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THE SEPARATION OF AIRBORNE DUST AND PARTICLES*

This paper is a study of the physical theory underlying various methods of separating particles from gases. Emphasis is entirely on fundamental considerations which lead to formulae of value to those concerned with the design and engineering of separators. New theories of the cyclone and of fibrous filtration are put forward together with graphs and tables which enable them to be applied to practical problems without difficulty.

1. Methods of Filtration. Dust suppression aims at purifying air breathed by industrial workers either by modification of the dust generating process, so that much less is disseminated while little interference with the prime purpose of the operation is inflicted, or else by subsequent removal of airborne material.

As a rule it is much better to deal with the source of dust; for example, in mining, the best suppression during rock drilling is achieved by seeing that the drill operates on wet rock. When this is not possible, dust must be separated from air which involves an air draught to carry the particles away to a place where they can be discharged harmlessly, or else to a separator of some kind which purifies the air.

This paper deals with the physical principles of different kinds of separator so that their design can be understood and applied appropriately. Separation is achieved by the agency of mechanical, electrical or thermal forces. The mechanical methods involve settling tanks and elutriators, depending on gravity, and also cyclones, scrubbers and the like, which operate because of the inertia of the particles. Such processes overlap with the many types of filter, since in filter beds of granular, metallic or other packings, the forces of gravity and particle inertia are of greatest importance, although fibrous filters, used especially for very fine particles, depend partly on thermal motion of the particle due to molecular agitation in the air.

Separation by thermal diffusion or radiometer force is another possibility, again for small sizes. The separation of fine dust is very important in the personal respirator and in dust sampling devices.

Electrical forces operate in certain kinds of fibrous filter in the absence of obvious electrical apparatus. Their effect, however, is just as striking as that produced by the complicated high tension gear of electrostatic precipitators which, while very good for coarse dust, are not usually effective for smaller sized particles.

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2. Settlement and Elutriation. Separation by settlement depends on the rate of fall of particles, which can often be calculated with sufficient accuracy by the methods which have been described by the author (1947). Sedimentation rates for individual, spherical particles of unit density, falling in air at one atmosphere pressure and 15° C are shown in Table I.

TABLE I.

Rates of fall of individual spherical particles of unit density in air at one atmosphere pressure and 20° C.

Diameter	Terminal Velocity
.2 μ	.000225 cm/sec.
.3	.00042
.5	.0010
1	.0035
2	.0128
3	.0275
5	.078
10	.30
20	1.2
30	2.7
50	7.2
100	25
200	70
300	115
500	200
1000	385

Large particles are easily removed from an air stream by simply expanding the ducting through which it flows so that a sufficiently low velocity is maintained across the wider section. The entrainment of dust in ducts has been studied by Baliff, Greenburg and Stern (1947) who give a list of the air pick-up velocities of dust ranging from about 3 to 8 metres per second for a wide variety of materials. As long as the air velocity is lower than this, settled dust is not disturbed. The particle sizes ranged from 15 μ to 540 μ but the finest dust is not necessarily the most easily blown away. Attempts to generalise these results by forming dimensionless groups of variables have been unsuccessful, in fact, fine particles often tend to pack into a secure sediment and other factors influence dispersal.

Roughly speaking, settling chambers across which the air moves horizontally are suitable for the elimination of particles exceeding 100 μ in size. These will, of course, entrain a lot of fine dust if this happens to be present.

From 100 μ down to 10 μ vertical chamber is best with a rising air flow. The chamber is usually conical and expands upwards to promote a uniform air flow across its section. It is, however, inevitable that the air should rise more slowly near the walls than on the axis. The particle paths in an elutriator like this have a radial component and dust settles out on the walls from which it is removed by shaking or rapping.

The sedimentation of the smaller sizes is so slow that the area of cross section of a separator would be considerable if a large air flow had to be dealt with. If the tube is made too large, convection currents invariably develop which are as great as or even exceed the air velocity. Under these circumstances the laws of settlement are more complicated and the method is unsatisfactory.

Even the most extravagant precautions cannot eliminate stray temperature gradients in large apparatus, and as rough guide it can be stated that the gravitational separation of particles of 10 μ to 20 μ diameter can only be achieved with an air flow of less than three litres per minute per tube, which means that the elutriation tube must not exceed some 10 cm diameter.

The elutriation of smaller particles necessitates the use of even smaller tubes and is only of interest for laboratory work or dust sampling on a small scale.

3. Inertia Separators. There are a number of different devices operating on this principle. Scrubbers depend on the collisions between dust particles and water spray droplets which are heavier and consequently settle out more rapidly. In cyclones, an air stream is blown tangentially into a cylindrical chamber inside which the dusty air spins round a number of times before being exhausted from an axial orifice. Owing to their mass, the dust particles drift outwards and downwards and a deposit forms in the bottom.

Inertia filters with beds of randomly arranged rings or other shapes, turnings or gauzes, depend, like scrubbers, on the dynamics of collision between particle and collecting obstacle. These have been developed to include oriented collectors of specially favourable shape. Baffled or corrugated tubes, or labyrinths, in which the air frequently changes its course, each time impinging some of its load of dust particles against the wall or baffle, come into the same category.

In all inertia separators, the efficiency with which they function depends on three things. Firstly there is the field of air flow, or velocity distribution, around the collecting obstacle, whether this is the moving drop in the scrubber or the baffle in the labyrinth. Secondly, there is the trajectory of the particle, which in a given geometrical distribution of air flow depends on the mass of the particle, its air resistance, the size of the obstacle and the rate of flow. Finally, the elimination of particles, after collision with the collector, may not be certain, a difficulty sometimes circumvented by wetting or oiling the collecting surfaces.

The last point is primarily a matter of engineering and its consideration from that angle will not be attempted, but the fundamental problems of flow field and particle trajectory are worth discussing on the assumption that the phrase 'efficiency of dust collection' implies the elimination of all particles which strike the target.

Other things being equal, high speeds favour the efficient impingement of fine particles upon obstacles. This arises in two ways. Firstly, there is the evident increase in the momentum of the particle which makes it less subject to deviation from its course when the streamlines of flow spread sideways past the obstacle. Secondly there is the distinction between the flow patterns at high and low values of the Reynolds' number, *Re*, with reference to the obstacle.

This is equal vx/ν where v is the velocity of the approaching air stream, ν the kinematic viscosity of air and x a characteristic dimension of the obstacle. When *Re* is high, the flow upstream of an obstacle is governed only by the pressures developed and, except near solid surfaces, possesses a flow pattern which is often very near to that of an ideal fluid without viscosity. At low Reynolds' numbers, that is at low speeds and on a small scale of dimensions, flow is governed entirely by viscosity.

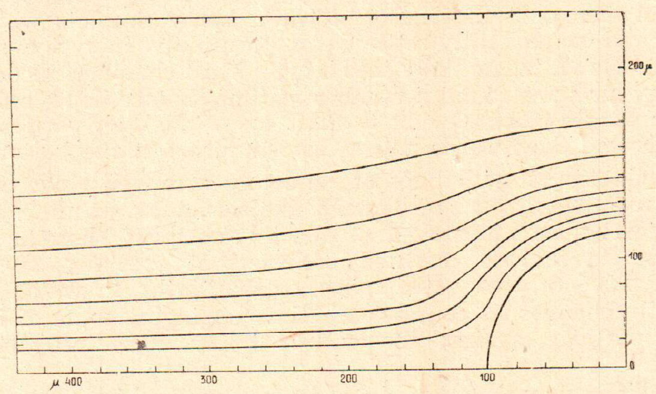

Fig. 1 a — Streamlines of flow transverse to a cylinder
u = 1500 cm/sec R = 0.1 cm Re = 10

The effect of Reynolds' number is exemplified by the air flow transverse to a cylindrical rod. In figure 1 (a—c) the streamlines of flow have been drawn for high moderate and low values of the Reynolds' number. The figures quoted were selected because they are typical of (a) flow through a grill, (b) flow through a wire gauze air cleaner and (c) flow in a fibrous air filter.

It will be noticed that the streamlines spread outwards to pass around the cylinder much more suddenly at high values of Re than they do at low ones. In fact, the effect of the obstacle is hardly apparent a couple of diameters upstream in the first case, but when $Re = 0.2$ there is a 3% disturbance as far as 100 diameters ahead.

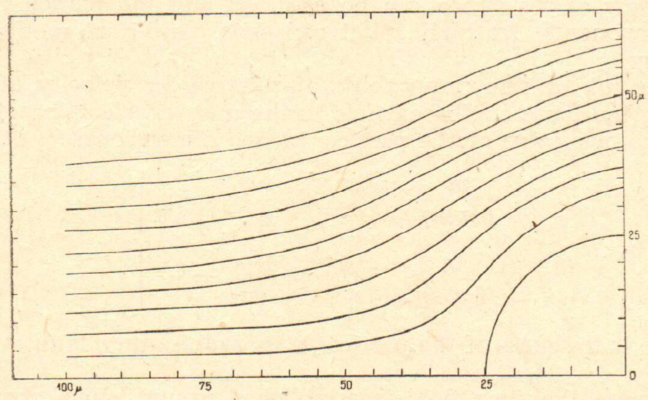

Fig. 1 b — Streamlines of flow transverse to a cylinder
u = 300 cm/sec R = 0.0025 cm Re = 10

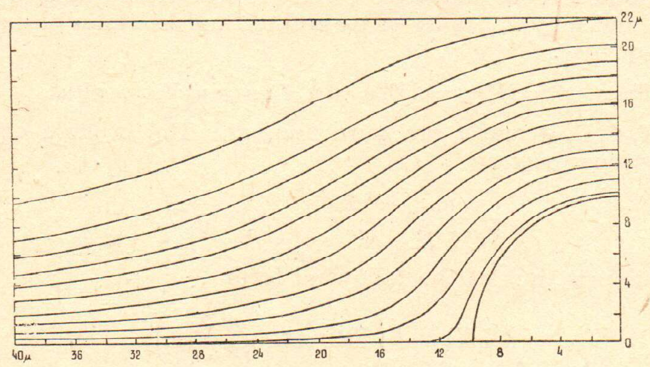

Fig. 1 c — Streamlines of flow transverse to a cylinder
u = 15 cm/sec R = 0.001 cm Re = 0.2

Thus, with viscous flow, particles approaching in line with the cylinder begin to experience a slowing down and a gradual lateral displacement long before they are near and are much more liable to be displaced sideways enough to miss it altogether than is the case, under equivalent circumstances, at high speeds. Particle inertia is less effective to ensure impingement against a gradually applied displacing force. The same kind of thing happens whatever the shape of the obstacle.

Let us consider trajectories of spherical particles in more detail. If the relative velocity between particle and air is not too great, Stokes' law of resistance can be assumed and the motion resolved into components. At high relative speeds, empirical methods are necessary.

If (u, v) are the components of relative air velocity at (x, y), equating the force on the particle to the mass-acceleration gives the equations of motion of the particle in two dimensions:

$$\left.\begin{aligned} \frac{m}{6\pi a\eta}\,\frac{\partial^2 x}{\partial t^2} + \frac{\partial x}{\partial t} &= u \\ \frac{m}{6\pi a\eta}\,\frac{\partial^2 y}{\partial t^2} + \frac{\partial y}{\partial t} &= v \end{aligned}\right\}, \qquad (1)$$

where m is the mass of the particle, a its radius and η is the viscosity of air.

Suppose the flow field is established round some obstacle of characteristic dimension l, for example the radius of the cylinder. Let u_0 be the wind speed at a distance from the obstacle.

Replace the values of x and y in the original equations by lengths measured as multiples of the characteristic length, l; substitute u and v by multiples of u_0 and v_0, and let times be expressed in units of l/u_0.

Then, $x = lX, y = lY, u = u_0 U, v = u_0 V, t = \tau l/u_0$,

where X, Y, U, V and τ are pure numbers. The original equations of motion now become

$$\frac{m u_0}{6\pi a\eta l}\,\frac{\partial^2 X}{\partial \tau^2} + \frac{\partial X}{\partial \tau} = U, \quad \text{etc.} \qquad (2)$$

The solution is thus of the following form

$$(X, Y) = f\left(X_0, Y_0, \tau, \frac{m u_0}{6\pi a\eta l}\right), \qquad (3)$$

where (X, Y) is the position at time τ of a particle which enters the system at (X_0, Y_0) with velocity u_0 when $\tau = 0$. The function f depends on the boundaries surrounding the flow and on the type of flow.

In geometrically similar systems operating at the same Reynolds' number the flow patterns are dynamically similar. This means that the sets of streamlines are geometrically similar though the speeds involved may be different. In such systems the particle trajectories will be identical for the same value of the particle parameter, $P = mu_0 / 6 \pi a \eta l$, when plotted on coordinates (X, Y). This assumes that hydrodynamic interference between particle and obstacle is negligible, which is reasonable providing the particles are small in comparison. The actual velocities, (u, v) and positions (x, y) will usually differ, of course, so it is much more convenient to reduce all variables to dimensionless form and so generalise the problem.

Direct mathematical evaluation of solution (3) is impossible, as a rule, so particle trajectories must be obtained by approximate, stepwise solutions of equations (1) or (2). Unfortunately there are a number of inconsistencies between solutions of this kind which have been published and as the labour of checking them is considerable they are likely to persist.

Having established a number of particle trajectories, the next step is to ascertain the proportion of particles which will strike the obstacle. Suppose an air stream of width considerably greater than a cylindrical rod is blowing across it and carrying along a number of particles which are uniformly distributed. Were it not for the pseudo repulsion of particles from the rod, due to their tending to follow the streamlines of flow by virtue of air viscosity, the number of particles hitting the rod would be

$$2Nu_0 (R + a)$$

particles per second on each cm of its length, where N is the concentration of airborne particles (number per cm^3), R is the radius of the rod and a is the radius of the particles (figure 2a). This would hold in reality only for very heavy particles for which $P \rightarrow \infty$

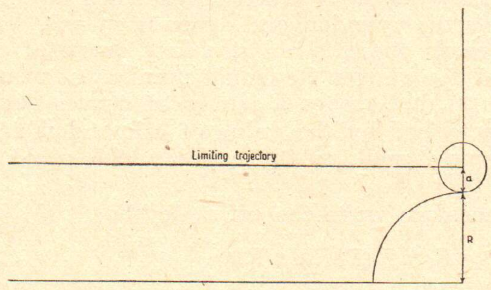

Fig. 2 a — Illustration of efficiency of collection
P Particle undeflected

In fact, a smaller quantity strike the wire, depending on P, the ratio of particle radius to wire radius and on the type of flow. The fraction striking is called the efficiency of collection, E, on the assumption that they all adhere. It is calculated by plotting a series of particle trajectories on the flow field, by a stepwise method, each of which starts at a different distance from the centre streamline that meets the cylinder at the forward stagnation point. When the distance from the axis exceeds a certain amount, particles no longer hit the cylinder. The limiting trajectories are shown in figure 2b.

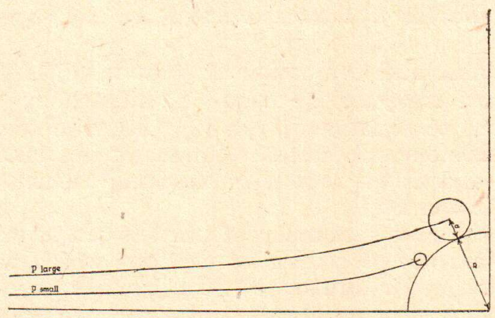

Fig. 2 b — Illustration of efficiency of collection. Limiting trajectories for for finite values of P

The efficiency of collection depends not only on the parameter P, which characterises the path of a particle starting at a certain point, but also on the ratio of particle to cylinder radius. As a matter of fact this ratio must also affect the flow field, causing the particle trajectory to curve away from the cylinder to a greater extent than is found from the stepwise calculation, taking them as massive points. However this is less important than the interception factor which allows for collision occuring when the surface of the particle, not its centre, meets the cylinder, and depends on a/R.

For this reason there is not a unique curve of collection efficiency against P for a given Reynolds' number, as would be deduced from equation (3), but rather a family of curves each member of which is characterised by the value of a/R and which tend, when this ratio becomes small, to the curve which could be calculated from equation (3).

We can therefore write the efficiency as

$$E = F\,(P, Re, a/R)$$

$$= F\left(\frac{2\,u_0\,a^2\,\varrho}{9\,\eta\,R}, \frac{2\,u_0\,R}{\nu}, \frac{a}{R}\right) \tag{4}$$

where the cylinder radius, R, is taken as the characteristic length, l.

A large velocity increases both the Reynolds' number and the particle parameter, so that on both counts the efficiency of collection is increased. However, an increase in the radius of the cylinder, R, other things being equal, has an adverse effect on both P and a/R, although it increases the Reynolds' number. As a rule the latter variation is less important so that fine wires collect particles better than coarse ones.

The preceding discussion has been rather abstract, but the general conclusions are quite valuable because they apply to all types of direct impingement systems. The only point which requires watching is the occurrence of breakaway of flow from the obstacle at high Reynolds' numbers with a region of uniform pressure and turbulence astern, which may modify the upstream flow, making the obstacle act as if it were bigger than is really the case.

By way of illustration we will consider further the impingement of particles upon rods taking the three flow regimes of figure 1.

The efficiency of impaction as a function of the particle parameter, P, is shown on figure 3 for particles which are negligibly small compared with the cylinder. The curve for high values of the Reynolds' number is taken from an average of the computations of *Albrecht* (1931), *Sell* (1931) and *Glauert* (1940). These are not in exact agreement, but the divergences are not significant for the present purpose. The streamlines of flow assumed for the calculations were those of an ideal fluid which approximates sufficiently closely to a real fluid upstream of the cylinder, providing the Reynolds' number is high and conditions near the surface are not important. These limitations can reasonably be accepted for $Re > 1000$ in this problem.

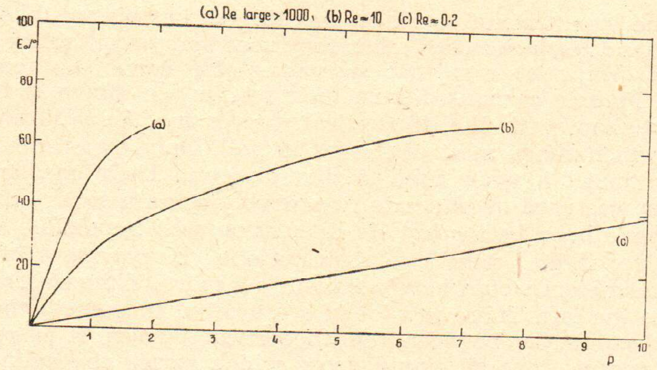

Fig. 3 — Impingement of particles on cylinders taken when particle is small compared with radius of cylinder

For medium Re, upstream velocities calculated from the flow data of *Thom* (1933) were used and particle trajectories were computed in the usual way.

For low Reynolds' numbers the flow formula of the present author (1950) was employed.

The curves on figure 3 emphasise the importance of the type of flow in affecting impingement efficiency. This factor, however, can be offset by the a/R term which is important at low Reynolds' numbers with very small cylinders, as shown in figure 4. The curves of this figure were plotted without allowance for the hydrodynamic interference between particle and cylinder which must be appreciable for the larger particles. Hence the curves are too optimistic concerning efficiency for the higher values a/R. Nevertheless, fine wires are very much better at catching particles than thick ones.

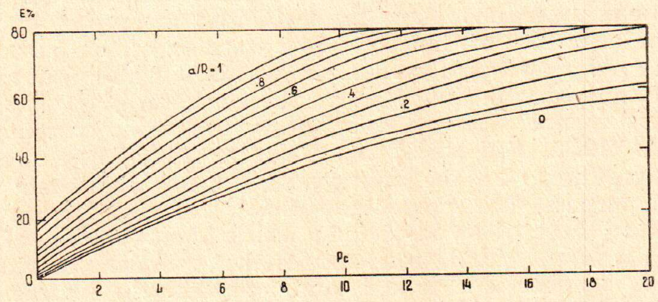

Fig. 4 — Impingement of particles on cylinders for Re = 0.2 with different values of the ratio of particle to cylinder radius

Some experimental work on this subject has been published by *Landahl* and *Hermann* (1949), but they used sprays containing a wide range of droplet sizes so that the values of P cannot be accurately defined. Figures calculated from their results are shown in figure 5 in comparison with the theoretical curves for $Re = 10$ and 1000 within which range most of the points fall. Only experiments with a/R less than 0.1 were used in this diagram. Their median mass diameter was used to calculate P; probably a mean square diameter would have been better but its calculation was impossible without knowledge of the drop size distributions. Employment of mean square diameters would have resulted in a move of the experimental points to the left. There is a lot of scatter, but in general the experimental points lie between the theoretical curves. It is probably safe to assume that all liquid drops which struck the cylinder remained adhering to it.

Precisely similar conclusions to those relating to the impingement of particles upon cylinders govern collisions with spheres, although exact data have not yet been worked out in detail. Particularly, results for flow past spheres at moderate Reynolds' numbers are not available, and it is these figures which are needed for accurate assessment of the performance of spray scrubbers. It is evident that the figures for spheres will run more or less parallel with those for cylindrical obstacles, while the efficiency of impaction on a sphere is rather lower than on a cylinder of the same diameter. Bearing this in mind, we can proceed to make a rough estimate of the efficiency of spray scrubbers.

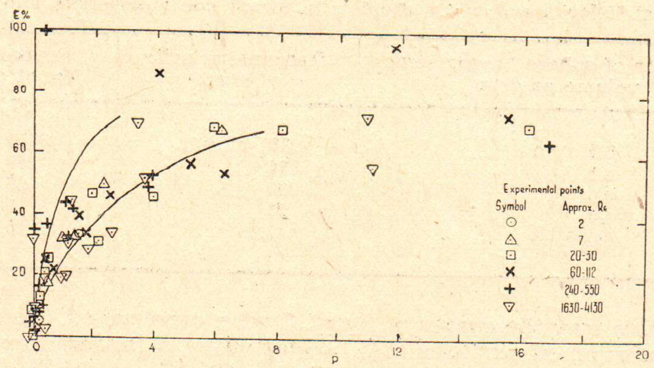

Fig. 5 — Impingement of particles on cylinders. Comparison of experiments of Landahl and Hermann with theoretical results for Re = 1000 and Re = 10 when a/R is negligible

In the simple scrubber a cloud of water droplets is allowed to fall under gravity through dusty air. If one assumes the droplets to be far enough apart to act independently, the efficiency of the scrubber can be calculated from the total number of water drops released and the collection efficiency of the individual drops. If spraying is continued for a long period through nearly static air the dust concentration will fall exponentially but the coefficient of decay will depend on particle size, owing to the variation of impingement efficiency. The size distribution of the dust cloud will therefore become finer as spraying proceeds. We will confine ourselves, in this paper, to consideration of the fundamental collection efficiency of individual drops, leaving the application of the results to the engineers.

Water drops smaller than about 1.5 mm diameter are kept spherical while falling by the force of surface tension. Larger sizes deform by virtue of the air pressure due to motion and of centrifugal forces set up by circulation within the drop.

In Table II the velocities of free fall and the corresponding Reynolds' numbers are tabulated. From this data the dust particle parameters, *P*, were worked out. The only calculations of the impingement efficiencies of spheres of which we were aware were those of *Sell* (1931) and *Vasseur* (1949) for ideal fluid flow. Unfortunately these are not in agreement, the French worker predicting a lower efficiency. For the purpose of our estimate we have been guided by Vasseur's results at Reynolds' numbers exceeding 100 and by curves extrapolated from the data for cylinders at lower values.

TABLE II.

Terminal Velocities of water droplets in air at one atmosphere and 20° C.

Diameter of sphere having same volume as drop	Terminal velocity of drop.	Reynolds' No, of drop.
0.5 cm	910 cm/sec.	3030
0.1	385	257
0.05	200	67
0.01	25	1.67
0.005	7.2	0.24

In this way the curves of figure 6 were constructed. These show the proportion of particles ($E\%$) collected by a falling drop out of all those lying in its path, while it falls vertically, which has a cross sectional area of πa^2.

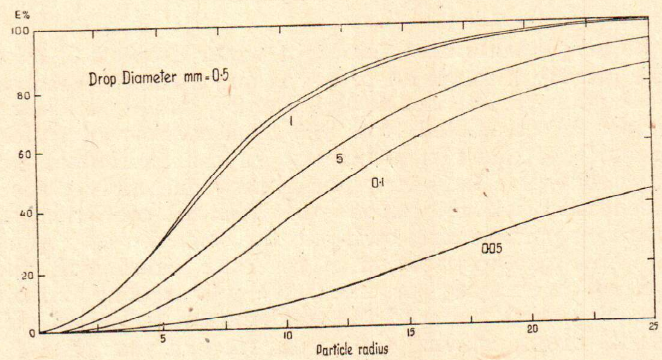

Fig. 6 — Efficiency of collection of dust particles by freely falling water droplets in a spray scrubber

It is clear that the optimum size of water drop has a diameter between 0.5 and 1 mm. Smaller drops are poorer because thay fall so slowly that the dust particles are pushed sideways, as the drops fall past them, without colliding. The lower efficiency of large drops is explained as follows. The particle parameter P is inversely proportional to the drop diameter so that particles of a given size have a smaller value of P relative to the larger drops. Impaction is of course less efficient for smaller P. This effect outweighs the increase in terminal velocity which is very slight for drops greater than 4 mm diameter, so that impaction upon large drops decreases as drop size increases above the optimum value.

The curves show that a substantial gain in efficiency of dust removal can be achieved by using spray droplets of the right size. Even so, suppression of dust particles having radii below 5 microns is very poor and gravitational scrubbers are obviously no use for fine dust.

Improvement in this direction can only be achieved by increasing the relative velocity between dust particle and spray droplet. This has actually been done by *Kleinschmidt* and *Anthony* (1941) by placing radially directed sprays on the axis of a cyclone, but the advantages accruing from the employment of the optimum size were not appreciated by the designers, nor are any data given for particle size efficiency.

We will now pass on to consider the separation of dust in cyclones. An account of the distribution of air velocities in a cyclone has been given by *ter Linden* (1949) who also describes the effect of varying certain relative dimensions upon the efficiency of dust collection.

A diagram of an efficient cyclone is shown in figure 7. Air enters tangentially with velocity V_0 at the top of the cylindrical section of height h and radius R_2 and spirals down through a vertical distance H to the bottom of the cone, making several revolutions on the way. It flows upwards along the axis of the cyclone dissipating most of its spin into a confused turbulence in a central core which expands upwards and exhausts through the vertical outlet in the centre of the top. This has radius R_1.

Ter Linder discusses the shape of the inlet, the depth to which the outlet duct is carried below the top, the ratio of the radii R_1 and R_2, and the height H.

The tangential component of air velocity increases towards the bottom of the cone, and also towards the axis until the ascending turbulent core of the vortex is reached. The faster the air spins, the lower is the pressure on the axis of the cyclone. Owing to conservation of angular momentum, the spin increases towards the vertex of the inverted cone so the pressure is a minimum in this region and induces the downwards component of flow in the periphery. Near the vertex, the boundary layer of fluid, adjacent to the wall of the cyclone, is being very rapidly sheared and breaks down into eddies. A little consideration makes it evident that the rotation in these eddies is in the opposite direction to the spin of the main vortex. These eddies are discharge up the central core, where they blend with the main flow and the rotations tend to cancel out; this explains the comparatively small spin found in the emergent stream.

Particles are flung towards the wall by centrifugal action and are then swept down to the dust bunker which must be airtight because the pressure all along the axis of the body is below inlet pressure; this is usually atmospheric. Leakage of air into the bunker would redisperse the dust.

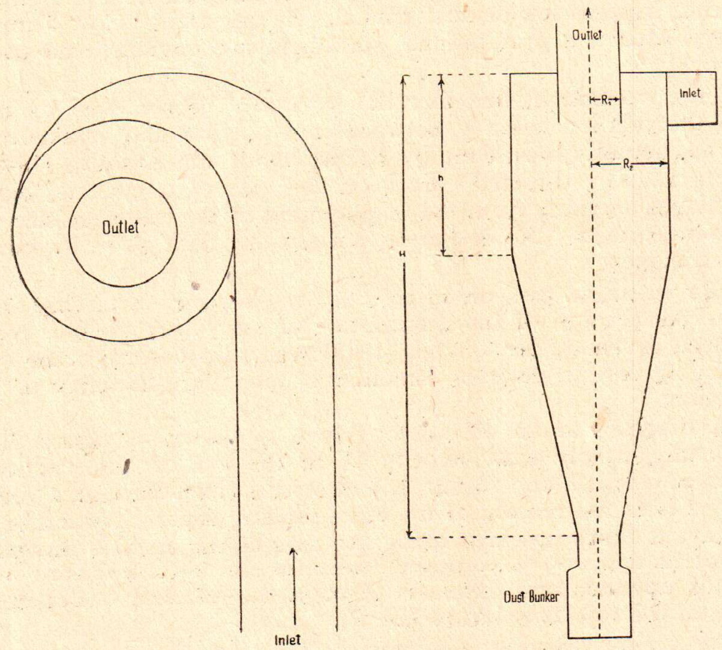

Fig. 7 — Cyclone Dust Collector

There is a certain amount of radial flow into the upwards moving core, but owing to the much higher tangential velocity particles are not readily lost in this way.

In a free vortex of ideal fluid the tangential velocity is inversely proportional to the radius, but in cyclones the increase in the rate of spin, going inwards, is rather slower owing to the viscous transfer of energy to the turbulent, relatively non-rotating core.

To obtain some insight into the separation process it is necessary to consider the equations of motion of a dust particle inside the cyclone. For this purpose we will first study the particle trajectory in a free vortex in which the air moves in concentric circles and the speed is V at radius R, so that

$$VR = V_0 R_2. \tag{5}$$

Suppose the coordinates of the particle are (R, Θ), then the velocity component of the particle directed outwards from the axis of the cyclone is

$$u' = dR/dt$$

and the component tangential to the air flow is

$$v' = R d\Theta/dt.$$

It is shown in text books of mechanics that the radial and tangential accelerations are equal to

$$d^2R/dt^2 - R\,(d\Theta/dt)^2 \text{ and } R d^2\Theta/dt^2 + 2\,(dR/dt)\cdot(d\Theta/dt),$$

respectively. We can now write down the equations of motion of a particle corresponding to equations (1):

$$\left.\begin{aligned} &\frac{m}{6\pi a\eta}\left\{\frac{d^2R}{dt^2} - R\left(\frac{d\Theta}{dt}\right)^2\right\} = -u' = -\frac{dR}{dt} \quad \text{(radial)} \\ &\frac{m}{6\pi a\eta}\left(R\frac{d^2\Theta}{dt^2} + 2\frac{dR}{dt}\cdot\frac{d\Theta}{dt}\right) = V - v' = \frac{V_0R_2}{R} - R\frac{d\Theta}{dt} \\ &\qquad\qquad\qquad\qquad\qquad\qquad \text{(tangential)} \end{aligned}\right\} \quad (6)$$

It is convenient to render these equations dimensionless, as was done before, by measuring radii in units of R_2, velocities in units of V_0 and times in units of R_2/V_0. Hence,

$u' = uV_0$, $v' = vV_0$, $R = rR_2$, $t = \tau R_2/V_0$, where u, v, r, and τ are dimensionless variables. The equations now reduce to

$$\left.\begin{aligned} &P\left\{\frac{d^2r}{d\tau^2} - r\left(\frac{d\Theta}{d\tau}\right)^2\right\} = -\frac{dr}{d\tau} \\ &P\left(r\frac{d^2\Theta}{d\tau^2} + 2\frac{dr}{d\tau}\cdot\frac{d\Theta}{d\tau}\right) = \frac{1}{r} - r\frac{d\Theta}{d\tau} \end{aligned}\right\} \quad (7)$$

where P is the particle parameter, as defined previously, using R_2 as the characteristic length and V_0 as the velocity,

$$P = \frac{mV_0}{6\pi a\eta R_2} = \frac{2a^2(\varrho - \sigma)V_0}{9\eta R_2} \qquad (8)$$

where ϱ is the density of the particle, and σ that of the fluid in the cyclone.

If we assume that the tangential component of particle velocity is equal to the air velocity, which is the case if the particles are not too large, the right hand side of the tangential equation becomes equal to zero which gives

$$\frac{P}{r}\frac{d}{d\tau}\left(r^2\frac{d\Theta}{d\tau}\right) = 0$$

Hence $r^2\, d\Theta/d\tau$ = constant.

Since $V = R\, d\,\Theta/d\,t = v'$, and by (5) $R^2\, d\,\Theta/d\,t = V_0 R_2$, we see that

$$r^2\, d\,\Theta/d\,\tau = 1$$

Substituting this in the radial equation gives

$$\frac{d^2 r}{d\tau^2} + \frac{1}{P}\frac{d r}{d\tau} - \frac{1}{r^3} = 0$$

This equation is non-linear and is not readily solvable except by numerical methods. However, again for particles which are not too large, it is permissible to neglect the second differential coefficient in comparison with the other terms so that we are with

$$\frac{d r}{d\tau} = P/r^3$$

giving $\tau_2 - \tau_1 = \dfrac{{r_2}^4 - {r_1}^4}{4P}$. (9)

Returning to ordinary units, this gives the time taken by a particle to travel from R_1 to R_2 as

$$\Delta t = \frac{{R_2}^4 - {R_1}^4}{4 P V_0 {R_2}^3} \quad (10)$$

This is the time required by the most unfavourably located particle to reach the outer wall of the cyclone. That is to say, a particle initially coming from near the periphery of the outlet tube.

Such a distance can be covered only if the time of descent through the cyclone is as long as or greater than $\Delta\, t$. For cyclones of normal design the time of descent from the inlet to the bottom is nearly equal to H/V_0, hence, by equating this to the expression for $\Delta\, t$ a formula is obtained giving the radius of the minimum sized particle which can be caught.

Thus, after substituting for P from (8),

$$H/V_0 = \frac{9\,\eta\,({R_2}^4 - {R_1}^4)}{8\, a^2_{min}\,(\varrho - \sigma)\, {V_0}^2 {R_2}^2}$$

so that $a_{min} = \sqrt{\dfrac{9\,\eta\, R_2}{8\,(\varrho - \sigma)\, V_0\, H/R_2}\left\{1 - \left(\dfrac{R_1}{R_2}\right)^4\right\}}$ (11)

This is the particle size above which 100% efficiency of collection will result in the absence of entrainment of deposited dust. The formula shows us that it pays to have high inlet speed, small external radius, large ratio of height to radius and the ratio of external to outlet radius as near to amity as is possible. The effect of varying the latter is negligible beyond a ratio of the order of 3 : 1, though the permissible range is governed by its influence upon air flow.

Equation (11) agrees well with an experimental curve given by ter Linden in his paper. It has been used in the present work to construct figure 8 which shows how the efficiency depends on particle size, radius of cyclone and inlet air velocity for a cyclone of normal relative proportions having $R_1/R_2 = 1/2.5$ and $H/R_2 = 6$.

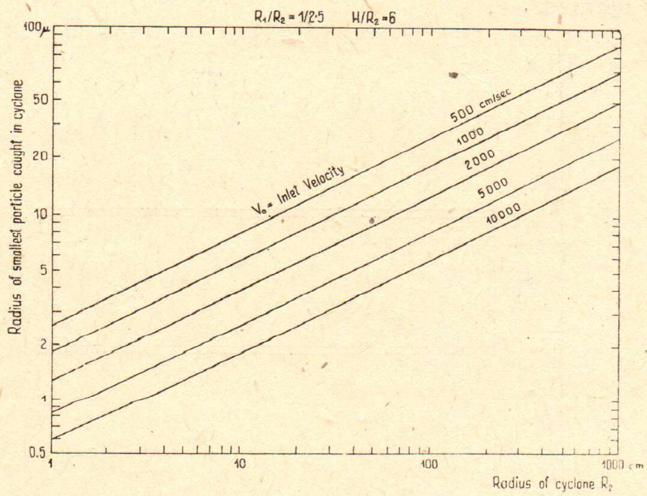

Fig. 8 — Performance data for spherical particles of unit density in cyclones using atmospheric air

Particles of unit density were assumed and the fluid was taken to be air at normal temperature and pressure. The formula above can, of course, be applied to any liquid or gas.

On the lines of equation (4) we could argue that the efficiency of collection of particles finer than the limiting size a_{min} would be a function of P alone since the influence of Reynolds' number of flow and particle radius, both relative to cyclone radius, must be small. However, a cyclone is not a good selective device, partly for fundamental reasons and partly because fine particles are caught up by coarser ones.

The most suitable method of selecting particles of a given size range from a wide distribution of sizes is by impingement from a jet which plays perpendicularly on to a plate.

Such a system can be made to collect large quantities of dust by trapping the particles after impact on a sticky, revolving drum, or under water, or by some other method of fixation which does not interfere too much with the aerodynamics of the jet.

The theory of impaction from such systems has been worked out by *Davies* and *Aylward* (*1951*). The jet should be a slit about ten times longer than its width and the entry should be nearly parallel for some distance upstream, well faired and smooth. There is an optimum distance of the mouth of the jet from the plate equal to about 0.3 of the slit width (figure 9). At other distances, especially greater ones, the particle size below which penetration starts is less sharply defined.

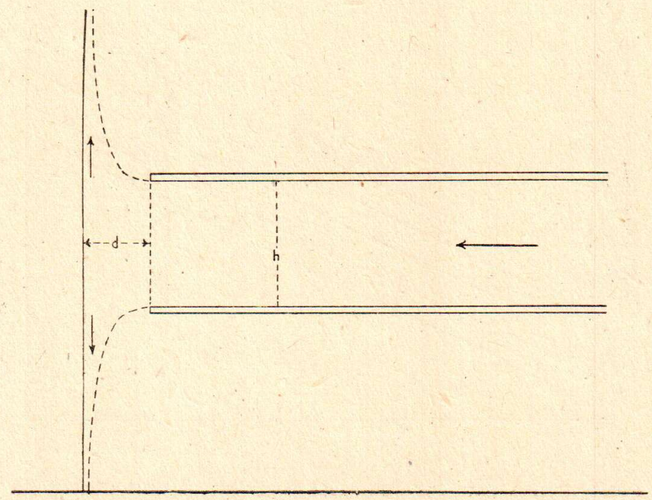

Fig. 9 — Impingement jet designed for maximum selectivity

For such a jet, operating with streamline flow, all particles having $P > 0.14$ are deposited and the range over which the efficiency of deposition varies from 0 to 100% is only $0.14 < P < 0.18$. In this case the characteristic length used in forming P is half the width of slit, h, and u_0 is the velocity upstream of the orifice, so

$$P = 4u_0 \ a^2\varrho / 9\eta h.$$

If the velocity exceeds about 100 metres per second compressibility effects become appreciable; actually a correction to the calculated velocity of 5% is needed at this speed and rapidly increases above it. At high speeds it is necessary to give the particles time to pick up the stream velocity before they reach the orifice.

In certain applications of dust separators there is no need to collect the precipitated dust. In this case appliances can be designed, on the foregoing principles, which are capable of dealing with considerable volumes of air at the expense of only a negligible pressure drop, a factor of great importance, for example, in air cleaners for engines. The method employed is to cause the flow lines to bend so that dust concentrates in the outer peripheral region which is subsequently allowed to blow away. By good aerodynamic design a high proportion of coarse dust can be removed with small loss of head, no troubles arise from lack of adhesion of particles to collecting obstacles and clogging and cleaning problems are avoided. Centrifugal action is sometimes induced by the use of radial vanes.

Figure 10 illustrates a cleaner of the type referred to; such devices have been the subjects of many patents.

4. Fibrous Filters.

Fibrous dust filters operating by sieve action are rarely used since thin, sieve-like materials rapidly become clogged and the filtration and air resistance characteristics are those of the cake of

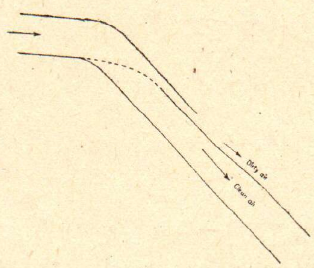

Fig. 10 — Inertia type air-cleaner

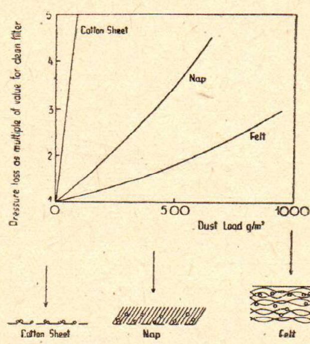

Fig. 11 — Clogging of various types of filter material

solid built up, rather than those of the clean material. Cloth with a nap raised on the side facing the flow is better, because some dust particles are entangled before reaching the finer pores of the base and the life of the filter is consequently extended. Better still are homogeneous fibrous pads of felt-like consistency. These can be made effective against fine particles, which deposit in depth, even though the pores are much greater in size than the particles (figure 11). The latter type of filter is universally employed where high efficiency against fine particles of dust and mist is demanded, and it is with the physical principles of its action that we shall now deal.

Suppose the filter consists of a homogeneous mass of fibres in the form of a pad of area A and thickness h, and that air flow Q passes through uniformly in a direction normal to its area. Let c be the packing density of the fibres which is equal to the volume of the fibres divided by the volume of the filter, Ah. Then the mean velocity of air inside the filter is v, where

$$v \approx Q/A\,(1 - c) \tag{12}$$

Let the concentration of particles of radius a in the incident air stream be n_0, while n is the concentration in the emergent air. It is easy to see, for a homogeneous filter and cloud, that

$$n = n_0 \; exp\;(-\gamma h) \tag{13}$$

where γ is the index of filtration efficiency. The elucidation of γ as a function of particle size, air flow, fibre diameter, etc. is the goal of the theory of filtration.

Suppose that an aggregate length L of fibres of radius R is packed in unit volume of the filter. Then

$$\pi R^2 L = c. \tag{14}$$

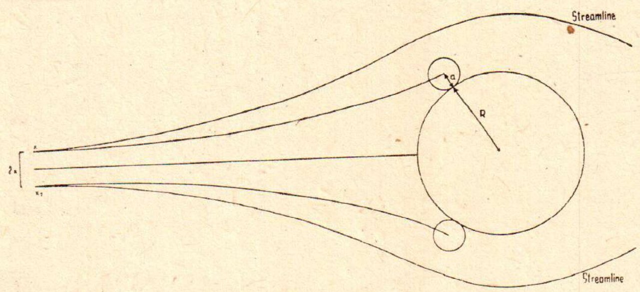

Fig. 12 — Theory of filter action

Consider the air entering an element of filter of area A and thickness dh in which the fibres are all dispersed transverse to the flow. We assume that all particles carried in the air approaching a particular fibre in a direction at right angles to its axis will strike and adhere to it if they enter a certain region of flow, $x_1 x_2 = 2x$, as shown in figure 12. The volume of air approaching the fibre in this region is equal to $2vx$ per unit length, so the total volume of air filtered per second in passing through the element dh is

$$2vxLA\;dh.$$

All the particles in this air are filtered, so the change in particle concentration due to passage through the element dh is equal to

$$dn = -\frac{n\,2vxLA\,dh}{Q}.$$

Substituting for v from equation (12) and for L from equation (14) gives

$$dn = -n\,\frac{c}{1-c}\,\frac{2x\,dh}{\pi R^2}$$

By comparison with equation (13) we now see that

$$\gamma = \frac{c}{1-c}\,\frac{2x}{\pi R^2} \tag{15}$$

Thus, if the packing density, c, and the fibre radius, R, are known it only remains to calculate x in order to obtain a theoretical evaluation of the index of filtration efficiency.

In deriving equation (15) we have supposed that the flow is predominantly normal to the axes of the fibres and that a region exists upstream of each fibre where the mean velocity of the approaching air is equal to v, as defined by equation (12), and is independent of x for a region rather wider than x_1x_2. Another difficulty arises in connection with R, the fibre radius.

Dispersal of the fibrous material is rarely so perfect that the fibres act individually. In fact, the effective fibre radius is usually greater than the actual radius measured under a microscope.

Some tendency to clumping exists and the air prefers to flow through the more open interstices, avoiding the fine spaces within clumps. In addition, a proportion of fibres is disposed parallel to the flow.

Errors due to these causes are diminished by interpreting R as an effective radius determined empirically in the manner to be described.

From dimensional analysis, assuming D'Arcy's law of flow through porous media, it is easy to see that a unique relationship must exist between c, the packing density, and the dimensionless group

$$\frac{4PA\varrho^2}{\eta Qh}$$

where ϱ is the mean fibre radius, P is the pressure drop across the pad in absolute units, and η is the viscosity of air. This holds for geometrically similar arrays of fibres.

Measurements on actual pads made of a wide variety of fibrous materials are exhibited on figure (13). It will be seen that the scatter about the curve

$$\frac{4\,P A\,\varrho^2}{\eta\,Q\,h} = 64\,c^{1.5}\,(1 + 56\,c^3) \tag{16}$$

is not very great in view of the enormous ranges of materials and packing densities which are covered. There is no appreciable dependence on length of fibre and the scatter represents the influence of clumping, orientation and type of fibre. It is governed by the material and the method of dispersion.

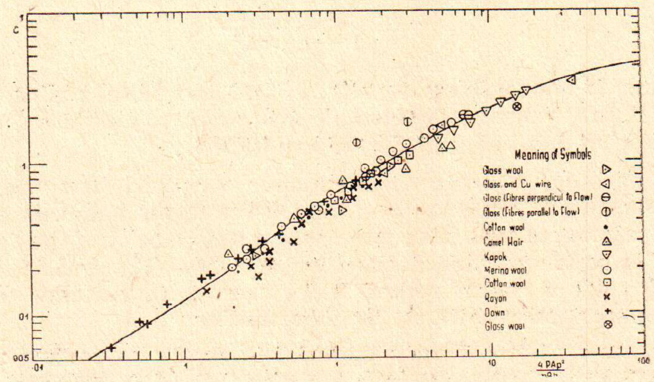

Fig. 13 — Fluid flow through fibrous materials

Knowing the other variables from experiment, equation (16) can be used to define the mean fibre radius, ϱ. Furthermore, the difference between this result and the actual, measured value, using the microscope, is a good indication of the efficiency of the dispersing process employed to make the filter pad, be this by carding, paper making, air flotation or some other technique.

If ϱ exceeds the measured radius it is due to aggregation of fibres, while if it is smaller the reason is probably that a fair proportion of fibres are parallel to the flow.

Other forms of equation (16) have been proposed for work with fibres by *Wiggins, Campbell* and *Maass* (1939), *Fowler* and *Hertel* (1940) and *Sullivan* and *Hertel* (1940), based on the Kozeny-Carman equation *(Carman,* 1937). However, the latter equation can only be adjusted to fit experimental data for high values of the packing density, *c*, and predicts too low a resistance for loosely packed filters. It is not possible to obtain better agreement by merely altering the Kozeny constant; the function itself is wrong.

It should be noted that when the Reynolds' number of flow with respect to the fibre ($2vR/\nu$) is greater than one, equation (16) is no longer valid because D'Arcy's law is not obeyed and turbulence develops within the interstices of the filter.

By comparison with experimental data on the efficiency of air filtration, it has been found that for low values of c the mean fibre radius, ϱ, obtained in this way is in fact identical with the radius, R, which is effective in filtration. However, when c exceeds about 0.02, that is in closely packed filters, the use of ϱ in filtration theory predicts too high an efficiency. The tendency for air flow to take place through the more open interstices therefore has a greater effect on filtration than on air resistance, and R is greater than ϱ.

Correlation of experimental results for filtration, using pads of wool, cotton, rayon, glass wool and steel wool, with radii measuring from 0.8 μ to 40 μ, has shown that a good approximation is given by the expression

$$\frac{4\,P\,R^2}{\eta\, v\,(1-c)\,h} = 70\,c^{1.5}\,(1 + 52\,c^{1.5}) \qquad (16a)$$

This formula gives ϱ and R in close agreement for small values of c, and $R > \varrho$, usually, for larger values. As a first approximation, the effective radius, R, calculated from (16a) must be used in the following theory of filtration. Deviations from (16a) will, of course, occur as a result of variations in the way the fibres lie in the pad. If filtration efficiency is determined for a few pads of a new material it is possible to check, and if necessary, modify (16a) for that material. This is the sense in which equation (16a) is to be regarded as giving a first approximation to R, the effective radius of the fibres. As in the case of resistance, R usually exceeds the measured radius owing to the presence of clumps of fibres. Occasionally, however, a lower value of R is found which is probably associated with fibre orientation parallel to the mean air flow.

Having thus defined the constants c and R, we will now return to equation (15) and consider the variable x.

A particle approaching a fibre, carried along by the air on a certain streamline of flow, x_1, just touches the fibre; particles outside the region x_1x_2 will miss, those inside will strike. We have already considered the effect of particle inertia expressed by the particle parameter which we will now write P_i. When $P_i = 0$ the trajectory x_1 is identical with a streamline of flow. However, inertia impingement is only important for particles above a certain size at sufficiently high speeds. Diffusion of particles towards the fibres is the principal mechanism of filtration for very fine particles. We shall now define P_d, a particle parameter expressing the tendency for diffusion towards the fibres.

When a cloud of fine dust particles blows past a fibre, which is a perfect absorber of particles, a concentration gradient is established around the fibre down which particles steadily diffuse while flowing past; near the surface of the fibre the concentration is practically zero and at a distance it is equal to that in the incident cloud.

The exact mathematical solution of this problem has not been obtained, but it is easy to show, just as was done for equation (4) in the case of impingement, that the efficiency of collection by diffusion in steady flow can be expressed as

$$E = F\left(\frac{\Delta}{vR}, \frac{2vR}{\nu}, \frac{a}{R}\right) \tag{17}$$

where Δ is the diffusion coefficient of the particles and E is the ratio of the number of particles collected by the fibre to the number which would have been collected if all those approaching in a direct line with it were caught (i. e. $P_d = \infty$ as in figure (2)).

By analogy with equation (4) we see that

$$P_d = \frac{\Delta}{vR}. \tag{18}$$

The region surrounding a fibre within which there is a deficit of concentration is not sharply defined. However, it is possible to define an arbitrary ordinate, x, at adistance upstream, so that the actual number of particles striking the fibre per second, per unit length, is equal to $2x$ multiplied by the particle concentration in the approaching air, n, and also multiplied by v, where n and v are almost independent of x at a suitable distance upstream of the fibre under consideration.

This being done, the distance x is in perfect analogy with the corresponding distance for impingement and we can say, whatever the mechanism of filtration, that in effect particles approaching outside x will not be filtered.

It is also possible to define a general particle parameter

$$P = P_i + P_d \tag{19}$$

as a measure of likelihood of filtration, though it will be found that when either of the components is appreciable, the other is always very small so that it usually suffices to take P as equal to either P_i or P_d. It is possible that it may eventually turn out best to weight P_d in (19) but there are no experimental results at present which show this to be desirable.

It is convenient at this stage to write

$$\Delta = \frac{2.10^{-16}}{a^2} \; cm^2/sec$$

for air at 20° C and one atmosphere. This varies a little too rapidly with particle radius but it is about right in the important range $0.05 < a < 0.15$ microns. Hence,

$$\left.\begin{aligned} P_i &= \frac{2\,\varrho\, a^2\, v}{9\,\eta\, R} \\ P_d &= \frac{2.10^{-16}}{a^2\, v\, R} \end{aligned}\right\} \qquad (20)$$

Both parameters are functions of a^2v, diffusion falling off as a^2v increases, while impingement increases in sympathy with it. These expressions are plotted on figure (14) for particles of unit density in air at 20° C and one atmosphere for a range of values of fibre radius, R. Knowing air speed, particle radius and fibre radius, figure (14) enables us to see whether diffusion, direct interception or impingement will constitute the main mechanism of filtration. It also shows that a filter will possess the minimum of efficiency when the fibre radius is large and

$$a^2\, v \approx 10^{-9.4}\ cm^3/sec \qquad (21)$$

In this region direct interception is the chief filtration process. Particles sizes for optimum penetration are shown in Table III.

TABLE III.

The sizes of particles which penetrate fibrous filters most readily. (Equation 21).

Mean air velocity, v, in filter. cm/sec.	Optimum particle radius for penetration. μ
100	.02
16	.05
4	.1
1.8	.15
1.0	.2
.64	.25
.44	.3
.25	.4
.16	.5

We now turn to hydrodynamic considerations and the critical ordinate, x. Neglecting the variation with Reynolds' number, which we have already seen to be small, especially when the packing density is high we can write

$$x/R = f\ (c, P, a/R) \tag{22}$$

An exact evaluation of this function is impossible at present owing to difficulties in the hydrodynamics, however, by making full use of results which have recently been obtained we can derive a first approximation.

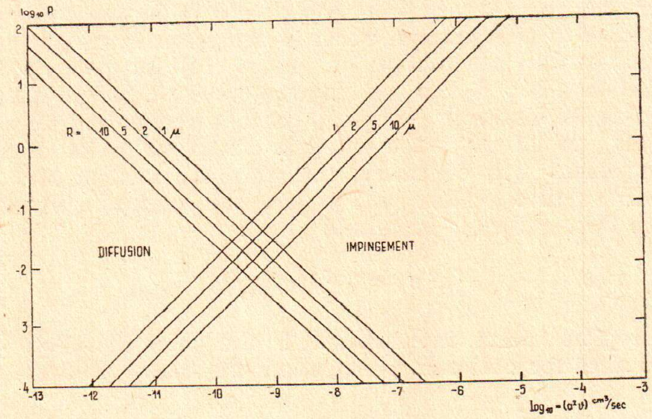

Fig. 14 — Particle filtration parameters

$$P_d = \frac{\Delta}{R\,v} \approx \frac{2.10^{-16}}{a^2\,v\,R},\quad P_i = \frac{2\,\varrho\,a^2\,v}{9\,\eta\,R}$$

It is at first noted that x/R becomes equal to zero only when P and a/R are both equal to zero, and that when c tends to zero we are dealing with an isolated fibre.

Consider the case when P is zero so that particles travel along streamlines of flow and deposit by direct interception. Using the hydrodynamic formulae of *Davies* (1950) for flow past an isolated cylinder ($c \rightarrow 0$) the graph of x/R against a/R on figure 15 has been plotted. There are no correspondingly exact hydrodynamic results for arrays of cylinders ($c > 0$) but *Kovasznay* (1947) has obtained an exact periodic solution to the Navier-Stokes equations in two dimensions from which it is possible to estimate the lateral displacement of streamlines passing through a grid of obstacles. Using this information the two other curves of figure 15 for $c = 0.08$ and 0.213 were estimated.

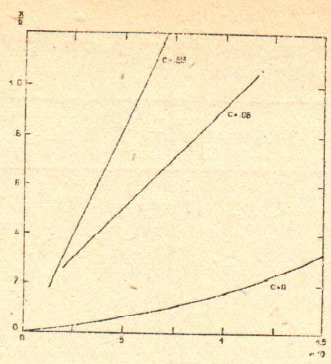

Fig. 15 — Interception of particles by a cylinder P = O,
Viscous flow c = o, R = 0.2.
Isolated cylinder. (Davies).
Transition. Grid of cylinders.
c = 0.08 } spacing { 3.84 R
c = 0.213 } of centres { 2 π R
(Kovasznay)

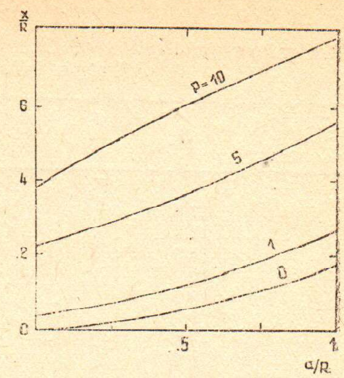

Fig. 16 — Effect of P (due to impingement or diffusion) on interception. From impingement calculations upon isolated cylinders (cf. fig. 14)

These curves, for $P = 0$ are approximately represented by

$$x/R = (a/r)\ (0.16 + 10.9c - 17c^2).$$

When $c = 0$ we have already plotted on figure (4) the relationship of efficiency of impingement to P for various values of a/R, these results being based on stepwise calculations of particle trajetories in an exact viscous flow field for $Re = 0.2$. From these curves we have constructed graphs of x/R against a/R for different values of P which is now interpreted according to equation (19). The graphs are shown on figure 16 and are fitted by the formula

$$x/R = 0.16\ \ [a/R + (0.25 + 0.4a/R)\ \ P - 0.0263aP^2/R]$$

when $c = 0$.

Combining the last two numerical equations to fulfil equation (22) yields the general expression for x/R, or rather a first approximation to it,

$$x/R = [a/R + (0.25 + 0.4a/R)\ \ P - 0.0263aP^2/R]\ \ (0.16 + 10.9c - 17c^2) \qquad (23)$$

This constitutes the best solution of the mechanical filter problem to date and in conjunction with (20), (15) and (13) enables the perfomance of filters composed of masses of fibres to be calculated. Figures 17,18 and 19 can be used to facilitate the calculation and Table IV enables the results to be easily converted into percentage penetration.

TABLE IV.

Percentage penetration of filters as a function of thickness, h, cm, and filtration index, γ, cm^{-1}. (Equation 13).

h	% Penetration, $100n/n_0$
13.9	.0001
12.2	.0005
11.5	.001
10.8	.002
9.9	.005
9.2	.01
7.6	.05
6.9	.1
5.3	.5
4.6	1.0
3.0	5
2.3	10
1.61	20
0.694	50
0.511	70
0.223	80
0.105	90

No allowance has been made for sedimentation inside the filter because with most of the fibres transverse to the flow this is not an important process of filtration. It is possible that the theory could be improved by taking a proportion of the fibres to be distributed parallel to the air stream. In this case the effect of impingement is diminished while diffusion and sedimentation become more important. Further improvement could perhaps be effected, especially for coarse dust, by allowing for a fraction of the collisions between particle and fibre which did not result in adhesion.

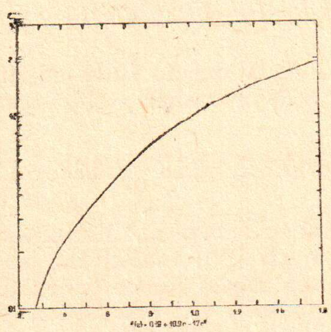

Fig. 17 — Calculation of filtration efficiency

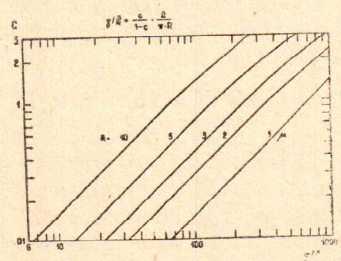

Fig. 19 — Calculation of filtration efficiency. Equ. 15

Another point which might be developed is the hydrodynamic repulsion between particle and fibre, when they have comparable radii, which was briefly mentioned in the preceding section. A theory on analogous lines could probably be developed for resin filters which, in addition to the three mechanical processes dealt with in this section, operate by electrostatic attraction of particles towards the resin coated fibres and are thereby rendered more efficient.

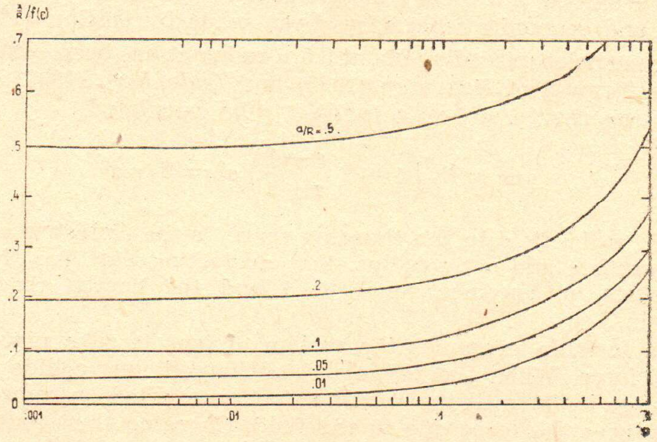

Fig. 18 — Calculation of filtration efficiency. Equ. 23

5. Electrostatic Precipitation.

The history of Electrical precipitation has been discussed by *Lodge* (1925); the process essentially arose from experiments by *Lodge* and *Clark* in 1884 and during the next 30 years was adopted on a large scale for purifying blast furnace gas. Many other applications in heavy industry followed, including both the recovery of valuable material and the mitigation of atmospheric pollution.

In all such installations the contaminated gas is passed between electrodes with a potential difference of 30—70 Kv. The positive electrode is earthed and has a large surface area, while the negative is usually a thin wire or rod. In the vicinity of the negative electrode the potential gradient is steep enough to maintain a corona discharge which provides a steady flow of negative ions towards the earthed electrode. Current is of the order of 1/3 ma per unit, and several hundred units are often parallel in a single installation.

The negative ions collide with dust particles, which therefore become charged. As the gas passes through the space between the electrodes the particles are swept towards the earthed positive plate, where they deposit, and a purified gas stream emerges.

A common type of electrostatic precipitator consists of a deep honeycomb of cells down the axis of each of which a wire passes. The wires are connected together and carefully insulated; they are made negative and the metal cell walls are positive. The gas passes up the cell spaces. Rapping or shaking gear is necessary on both wire and cell walls to clear the deposit from time to time. Two distinct physical processes take place, the charging of the particle and its subsequent transport to the earthed electrode.

A review of the theory of operation, as well as certain engineering and economic aspects has been made by *Onslow* (1941).

For particles exceeding about 0.5 μ radius it has been established that the formula of *Rohmann* (1923) and *Ladenberg* (1930) enables the limiting charge, ne, on a particle to be calculated.

$$ne = E_0 \left(1 + 2\,\frac{\varepsilon - 1}{\varepsilon + 2}\right) a^2 = E_0\, p\, a^2 \tag{24}$$

where E_0 is the field in the charging space, ε the dielectric constant of the particle and a its radius. For conducting particles the term in brackets, p, becomes equal to 3 and the charge reaches its maximum.

The formula presumes the motion of ions to take place along lines of force. When the particle is uncharged ions collide with it and charge it up until the repulsive action exerted on approaching ions balances the force due to the field. Charging then ceases. The supposition is made that diffusion of the ions and of the particles can be neglected and also that the attractive »mirror« force between a particle and an approaching ion, due to induction, is negligible.

The work of *Pauthenier* (1922, 1934) and of *Fuchs* (1936) shows that these assumptions are justified for sufficiently large particles for which the charging is very rapid.

For smaller particles the neglected factors become important and a full theory is lacking. *Ladenburg* (1930), however, found that with small currents and moderate field strength

$$ne = 9.5 \cdot 10^{-4}\, a. \tag{25}$$

This formula is incorrectly quoted by *Onslow* (1941).

If E is the field strength in the transport region, the velocity of drift of a charged particle obeying Stokes' law is

$$v = \frac{E\, n\, e}{6\, \pi\, a\, \eta}. \tag{26}$$

Combining this with (24) and (25) we have for large particles,

$$v = \frac{p\, E\, E_0\, a}{6\, \pi\, \eta}, \tag{27}$$

and for small,

$$v = \frac{9.5 \cdot 10^{-4} E}{6 \pi \eta}. \tag{28}$$

Hence, in the first case the velocity is proportional to the radius and to the product EE_0, whereas in the second it is proportional to E but independent of particle size.

For a field strength $E = E_0 = 10$ e. s. u. (3000 volts) per cm. the theoretical particle velocities are shown in Table V.

TABLE V.

Particle radius	Velocity	
100 μ	885 cm./sec.	
10 μ	88.5 cm./sec.	Equation (27)
1 μ	8.85 cm./sec.	
.1 μ	.885 cm./sec.	
All sizes	2.8 cm./sec.	Equation (28)

It will be seen that equation (28) predicts, in this typical example, a greater transport velocity for particles of about 0.4 μ radius and under than does equation (27), which is undoubtedly valid for larger sizes. This is because a large uncharged particle intercepts so many ions that the small increase in charge due to attraction by the induced charge and to diffusion is unimportant. These effects are decisive, however, when the number of ion paths covered by the cross section of the particle is small.

It is clear, therefore, that with such high transport velocities it is not a difficult matter to sweep our particles of 1 μ and over in a precipitator tube of reasonable length. Smaller sizes constitute a problem, however, which is worse than it appears from the above figures owing to their slow rate of charging. The limiting charge is reached almost instantaneously for particles above 1 μ. Smaller ones become charged by diffusion, which is a slow process, so that their transport velocities are less than the figure quoted. On the other hand, below 0.05 μ the thermal motion of the particles themselves becomes appreciable and the transport velocity is increased by Cunningham's slip factor.

These considerations lead us to expect that given a reasonable length of gas path it should be possible to precipitate completely particles over 0.5 μ radius. Very minute particles, also, should be readily removed, but in the region of 0.1 μ radius a fall in efficiency may be expected because of the difficulty of charging and the low limiting charge. There is scope for theoretical and experimental research work on the electrical precipitation of particles of this order of size.

In precipitators handling large quantities of dust it is necessary to consider the effect of the charged dust particles upon the field. The transport speed of these towards the collecting electrode is so much less than that of the ions that the current carried by the dust is negligible; it produces a space charge in the field, however, which diminishes the current considerably and renders precipitation less efficient.

Accumulation of dust on the deposit electrode may also lead to a fall in efficiency due to back ionization resulting from intense fields produced in local discontinuities of the deposit.

A number of small precipitators have been produced for sampling dust. *Drinker* (1932) demonstrated how ozone and oxides of nitrogen could be rendered negligible by working at a low voltage. *Barnes* and *Penney* (1936) and *Patrick* (1947) made instruments for measuring mass and number concentration of dust and *Lea* (1943) describes a battery operated particle outfit. No convincing data on efficiency as a function of particle size are given in any of these papers, however.

In a unit now being used for air purification, rather than the recovery of air-borne material, the funktions of particle charging and sweep-out are separated and a higher efficiency is obtained than in the more orthodox designs. Basic units handle 600 or 1200 cu. ft/min of air which passes first through an ionizer and then between parallel, charged plates. Positive ion current is used with a positive electrode of small radius and a large, earthed negative. This feature is claimed to decrease ozone production.

Elimination of particles is best for sizes over 1 micron at low speeds of flow and with the voltage just below sparking. There is improvement as the plates become coated with dust, up to a point. As with all electrostatic separators it is difficult to guarantee an extremely high degree of purification since a small proportion of particles seems to slip through unless the air flow is kept quite small. The method is ideal, however, for coping with large quantities of dust.

6. Thermal separation

This process, which is applied in the thermal precipitator for sampling dust, has been a subject of investigation for some time but a complete theory is not yet available. It is allied on the one hand to thermal diffusion, which is treated in the kinetic theory of mixed gases and accounts for a tendency for heavy molecules to concentrate in cool regions. On the other hand, the radiometer force of gas molecules upon a heated surface represents a limiting case for large particles. There is also the question of the influence of convection which is important in isotope separation.

A discussion of the motion of particles in a temperature gradient has been presented by *Rosenblatt* and *LaMer* (1946) who applied radiometer theory and carried out experiments with small temperature gradients. *Weber* (1947) has also published some measurements.

For pressures near to one atmosphere the thermal velocity of oil droplets was found to be only slightly dependent upon their radius over the range from 0.4 to 2 μ. The velocity under these conditions agrees closely with that calculated from an approximate theory due to Einstein, which fails at lower pressures where variation with particle radius also appears.

For designing particle separators the following formula may be of use:

$$v = \frac{0.036}{T} \frac{dT}{dx}, \tag{29}$$

where v cm/sec. is the thermal velocity of particles down the temperature gradient dT/dx (degrees absolute per cm.). The formula is a good approximation under the conditions quoted. Rosenblatt and LaMer worked with temperature gradients up to only about 50° per cm. Weber went up 500° per cm. but his transport velocity at the highest gradient is over double that given by equation (29), though this may be due to other causes. Convection may influence the velocity and also the material of the particles may be important.

In the thermal precipitator for dust sampling the gradient is some 4000° per cm., but experiments have suggested that equation (29) gives an indication at least of the order of magnitude of the particle velocity.

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SADRŽAJ

ODJELJIVANJE PRAŠINE I ZRAKOM PRENOŠENIH ČESTICA

U ovoj radnji je izložena fizikalna teorija, na kojoj se osnivaju različite metode za odjeljivanje čestica od plinova. Glavna pažnja je posvećena osnovnim razmatranjima. Ta razmatranja dovode do matematičkih izraza, koji mogu biti korisni onima, koji se bave konstrukcijom i pogonom separatora.

Odstranjivanje prašine iz radne atmosfere vrši se tako, da se modificira proizvodni proces, kod kojeg se stvara prašina, ali tako da se zrakom prenošene čestice naknadno odstrane. Bolje je ukloniti izvor prašine. Ako to nije moguće, treba upotrebiti struju zraka, koja prenosi čestice prašine na mjesto, gdje se one mogu istaložiti bez štete; druga je mogućnost da se upotrebi separator bilo koje vrste. Separatori se osnivaju na mehaničkim, električnim ili termičkim silama. Među mehaničke metode pripadaju posude za sedimentaciju i elutrijatori, koji se osnivaju na djelovanju sile teže, zatim cikloni, skraberi i slične aparature, koje djeluju zbog inercije čestica. Djelovanje tih aparatura slično je filtrovima, jer su i u slojevima za filtraciju sile gravitacije i inercije čestica od naročite važnosti, premda djelovanje fibroznih filtrova za odjeljivanje vrlo finih čestica zavisi djelomično i od gibanja čestica pod utjecajem termičkog kretanja molekula zraka.

Odjeljivanje spomoću termičke defuzije ili radiometarskih sila predstavlja također mogućnost za konstrukciju separatora za male čestice. Odjeljivanje malih čestica naročito je važno kod ličnih respiratora i kod izvedbe aparata za skupljanje uzoraka prašine u analitičke svrhe. U nekim fibroznim filtrovima dolaze do izražaja i električne sile. Na električnim silama osnivaju se i električni precipitatori, koji su vrlo djelotvorni za grube čestice.

Autor predlaže nove teorije ciklona i filtracija zraka na fibroznim filtrovima. Izlaganje je popraćeno grafovima i tablicama, koji omogućuju da se teorija bez poteškoća primijeni na praktične probleme.

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