Minimal structures, punctually m-open functions in the sense of Kuratowski and bitopological spaces

TAKASHI NOIRI* AND VALERIU POPA†

Abstract. *By using m-open functions from a topological space into an m-space, we establish the unified theory for several weak forms of open functions in the sense of Kuratowski between bitopological spaces.*

Key words: *m-structure, m-open, (i, j)-K-m-open function, bitopological space*

AMS subject classifications: 54A05, 54C10, 54E55

Received May 15, 2007 Accepted September 18, 2007

1. Introduction

Semi-open sets, preopen sets, α-open sets, β-open sets and δ-open sets play an important role in the research of generalizations of open functions in topological spaces and bitopological spaces. By using these sets, several authors introduced and studied various types of modifications of open functions in topological spaces and bitopological spaces. Maheshwari and Prasad [10] and Bose [1] introduced the concepts of semi-open sets and semi-open functions in bitopological spaces. Jelić [2], [4], Kar and Bhattacharyya [5] and Khedr et al. [6] introduced and studied the concepts of preopen sets and preopen functions in bitopological spaces. The notions of α-open sets and α-open functions in bitopological spaces were studied in [3], [12] and [7]. The notion of semi-preopen sets in bitopological spaces was studied in [6]. Recently, in [14] and [15] the present authors introduced the notions of minimal structures, m-spaces and m-continuity. Quite recently, in [13], they have introduced some generalizations of open functions in bitopological spaces.

In the present paper, we introduce the punctual notion of m-open functions in the sense of Kuratowski [8]. We obtain some characterizations of a function which is m-open at a point of the range of the function and also characterize the set of all points at which a function is not m-open.

*2949-1 Shiokita-Cho, Hinagu, Yatsushiro-Shi, Kumamoto-Ken, 8695142 Japan, e-mail: t.noiri@nifty.com

†Department of Mathematics, University of Bacău, 600 114 Bacău, Romaina, e-mail: vpopa@ub.ro

2. Preliminaries

Let (X, τ) be a topological space and A a subset of X. The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. Throughout the present paper, (X, τ) and (Y, σ) always denote topological spaces and (X, τ_1, τ_2) and (Y, σ_1, σ_2) denote bitopological spaces. The closure of A and the interior of A with respect to τ_i are denoted by $i\mathrm{Cl}(A)$ and $i\mathrm{Int}(A)$, respectively, for $i = 1, 2$.

Definition 1. *A subfamily m_X of the power set $\mathcal{P}(X)$ of a nonempty set X is called a* minimal structure *(or briefly m-*structure*) [14], [15] on X if $\emptyset \in m_X$ and $X \in m_X$.*

By (X, m_X) (or briefly (X, m)), we denote a nonempty set X with a minimal structure m_X on X and call it an m-space. Each member of m_X is said to be m_X-open (or briefly m-open) and the complement of an m_X-open set is said to be m_X-closed (or briefly m-closed).

Definition 2. *Let X be a nonempty set and m_X an m-structure on X. For a subset A of X, the m_X-*closure* of A and the m_X-*interior* of A are defined in [11] as follows:*

(1) m_X-$Cl(A) = \cap\{F : A \subset F, X - F \in m_X\}$,

(2) m_X-$Int(A) = \cup\{U : U \subset A, U \in m_X\}$.

Lemma 1. (Maki et al. [11]). *Let (X, m_X) be an m-space. For subsets A and B of X, the following properties hold:*

(1) m_X-$\mathrm{Cl}(X - A) = X - m_X$-$\mathrm{Int}(A)$ *and* m_X-$\mathrm{Int}(X - A) = X - m_X$-$\mathrm{Cl}(A)$,

(2) if $(X - A) \in m_X$, *then* m_X-$\mathrm{Cl}(A) = A$ *and if* $A \in m_X$, *then* m_X-$\mathrm{Int}(A) = A$,

(3) m_X-$\mathrm{Cl}(\emptyset) = \emptyset, m_X$-$\mathrm{Cl}(X) = X$, m_X-$\mathrm{Int}(\emptyset) = \emptyset$ *and* m_X-$\mathrm{Int}(X) = X$,

(4) if $A \subset B$, *then* m_X-$\mathrm{Cl}(A) \subset m_X$-$\mathrm{Cl}(B)$ *and* m_X-$\mathrm{Int}(A) \subset m_X$-$\mathrm{Int}(B)$,

(5) $A \subset m_X$-$\mathrm{Cl}(A)$ *and* m_X-$\mathrm{Int}(A) \subset A$,

(6) m_X-$\mathrm{Cl}(m_X$-$\mathrm{Cl}(A)) = m_X$-$\mathrm{Cl}(A)$ *and* m_X-$\mathrm{Int}(m_X$-$\mathrm{Int}(A)) = m_X$-$\mathrm{Int}(A)$.

Lemma 2. (Popa and Noiri [14]). *Let (X, m_X) be an m-space and A a subset of X. Then $x \in m_X$-$\mathrm{Cl}(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \in m_X$ containing x.*

Definition 3. A minimal structure m_X on a nonempty set X is said to have *property $\mathcal{B}$* [11] if the union of any family of subsets belonging to m_X belongs to m_X.

Lemma 3. (Popa and Noiri [16]). *Let (X, m_X) be an m-space and m_X satisfy property $\mathcal{B}$. Then for a subset A of X, the following properties hold:*

(1) $A \in m_X$ *if and only if* m_X-$\mathrm{Int}(A) = A$,

(2) A *is m-closed if and only if* m_X-$\mathrm{Cl}(A) = A$,

(3) m_X-$\mathrm{Int}(A) \in m_X$ *and* m_X-$\mathrm{Cl}(A)$ *is* m_X*-closed.*

3. Punctual m-openness in the sense of Kuratowski

Definition 4. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be* K-open at $y \in Y$ *[8] if for each open set U of X such that $y \in f(U)$, there exists an open set V of Y such that $y \in V \subset f(U)$.*

Definition 5. *A function $f : (X, \tau) \to (Y, m_Y)$ is said to be* K-m-open at $y \in Y$ *if for each open set U of X such that $y \in f(U)$, there exists $V \in m_Y$ such that $y \in V \subset f(U)$. The function f is said to be* K-m-open *if it is K-m-open at every point $y \in Y$.*

Theorem 1. *For a function $f : (X, \tau) \to (Y, m_Y)$, the following properties are equivalent:*

(1) f is K-m-open at $y \in Y$;

(2) for every open set G of X such that $y \in f(G)$, $y \in m_Y$-$\mathrm{Int}(f(G))$;

(3) if $y \in f(\mathrm{Int}(A))$ for $A \in \mathcal{P}(X)$, then $y \in m_Y$-$\mathrm{Int}(f(A))$;

(4) if $y \in f(\mathrm{Int}(f^{-1}(B)))$ for $B \in \mathcal{P}(Y)$, then $y \in m_Y$-$\mathrm{Int}(B)$;

(5) if $y \in m_Y$-$\mathrm{Cl}(B)$ for $B \in \mathcal{P}(Y)$, then $f^{-1}(y) \subset \mathrm{Cl}(f^{-1}(B))$.

Proof. (1) ⇒ (2): Let G be any open set of X such that $y \in f(G)$. Then, there exists $V \in m_Y$ such that $y \in V \subset f(G)$; hence $y \in m_Y$-$\mathrm{Int}(f(G))$.
(2) ⇒ (3): Let $A \in \mathcal{P}(X)$. Then $\mathrm{Int}(A)$ is open in X. Since $y \in f(\mathrm{Int}(A))$, by (2) $y \in m_Y$-$\mathrm{Int}(f(\mathrm{Int}(A))) \subset m_Y$-$\mathrm{Int}(f(A))$.
(3) ⇒ (4): Let $B \in \mathcal{P}(Y)$ and $y \in f(\mathrm{Int}(f^{-1}(B)))$. By (3), $y \in m_Y$-$\mathrm{Int}(f(f^{-1}(B))) \subset m_Y$-$\mathrm{Int}(B)$.
(4) ⇒ (1): Let U be an open set of X such that $y \in f(U)$. Then $U \subset f^{-1}(f(U))$ and hence $U \subset \mathrm{Int}(f^{-1}(f(U)))$. Therefore, $y \in f(\mathrm{Int}(f^{-1}(f(U))))$. By (4), $y \in m_Y$-$\mathrm{Int}(f(U))$. Hence, there exists $V \in m_Y$ such that $y \in V \subset f(U)$. Therefore, f is K-m-open at y.
(2) ⇒ (5): Let $y \in Y$ and $B \in \mathcal{P}(Y)$ such that $f^{-1}(y)$ is not contained in $\mathrm{Cl}(f^{-1}(B))$. Put $G = X - \mathrm{Cl}(f^{-1}(B))$. Then $f^{-1}(y) \cap G \neq \emptyset$. Hence $y \in f(G) = f(X - \mathrm{Cl}(f^{-1}(B))) \subset f(f^{-1}(Y - B)) \subset Y - B$. By (2), we have $y \in m_Y$-$\mathrm{Int}(Y - B) = Y - m_Y$-$\mathrm{Cl}(B)$. Hence $y \notin m_Y$-$\mathrm{Cl}(B)$.
(5) ⇒ (2): Let G be an open set of X and $y \in Y - m_Y$-$\mathrm{Int}(f(G)) = m_Y$-$\mathrm{Cl}(Y - f(G))$. Set $B = Y - f(G)$. Then $y \in m_Y$-$\mathrm{Cl}(B)$. Therefore, $f^{-1}(y) \subset \mathrm{Cl}(f^{-1}(B)) = \mathrm{Cl}(f^{-1}(Y - f(G))) \subset \mathrm{Cl}(X - f^{-1}(f(G))) \subset \mathrm{Cl}(X - G) = X - G$. Hence $f^{-1}(y) \cap G = \emptyset$ and $y \notin f(G)$. □

Theorem 2. *A function $f : (X, \tau) \to (Y, m_Y)$ is K-m-open if and only if m_Y-$\mathrm{Int}(f(U)) = f(U)$ for each open set U of X.*

Proof. *Necessity.* Let $y \in Y$ and U be an open set of X such that $y \in f(U)$. Then, by *Theorem 1* we have $y \in m_Y$-$\mathrm{Int}(f(U))$. Therefore, $f(U) \subset m_Y$-$\mathrm{Int}(f(U))$ and by *Lemma 1* $f(U) = m_Y$-$\mathrm{Int}(f(U))$.
Sufficiency. Let $y \in Y$ and U be an open set of X such that $y \in f(U)$. Then we have $y \in f(U) = m_Y$-$\mathrm{Int}(f(U))$. Therefore $y \in m_Y$-$\mathrm{Int}(f(U))$. By *Theorem 1*, f is K-m-open at y. □

Remark 1. Theorem 2 *shows that a function* $f : (X, \tau) \to (Y, m_Y)$ *is* K*-*m*-open if and only if it is* m*-open in [13].*

For a function $f : (X, \tau) \to (Y, m_Y)$ we denote

$$D^0(f) = \{y \in Y\colon f \text{ is not } K\text{-}m\text{-open at } y\ \}.$$

Theorem 3. *For a function* $f : (X, \tau) \to (Y, m_Y)$ *the following properties hold:*

$$D^0(f) = \cup_{G\in\tau}\{f(G) - m_Y - \mathrm{Int}(f(G))\} = \cup_{A\in\mathcal{P}(\mathcal{X})}\{f(\mathrm{Int}(A)) - m_Y - \mathrm{Int}(f(A))\}$$
$$= \cup_{B\in\mathcal{P}(\mathcal{Y})}\{f(\mathrm{Int}(f^{-1}(B))) - m_Y - \mathrm{Int}(B))\}.$$

Proof. Let $y \in D^0(f)$. Then, by *Theorem 2*, there exists an open set G_0 of X such that $y \in f(G_0)$ and $y \notin m_Y$-Int$(f(G_0))$. Hence we obtain

$$y \in f(G_0) - m_Y\text{-Int}(f(G_0)) \subset \cup_{G\in\tau}\{f(G) - m_Y\text{-Int}(f(G))\}.$$

Conversely, let $y \in \cup_{G\in\tau}\{f(G) - m_Y\text{-Int}(f(G))\}$. Then there exists an open set G_0 of X such that $y \in f(G_0) - m_Y$-Int$(f(G_0))$. Therefore, by *Theorem 2* f is not K-m-open at y and $y \in D^0(f)$.

The other equations are proved silimarly. □

4. Minimal structures on bitopological spaces

First, we shall recall some definitions of weak forms of open sets in a bitopological space.

Definition 6. *A subset* A *of a bitopological space* (X, τ_1, τ_2) *is said to be*

(1) (i, j)-semi-open *[10] if* $A \subset j\mathrm{Cl}(i\mathrm{Int}(A))$*, where* $i \neq j$*,* $i, j = 1, 2$*,*

(2) (i, j)-preopen *[2] if* $A \subset i\mathrm{Int}(j\mathrm{Cl}(A))$*, where* $i \neq j$*,* $i, j = 1, 2$*,*

(3) (i, j)-α-open *[3] if* $A \subset i\mathrm{Int}(j\mathrm{Cl}(i\mathrm{Int}(A)))$*, where* $i \neq j$*,* $i, j = 1, 2$*,*

(4) (i, j)-semi-preopen *[6] if there exists an* (i, j)*-preopen set* U *such that* $U \subset A \subset j\mathrm{Cl}(U)$*, where* $i \neq j$*,* $i, j = 1, 2$*.*

The family of (i, j)-semi-open (resp. (i, j)-preopen, (i, j)-α-open, (i, j)-semi-preopen) sets of (X, τ_1, τ_2) is denoted by $(i, j)\mathrm{SO}(X)$ (resp. $(i, j)\mathrm{PO}(X)$, $(i, j)\alpha(X)$, $(i, j)\mathrm{SPO}(X)$).

Remark 2. *Let* (X, τ_1, τ_2) *be a bitopological space and* A *a subset of* X*. Then* $(i, j)SO(X)$*,* $(i, j)PO(X)$*,* $(i, j)\alpha(X)$ *and* $(i, j)SPO(X)$ *are all* m*-structures on* X*. Hence, if* $m_{ij} = (i, j)SO(X)$ *(resp.* $(i, j)PO(X)$*,* $(i, j)\alpha(X)$*,* $(i, j)SPO(X)$*), then we have*

(1) m_{ij}-Cl$(A) = (i, j)$-sCl(A) *[10] (resp.* (i, j)-pCl(A) *[6],* (i, j)-αCl(A) *[12],* (i, j)-spCl(A) *[6]),*

(2) m_{ij}-Int$(A) = (i, j)$-sInt(A) *(resp.* (i, j)-pInt(A), (i, j)-αInt(A), (i, j)-spInt(A)*).*

Remark 3. *Let* (X, τ_1, τ_2) *be a bitopological space.*

(a) *Let* $m_{ij} = (i,j)SO(X)$ *(resp.* $(i,j)\alpha(X)$*). Then, by* Lemma 1 *we obtain the result established in Theorem 13 of [10] (resp. Theorem 3.6 of [12]).*

(b) *Let* $m_{ij} = (i,j)SO(X)$ *(resp.* $(i,j)PO(X)$, $(i,j)\alpha(X)$, $(i,j)SPO(X)$*). Then, by* Lemma 2 *we obtain the result established in Theorem 1.15 of [9] (resp. Theorem 3.5 of [6], Theorem 3.5 of [12], Theorem 3.5 of [6]).*

Remark 4. *Let* (X, τ_1, τ_2) *be a bitopological space.*

(a) *It follows from Theorem 2 of [10] (resp. Theorem 4.2 of [5] or Theorem 3.2 of [6], Theorem 3.2 of [12], Theorem 3.2 of [6]) that* $(i,j)SO(X)$ *(resp.* $(i,j)PO(X)$, $(i,j)\alpha(X)$, $(i,j)SPO(X)$*) is an* m*-structure on* X *satisfying property* $\mathcal{B}$.

(b) *Let* $m_{ij} = (i,j)SO(X)$ *(resp.* $(i,j)PO(X)$, $(i,j)\alpha(X)$, $(i,j)SPO(X)$*). Then, by* Lemma 3 *we obtain the result established in Theorem 1.13 of [9] (resp. Theorem 3.5 of [6], Theorem 3.6 of [12], Theorem 3.6 of [6]).*

5. K-m-open functions and bitopological spaces

Definition 7. *A function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be* (i,j)-semi-open *[1] (resp.* (i,j)-preopen *[5],* (i, j)-α-open *[7],* (i, j)-semi-preopen*) if for each* τ_i*-open set* U *of* X, $f(U)$ *is* (i,j)*-semi-open (resp.* (i,j)*-preopen,* (i,j)*-*α*-open,* (i,j)*-semi-preopen) in* Y.

Remark 5. *By Remark 4(a)*

$$(i,j)SO(Y),\ (i,j)PO(Y),\ (i,j)\alpha(Y) \text{ and } (i,j)SPO(Y)$$

are all m*-structures on* Y *satisfying property* $\mathcal{B}$*. Therefore, a function*

$$f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$$

is (i,j)*-semi-open (resp.* (i,j)*-preopen,* (i,j)*-*α*-open,* (i,j)*-semi-preopen) if and only if* $f : (X, \tau_i) \to (Y, m_{ij})$ *is* m*-open, where* $m_{ij} = (i,j)SO(Y)$ *(resp.* $(i,j)PO(Y)$, $(i,j)\alpha(Y)$, $(i,j)SPO(Y)$*).*

Definition 8. *Let* (Y, σ_1, σ_2) *be a bitopological space and* m_{ij} *an* m*-structure on* Y *determined by* σ_1 *and* σ_2*. A function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be* (i,j)-K-m-open at $y \in Y$ *if* $f : (X, \tau_i) \to (Y, m_{ij})$ *is* K*-*m*-open at* y*. The function* f *is said to be* (i, j)-K-m-open *if it is* (i,j)*-*K*-*m*-open at each* $y \in Y$.

Remark 6. *By* Definition 5.2*, it follows that a function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is* (i,j)*-*K*-*m*-open at* $y \in Y$ *if and only if for each* τ_i*-open set* U *such that* $y \in f(U)$*, there exists an* m_{ij}*-open set* V *of* Y *such that* $y \in V \subset f(U)$.

By *Definition 8* and *Theorems 1* and *2*, we obtain the following theorems.

Theorem 4. *Let* (Y, σ_1, σ_2) *be a bitopological space and* m_{ij} *an* m*-structure on* Y *determined by* σ_1 *and* σ_2*. For a function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) *f is (i, j)-K-m-open at* $y \in Y$;

(2) *for every* $G \in \tau_i$ *such that* $y \in f(G)$, $y \in m_{ij}\text{-Int}(f(G))$;

(3) for every $A \in \mathcal{P}(X)$ such that $y \in f(i\mathrm{Int}(A))$, then $y \in m_{ij}\text{-}\mathrm{Int}(f(A))$;

(4) for every $B \in \mathcal{P}(Y)$ such that $y \in f(i\mathrm{Int}(f^{-1}(B)))$, then $y \in m_{ij}\text{-}\mathrm{Int}(B)$;

(5) for $B \in \mathcal{P}(Y)$ such that $y \in m_{ij}\text{-}\mathrm{Cl}(B)$, $f^{-1}(y) \subset i\mathrm{Cl}(f^{-1}(B))$.

For example, put $m_{ij} = (i,j)\mathrm{PO}(Y)$, then we obtain the following characterizations:

Corollary 1. *Let (Y, σ_1, σ_2) be a bitopological space and $m_{ij} = (i,j)PO(Y)$. For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is (i, j)-K-preopen at $y \in Y$;

(2) for every $G \in \tau_i$ such that $y \in f(G)$, $y \in (i,j)\text{-}\mathrm{pInt}(f(G))$;

(3) for every $A \in \mathcal{P}(X)$ such that $y \in f(i\mathrm{Int}(A))$, then $y \in (i,j)\text{-}\mathrm{pInt}(f(A))$;

(4) for every $B \in \mathcal{P}(Y)$ such that $y \in f(i\mathrm{Int}(f^{-1}(B)))$, then $y \in (i,j)\text{-}\mathrm{pInt}(B)$;

(5) for $B \in \mathcal{P}(Y)$ such that $y \in (i,j)\text{-}\mathrm{pCl}(B)$, $f^{-1}(y) \subset i\mathrm{Cl}(f^{-1}(B))$.

Theorem 5. *Let (Y, σ_1, σ_2) be a bitopological space and m_{ij} an m-structure on Y determined by σ_1 and σ_2. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (i,j)-K-m-open if and only if $f(U) = m_{ij}\text{-}\mathrm{Int}(f(U))$ for every $U \in \tau_i$.*

Remark 7. *By* Theorem 5 *and* Remark 5, *we obtain* Definition 7.

For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, we denote

$D^0_{ij}(f) = \{y \in Y$: *f is not (i, j)-K-m-open at y* $\}$,

then by *Definition 8* and *Theorem 3* we obtain the following theorem:

Theorem 6. *For a function $f : (X, \tau) \to (Y, m_Y)$, where m_{ij} is a minimal structure on Y determined by σ_1 and σ_2, the following properties hold:*

$$D^0_{ij}(f) = \bigcup_{G \in \tau_i} \{f(G) - m_{ij} - \mathrm{Int}(f(G))\} = \bigcup_{A \in \mathcal{P}(\mathcal{X})} \{f(i\mathrm{Int}(A)) - m_{ij} - \mathrm{Int}(f(A))\}$$
$$= \bigcup_{B \in \mathcal{P}(\mathcal{Y})} \{f(i\mathrm{Int}(f^{-1}(B))) - m_{ij} - \mathrm{Int}(B)\}.$$

For example, if we put

$m_{ij} = (i,j)\mathrm{SO}(Y)$ and $D^{SO}_{ij}(f) = \{y \in Y$: *f is not (i, j)-semi-open at y* $\}$,

then we obtain the following properties:

Corollary 2. *For a function $f : (X, \tau) \to (Y, m_Y)$, the following properties hold:*

$$D^{SO}_{ij}(f) = \bigcup_{U \in \tau_i} \{f(U) - (i,j) - \mathrm{sInt}(f(U))\} = \bigcup_{A \in \mathcal{P}(\mathcal{X})} \{f(i\mathrm{Int}(A)) - (i,j) - \mathrm{sInt}(f(A))\}$$
$$= \bigcup_{B \in \mathcal{P}(\mathcal{Y})} \{f(i\mathrm{Int}(f^{-1}(B))) - (i,j) - \mathrm{sInt}(B)\}.$$

References

[1] S. BOSE, *Semi-open sets, semi-continuity and semi-open mappings in bitopological spaces*, Bull. Calcutta Math. Soc. **73**(1981), 237–246.

[2] M. JELIĆ, *A decomposition of pairwise continuity*, J. Inst. Math. Comput. Sci. Math. Ser. **3**(1990), 25–29.

[3] M. JELIĆ, *Feebly p-continuous mappings*, Suppl. Rend. Circ. Mat. Palermo **24**(1990), 387–395.

[4] M. JELIĆ, *On some mappings of bitopological spaces*, Suppl. Rend. Circ. Mat. Palermo **29**(1992), 483–491.

[5] A. KAR, P. BHATTACHARYYA, *Bitopological preopen sets, precontinuity and preopen mappings*, Indian J. Math. **34**(1992), 295–309.

[6] F. H. KHEDR, S. M. AL-AREEFI, T. NOIRI, *Precontinuity and semi-precontinuity in bitopological spaces,* Indian J. Pure Appl. Math. **23**(1992), 625–633.

[7] S. SAMPATH KUMAR, *Pairwise α-open, α-closed and α-irresolute functions in bitopological spaces*, Bull. Inst. Math. Acad. Sinica, **21**(1993), 59–72.

[8] K. KURATOWSKI, *Topologia*, tom. 1, Mir, Moskva, 1966.

[9] T. LIPSKI, *Quasicontinuous multivalued maps in bitopological spaces*, Slupskie Prace Mat. Przyrodnicze Slupsk **7**(1988), 3–31.

[10] S. N. MAHESHWARI, R. PRASAD, *Semiopen sets and semi continuous functions in bitopological spaces*, Math. Notae **26**(1977/78), 29–37.

[11] H. MAKI, K. C. RAO, A. NAGOOR GANI, *On generalizing semi-open and preopen sets*, Pure Appl. Math. Sci. **49**(1999), 17–29.

[12] A. A. NASEF, T. NOIRI, *Feebly open sets and feeble continuity in bitopological spaces*, An. Univ. Timişoara Ser. Mat.-Inform. **36**(1998), 79–88.

[13] T. NOIRI, V. POPA, *Minimal structure, m-open functions and bitopological spaces*, submitted.

[14] V. POPA, T. NOIRI, *On M-continuous functions*, Anal. Univ. "Dunarea de Jos" Galaţi, Ser. Mat. Fiz. Mec. Teor. **18**(**23**)(2000), 31–41.

[15] V. POPA, T. NOIRI, *On the definitions of some genralized forms of continuity under minimal conditions*, Mem. Fac. Sci. Kochi Univ. Ser. A Math. **22**(2001), 9–18.

[16] V. POPA, T. NOIRI, *A unified theory of weak continuity for functions*, Rend. Circ. Mat. Palermo **51**(2002), 439–464.