

# DEVELOPING A MODEL OF A STRAIN (DEFORMATION) OF A DAMAGED REINFORCED CONCRETE PILLAR IN RELATION TO A LINEAR LOAD CAPACITY

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**Abstract:** Most research is mainly focused on the border load strength of a reinforced concrete pillar made of a high strength concrete, but what about the bearing capacity of the damaged AB columns of concrete with a normal strength from the last century. In this article, an experiment of a strain test (deformation) was described in relation to the amount of the axial eccentric load, to the breakdown of the damaged reinforced concrete pillar. Through real values, a model has been developed which can predict the straining of the deflection in relation to the size of the axial eccentric load. After testing the function of strain stress dependence and the axial eccentric load, a correlation test using the  $\chi^2$  test was performed. Through testing, it was found that there was no statistically significant difference between the real values and the values obtained by the mathematical model.

**Keywords:** load; strain; reinforced concrete pillar

## 1 INTRODUCTION

A pillar as a support structural element of a construction has a very important impact on the ductile effect and the combined effectiveness of the whole construction, which is why it is essential to know its performance completely. There has been much behavioural research of reinforced concrete pillars in the last few decades for different types of strain, together with the research of the aspect and limitations of armature given to cracking and the pillar's load capacity. All previous research was primarily focused on the linear load capacity of a construction element. Due to the greater demands of load capacity and a longer lifetime, together with the endurance of pillars, especially in the seismological unstable areas, research is conducted mainly on reinforced steel pillars made from high strength concrete ( $\geq 50$  MPa). There is an open question of damaged reinforced pillar load capacity on the buildings built in the last century from the normal strength concrete ( $< 50$  MPa) [7]. The question is what threatens the building structure as a whole, what the remaining time of service of the damaged pillars is and whether there is a possible and payable way for the recovery of the damaged pillars. Based on the above-mentioned questions, what was approached was the planning and realization of the experiment of exploring the strain stress (deformation) in relation to the size of the axial eccentric load, all the way to the breakdown of the damaged reinforced concrete pillar. Through real values, a model has been developed which can predict the straining (deformation) of the deflection in relation to the size of the axial eccentric load. The model development process has no strict rules, thus, modelling is first and foremost art. The only rule is that the model must provide a realistic picture of the structure and functions of the actual system and show the behaviour of the system in different states, all the way from working under normal conditions to testing its work in critical situations, respectively examining the system's adaptability to the changes in key parameters.

Modelling can be explained as a process of building a representative of a real system and its use to solve the perceived problem, where only those characteristics of the real system that contribute to the problem solving are important. The use of a well-designed model provides valuable information about the problem-solving possibilities, as well as about all variants of a possible solution, and about the possible adverse impacts and states in which the actual system can be found by changing certain features of the system [2].

The contemporary modelling of complex systems assumes the use of computers in defining the requirements and model building, although the entire modelling process is still based on the knowledge, logic, abilities and experience of the modelling person and cannot be fully automated. The use of computers in the modelling process applies primarily to the mathematical calculations of the value of individual characteristics, because the complexity and the amount of such calculations is increasing with the geometric progression in dependence on the increase in the number of elements of the model [3].

The purpose of the model developed for this work is prediction. It involves evaluating the value (quantitatively or qualitatively) of system variables over a given time, based on the knowledge of other systems, respectively changes in the value of their variables in the same period. Models are often developed to anticipate the effect of changing the system's driving factors or system results. Prediction models can be very simple (often empirical) but may be more complicated. They must have a certain level of accuracy in the reproduction of previous measurements and thus require calibration data and other independent verification data [1].

The prediction is based mainly on quantitative data, although qualitative data can be used with them. Quantitative data refer to the measurable characteristics or flows across the system and can be collected as time series, as parts of a given area, or by surveying a particular

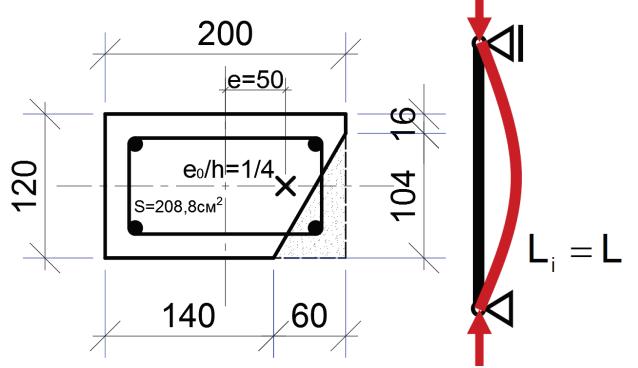
population. Qualitative data or information include professional opinions, beliefs, or information obtained from surveys and conversations. This information can be categorized (yes/no, high/middle/low ...), but may also be descriptive or specific by regulation.

Almost every model development relies on quantitative and qualitative information. However, models based on quantitative data rely mainly on the theory or knowledge gained by the iterations in the development of their conceptual model.

## 2 PREPARING OF A RESEARCH

### 2.1 Research of the damaged reinforced concrete pillar

The choice of an experimental pattern for measuring the strain-deformation stress at the deflection is largely dependent on the area of the slenderness of the reinforced concrete pillar, which depends on the geometric size of the model (Fig. 1) and on the way of fixing of the ends of the pillar of the so-called edge conditions (Fig. 2).



According to the European norms (EC-2, EN) [5], pillars are considered short and no budget is required for the theory of the 2<sup>nd</sup> order if the increase of the bending moment determined by the 1<sup>st</sup> order theory due to the deformation is not greater than 10%. This is fulfilled when the condition is met:

$$\lambda = 25(\omega + 0.9) \cdot \left( 2 - \frac{M_{01}}{M_{02}} \right), \quad (1)$$

where:

$$\omega = \frac{A_s \cdot f_{yd}}{A_s \cdot f_{cd}} - \text{If } A_s \text{ is unknown, } \omega \text{ is taken 0.1}$$

$A_s$  - The surface of the armature in the transverse section of the element

$\lambda$  - Coefficient of the element slenderness

$M_{01}, M_{02}$  - First-order moments at the ends - if moments are 0, ratios are taken 1.0.

Review of slenderness of the reinforced concrete pillar length of 175 cm, cross section 20 × 12 cm.

$$\lambda = \frac{l_0}{i} \leq \lambda_{\max}, \quad (2)$$

where:

$l_0$  - Element deflection length

$i = \sqrt{\frac{I}{A_c}}$  - The radius of the inertia in the direction of the smaller deflection resistance

$I$  - Moment of inertia

$A_c$  - Cross-section surface.

What is selected is a 20 × 12 cm cross section, 175 cm long research form (pillar) with the length of the damaged part of 40 cm in the middle of the height of the pillar, i.e. the damage is in the range of 67.5 cm to 107.5 cm height of the research form. The research form is made of concrete C 25/30 with longitudinal bars of B500 4 × 10 mm and forks B360 of 6 mm / 15 cm (/ 10 cm at both ends of the research form). By setting the basic parameters of the selected reinforced concrete pillar, not taking into account the damage of the cross section of the research form, the slenderness was  $\lambda = 50.52$ , which is more than the calculated  $\lambda_{\max} = 29.61$  limit. Therefore, the reinforced concrete pillar for the analysis has slenderness greater than the boundary and a calculation is required by the 2<sup>nd</sup> order theory (calculation for slender pillars).

### 2.2 Research form reinforced concrete pillar cross-section damage

According to the investigations of the undamaged samples of the pillar [6] after the breakdown, what was noticed was a localized type of damage of the examined samples, "wedged" shape approximately at the centre of the column, the depth of damage of 1/4 to 1/2 of the horizontal section length and the damage length of 20÷30 % of the total column length. This type of damage also occurs on the buildings of the last century; thus, for the experiment, a similar damage was made in the shape of the rectangular triangle cross section which was placed in one of the corners of the rectangular cross section. The cathetus of the triangle are given by the  $b$  side of the cross section 10.4 cm and by the  $h$  side 6 cm (Fig. 1).

### 2.3 Eccentric axial force

Since the eccentric loading of columns is often in the constructed structures, and because of the assumption of increasing the deformation of the deflection in the direction of the larger inertia torque of the cross section, the eccentricity of 1/4 h of the axial force is determined on the half section of the column where damage is present. In the direction of the lower inertia torque of the cross-sectional area, the position of the axial force action was determined with no eccentricity (Fig. 3) for the reason that the impact of the damage to the pillar curvature would be more noticeable.

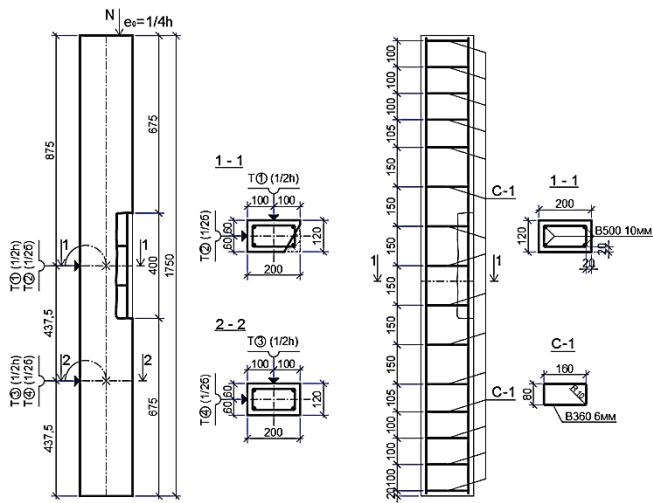


Figure 3 Research form

### 3 DESCRIPTION OF MEASURING AND RESULTS

The load on both ends of the research form is applied by a hydraulic press over a metal plate with a surface of a half-size of the cross-section ( $h/2 \times b$ ) of the research form, located on the cross section at which damage is present. Thus, the axial force, eccentric to the height ( $h$ ) of the cross section of the research form, was gradually applied and the research of the research form was measured every 50 kN at four measurement points using a 0.01 mm precision measuring instruments.

The measuring points shown in Figure x were placed on two undamaged sides of the research form, with a right angle between, two placed at the centre and two in a quarter of the height of the research pattern, planar on the half of the "b" and  $h$  sections of the research pattern. The expected deflection of the  $b$  and  $h$  sides outwards, relative to the vertical axis of the research form was achieved. Because of the eccentric loading and damage on the opposite side of the measurement, a significant bending of the side  $b$  was noted, i.e. a larger deflection was recorded in the direction of the maximum inertia torque of the cross section, while in the direction of the minimum inertia torque moment of the cross section was markedly less deflection but in the expected direction, or side  $h$  opposite to the damage, it is also moved outward in relation to the vertical axis of the research form. The results obtained are shown in Tab. 1.

Table 1 Result of stress strain changes

F/Fult	$\varepsilon_1$	$\varepsilon_2$	$\varepsilon_3$	$\varepsilon_4$
0	0	0	0	0
0.122	0	0.043	0	0
0.244	0.180	0.011	0.126	0
0.366	0.460	0.057	0.320	0
0.488	0.586	0.140	0.406	0.006
0.610	0.666	0.286	0.469	0.103
0.732	0.754	0.549	0.509	0.234
0.854	0.891	1.286	0.514	0.571
0.976	1.071	2.971	0.520	1.183

With the ver. 8st Stat Soft Inc. application *Statistica*, the stress dependency function ( $\varepsilon$ ) is tested at each load

point (F). According to the results obtained, in the  $T_1$  and  $T_3$  functions the strain corresponded to the logarithmic function (Fig. 4 and Fig. 5) and this:

$$T_1: f(x) = 0.962 + 1.1422 \cdot \log x, \quad (3)$$

$$T_3: f(x) = 0.5723 + 0.634 \cdot \log x. \quad (4)$$

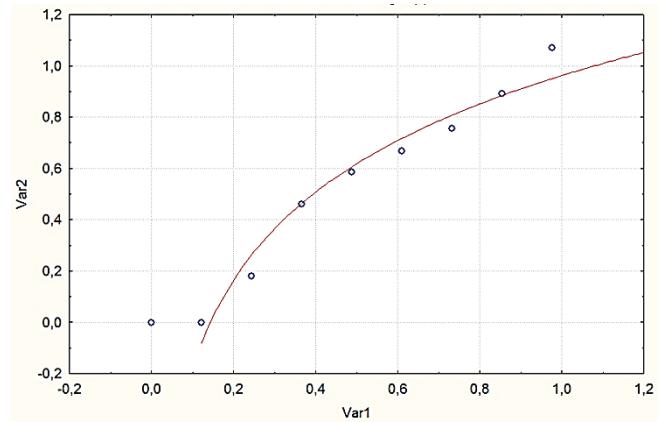


Figure 4 Load stress dependence in item 1

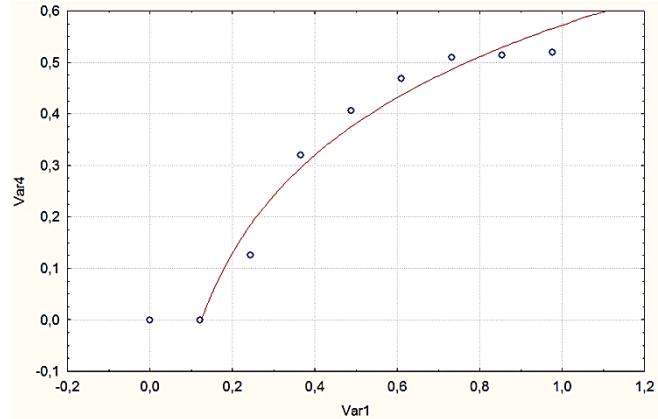


Figure 5 Load stress dependence in item 3

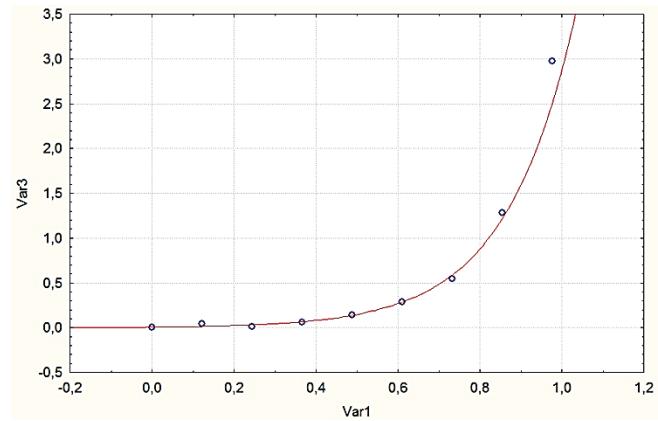


Figure 6 Load stress dependence in item 2

Unlike in the points 1 and 3, in the points 2 and 4 the functions of the reluctance were exponential functions (Fig. 6 and Fig. 7):

$$T_2: f(x) = 0.0075 \cdot e^{5.9489 \cdot x}, \quad (5)$$

$$T_4: f(x) = 9.9514 \times 10^{-5} \cdot e^{10.0662 \cdot x}. \quad (6)$$

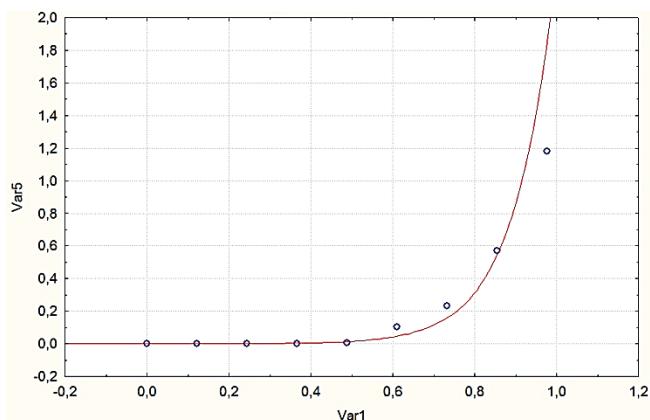


Figure 7 Load stress dependence in item 4

With the  $\chi^2$  test, the existence of statistically significant correlations in the actual frequency and value of the computed ones was calculated. This test is used to test the significance of the difference between the obtained ( $f_d$ ) and expected ( $f_0$ ) frequencies. Defined by the formula [4]:

$$T_2: \chi^2 = \sum \frac{(f_d - f_0)^2}{f_0}. \quad (7)$$

Table 2 Values derived from dependency functions

F/Fult	$\varepsilon_1$	$\varepsilon_2$	$\varepsilon_3$	$\varepsilon_4$
0	-	0.0075	-	0.000099514
0.122	-0.0816	0.0155	-0.0069	0.000339806
0.244	0.26228	0.03202	0.18391	0.00116032
0.366	0.46341	0.06617	0.29555	0.003962092
0.488	0.60611	0.13672	0.37476	0.013529178
0.610	0.7168	0.28252	0.4362	0.046197471
0.732	0.80724	0.58376	0.4864	0.157748416
0.854	0.88371	1.20624	0.52884	0.538656383
0.976	0.94995	2.49247	0.56561	1.839325595

$\chi^2$  test results are shown in Tab. 3.

Table 3 Values of the  $\chi^2$  test

	Function 1	Function 2	Function 3	Function 4
$\chi^2$	0.026	0.374	0.017	0.136
Degrees of freedom	6	8	6	8
P	0.99999964	0.99995611	0.9999999	0.99999916
Conclusion	There is no statistically significant difference			

The differential equation system is used to show the change of tension ( $d\varepsilon$ ) of the load change ( $dF$ ):

$$T_1: d\varepsilon = 0.962 + 1.1422 \cdot \log(dF), \quad (8)$$

$$T_2: d\varepsilon = 0.0075 \cdot e^{5.9489 \cdot dF}, \quad (9)$$

$$T_3: d\varepsilon = 0.5723 + 0.634 \cdot \log(dF), \quad (10)$$

$$T_4: d\varepsilon = 9.9514 \times 10^{-5} \cdot e^{10.0662 \cdot dF}. \quad (11)$$

#### 4 CONCLUSION

The experiment showed that at the linear increase of the load on the research form, due to the eccentric position of the axial force and the damage on the opposite side of the measurement, a significant curvature of side  $b$  was recorded, that is, the larger deflection was recorded in the direction of the maximum moment of inertia. The action of the eccentric axial force is applied perpendicularly to the part of the cross section where damage is present. The assumption of the experiment is that, in such an unfavourable position of the axial force, a significant role in the curvature other than eccentricity also has a damage of the research form cross section. In the direction of the minimum moment of inertia, there was significantly less deflection, but in the expected direction, or side  $h$  opposite to the damage, it also turned outward in relation to the vertical axis of the research form.

The expected direction of the curvature of the research form in the direction of the minimum moment of inertia confirms the influence of cross-sectional defects, while the main cause of the significant curvature of the research form in the direction of the maximum moment of inertia is the eccentricity itself. In order to fully confirm the impact of the cross-sectional defects of the research form on bending, and most importantly determine the magnitude of that impact, it is necessary to extend the experiment to a larger number of research forms.

The test patterns should contain different sizes and geometric forms of cross-sectional defects. Finally, such an expanded experiment should result in finding out the method of calculating the remaining firmness of the reinforced concrete columns with the damaged cross-section.

Compared to the linear increase of the eccentric axial force, the deformation of the curvature took place in the form of the exponential function at the points  $T_2$  and  $T_4$  and according to the logarithmic function, at the points  $T_1$  and  $T_3$ . Thus, the deflection of side  $b$ , although in the direction of the maximum inertia moment, takes place in the exponential function, while the deflection of the page  $h$  takes place according to the logarithmic function.

By performing a  $\chi^2$  test, it is proved that the functions describe the actual data statistically, that is, that there is no statistical difference between the actual data set and the number of data obtained by the function. The equations obtained were translated and merged into the differential equation system and a mathematical model of the behaviour of a damaged reinforced concrete pillar made of concrete of normal strength (<50 MPa) due to the axial eccentric load was conceived.

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