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Original scientific paper

STRAIN DIAGRAMS OF CROSS-SECTION OF REINFORCED CONCRETE BENDING ELEMENTS

Anatolii KOVROV, Aleksei KOVTUNENKO, Nina VYSOCHAN

Abstract: The article is dedicated to the construction of the "bending moment – curvature" diagrams for rectangular cross-sections of reinforced concrete bending elements. Assumptions and equilibrium equations used in the construction of diagrams are given in the article. The algorithm for the construction of the "bending moment – curvature" diagram is shown. Diagrams constructed by the proposed algorithm are compared with experimental data and with the results obtained by the methods proposed by other authors. The equations for determination of key points of linearized "bending moment – curvature" diagrams for cross-section with different reinforcement ratio are proposed.

Keywords: bending moment; curvature; diagram; reinforced concrete; strain

1 INTRODUCTION

Determination of the stress-strain state of reinforced concrete bending elements using the deformation theory is regulated by the current normative documents [1, 2] of Ukraine. In this case, to describe the diagram of deformation of concrete it is recommended to use the formula:

$$\frac{\sigma_c}{f_{cd}} = \frac{k\,\varepsilon_c/\varepsilon_{c1} - \left(\varepsilon_c/\varepsilon_{c1}\right)^2}{1+\left(k-2\right)\varepsilon_c/\varepsilon_{c1}} \tag{1}$$

or

$$\sigma_c = f_c \sum_{k=1}^{5} a_k \left(\frac{\varepsilon_c}{\varepsilon_{c1}}\right)^k . \tag{2}$$

The works of L. Mailyan [3], I. Prokopovich [4], A. Kovrov [5] and other scientists [6÷9] are devoted to the development of methods for determining the stress-strain state of rod-shaped reinforced concrete elements on the basis of the deformation model.

It is necessary to use the "bending moment – curvature" diagrams that fully take account of the work of materials and which can be used in practical analysis to more fully take into account the processes occurring in statically indeterminate reinforced concrete structures with changing loads up to destruction.

The aim of this work is to study the influence of the degree of reinforcement on the "bending moment – curvature" diagrams of the cross-sections of reinforced concrete bending elements and the development of a practical methodology for their construction.

2 BASIC ASSUMPTIONS

The following assumptions are accepted in the paper to describe the stress-strain state of reinforced concrete bending elements:

1) Bernoulli's conjecture (the hypothesis of flat sections) is considered valid – the deformations vary linearly with respect to the cross-sectional height.
2) There is cohesion between the reinforcement and the surrounding concrete, such that the deformations in the reinforcement and concrete are equal.
3) The dependence of "stress-strain" upon compression of concrete is described, in accordance with the proposals of A. Bambura [6], using the Eq. (2).
4) The relationship between stresses and strains upon tension of concrete is described using the Prandtl diagram.
5) The relationship between stresses and strains in tension and compression of reinforcement is described using the Prandtl diagram.
6) Resistance of the design section is considered exhausted when deformations of the extreme compressed concrete fiber or tension reinforcement reach the limit values, respectively, ε_{bu} and ε_{su}.

Equilibrium equations having the following form can be written for the cross-section of the reinforced concrete bending element:

$$\sum X = 0:$$

$$b\left[f_{cd}\sum_{k=1}^{5}\frac{a_k}{k+1}\frac{\varepsilon_c^{k+1}}{\aleph\,\varepsilon_{c1}^{k}} - f_{ctd}\left(\frac{\varepsilon_{ctu1}}{\aleph} - \frac{\varepsilon_{ct1}}{2\aleph}\right)\right] + \\ + E_s\left(A_{sc}\varepsilon_{sc} - A_{st}\varepsilon_{st}\right) = 0, \tag{3}$$

$$\sum m = 0:$$

$$b\left[f_{cd}\sum_{k=1}^{5}\frac{a_k}{k+2}\frac{\varepsilon_c^{k+2}}{\aleph^2\varepsilon_{c1}^{k}} + \frac{f_{ctd}}{2}\left(\frac{\varepsilon_{ctu1}^2}{\aleph^2} - \frac{\varepsilon_{ct1}^2}{3\aleph^2}\right)\right] + \\ + E_s\left[A_{sc}\varepsilon_{sc}\left(\frac{\varepsilon_c}{\aleph} - a_{sc}\right) + A_{st}\varepsilon_{st}\left(d - \frac{\varepsilon_c}{\aleph}\right)\right] - M = 0 \tag{4}$$

where $\aleph$ – a beam curvature; b – a cross-section width; d – effective depth of a cross-section; a_{sc} – distance from the most compressed face to the center of gravity of the compressed reinforcement; ε_c, ε_{sc}, ε_{st} – strain of the most compressed concrete fiber, compression reinforcement layer and tension reinforcement layer, respectively; A_{sc}, A_{st} – area of the compression and tension reinforcement layer, respectively.

3 CONSTRUCTION OF THE "BENDING MOMENT – CURVATURE" DIAGRAMS BY STEP ITERATION METHOD

The authors propose the following algorithm for determining the stress-strain state of rectangular cross-sections of reinforced concrete bending elements with increasing load up to the limit state with the construction of the "bending moment – curvature" diagram:

1) The initial data for the construction of the "bending moment-curvature" diagram for the rectangular section of reinforced concrete bending elements are as follows:
- design values of compressive f_{cd} and tensile f_{ct} strength of concrete;
- design value of modulus of elasticity of concrete E_{cd};
- ultimate compressive ε_{c1}, ε_{cu1} and tensile ε_{ct1}, ε_{ctu1} strain of concrete;
- design value of reinforcement strength f_y;
- design value of modulus of elasticity of reinforcement E_s;
- ultimate strain of reinforcement f_y/E_s and ε_{ud};
- dimensions of cross-section h and b;
- area of compression and tension reinforcement A_s and A_s';
- values of concrete covers for tension and compression reinforcement a and a';

1) Strain of the extreme tensile fiber $\varepsilon_{bt} = \Delta\varepsilon_{bt}$ and curvature $\aleph$ of the bending element is set in the first stage of work of the cross-section of the reinforced concrete bending element, in accordance with the proposals of L. Maillian [3]. It is assumed that $\Delta\varepsilon_{bt} = 0.05\varepsilon_{btu}$, and curvature of bending elements is proposed $\aleph = 2\varepsilon_{ct}$.
2) Strain of the extreme compressed fiber ε_c, strain of the compressive ε_{sc}, and the tensile reinforcement ε_s are determined from the accepted values of strain of the extreme tensile fiber and the curvature of the element based on the similarity of the triangles on the strain diagram of the cross-section and are substituted into Eq. (3).
3) If the equation of equilibrium (3) is not satisfied, then it is necessary to clarify the curvature and repeat the calculation on points 3 and 4 until the specified accuracy is reached. In this case, if the left side of Eq. (3) is greater than zero, then the curvature at the subsequent iteration is assumed $\aleph_{i+1} = 0.9999\aleph_i$. If the left side of the equation is less than zero, then the height of the compressed zone at the subsequent iteration is assumed $\aleph_{i+1} = 1.0001\aleph_i$.
4) The value of the bending moment perceived by the cross-section for a given strain of the extreme tensile fiber of concrete is determined by formula (4).
5) At the next step, strain of the extreme tensile fiber of concrete is increased. The calculation of points 2...6 is repeated until the strain of the extreme tensile fiber of concrete reaches the ultimate value.
6) The crack is formed in the section after the deformations of the extreme tensile fiber of concrete have reached the ultimate value. Strain of the extreme compressed fiber is increased for further construction of the diagram, in accordance with the proposals of L. Maillian [3].
7) Strain of the compressive ε_{sc} and the tensile reinforcement ε_s are determined from the accepted values of strain of the extreme compressed fiber and the curvature of the element based on the similarity of the triangles on the strain diagram of the cross-section and are substituted into Eq. (3).
8) If the equation of equilibrium (3) is not satisfied, then it is necessary to clarify the curvature and repeat the calculation on points 8 and 9 until the specified accuracy is reached. In this case, if the left side of Eq. (3) is greater than zero, then the curvature at the subsequent iteration is assumed $\aleph_{i+1} = 0.9999\aleph_i$. If the left side of the equation is less than zero, then the height of the compressed zone at the subsequent iteration is assumed $\aleph_{i+1} = 1.0001\aleph_i$.
9) The value of the bending moment perceived by the cross-section for a given strain of the extreme compressed fiber of concrete is determined by Eq. (4).
10) The calculation of points 8...10 is repeated until the strain of the extreme compressed fiber of concrete or tensile reinforcement reaches the ultimate value.

The program for determining the stress-strain state of rectangular cross-sections of reinforced concrete bending elements using the constructed "bending moment – curvature" diagrams was developed in the computer mathematics system MATLAB in accordance with the above algorithm.

Let us compare the "bending moment – curvature" diagrams, constructed from experimental data, to the formulas SNiP 2.03.01-84* "Concrete and reinforced concrete structures" [8], to the practical method based on I. Prokopovich's proposals, and the proposed method.

In the experiments of R. Asaad [9], studies were carried out with the beam of the following characteristics: dimensions of the cross-section $b \times h = 12 \times 20$ cm, strength of concrete $f_c = 18.5$ MPa, tension reinforcement 2Ø28 of class A-400 with area $A_s = 9.852$ cm^2, compression reinforcement 1∅6 of class A-400 with area $A_s' = 0.338$ cm^2, reinforcement ratio $\mu = 4.83$ %. "Bending moment – curvature" diagrams are shown in Fig. 1.

In the experiments of Chin Kim Dam [10], studies were carried out with the beam of the following characteristics: dimensions of the cross-section $b \times h = 10 \times 16$ cm, strength of concrete $f_c = 21.2$ MPa, tension reinforcement 2∅12 of class A-400 with modulus of elasticity $E_s = 2.05\times10^5$ MPa and yield strength $f_y = 490$ MPa, area $A_s = 2.26$ cm^2, reinforcement ratio $\mu = 1.67$ %. "Bending moment – curvature" diagrams are shown in Fig. 2.

A statistical estimate of the distribution of the ratio of theoretical and experimental curvatures is given in Tab. 1.

Table 1 Statistical estimate of the distribution of the ratio of theoretical and experimental curvatures

	Experiment of R. Asaad	Experiment of Chin Kim Dam
Sample mean M_x	0.9502	0.9942
Sample variance S_x	0.0944	0.1136
Selective coefficient of variation v_x	0.0994	0.1142
Confidence interval (P=0.95)	0.8847	0.9299
	1.0156	1.0585

4 CONSTRUCTION OF LINEARIZED "BENDING MOMENT – CURVATURE" DIAGRAMS

The type of diagrams for different sizes of cross-sections, strength of concrete and reinforcement of reinforced concrete elements is studied using the proposed algorithm for constructing the "bending moment – curvature" diagrams for cross-sections of reinforced concrete bending elements.

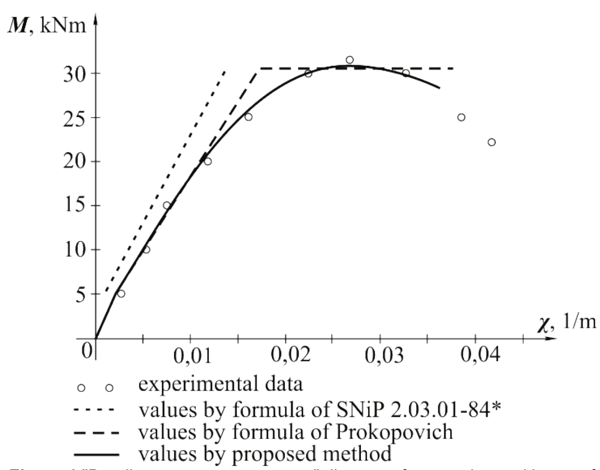

Figure 1 "Bending moment – curvature" diagrams for experimental beam of R. Asaad [9]

A family of "bending moment – curvature" diagrams for a rectangular section with dimensions 200 × 400 mm from concrete with strength of $f_c = 15$ MPa reinforced by reinforcement of class A-400 with yield strength of $f_y = 390$ MPa and the modulus of elasticity $E_s = 2\times10^5$ MPa with a change of reinforcement ratio in the range from 0.315 % to 2.58 % is shown on Fig. 3.

A family of "bending moment – curvature" diagrams for a rectangular section with dimensions 200 × 400 mm from concrete with strength of $f_c = 20$ MPa constructed with the help of a program compiled in the computer mathematics system MATLAB is shown in Fig. 4.

For practical purposes, it is convenient to use linearized deformation diagrams.

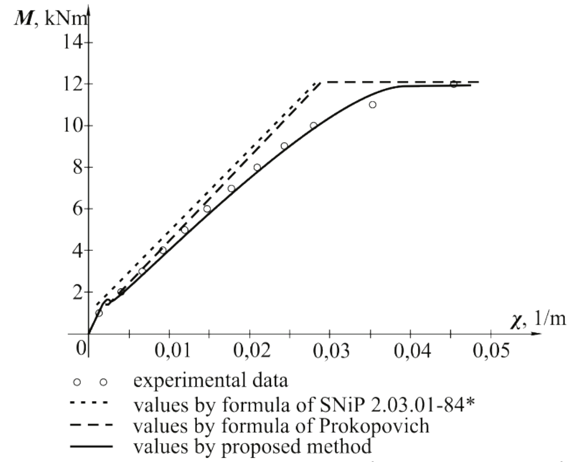

Figure 2 "Bending moment – curvature" diagrams for experimental beam of Chin Kim Dam [10]

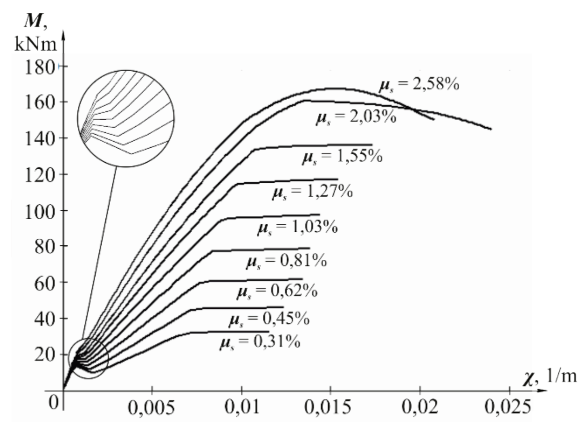

Figure 3 "Bending moment – curvature" diagrams with a change of reinforcement ratio (f_c = 15 MPa)

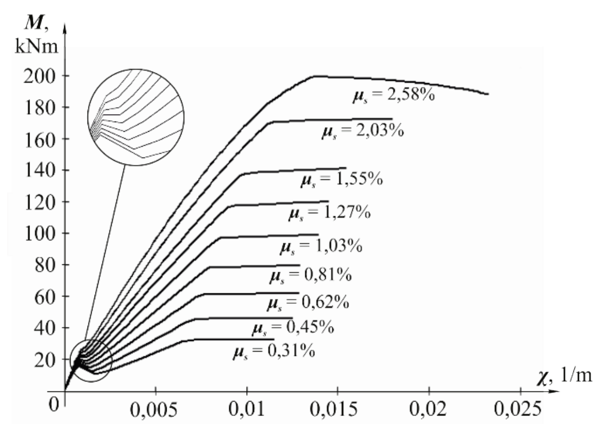

Figure 4 "Bending moment – curvature" diagrams with a change of reinforcement ratio (f_c = 20 MPa)

Studies in the families of the "bending moment – curvature" diagrams shown in Fig. 3 and Fig. 4 carried out by the authors allowed to conditionally distinguishing the following three characteristic types of linearized diagrams:

1. For weakly reinforced sections (Fig. 5)

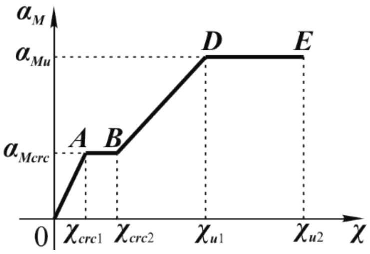

Figure 5 "Bending moment – curvature" diagram for weakly reinforced section

2. For normally reinforced sections (Fig. 6)

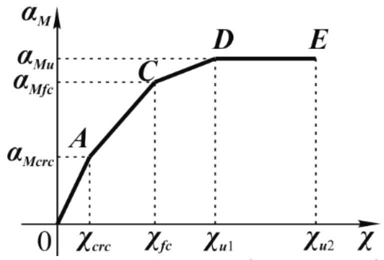

Figure 6 "Bending moment – curvature" diagram for normally reinforced section

3. For heavily reinforced sections (Fig. 7)

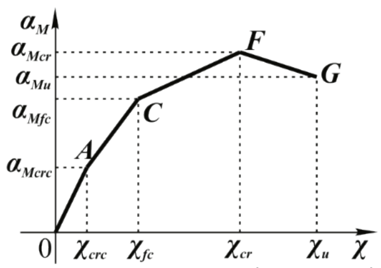

Figure 7 "Bending moment – curvature" diagram for heavily reinforced section

Each of these types of diagrams has characteristic points associated with the physical processes that occur during the deforming process of the bending elements.

It is recommended to reduce the equilibrium Eqs. (3) and (4) to the dimensionless form to exclude the necessity of solving the problem of constructing a diagram for each section. To this end, we divide the above equations by $f_{cd}bh$ and $f_{cd}bh^2$, respectively. As a result, we get:

$$\sum_{k=1}^{5}\frac{a_k}{k+1}\frac{\varepsilon_c^{k+1}}{\chi\varepsilon_{c1}^{k}}-\varphi_{ct}\left(\frac{\varepsilon_{ctu1}}{\chi}-\frac{\varepsilon_{ct1}}{2\chi}\right)+ +\varphi_s\left(\mu_{sc}\varepsilon_{sc}-\mu_{st}\varepsilon_{st}\right)=0, \tag{5}$$

$$\sum_{k=1}^{5}\frac{a_k}{k+2}\frac{\varepsilon_c^{k+2}}{\chi^2\varepsilon_{c1}^{k}}+\frac{\varphi_{ct}}{2}\left(\frac{\varepsilon_{ctu1}^2}{\chi^2}-\frac{\varepsilon_{ct1}^2}{3\chi^2}\right)+ +\varphi_s\left[\mu_{sc}\varepsilon_{sc}\left(\frac{\varepsilon_c}{\chi}-\xi_{sc}\right)+\mu_{st}\varepsilon_{st}\left(\xi_{c0}-\frac{\varepsilon_c}{\chi}\right)\right]- -\alpha_M=0 \tag{6}$$

where $\chi=\aleph h$ – curvature reduced to the section height (reduced curvature); $\varphi_{ct}=f_{ctd}/f_{cd}$; $\varphi_s=E_s/f_{cd}$; $\mu_{sc}=A_{sc}/(bh)$ – compressive reinforcement ratio; $\mu_{st}=A_{st}/(bh)$ – tensile reinforcement ratio; $\xi_{sc}=a_{sc}/h$ – the relative height of the concrete cover for compressed reinforcement; $\xi_{c0}=h_0/h$ – the relative effective depth of section; $\alpha_M=M/\left(f_{cd}bh^2\right)$ – bending moment reduced to dimensionless form (reduced bending moment).

The point **A** in the given diagrams corresponds to the beginning of the formation of a crack in the cross-section. Strain of the extreme tensile fiber of concrete reaches the value ε_{ctu1}. The following equation for determining the reduced curvature is obtained by substituting the strains of the extreme compressed fiber of concrete, compressed, and tensile reinforcement, expressed through strain of the extreme tensile fiber of concrete, into Eq. (5) and performing equivalent mathematical transformations:

$$A_1\chi^6+A_2\chi^5+A_3\chi^4+A_4\chi^3+A_5\chi^2+A_6\chi+A_7=0, \tag{7}$$

where $A_1=10a_5$; $A_2=12\left(a_4\varepsilon_{c1}-5a_5\varepsilon_{ctu1}\right)$;

$$A_3=15\left(a_3\varepsilon_{c1}^2-4a_4\varepsilon_{c1}\varepsilon_{ctu1}+10a_5\varepsilon_{ctu1}^2\right);$$

$$A_4=20\left(a_2\varepsilon_{c1}^3-3a_3\varepsilon_{c1}^2\varepsilon_{ctu1}+6a_4\varepsilon_{c1}\varepsilon_{ctu,1}^2-10a_5\varepsilon_{ctu1}^3\right);$$

$$A_5=30\left\{a_1\varepsilon_{c1}^4-2a_2\varepsilon_{c1}^2\varepsilon_{ctu1}+3a_3\varepsilon_{c1}^2\varepsilon_{ctu1}^2-4a_4\varepsilon_{c1}\varepsilon_{ctu1}^3+ +5a_5\varepsilon_{ctu1}^4+2\varphi_s\varepsilon_{c1}^5\left[\mu_{sc}\left(1-\xi_{sc}\right)+\mu_{st}\left(1-\xi_{c0}\right)\right]\right\};$$

$$A_6=-60\varepsilon_{ctu1}\left[a_1\varepsilon_{c1}^4-a_2\varepsilon_{c1}^3\varepsilon_{ctu1}+a_3\varepsilon_{c1}^2\varepsilon_{ctu1}^2- -a_4\varepsilon_{c1}\varepsilon_{ctu1}^3+a_5\varepsilon_{ctu1}^4+\varphi_s\varepsilon_{c1}^5\left(\mu_{sc}+\mu_{st}\right)\right];$$

$$A_7=\varepsilon_{ctu1}^2\left(30a_1\varepsilon_{c1}^4-20a_2\varepsilon_{c1}^3\varepsilon_{ctu1}+15a_3\varepsilon_{c1}^2\varepsilon_{ctu1}^2- -12a_4\varepsilon_{c1}\varepsilon_{ctu1}^3+10a_5\varepsilon_{ctu,1}^4\right)- -30\varphi_{ct}\varepsilon_{c1}^5\left(2\varepsilon_{ctu1}-\varepsilon_{ct1}\right).$$

It is necessary to substitute in Eq. (6) the curvature value obtained from Eq. (7) and the strain of the extreme compressed fiber of concrete, compressed, and tensile reinforcement to determine the reduced bending moment of cracking.

The horizontal section AB in the diagram shown in Fig. 5 corresponds to the development of a crack in the cross-section with an invariable value of the bending moment. The following system of equations for the unknown strain of the extreme compressed fiber of concrete and reduced curvature is obtained by substituting the strains of the compressed and tensile reinforcement, expressed through strain of the extreme compressed fiber of concrete, into Eqs. (5) and (6) and after equivalent mathematical transformations:

$$\begin{cases} B_{1,1}\varepsilon_c^6+B_{1,2}\varepsilon_c^5+B_{1,3}\varepsilon_c^4+B_{1,4}\varepsilon_c^3+B_{1,5}\varepsilon_c^2+ \\ \quad +B_{1,6}\chi^2+B_{1,7}\varepsilon_c\chi+B_{1,8}=0; \\ B_{2,1}\varepsilon_c^7+B_{2,2}\varepsilon_c^6+B_{2,3}\varepsilon_c^5+B_{2,4}\varepsilon_c^4+B_{2,5}\varepsilon_c^3+ \\ \quad +B_{2,6}\chi^3+B_{2,7}\varepsilon_c^2\chi+B_{2,8}\varepsilon_c\chi^2+ \\ \quad +B_{2,9}\alpha_M^{crc}\chi^2+B_{2,10}=0; \end{cases} \tag{8}$$

where $B_{1,1}=10a_5$; $B_{1,2}=12a_4\varepsilon_{c1}$; $B_{1,3}=15a_3\varepsilon_{c1}^2$;
$B_{1,4}=20a_2\varepsilon_{c1}^3$; $B_{1,5}=30a_1\varepsilon_{c1}^4$;
$B_{1,6}=-60\varphi_s\varepsilon_{c1}^5\left(\mu_{sc}\xi_{sc}+\mu_{st}\xi_{c0}\right)$; $B_{1,7}=60\varphi_s\varepsilon_{c1}^5\left(\mu_{sc}+\mu_{st}\right)$;
$B_{1,8}=-30\varphi_{ct}\varepsilon_{c1}^5\left(2\varepsilon_{ctu1}-\varepsilon_{ct1}\right)$; $B_{2,1}=60a_5$; $B_{2,2}=70a_4\varepsilon_{c1}$;
$B_{2,3}=84a_3\varepsilon_{c1}^2$;
$B_{2,4}=105a_2\varepsilon_{c1}^3$; $B_{2,5}=140a_1\varepsilon_{c1}^4$;
$B_{2,6}=420\varphi_s\varepsilon_{c1}^5\left(\mu_{sc}\xi_{sc}^2+\mu_{st}\xi_{c0}^2\right)$;
$B_{2,7}=420\varphi_s\varepsilon_{c1}^5\left(\mu_{sc}+\mu_{st}\right)$;
$B_{2,8}=-840\varphi_s\varepsilon_{c1}^5\left(\mu_{sc}\xi_{sc}+\mu_{st}\xi_{c0}\right)$; $B_{2,9}=-420\varepsilon_{c1}^5$;
$B_{2,10}=70\varphi_{bt}\varepsilon_{c1}^5\left(3\varepsilon_{ctu1}^2-\varepsilon_{ct1}^2\right)$.

Point C on the diagrams shown in Fig. 6 and 7, corresponds to the reaching the ultimate value ε_{c1} by strain of the extreme compressed fiber of concrete. The equation for determining the unknown reduced curvature is obtained by substituting the strain of compressed and tensile reinforcement, expressed through strain of the extreme compressed fiber of concrete, into Eq. (5) and performing equivalent mathematical transformations:

$$C_1\chi^2+C_2\chi+C_3=0, \quad (9)$$

where $C_1=-60\varphi_s\left(\mu_{sc}\xi_{sc}+\mu_{st}\xi_{c0}\right)$;
$C_2=60\varphi_s\varepsilon_{c1}(\mu_{sc}+\mu_{st})$;
$C_3=\varepsilon_{c1}\left(30a_1+20a_2+15a_3+12a_4+10a_5\right)-$
$-30\varphi_{ct}\left(2\varepsilon_{ctu1}-\varepsilon_{ct1}\right)$

It is necessary to substitute in Eq. (6) the reduced curvature value obtained from Eq. (9) and the strain of the compressed and tensile reinforcement to determine the value of the reduced bending moment corresponding to the point C.

Point D on the diagrams shown in Fig. 5 and 6, corresponds to the reaching the ultimate value f_y/E_s by strain of the tensile reinforcement. The equation for determining the unknown reduced curvature is obtained by substituting the strain of the extreme compressed fiber of concrete and compressed reinforcement, expressed through strain of tensile reinforcement, into Eq. (5) and performing equivalent mathematical transformations:

$$D_1\chi^6+D_2\chi^5+D_3\chi^4+D_4\chi^3+D_5\chi^2+D_6\chi+D_7=0, \quad (10)$$

where $D_1=10a_5\xi_{c0}^6$; $D_2=12\xi_{c0}^5\left(a_4\varepsilon_{c1}-5a_5\varepsilon_{s0}\right)$;
$D_3=15\xi_{c0}^4\left(a_3\varepsilon_{c1}^2-4a_4\varepsilon_{c1}\varepsilon_{s0}+10a_5\varepsilon_{s0}^2\right)$;
$D_4=20\xi_{c0}^3\left(a_2\varepsilon_{c1}^3-3a_3\varepsilon_{c1}^2\varepsilon_{s0}+6a_4\varepsilon_{c1}\varepsilon_{s0}^2-10a_5\varepsilon_{s0}^3\right)$;
$D_5=30\Big[\xi_{c0}^2\left(a_1\varepsilon_{c1}^4-2a_2\varepsilon_{c1}^3\varepsilon_{s0}+3a_3\varepsilon_{c1}^2\varepsilon_{s0}^2-\right.$
$\left.-4a_4\varepsilon_{c1}\varepsilon_{s0}^3+5a_5\varepsilon_{s0}^4\right)+2\varphi_s\mu_{sc}\varepsilon_{c1}^5\left(\xi_{c0}-\xi_{sc}\right)\Big]$;

$D_5^{II}=30\xi_{c0}^2\left(a_1\varepsilon_{c1}^4-2a_2\varepsilon_{c1}^3\varepsilon_{s0}+3a_3\varepsilon_{c1}^2\varepsilon_{s0}^2-\right.$
$\left.-4a_4\varepsilon_{c1}\varepsilon_{s0}^3+5a_5\varepsilon_{s0}^4\right)$;
$D_6=-60\varepsilon_{s0}\Big[\xi_{c0}\left(a_1\varepsilon_{c1}^4-a_2\varepsilon_{c1}^3\varepsilon_{s0}+a_3\varepsilon_{c1}^2\varepsilon_{s0}^2-\right.$
$\left.-a_4\varepsilon_{c1}\varepsilon_{s0}^3+a_5\varepsilon_{s0}^4\right)+\varphi_s\varepsilon_{c1}^5\left(\mu_{sc}+\mu_{st}\right)\Big]$;
$D_6^{II}=-60\Big[\varepsilon_{s0}\xi_{c0}\left(a_1\varepsilon_{c1}^4-a_2\varepsilon_{c1}^3\varepsilon_{s0}+a_3\varepsilon_{c1}^2\varepsilon_{s0}^2-\right.$
$\left.-a_4\varepsilon_{c1}\varepsilon_{s0}^3+a_5\varepsilon_{s0}^4\right)+\varphi_{sy}\varepsilon_{c1}^5\left(\mu_{st}-\mu_{sc}\right)\Big]$;
$D_7=\varepsilon_{s0}^2\left(30a_1\varepsilon_{c1}^4-20a_2\varepsilon_{c1}^3\varepsilon_{s0}+15a_3\varepsilon_{c1}^2\varepsilon_{s0}^2-\right.$
$\left.-12a_4\varepsilon_{c1}\varepsilon_{s0}^3+10a_5\varepsilon_{s0}^4\right)-30\varphi_{ct}\varepsilon_{c1}^5\left(2\varepsilon_{ctu1}-\varepsilon_{ct1}\right)$.

here $\varphi_{sy}=f_{cd}/f_{yd}$.

The values of the coefficients D_5^{II} and D_6^{II} are used in constructing the linearized diagram shown in Fig. 6. The ultimate value of the reduced bending moment for diagrams shown in Fig. 5 and 6 is determined by substituting in Eq. (6) the reduced curvature value obtained from Eq. (10) and the strain of the extreme compressed fiber of concrete and compressed reinforcement.

The horizontal section DE on the diagrams shown in Fig. 5 and 6 corresponds to the formation of a conditional plastic hinge in the cross-section (an increase in the curvature at a constant value of the bending moment until destruction). The criterion for failure element is the achievement of ultimate values ε_{cu1} and ε_{sud} respectively by strain of extreme compressed fibers of concrete or tensile reinforcement. The ultimate value of reduced curvature is determined by solving one of the following equations in accordance with the failure criterion:

- in case of failure due to rupture of tensile reinforcement:

$$E_1^s\chi^6+E_2^s\chi^5+E_3^s\chi^4+E_4^s\chi^3+E_5^s\chi^2+E_6^s\chi+E_7^s=0, \quad (11)$$

where $E_1^S=10a_5\xi_{c0}^6$; $E_2^S=12\xi_{c0}^5\left(a_4\varepsilon_{c1}-5a_5\varepsilon_{sud}\right)$;
$E_3^S=15\xi_{c0}^4\left(a_3\varepsilon_{c1}^2-4a_4\varepsilon_{c1}\varepsilon_{sud}+10a_5\varepsilon_{sud}^2\right)$;
$E_4^S=20\xi_{c0}^3\left(a_2\varepsilon_{c1}^3-3a_3\varepsilon_{c1}^2\varepsilon_{sud}+6a_4\varepsilon_{c1}\varepsilon_{sud}^2-10a_5\varepsilon_{sud}^3\right)$;
$E_5^S=30\Big[\xi_{c0}^2\left(a_1\varepsilon_{c1}^4-2a_2\varepsilon_{c1}^3\varepsilon_{sud}+3a_3\varepsilon_{c1}^2\varepsilon_{sud}^2-\right.$
$\left.-4a_4\varepsilon_{c1}\varepsilon_{sud}^3+5a_5\varepsilon_{sud}^4\right)+2\varphi_s\mu_{sc}\varepsilon_{c1}^5\left(\xi_{c0}-\xi_{sc}\right)\Big]$;
$E_6^S=-60\Big[\varepsilon_{sud}\xi_{c0}\left(a_1\varepsilon_{c1}^4-a_2\varepsilon_{c1}^3\varepsilon_{sud}+a_3\varepsilon_{c1}^2\varepsilon_{sud}^2-\right.$
$\left.-a_4\varepsilon_{c1}\varepsilon_{sud}^3+a_5\varepsilon_{sud}^4\right)+\varepsilon_{c1}^5\left(\varphi_s\mu_{sc}\varepsilon_{sud}+\varphi_{sy}\mu_{st}\right)\Big]$;
$E_7^S=\varepsilon_{sud}^2\left(30a_1\varepsilon_{c1}^4-20a_2\varepsilon_{c1}^3\varepsilon_{sud}+15a_3\varepsilon_{c1}^2\varepsilon_{sud}^2-\right.$
$\left.-12a_4\varepsilon_{c1}\varepsilon_{sud}^3+10a_5\varepsilon_{sud}^4\right)-30\varphi_{ct}\varepsilon_{c1}^5\left(2\varepsilon_{ctu1}-\varepsilon_{ct1}\right)$;

- in case of failure due to crushing of compressed concrete:

$$E_1^C\chi+E_2^C=0, \quad (12)$$

where $E_1^C = 60\varphi_{sy}\varepsilon_{c1}^5\left(\mu_{sc} - \mu_{st}\right)$;

$$E_2^C = \varepsilon_{cu1}^2\left(30a_1\varepsilon_{c1}^4 + 20a_2\varepsilon_{c1}^3\varepsilon_{cu1} + 15a_3\varepsilon_{c1}^2\varepsilon_{cu1}^2 + 12a_4\varepsilon_{c1}\varepsilon_{cu1}^3 + 10a_5\varepsilon_{cu1}^2\right) - 30\varphi_{ct}\varepsilon_{c1}^5\left(2\varepsilon_{ctu1} - \varepsilon_{ct1}\right).$$

The point F in the diagram shown in Fig. 7 corresponds to the maximum value of the reduced bending moment perceived by the cross-section. Deformation of the extreme compressed concrete fiber is also an unknown value in determining the values of the reduced curvature and the reduced bending moment corresponding to the point F. In this connection, the two equilibrium equations must be supplemented by the equation obtained from the extreme condition of the reduced bending moment:

$$\frac{\mathrm{d}\alpha_M}{\mathrm{d}\chi} = 0. \tag{13}$$

The derivation of an additional equation from the extremality condition (13) requires cumbersome mathematical transformations due to the existence of a relationship between the values of strain of the extreme compressed fiber of concrete and the reduced curvature that are entered into Eq. (6). In this connection, the determination of the parameters corresponding to point F is proposed to be performed by using the step-iteration method. For this, it is necessary to specify a series of values of strain of the extreme compressed fiber of concrete for which the condition $\varepsilon_{c1} < \varepsilon_c < \varepsilon_{cu1}$ is satisfied, and substitute them in the equilibrium Eq. (5). The following equation for determining the values of curvature corresponding to the given values of strains of the extreme compressed fiber of concrete is obtained as a result of equivalent transformations:

$$F_1\chi_i^2 + F_{2(i)}\chi_i + F_{3(i)} = 0, \tag{14}$$

where $F_1 = -60\varphi_s\varepsilon_{c1}^5\mu_{st}\xi_{c0}$;

$$F_{2(i)} = 60\varepsilon_{c1}^5\left(\varphi_{sy}\mu_{sc} + \varphi_s\mu_{st}\varepsilon_{c(i)}\right);$$

$$F_{3(i)} = \varepsilon_{c(i)}^2\left(30a_1\varepsilon_{c1}^4 + 20a_2\varepsilon_{c1}^3\varepsilon_{c(i)} + 15a_3\varepsilon_{c1}^2\varepsilon_{c(i)}^2 + 12a_4\varepsilon_{c1}\varepsilon_{c(i)}^3 + 10a_5\varepsilon_{c(i)}^2\right) - 30\varphi_{ct}\varepsilon_{c1}^5\left(2\varepsilon_{ctu1} - \varepsilon_{ct1}\right).$$

The values of the reduced bending moments corresponding to the accepted values of strains of the extreme compressed fiber of concrete are determined by substituting in Eq. (6) the values of reduced curvature determined as a result of solving of Eq. (14).

The maximum value of the reduced bending moment and the corresponding value of the reduced curvature are selected from a series of values of the reduced curvature and reduced bending moment obtained with the assumed values of strain of the extreme compressed fiber of concrete for plotting the point F.

The point G in the diagram in Fig. 7, corresponds to the achievement of the ultimate reduced curvature with a decrease of the reduced bending moment perceived by the cross-section. The value of reduced curvature corresponding to point G is determined by solving Eq. (14) with strain of the extreme compressed fiber of concrete $\varepsilon_c = \varepsilon_{cu1}$. The obtained value of reduced curvature is substituted into Eq. (6) to determine the value of the reduced bending moment.

Equations of equilibrium (5) ÷ (14) allow us to construct linearized "bending moment – curvature" diagrams in the reduced values for reinforced concrete bending elements of rectangular cross-section with arbitrary dimensions. The bending stiffness of a reinforced concrete element of rectangular cross-section at any stage of the section work can be determined, according to the accepted designations, by the formula:

$$B_M = \frac{M}{\aleph} = \frac{\alpha_M}{\chi} f_{cd}bh^3. \tag{15}$$

5 CONCLUSION

The "bending moment – curvature" diagrams, constructed according to the step-iteration method based on the complete material strain diagrams, describe the work of reinforced concrete bending elements with sufficient accuracy. The sample mean of the ratio of the theoretical values of the curvatures to the experimental values is from 0.95 to 1.01.

The type of the "bending moment – curvature" diagrams constructed for cross-sections of reinforced concrete bending elements depends on the ratio of reinforcement and strength of concrete.

The equations that allow determining the key points of the linearized "bending moment – curvature" diagrams of strain of rectangular cross-section of the reinforced concrete bending elements have been proposed.

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Authors' contacts:

Anatolii KOVROV, PhD, Professor
Odessa State Academy of Civil Engineering and Architecture,
Strength of Materials Department
65026, Ukraine, Odessa, Didrikhsone St., 4
E-mail: akovrov@inbox.ru

Aleksei KOVTUNENKO, PhD, Assistant Professor
Odessa State Academy of Civil Engineering and Architecture,
Strength of Materials Department
65026, Ukraine, Odessa, Didrikhsone St., 4
E-mail: kovtunenko.aleks@mail.ru

Nina VYSOCHAN, Research Assistant
Odessa State Academy of Civil Engineering and Architecture,
Strength of Materials Department
65026, Ukraine, Odessa, Didrikhsone St., 4
E-mail: ninokli@mail.ru