AN OVERVIEW OF THE APPLICATIONS OF WAVELET TRANSFORM FOR DISCHARGE AND SUSPENDED SEDIMENT ANALYSIS

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Analysis and modelling of discharge and suspended sediment time series is of great importance in hydrology and water management. Discharge and suspended sediment time series are the result of complex physical processes and characterized by non-stationarity. Wavelet transform enables representation of non-stationarities in time-frequency domain, deconstruction and reconstruction of series and de-noising of series, and therefore represents powerful tool for analysis of hydrological time series. Overview of mentioned wavelet transform advantages is given in this paper with focus on discharge and suspended sediment time series, presented through: (i) multi-temporal scale analysis of series variability; (ii) multi-scale trend analysis; (iii) prediction and forecasting of series with wavelet based hybrid black-box models; and (iv) wavelet-aided simulation of synthetic series.

Keywords: discharge; suspended sediment; wavelet transform

Pregled primjene valične transformacije u analizi protoka i suspendiranog nanosa

Anaža i modeliranje vremenskih serija protoka i suspendiranog nanosa je vrlo važan zadatak u hidrologiji i gospodarenju vodama. Vremenske serije protoka i suspendiranog nanosa se ovdje uvažaju u svrhe svakodnevnog provođenja gospodarenja vodama. Za ove dvije vrste serija više faktora utiče na njihov izgled i varijabilnost. U ovom radu lako se vidi da je analiza i modeliranje vremenskih serija protoka i suspendiranog nanosa vrlo važan zadatak u hidrologiji i gospodarenju vodama.

Ključne riječi: protok; suspendirani nanos; valična transformacija

1 Introduction

Information about discharge and suspended sediment time series is of great importance in hydrology and water management because it reflects variations in hydrological processes. Analysis, forecasting and simulation of discharge and sediment time series are complex procedures because of their nonlinear and time-varying autocorrelation properties. Traditional methods for time series analysis, such as analysis of serial correlation in time domain and Fourier transform in frequency domain, are dealing with stationary time series. In contrast to the above methods, wavelet transform (WT) enables multi-temporal scale analysis and representation of non-stationarity and therefore has become powerful tool in hydrological time series analysis. In some aspects, wavelet transform is similar to Fourier transform and represents its upgrade with ability to present non-stationary time series in time-frequency space with a good resolution. Advantage of WT over short time Fourier transform is the use of shifting window of variable width and that enables enhanced resolution along all frequency (i.e. scale) bands [1, 2].

Wavelet transform originated from the field of applied mathematics and has been adjusted for application in geophysics by Grossman and Morlet [3, 1]. Introduction to the wavelet transform can be found in Mallat [4], statistical significance tests for wavelet power spectra are introduced by Torrence & Compo [5] and application of method for hydrology and water resources is elaborated by Labat [6, 7]. Although some early studies with applications of wavelet analysis (WA) on discharge, and suspended sediment time series are from late 1990s and early 2000s [8], WA with all its advantages in filtering series, revealing information on different scales, dealing with noise and non-stationarity has started to be fully exploited in the last decade. Sang [9] offered general overview of WT applications in hydrology and Nourani et al. [10] gave overview with emphasis on WT combined with artificial intelligence (AI) models for forecasting of different hydrological variables.

Overview of main groups of WT applications focusing on discharge and suspended sediment time series is offered here, as extension to the referred review papers. Other published papers with that focus, of which authors have personal knowledge, have not been found. The subject is presented in two parts. First part gives a theoretical overview of the wavelet transform method with its properties and types. Second part presents four group of applications of wavelet transform for discharge and suspended sediment time series: (i) multi-temporal scale analysis of series variability; (ii) multi-scale trend analysis; (iii) prediction and forecasting of series with wavelet based hybrid black-box models; and (iv) wavelet-aided simulation of synthetic series.

2 Wavelet transform – theoretical background

There are two types of wavelet transform: continuous wavelet transform (CWT) and discrete wavelet transform (DWT). CWT is mostly used for detecting variations in time series and their simultaneous representation in time-frequency space. On the other hand, DWT is mostly used for decomposition of time series in subseries of different scales that enables further analysis, such as: noise reduction, trend analysis and wavelet aided modelling of hydrological time series with hybrid models [9].
2.1 Continuous wavelet transform

For good localisation of time series i.e. signal $x(t)$ in both, time and frequency domain, wavelet function $\Psi(t)$ can be used:

$$\Psi_{a,r} = \frac{1}{\sqrt{|a|}} \Psi \left( \frac{t-r}{a} \right).$$  \hspace{1cm} (1)

Function $\Psi(t)$ can be shifted in time for translation factor $r$, $r \in \mathbb{R}$, and it can be scaled in size with scale $a$, $a \in \mathbb{R}^*$, $a \neq 0$ (where dilation is for $a > 1$, and contraction is for $a < 1$). Shape of function $\Psi(t)$ is similar to small wave, and therefore it is called "mother wavelet" [11].

Function $\Psi(t)$ has to satisfy the following conditions:

(i) admissibility condition that defines function wave shape with function mean in zero:

$$\int_{-\infty}^{\infty} \Psi(t) \, dt = 0,$$  \hspace{1cm} (2)

and (ii) regularity condition that defines function damping in order to achieve its localization in time domain:

$$\int_{-\infty}^{\infty} \tau^4 \Psi(t) \, dt = 0$$  \hspace{1cm} (3)

where $k = 1, 2,\ldots, N-1$. Together, both conditions enable total reconstruction of time series [4].

Wavelet transform, based on function that satisfies aforementioned conditions, is therefore defined through transformation pair:

$$X(\tau, a) = \frac{1}{\sqrt{|a|}} \int x(t) \cdot \Psi^* \left( \frac{t-\tau}{a} \right) \, dt,$$

$$x(t) = \int \int X(\tau, a) \frac{1}{\sqrt{|a|}} \Psi \left( \frac{t-\tau}{a} \right) \, d\tau \, da,$$  \hspace{1cm} (4)

where $*$ is complex conjugate symbol [11].

Continuous time series are sampled at discrete time steps $\Delta t$. CWT can be applied on them and then Eq. (1) becomes:

$$X(\tau, a) = \sum_{q=1}^{N-1} x(q) \cdot \Psi^* \left( \frac{q-n\Delta t}{a} \right) \frac{1}{\sqrt{|a|}},$$  \hspace{1cm} (5)

where $*$ is complex conjugate symbol, $N$ is number of sampled measurements in time series and $q$ is localised time index. Eq. (5) represents convolution of time series $x(t)$ that is conducted $N$ times for every scale. Calculation of Eq. (5) is faster with application of discrete Fourier transform (DFT):

$$X_k(a) = \sum_{q=1}^{N-1} x_q(k) \cdot \Psi^* (a \omega_q) e^{-i \omega_q \tau}$$  \hspace{1cm} (6)

where $\Psi^*(a \omega_q)$ is DFT representation of function $\Psi(t)$, $\omega$ is angular frequency and $k = 1, 2,\ldots, N-1$ is frequency index [11]. In CWT time series sampling is discrete but it is calculated for continuous scale and time [1].

2.2 Discrete wavelet transform

CWT can be also calculated for discrete values of scale $a = a_0^j$, $a_0 > 0$, $a \neq 1$, $j \in \mathbb{Z}$, and $\tau = k \tau_0 a_0^j$, $\tau_0 > 0$, $k \in \mathbb{Z}$, and that gives discrete wavelet function:

$$\Psi_{j,k}(t) = a_0^{-j/2} \cdot \Psi(a_0^{-j} t - k \tau_0)$$  \hspace{1cm} (7)

Discrete wavelet transform is defined through transformation pair:

$$X(j, k) = \frac{1}{a_0^{j/2}} \int x(t) \cdot \Psi_{j,k}^*(t) \, dt,$$

$$x(t) = \sum_k X(j, k) \cdot \tilde{\Psi}_{j,k},$$  \hspace{1cm} (8)

where $\tilde{\Psi}_{j,k}$ is reconstruction function [12].

Wavelet function in dyadic scale ($a_0 = 2^{-1} \rightarrow a = 2^j$) is defined as:

$$\Psi_{j,k}(t) = 2^{j/2} \cdot \Psi(2^j t - k).$$  \hspace{1cm} (9)

The calculation of Eq. (8) together with Eq. (7) is numerically demanding and for that reason wavelets are coupled with the multi-resolution analysis (MTA). MTA introduces scaling function $\Phi(t)$, which has similar properties to wavelet function $\Psi(t)$ and it can be translated and scaled in time [1]:

$$\Phi_{a,r}(t) = \frac{1}{\sqrt{|a|}} \Phi \left( \frac{t-r}{a} \right).$$  \hspace{1cm} (10)

Scale function $\Phi(t)$, instead of Eq. (2), needs to satisfy the following condition:

$$\int_{-\infty}^{\infty} \Phi(t) \, dt = 1$$  \hspace{1cm} (11)

and then scaling function in dyadic scale is:

$$\Phi_{j,k}(t) = 2^{j/2} \cdot \Phi(2^j t - k).$$  \hspace{1cm} (12)

Wavelet transform in combination with MTA can be considered as process of filtering signal (time series) with series of low band and high band filters and signal $x(t)$ can be then presented as:

$$x(t) = \sum_k a_{j_0}(k) \Phi_{j_0,k}(t) + \sum_{j=j_0}^{\infty} \sum_k d_j(k) \Psi_{j,k}(t),$$  \hspace{1cm} (13)

where $j_0$ is arbitrary initial scale value, $a_{j_0}(k)$ value is scale coefficient or approximation coefficient and $d_{j_0}(k)$
is wavelet or detail coefficient. Approximation and detail coefficients are obtained with application of discrete convolution and down sampling. Simplifying Eq. (13) for reconstruction of signal in original units, reconstruction of approximation coefficients \( A \) and detail coefficients \( D \) is introduced:

\[
x(t) = A_j(t) + \sum_{j=1}^{J} D_j(t),
\]

where \( D_j(t) \) is signal detail or high frequency component on level \( j = 1, 2, \ldots, J \), and \( A_j(t) \) is approximation or low frequency component of signal on level \( J \). Maximum level of signal decomposition is defined as [12]:

\[
J \leq \lfloor \log_2 N \rfloor,
\]

where \( \lfloor \cdot \rfloor \) indicates integer and \( N \) is number of elements in signal \( x(t) \), \( t = 0, 1, 2, \ldots, N \). Eq. (14) represents inversed DWT in its condensed form [11].

There are different families of wavelet functions i.e. mother wavelets that meet the condition of Eq. (11) and enable reconstruction of the original series: Haar (haar), Daubechies (db), Coiflets (coif), Symlets (sym), BiorSplines (bior), ReverseBior (rbio) and Dmeyer (dmeiy). Other families that do not meet Eq. (11), such as Morlet, Meyer, Mexican hat and Gaussian wavelet are used for CWT [1]. Daubechies and Morlet wavelets are represented in the largest number of studies dealing with analysis of discharge and suspended sediment series.

3 Application of wavelet transform for discharge and suspended sediment time series

Based on the literature overview, application of the WT and its advantages to the discharge and suspended sediment series is here divided in four groups: (i) multi-temporal scale analysis of series variability; (ii) multi-scale trend analysis; (iii) prediction and forecasting of series with wavelet based hybrid black-box models and (iv) wavelet-aided simulation of synthetic series. The majority of the published papers dealing with the topics of trend analysis and simulation of synthetic series are done with discharge series. On the other hand, topics dealing with multi-temporal scale analysis of series variability along with prediction and forecasting of series with wavelet based hybrid black-box models are in greater number concerned with both discharge and suspended sediment (SS) series. Discharge series are represented in larger number of studies than SS series because of greater availability of its measurements. SS series are presented in the form of SS concentration, SS transport and suspended sediment load (SSL). SS series are available in a smaller number of long-term measurements at equidistant time intervals than discharge. Long-term measurements at equidistant time intervals are prerequisite for full application of all the advantages of WA. Examples of some papers with four different applications of WT for analysis, prediction and forecasting of discharge and SS series are provided in Tab. 1. Furthermore, short overview of each application with some additional studies is provided in the following sections.

3.1 Multi-temporal scale analysis of discharge and suspended sediment series variability

Wavelet analysis in this group of research is used for detecting multi-temporal scale patterns in discharge and SS series, i.e. variations of high and low energy in their wavelet power spectra. For that purpose, wavelet power spectrum (WPS) obtained with CWT based on Morlet mother wavelet is mostly used in univariate analysis. Also, global wavelet spectrum (GWS) is used and it represents averaged wavelet spectrum over time. In cross-wavelet analysis, wavelet cross-spectrum and wavelet coherence spectrum enable correlation and phase analysis between discharge and/or SS series and/or some other time series of interest, such as meteorological, climate or some other indices. Studies are made on different spatial scales, from smaller basins to the continental scale [7, 13, 14, 15]. Historically multi-temporal scale analysis was first and most widespread application of WT for analysis of discharge and SS series. Overview with examples of some research papers dealing with this group of problems is given below and in Tab. 1.

Univariate wavelet and cross-wavelet analysis is used in study made by Labat [15] in order to examine time-scale fluctuations of annual discharge from the world’s 55 largest rivers on the five continents with basins greater than \( 10^4 \) km\(^2 \) and with continuous measurements longer than 30 years. Additionally, cross-wavelet analysis of annual continental fresh water discharge to oceans is made with different climate indices: North Atlantic Oscillation, Arctic Oscillation, Southern Oscillation Index, Pacific Decadal Oscillation and NINO3.4 – Pacific mean Sea Surface Temperature (SST). Three main bands of temporal correlation between variability of discharge and indices are identified and analysed: 2–10 year, 10–20 year and 20–30 year band [16].

Rossi et al. [17] investigated intra- and inter- annual variability of daily discharge and SS concentration on the Mississippi River with the help of CWT. Non-stationary changes in wavelet spectra were detected in both time series. Detected intra- and inter- annual fluctuations of time series were compared with climatic and anthropogenic influences. Influence of the anthropogenic impact on SSL were examined with sinusoidal parametric modeling and compared with observed series. It is found that anthropogenic influences are mainly observed in SS series and could be connected with construction of large reservoirs. Further, it is found that climate influences could be connected with changes in precipitation and different climate patterns indices such as El-Niño/Southern Oscillation (ENSO).

Liu et al. [18] analyzed period variations of The Yellow River monthly discharge and SSL into the sea with the WA. Also, influencing factors for periodic and trend changes were examined with comparison with ENSO, precipitation and human activities. Cross-wavelet and coherence were used for comparison with ENSO series. Impact of discharge and SSL changes on the estuary were investigated by comparison with the measured coastline and bathymetry data.
An overview of the applications of wavelet transform for discharge and suspended sediment analysis

K. Potočki et al.

Potočki et al. [19] analyzed variations in WPS and GWS of mean and maximum monthly discharge and SS series on the Sava River in Croatia and statistically significant periods against the red noise in the WPS were determined. More significant differences between high and low energy in time for inter-annual and intra-annual scales up to 56 months (4.6 years) were noticed in GWS and WPS of maximum monthly series. SSL had GWS peaks in higher scales in comparison to the GWS peaks in discharge series. Peaks in larger scales (i.e. periods) are even more obvious in the suspended concentration series and this could be caused by the climate indices that influence SS production on observed scales. Also, spatial changes in discharge and SS spectrum have been analyzed at three gauging stations (GS) along the Sava River in upper, middle and lower course [20]. Example of the WPS and GWS presentation is given for the maximum monthly series in Fig. 1.

Figure 1 On the left from top down, continuous wavelet power spectrum (WPS) is shown for maximum monthly time series: (a) discharge (m$^3$/s), (b) suspended sediment load (tons) and (c) suspended sediment concentration (g/m$^3$) at GS Podsused on the Sava River. Significant wavelet spectra are shown with thick black contour within the cone of influence (COI) presented as U-shape line (dashed thick line). On the right at each figure is global wavelet spectrum (GWS) with 95% confidence level against red noise (dashed line) [19].

Although this application of the WT is present for many years, representation of signal in time-frequency domain still continues to be of great benefit for revealing discharge and SS series alternation through dry and wet seasons, possible temporal correlations with climate indices and anthropogenic interventions in basins. It is recommended to use this method at least in its simplest univariate form as preliminary step for analysis and revealing of the series main characteristics. Sang [9] notes that there is a need for further exploration of CWT results stability because small shift in the input series causes a big difference in the output. Majority of the published papers with discharge and SS series alteration through dry and wet seasons, possible temporal correlations with climate indices and anthropogenic interventions in basins. The main idea in multi-scale trend analysis is to use the abilities of WA to reveal non-stationarities and different cycles in hydro-meteorological series and analyse it in combination with trend tests. Choice of appropriate mother wavelet and decomposition level for each regional discharge or SS series should be examined before continuing to the trend analysis. Example of DWT decomposition of monthly discharge series from the river Sava on the six levels with db7 wavelet prepared for the trend analysis is given in Fig. 2.

Use of nonparametric Mann-Kendall test together with DWT is first made by Partal & Kucuk [21] on annual total precipitation series in Turkey. Different wavelet components, approximations and details, were used for trend analysis of precipitation series. Developed method was called “wavelet aided trend (W-T) analysis” and it was used also for studies with discharge and SS series.

3.2 Multi-scale trend analysis of discharge and suspended sediment time series

Generally, trend can be regarded as a cycle with large temporal scale, i.e. small frequency. Trend detection and trend analysis are providing important information about discharge and SS series on large temporal scales. Some of most widespread tests for analysis of trend in hydrological series are parametric least square linear trend test and rank based non parametric Mann-Kendal trend test. However, these tests can't manage non-stationaries and nonlinear features of hydrological series. The main idea in multi-scale trend analysis is to use the abilities of WA to reveal non-stationarities and different cycles in hydro-meteorological series and analyse it in combination with trend tests. Choice of appropriate mother wavelet and decomposition level for each regional discharge or SS series should be examined before continuing to the trend analysis. Example of DWT decomposition of monthly discharge series from the river Sava on the six levels with db7 wavelet prepared for the trend analysis is given in Fig. 2.
Generally, two approaches in application of multi-scale trend analysis are present. First approach is based on the decomposition of original series with series of DWT filters. Testing for trend on different time scales is done next, focusing on the remaining approximation (low frequency i.e. long time scale). In the second approach, different combination of the details added together with approximation are being tested for trend in order to establish contribution of the different time scales to the trend. Wavelet aided trend analysis can be made with both parametric and nonparametric trend tests.

Figure 2 Example of DWT decomposition of original monthly discharge time series on the six levels with db7 wavelet from GS Podsused on the Sava River. Trend test of choice can be then done on reconstructed details ($D_i, i = 1, 2, \ldots, 6$) of different time scales ($2^i$ months, $i = 1, 2, \ldots, 6$ ) and remaining approximation ($A_6$) and/or on different combinations of components [20].

Adamowski et al. [22] employed linear trend test with CWT on monthly minimum discharge series obtained from rivers in different climate regions in Canada. The study showed already known CWT advantages over DWT which enable analysis, detection and extraction of all potential scales or frequencies, not just multiples of the power two of the sampling interval. Analysis showed that small scales have decreasing, while larger multi-annual scales have increasing trends. However, there were present some uncertainties in trend results due to edge effects, especially in large scales.

Nalley et al. [23] used "W-T analysis", Mann-Kendall test in combination with DWT, on monthly, seasonally-based and mean annual discharge and total precipitation series in Canada. Moreover, influence of the mother wavelet choice, decomposition level choice and the extension level of signal borders used in the WT on trend identification is investigated. The results displayed that, in general, scales up to 4 years had the greatest impact on observed trends.

Sang et al. [24] elaborated wavelet based test for trend identification based on comparison of the energy difference between hydrologic time series and the noise. Furthermore, influence of the mother wavelet choice, decomposition level choice and noise content on trend identification method is investigated. Method is conducted on synthetic data and monthly temperature series, and therefore should be further examined on larger number of observed discharge and SSL series.

Examination of trend on different time scales with the additional help of WT is mostly done on discharge data. This could be caused by the earlier mentioned fact that there are less available long term continuous measurements of SS series data on rivers. Future exploration of wavelet aided trend tests on the larger number of studies with SS data is needed and then comparison of this trend on different time scales is possible with other series that are connected with sediment production and transport processes, such as stream flow and meteorological data. Wang et al. [25] suggested removal of noise from series with WA before conducting trend test in order to improve the precision of the wavelet based trend tests in future. Also, incorporation of other nonparametric trend test is possible and further exploration of application of CWT for extraction of specific seasonal time scales.

3.3 Prediction and forecasting of discharge and suspended sediment with wavelet based hybrid black-box models

WT overcomes the disadvantages of certain black-box models and therefore is used for pre-processing of input data – this combination results in WA based hybrid black-box models. AI models have shown advantages, over traditional regression and time series black-box models, in better forecasting performances of hydrological series, characterized with nonlinear
relationships. Consequently, a large number of published papers are concerned with coupling WA with AI models. AI models based on Artificial Neural Networks (ANN), Fuzzy logic (FL), Support Vector Machine (SVM), Genetic Algorithm (GA) and Genetic-Programming (GP) are most prevalent in published papers dealing with modelling hydro-meteorological series. GA and GP are mainly used for dealing with optimisation problems and ANN, FL based Neuro-Fuzzy (NF) models, and SVM are mainly used as an alternative to traditional regression and time series models.

Detailed overview of WA-AI hybrid models summarized by different hydrological variables is made by Nourani et al. [10]. Few examples of papers with WA-AI hybrid models dealing with discharge and SS series are presented in Tab. 1. Although there are some rainfall-runoff models [26], models presented here are largely using discharge input data from previous time steps for discharge prediction and forecasting; and using previous time step of SS and/or discharge for prediction and forecasting of SS because they are most represented. These models outperformed single AI, multilinear regression and auto-regressive models.

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<tr>
<td>Multi-temporal scale analysis of series variability</td>
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Pre-processing of the input data for WA-AI models is mostly done with DWT. Generally, the pre-processing of input data with DWT for AI/black-box models can be considered as de-noising because one part of information is filtered out from series. In most studies this is done in the following steps. First decomposition of input series to DWT components of different scales is made (refer to Fig. 2). In the second step there are two approaches: (a) where determination of significant components is made by statistical analysis (e.g. correlation) and constitution of a new series is made for each variable by adding together selected components; and/or (b) setting appropriate thresholds to each component is made, part of data from components is removed and constitution of a new series for each variable is made by adding together components. First approach is more prevalent in published papers and
it can be considered as special case of the second where threshold is set equal to cover the full range of values for chosen component (Fig. 3). Second case is iterative procedure where different combinations of thresholds are examined. Global soft threshold proved to be more appropriate for discharge and SSL series than the hard threshold [27, 28]. In general, statistically significant DWT components are added together to related variable and used as inputs in WA-AI models. On the other hand, models built with individual DWT components increase the number of input variables and it is necessary to use additional techniques (e.g. cross-validation) to avoid overfitting [29].

In the process of improvement of existing and development of new WA-AI hybrid models there is a need to carefully examine the adequacy of these models for modelling of discharge and SS series for different time steps. Results for monthly series time step give contradictory results in some papers. Although, application of WT pre-processing generally improves results in comparison to single AI model, sometimes that is not the case and the property of weaker autocorrelation relationship between monthly series in comparison to daily series could be the main cause for that. Additionally, performance of WA based hybrid models is connected with appropriate choice of wavelet mother type and level of decomposition. Although, Sang & Wang [30] have proposed method for statistical evaluation of chosen mother type, in most papers this choice is either arbitrary or examined only for smaller number of mother wavelets (mostly db wavelet family for DWT) and decomposition levels. This is inherent part and the “weakest link” of the WT based models and future exploration in this direction is needed. Moreover, application and research of different thresholds for discharge and SS series de-noising needs to be done in greater extent. WT based hybrid black-box models for SS forecasting and prediction are still not fully exploited in comparison to the modelling of discharge.

This is even more present in models that contain not only discharge and/or SS, but also other input variables that are related to the SS production and transport processes.

### 3.4 Wavelet-aided simulation of synthetic discharge and suspended sediment series

Simulation of synthetic hydrological sequences with similar statistical properties to the observed series is the main goal of stochastic hydrology. Simulated synthetic discharge and SS series of different time resolutions are used in hydrology and water resources management (e.g. for hydropower generation scheduling, for reservoir design and optimization, for testing of existing hydrological and hydraulic models). Traditional parametric stochastic models (e.g. Thomas-Fiering model, autoregressive moving average model) cannot fully represent complex hydrological mechanisms of discharge dynamic. Therefore, different nonparametric models have been developed, such as threshold auto-regression, data driven (e.g. ANN) and chaos theory based models. Their main advantage over parametric models is that they can avoid a priori assumptions about probability distribution [31]. Wavelet transform is mostly examined for synthetic series simulation with nonparametric approach where WT ability is used to decompose hydrological series on different time scales that are mutually independent of each other.

A simple nonparametric stochastic model based on WT and random combinations for simulation of monthly and annual discharge series from Turkey and USA was introduced by Bayazit & Aksoy [32]. Method is called \textit{BA algorithm} and it is based on \textit{Haar} wavelet. Correspondingly, similar model for synthetic generation of SS series based on \textit{Haar} wavelet is examined and results are compared with model based on moving-average process [33].

![Figure 4 Example of wavelet-aided approach to simulation of synthetic discharge time series. Series DWT components are divided on sections with similar properties and if $i$, $j$, $k$ and $l$ are not equal simultaneously then the reconstructed series presents newly simulated sequence (adapted from [34]).](image)

Wang et al. [34] introduced modified wavelet based method for generating synthetic daily discharge sequences. Method is founded on B3 spline discrete low-pass filters and random variations of wavelet coefficients.
Results showed advantage over BA algorithm in its insensitivity to series length and normality of hydrological series distribution. Example of the wavelet aided approach for the simulation of discharge sequence is given in Fig. 4.

Niu & Sivakumar [35] developed a method for generation of synthetic discharge series based on CWT with Morlet mother wavelet. Introduced method overcame disadvantages of dyadic time scale decomposition presented in DWT based methods. Case study with scale-controlled generation of daily discharge series in the Pearl River Basin showed preservation of spectral properties present in the original series.

Generally, advantage of WT based stochastic models is preserving series mean and correlation structure in simulation process. Disadvantage is its inability to reproduce coefficient of asymmetry and therefore is recommended for original data or on transformed data with symmetrical distribution [34]. It has been shown that WT based method is relatively simple in procedures and suitable for any hydrological sequences, such as stationary hydrological series, non-stationary hydrological series, normally distributed hydrological series, and skew distribution hydrological series. An impact of different DWT filters on statistical properties of generated discharge and SS series has not been fully examined. Also, possible combination of CWT based data generation combined with advantages of other stochastic models has the potential for future research.

4 Conclusion

Discharge and suspended sediment time series are characterized with non-stationarity that presents challenge to traditional hydrological models and analysis. Their accurate forecasting, estimation and representation is usually the primary goal for development of hydrological models. Largest body of research on discharge and suspended sediment time series is dealing with multi-temporal scale analysis and with wavelet based hybrid black-box models (especially AI). Future research should further exploit advantages of Derivative of Gaussian based and Paul based CWT in multi-temporal scale analysis of time series. This is especially the case for analysis of sharp features, such as peaks in discharge and suspended sediment transport time series, which correspond to the extreme events endangering infrastructure and impeding water management practice.

Multi-scale trend analysis of suspended sediment time series should be accompanied with other variables that are associated with sediment production and transport processes in order to deepen the knowledge of river morphodynamics. Exploration of greater number of wavelet-aided black-box models on monthly data is recommended because there is no unanimous conclusion of model improvement with WT pre-processing up to date. Similar to the multi-scale trend analysis, wavelet-aided black-box models need to incorporate additional input variables related to the suspended sediment production and transport processes. Investigation of different DWT filters and their impact on statistical properties of simulated synthetic discharge and suspended sediment time series is also needed. Generally, in all presented applications of WT most important step is appropriate choice of wavelet function with similar features to the analysed variables and appropriate level of series decomposition. Extensions and more generalized forms of WTs, such as stationary wavelet transform and wavelet packet decomposition, with their advantages in down sampling, reconstructing, filtering and time-frequency representations of the series should be employed for discharge and suspended sediment time series analysis and forecasting.

5 References


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The conference will address problems of great importance discussing more constructive and progressive approaches to promote sustainability. A major motivation for the meeting is to learn from past failures, to avoid repeating similar mistakes, while attempting to prevent emerging threats to environmental and ecological systems. Fundamental to these concepts are the analysis of the inherent risk and the development of appropriate strategies. All published papers from previous meetings are permanently archived in the WIT Online Library (www.witpress.com/online-library) where they are freely available to the international community.

Conference Topics

The following list covers the topics to be presented at the meeting. Papers on other subjects related to the objectives of the conference are also welcome.

- Environmental policies and planning
- Environmental assessments
- Development issues
- Sustainable cities
- Economic impact
- Natural resources management
- Energy and the environment
- Food production systems and policies
- Ecosystems health
- Soil contamination
- Remediation
- Decommissioning of hazardous plants
- Brownfield rehabilitation
- Water resources management
- Air and water pollution
- Toxicity studies
- Pollution and public health
- Environmental health risk
- Community participation
- Legislation and regulations

Location

Situated in the southern Italian region of Campania, Naples is one of Italy’s oldest and most important cities. The birthplace of some of Italy’s most iconic foods and drinks, good restaurants and cafés are plentiful in Naples, serving fresh local seafood and regional specialties. The historic centro is recognized as a UNESCO World Heritage site and offers a wealth of interesting architecture, monumental castles and museums, as well as galleries exhibiting works by some of Italy’s most distinguished Renaissance artists. The surrounding area and coastline provides picturesque views of Capri and numerous small islands, while nearby Mount Vesuvius and the Roman remains of Pompeii and Herculaneum remain some of the most notable sites in southern Italy.

Conference Venue

The conference will take place at Villa Doria d’Angri, a most beautiful palace built in 1890 on commission of the Prince Mecenate Donasi. The palace is the expression of the greatest artists and craftsmen of the time and retains much of its former glory. The villa situated in the Posilippo area has recently been restored by the University with the help of the European Union to make it suitable to host international meetings, seminars and cultural events of particular importance.

Submission Information

Abstracts of no more than 300 words should be submitted as soon as possible. Abstracts should clearly state the purpose, results and conclusions of the work to be described in the final paper. Final acceptance will be based on the full-length paper, which, if accepted for publication, must be presented at the conference.

The language of the conference will be English.

Online submission: www.witconferences.com/impact2018

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