265	24,300
X26	Cutting of the hollow-plates 400	37,300
X27	Production of the prefabricated-hall 16m/8/7m	
X28	Production of the prefabricated-hall 16m/8/9m	
X29	Production of the prefabricated-hall 16m/12/7m	
X30	Production of the prefabricated-hall 16m/12/9m	
X31	Production of the prefabricated-hall 18m/8/9m	
X32	Production of the prefabricated-hall 18m/12/9m	

Table 2. *Production activities*

In the first stage of optimisation by Model II, the optimal production quantities of the element E23 for the fixed number of months T, $T = 3$, are obtained by using the dynamic programming and assuming that the production and storage capacities are fixed and that the demand for each month t, $t = 1, 2, 3$, is known and should be satisfied.

Let us assume that 10, 20 or 30 pieces of element E23 can be produced, which causes 400, 550 and 800 monetary units of the production costs. If the production quantity equals 0, the fixed costs of 200 monetary units arise. Each piece of the element E23 being in the storehouse one month causes storage costs of 6 monetary units. Fixed storage costs are 50 monetary units per month. Maximal possible stock is 20 pieces. The market demand for the element E23 at the end of the first month equals 30, at the end of the second month 20 and at the end of the third month 35 pieces. Since the maximal produced quantity equals 30, the demand in the third month could not be satisfied without previously produced quantity of the element E23, therefore the need for dynamic programming to find the optimal producing quantity of the element for each period with minimal producing and storage costs. The stock at the beginning of the first month equals 20 pieces and the stock at the end of the third month should be minimal.

By using the methodology of discrete dynamic deterministic programming, briefly described in *Section 3*, the following results are obtained for production quantities of the element E23 for each month: 1. month 20 pieces, 2. month 20 pieces and 3. month 30 pieces.

The stock at the end of each month will be 10, 10 and 5 pieces of the element E23. The first stage of the Model II is therefore finished.

In the second stage, the optimal selling and production quantities of other products and optimal purchasing quantities of inputs are to be obtained for each month. First for month 1. Since the production centre/activity X8 where the element E23 is produced does not need any product from the other production centres/activities, this problem can be easily solved by the linear mixed integer optimisation model

$$
\begin{aligned}
\max\quad & (69z_{19} + 25z_{20} + 53z_{21} + 171.45z_{22} + 192.9z_{23} + 0.813z_{27} + 2.437z_{28} \\
& +5.425z_{29} + 7.712z_{30} + 9.890z_{31} + 0.520z_{32} + 0.945z_{33} + 0.454z_{34} + 0.99z_{35} \\
& +2.661z_{36} + 6.190z_{37} + 7.691z_{38} + 3763.2z_{44} + 4800z_{45} + 5145.6z_{46} \\
& +5683.2z_{47} + 7300.8z_{48} + 8337.6z_{49} - 0.42y_8 - 1.6y_{10} - 3y_{11} - 1.8y_{12} \\
& -2.7y_{13} - 3.2y_{14} - 3.5y_{15} - 5.04y_{41} - 6.24y_{42} - 8.4y_{43} - 8.78y_1 - 9.3y_2 \\
& -10.88y_3 - 14.21y_4 - 0.01y_5 - 0.011y_6 - 0.012y_7 - 0.35y_{39} - 0.15y_{40} \\
& -218x_1 - 220x_2 - 340x_3 - 6.9x_4 - 2.5x_5 - 5.36x_6 - 19.29x_7 - 17.145x_8 \\
& -1.295x_9 - 1.01x_{10} - 0.023x_{12} - 0.034x_{13} - 0.045x_{14} - 0.155x_{15} \\
& -0.233x_{16} - 0.295x_{17} - 0.420x_{18} - 0.545x_{19} - 0.110x_{20} - 0.140x_{21} \\
& -0.180x_{22} - 0.205x_{23} - 19.1x_{24} - 24.3x_{25} - 37.3x_{26})
\end{aligned}
$$

subject to

$$
\begin{array}{llll}
z_{19} \le 100 & z_{20} \le 100 & z_{21} \le 100 & z_{22} \le 10 \\
z_{27} \le 360 & z_{28} \le 350 & z_{29} \le 400 & z_{30} \le 420 \\
z_{31} \le 550 z_{32} \le 190000 & z_{33} \le 500 & z_{34} \le 500 z_{35} \le 5 & z_{36} \le 5 \\
z_{37} \le 5 & z_{38} \le 7 & y_{10} \le 1600 & y_{11} \le 360 \\
y_{12} \le 360 & y_{13} \le 360 & y_{14} \le 180 & y_{15} \le 180
\end{array}
$$

$$y_1 + y_2 + y_3 + y_4 \le 18000 \quad y_1 - 132x_1 - 192x_2 - 336x_3 \ge 0$$

$$
\begin{aligned}
& y_2 - 0.01x_{12} - 0.0125x_{13} - 0.0375x_{14} - 0.1x_{15} - 0.15x_{16} - 0.2x_{17} - 0.31x_{18} \\
& \quad -0.42x_{19} \ge 0 \\
& y_3 - 2x_4 - 0.45x_5 - 1.56x_6 - 0.03x_{20} - 0.08x_{21} - 0.13x_{22} - 0.21x_{23} \ge 0 \\
& y_4 - 4.6x_7 - 4.10x_8 - 0.6x_9 - 0.48x_{10} \ge 0 \\
& y_5 - 56000x_1 - 58000x_2 - 60000x_3 - 104x_4 - 24x_5 - 82x_6 - 460x_7 - 409x_8 \\
& \quad -114x_9 - 91x_{10} - 7x_{20} - 12x_{21} - 25x_{22} - 35x_{23} \ge 0 \\
& y_6 - 34000x_1 - 37000x_2 - 40000x_3 - 290x_4 - 64x_5 - 225x_6 - 160x_7 - 142x_8 \\
& \quad -82x_9 - 65x_10 \ge 0
\end{aligned}
$$

$$y_7 - 210x_7 - 187x_8 \geq 0$$
$$y_8 - 5x_4 - 2x_5 - 3.8x_6 - 31x_7 - 28x_8 - 4.75x_9 - 3.8x_{10} - 0.3x_{12} - 0.2x_{13} - 0.3x_{14} - 0.1x_{20} - 0.4x_{21} - 0.95x_{22} - 1.35x_{23} \geq 0$$
$$y_{10} - 46.5x_{27} - 46.5x_{28} - 46.5x_{29} - 46.5x_{30} - 46.5x_{31} - 46.5x_{32} \geq 0$$
$$y_{11} - 0.2x_4 - 0.15x_5 - 0.2x_6 - x_7 - x_8 - 0.15x_9 - 0.15x_{10} \geq 0$$
$$y_{12} - 0.01x_{12} - 0.08x_{13} - 0.09x_{14} - 0.11x_{15} - 0.12x_{16} - 0.13x_{17} - 0.16x_{18} - 0.2x_{19} - 0.01x_{20} - 0.05x_{21} - 0.25x_{22} - 0.35x_{23} \geq 0$$
$$y_{13} - 0.12x_{15} - 0.21x_{16} - 0.25x_{17} - 0.3x_{18} - 0.35x_{19} \geq 0$$
$$y_{14} - 36x_1 - 36x_2 - 36x_3 \geq 0$$
$$y_{15} - 20x_{24} - 30x_{24} - 40x_{26} \geq 0$$
$$x_1 - x_{24} \geq 0$$
$$x_2 - x_{25} \geq 0$$
$$x_3 - x_{26} \geq 0$$
$$x_4 - z_{19} - 48x_{28} - 32x_{30} - 32x_{32} - 32x_{32} \geq 0$$
$$x_5 - z_{20} \geq 0$$
$$x_6 - z_{21} - 48x_{27} - 32x_{29} \geq 0$$
$$x_7 - z_{22} - 4x_{31} - 3x_{32} \geq 0$$
$$x_8 - z_{23} - 4x_{27} - 4x_{28} - 3x_{29} - 3x_{30} \geq 0$$
$$x_9 - 16x_{29} - 16x_{30} - 16x_{32} \geq 0$$
$$x_{10} - 24x_{27} - 24x_{28} - 24x_{31} \geq 0$$

$$x_{15} - z_{27} \geq 0 \quad x_{16} - z_{28} \geq 0 \quad x_{17} - z_{29} \geq 0$$
$$x_{18} - z_{30} \geq 0 \quad x_{19} - z_{31} \geq 0 \quad x_{12} - z_{32} \geq 0$$
$$x_{14} - z_{33} \geq 0 \quad x_{15} - z_{34} \geq 0 \quad x_{20} - z_{35} \geq 0$$
$$x_{21} - z_{36} \geq 0 \quad x_{22} - z_{37} \geq 0 \quad x_{23} - z_{38} \geq 0$$

$$y_{39} - 432x_{27} - 432x_{28} - 432x_{29} - 432x_{30} - 480x_{31} - 480x_{32} \geq 0$$
$$y_{40} - 48x_{27} - 48x_{28} - 48x_{29} - 48x_{30} - 48x_{31} - 48x_{32} \geq 0$$
$$1200x_{24} + y_{41} - 336x_{27} - 432x_{28} - 336x_{29} - 432x_{30} - 432x_{31} - 432x_{32} \geq 0$$
$$1200x_{25} + y_{42} - 224x_{27} - 288x_{28} - 224x_{29} - 288x_{30} - 324x_{31} - 324x_{32} \geq 0$$
$$1200x_{26} + y_{43} - 384x_{27} - 384x_{28} - 384x_{29} - 384x_{30} - 432x_{31} - 432x_{32} \geq 0$$

$$x_{27} - z_{46} \geq 0 \quad x_{28} - z_{47} \geq 0 \quad x_{29} - z_{44} \geq 0$$
$$x_{30} - z_{45} \geq 0 \quad x_{31} - z_{49} \geq 0 \quad x_{32} - z_{48} \geq 0$$
$$x_8 \geq 20 \quad z_{23} = 20 \quad x_8 \leq 30$$

$$\text{int } x_{27} \quad \text{int } x_{28} \quad \text{int } x_{29} \quad \text{int } x_{30} \quad \text{int } x_{31} \quad \text{int } x_{32}\,.$$

It is taken into account that the market demand for 30 pieces of element E23 in the first month will be satisfied by 20 produced pieces and 10 pieces will be taken from the warehouse. The result is

OBJECTIVE FUNCTION VALUE 24139.4900

$X_1 = 1.0$	$Y_6 = 200306.70$	$Z_{19} = 100.0$
$X_2 = 0.73$	$Y_7 = 8058.00$	$Z_{20} = 100.0$
$X_3 = 1.04$	$Y_8 = 13838.80$	$Z_{21} = 100.0$
$X_4 = 164.0$	$Y_{10} = 139.5$	$Z_{22} = 10.0$

$X_5 = 100.0$	$Y_{11} = 128.0$	$Z_{23} = 20.0$
$X_6 = 148.0$	$Y_{12} = 360.0$	$Z_{32} = 35995.0$
$X_7 = 17.0$	$Y_1 = 620.96$	$Z_{35} = 5.0$
$X_8 = 24.0$	$Y_2 = 359.95$	$Z_{46} = 1.0$
$X_9 = 16.0$	$Y_5 = 216001.70$	$Z_{48} = 1.0$
$X_{10} = 48.0$	$Y_{14} = 99.6$	$Z_{49} = 1.0$
$X_{12} = 35995.0$	$Y_{15} = 91.6$	
$X_{20} = 5.0$	$Y_{40} = 144.0$	
$X_{24} = 1.0$	$Y_3 = 604.03$	
$X_{25} = 0.73$	$Y_4 = 209.24$	
$X_{26} = 1.04$	$Y_{39} = 1392.0$	
$X_{27} = 1.0$		
$X_{31} = 1.0$		
$X_{32} = 1.0$		

All other variables are 0.

This procedure must be repeated for the second and third month, where also all possible changes of market limitations (demand, prices, ...) can be taken into account.

5. Conclusion

The two-stage optimisation model discussed in this paper enables even higher degree of rationalisation in the business process, resulting in lower storage, production and purchasing costs.

This model has already been used also in the aluminium industry in Slovenia. It is suitable for every multiphase production, regardless of the number of production centres, especially in situations where storage costs represent an important part of business costs.

References

[1] J. Artenjak, *Two-stage Optimisation of the Multiphase Production*, AIRO '96, Volume of papers, Perugia, Italy, 1996, p.249.

[2] I. Mesko, *Mixed integer models of the business process and disjunctive goals*, Operations Research Proceedings 1988, Springer-Verlag, Berlin, 1989, 60–64.