OUTPUT GAP IN TRANSITION ECONOMIES USING UNOBSERVED COMPONENT METHOD: THE CASE OF CZECH REPUBLIC, ESTONIA AND KOSOVO

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Abstract

This paper investigates the concept and estimation of the output gap in transition economies, with special reference to the Czech Republic, Estonia and Kosovo. The motivation for investigating this phenomenon lies in the macroeconomic imbalances characterizing many transition economies, such as relatively sluggish growth, chronic balance of payments deficits and structural deficiencies, while continuously operating in the presence of relatively large underutilized resources. Given that the potential output and the corresponding output gap concepts are mainly discussed in the light of mainstream theories, the novelty of this paper stands in examining the relevance of the output gap in transition context. In order to reflect persistent underutilised resources as well as several structural breaks, the Unobserved Components model operationalized

* The views expressed in this paper are solely those of the authors. We would like to thank Professor Nick Adnett and SINCERE Conference participants, especially Zoran Aralica for helpful comments.
via the Kalman filter was employed as a the appropriate estimation method for transition economies. Another novelty of this study is the textual explanation of the technicalities underpinning the Kalman filtering procedure. While causing the output to fall below its potential, the results suggest that the Global Financial Crisis (GFC) had a significant but transitory impact in the Czech Republic and Estonia cases. Due to relatively low external exposure and domestically funded banking system, the GFC caused no recession in Kosovo, but rather slowed the pace of growth mainly via the external sector channels and the uncertainties perceived by the banking sector. Last, the negative relationship between inflation and output gap was informative in the case of the Czech Republic and Estonia because it suggested a presence of inflation inertia in these countries, whereas the impact of the output gap on the inflation rate in Kosovo proved insignificant.

Keywords: Output Gap, Unobserved Components Model, Kalman Filter, Transition Economies

1. INTRODUCTION

Although potential output and, therefore, the output gap are central to much policy making (e.g. inflation targeting and fiscal policies), they have been mainly discussed in the light of mainstream theories (e.g. Okun’s law, the Phillips curve and the production function). Mainstream theories are largely constructed based on the economic conditions in developed countries. Conventional theory defines the business cycle as fitting a mean trend to the actual output, while the cycle, as proxied by the output gap, will symmetrically fluctuate above or below that mean over time. The standard definition of the business cycle is that an output gap is present when economic activity is either above or below the potential output, caused typically by variations in actual output. In the context of transition economies (TEs), the output gap is subject to large and ongoing structural changes and adjustments, reflected in high structural unemployment rates, imported inflation and widespread market failures that continuously affect and typically limit the potential output. In TEs, therefore, the output gap may reflect not only the changes in actual output but also in potential output. This issue, coupled with the typically poor quality and short time-span of data, makes standard theories and empirical approaches non-applicable.

Some of the reasons for the lack of previous studies of the output gap in TEs may lie primarily in the past economic developments in these countries. For example, many TEs have undergone and are still undergoing a reform process, with the aim of building a market-oriented economy with functional institutions (Svejnar, 2002, Estrin et al., 2009; Roaf et al., 2014). Amongst the first problems to overcome were the sudden and mass obsolescence of the existing capital stock, especially in the early years of transition due to the old technology prevailing in existing enterprises, changes in relative prices, the neglect of capital depreciation
in the central planning system and a consequent large scale write down of the value of the existing capital stock (Pistor et al., 2000; Pyo, 2008).

In the context of ongoing reforms, the growth path and the growth variations in TEs are usually characterized by frequent and large structural breaks and shifts (Jones and Olken, 2005), making research of the business cycle and output gap in these countries during this period extremely complicated (Jagric and Ovin, 2004). This may particularly be the case in less developed TEs, such as the South Eastern European countries, including Kosovo. These countries are some of the late-comers to the transition process, experiencing a switch in regimes usually in the early 1990s and then, during 1997-2001, undergoing a period of large reversals in growth patterns (shifts in the growth path or actual output level). Even in more developed TEs, such as the Czech Republic and Estonia, which nowadays have almost caught-up with the western EU countries, some reversals in growth rates have been identified during the 1990s. Therefore, actual output, potential output and the output gap in TEs may be expected to experience large fluctuations and shifts, rather than conform to regular business cycle patterns (Neumeyer and Perri, 2004). It may therefore be inferred that TEs are unique in their economic experiences and, thus, it is challenging to conduct business cycle analysis for these countries.

The output gap in TEs has to incorporate structural as well as the cyclical effects. That is, while measuring the cycle of an economy, such as the aggregate movement of actual output towards expansion or contraction, in TEs the ‘cycle’ also reflects structural effects on potential output that may be reflected in the persistent underutilisation of resources, i.e. a negative output gap. While the cyclical effect may capture the temporary movements of the economy, the structural output gap reflects the more enduring, embedded imbalances in these economies (skill mismatches, chronic current account deficits, high dependence on imports, high and structural unemployment, rigidities in the market, etc.) that are present, regardless of the cyclical position in the economy. Thus, what is typically seen as an easy concept to define and apply, the output gap in the transition context becomes complex and relatively difficult to define and measure. Hence, fitting a mean-trend onto the actual data, say with the Hodrick-Prescott filter, whereby the cycle would be below the potential output as much as above over time at least theoretically (Mohr, 2005), may not be appropriate in the transition context. Given the structural problems and the persistence of underutilised resources, other methods that would allow fitting a different mean are needed. This is why it was decided to use the Unobserved Components (UC) model operationalized by the Kalman filter, as a method that accounts for structural changes in potential output (Harvey, 1989; Boone, 2000) as well as allowing fitting of the cycle reflecting persistent underutilised resources, i.e. a negative output gap.

The persistence of large negative output gaps, associated with structural changes in early transition, may also have been associated with more firms exiting the market than new firms entering, leading to a reduction in potential
output. Furthermore, due to a change in regime (i.e. centrally planned to market-oriented economy), the sectors that were previously vital in these economies (e.g. agriculture) gradually contracted and economic activity shifted towards other sectors. Therefore, previous labour skills were no longer required, generating a skill mismatch in the market. The skill mismatch, together with relatively high reservation wage (because of remittances) and other market rigidities meant that the NAIRU was extremely high in most TEs, whereas the actual unemployment rate could be much lower. In addition, given the widespread obsolescence of the capital stock (especially in the early years of transition), even where significant physical capital was present, its value was fairly low. Hence, given that obsolete capital does not directly enter into the calculation of the potential output, this may indicate that the potential output in these countries was low relative to the factors of production apparently available, hence the output gap was relatively small, which is reflected in the output gaps estimated and reported in this paper for the Czech Republic, Estonia and Kosovo. Therefore, the output gap in TEs may to a large extent reflect the structural changes and to a smaller extent the cyclical component.

Two main reasons led us to select three representative countries in Europe, namely Czech Republic, Estonia and Kosovo. Firstly, these respective countries may be homogenous in the sense that they all have departed the transition process in a post-communist era. Secondly, these three countries have undergone (and still undergoing to some extent) structural reforms, such as the build-up of the institutional setting, the establishment of a market based economy, the development of the banking system, the undergoing privatization process, encouraging trade and attracting investments, etc.; all experienced a number of shocks, such as change in regime, war, financial crisis, etc.; all share similar macroeconomic imbalances (excess liquidity in the banking system, chronic current account deficits, relatively low levels of credit to GDP compared to Eurozone countries, etc) and have market deficiencies (issues with contract enforcement, reporting forms and asymmetry of information and relatively low strength of legal rights).

Further, since we could not exercise the Unobserved Components method to every European transition economy, we have selected three different countries that would represent this region: the Czech Republic as one of the most developed countries in the region, representing the group with Poland and Hungary, Estonia a small and open economy as a representative for the Baltic countries, and Kosovo also a small and open economy as a representative for the Western Balkan countries.

2. Methodology
2.1. Unobserved Components Model

Fitting a mean-trend onto the actual data, say with the Hodrick-Prescott filter, whereby the cycle would be below the potential output as much above, may
not be appropriate in the transition context. Given the structural problems and the persistence of underutilised resources, the UC model operationalized by the Kalman filter is used, that accounts for structural changes as well as allowing fitting of the cycle reflecting persistent underutilised resources, i.e. a negative output gap.

The main objective of the UC model also lies in decomposing and estimating unobserved components (variables) such as potential output, output gap, natural rate of unemployment, etc. using all the information available from the observed variables (e.g. actual output, unemployment rate, inflation rate etc.); in other words, extracting the signal from the noise (signal processing technique). The UC approach is much richer and complex in structure, and models like least-squares or instrumental variables cannot be used to solve the model. Hence, the more advanced framework of the UC model enables the estimation of unobserved variables with less error. The UC method can be implemented using the Kalman filter, with the trend modelled as a random walk with drift and the cycle as an AR(2) process, once it is written in a state space notation (see Appendix 1). A state space representation of the UC approach means formulating all the components of interest, that in our case are potential output, output gap, parameters and error terms as a system of matrices and vectors. Afterwards, each component is separately modelled by an appropriate dynamic stochastic process, which usually depends on normally distributed disturbances (Koopman et al., 2008).

One of the earliest forms of the UC model is the Watson (1986) model, where the UC representation takes the following form:

\[ y_t = g_t + c_t \]  

where the trend is modelled as a random walk with drift:

\[ g_t = g_{t-1} + \mu_g + \lambda^* d_{2008} + \varepsilon_t \]  

(2)

\[ c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \nu_t \]  

(3)

In this representation of the UC model, the vector of observed variables \( Y_t \) contains only actual output, whereas the vector of unobserved variables contains potential output (\( g_t \)), the cyclical component (\( c_t \)) and the first two lags of the cyclical component (\( c_{t-1}, c_{t-2} \)), as well as the \( \lambda \) coefficient. Thus, in a state space representation we are faced with two groups of unknown parameters that need to be estimated, for a given model specification. The first group represents the trend (\( g_t \)), the cycle (\( c_t \)) and the drift (\( \mu_g \)) i.e potential output growth rate, whereas the second group are the initial parameters, \( \phi_1, \phi_2 \) and the error variances (\( \sigma^2_\varepsilon \) and \( \sigma^2_\nu \)) and last the \( \varepsilon_t \) and \( \nu_t \) represent the error terms of the trend and the cycle, respectively. The model is augmented with a dummy variable \( d_{2008} \) controlling for the impact of the GFC in the trend of each country, namely the

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1 Welch and Bishop (2006) explain that the filter is very powerful in several aspects, since it can handle past, present and future states even when the nature of the system is unknown.
Czech Republic, Estonia and Kosovo. While there are thus seven unknown components that need to be estimated, there is only one piece of information from which to extract them, which is actual output (real GDP).

The Phillips curve represents an essential structural relation that may convey information about the potential output and the output gap in the economy. Shepherd and Driver (2003) argue that the Phillips curve can be expressed as a relationship between price inflation and cost inflation or after some manipulation, a relationship between inflation and its own lagged values. As argued by Gordon (1997), in an amended Phillips curve equation we can capture supply and demand-side shocks that may impact the unobserved variables. The modified Phillips curve, (equations 4, 5 and 6 below), includes the demand side pressures $D_t$, usually captured by the output gap, the unemployment and the capacity utilization rates. The $\pi_t$ term stands for inflation, $\pi_{t-1}$ stands for the lagged effect during the adjustment process of prices (inertia); $L$ is for the lag polynomial, since each variable can be modelled with a different lag structure. The $a$ and $b$ terms are the parameters of the lagged inflation and demand side variables.

As argued in orthodox theory, changes in demand variables are those that affect the inflation rate, thus they need to enter into the equation as first differences or as the first lag of the levels. Further, we include exogenous supply-side shocks variables $z_t$ which are usually proxied through the change in import prices, change in food prices, change in oil prices, change in the real exchange rate, etc. (Gordon, 1997); $c$ is the parameter on the supply-side variables (in our case the output gap) and $\lambda$ is the parameter of the dummy variable that controls for the GFC effect.

In order to formalise the Phillips curve equation, we choose the output gap as a proxy for demand side pressures (Basarac et al., 2011), and the changes in food and oil prices as supply side shocks. The inflation equations for the Czech Republic, Estonia and Kosovo are respectively as presented in section 4. Once written in a state space form, that is in the form of the matrices and vectors in the measurement and transition equations (see Appendix 1), the UC model can be operationalized using the Kalman filter.

### 2.2. The Kalman Filter

The Kalman filter represents a recursive process that generates optimal estimates from indirect, inaccurate and noisy observations. In the case of stochastic processes which involve noisy data, the Kalman filter ‘cleans up the noise’ in order to extract the true value of the desired variable together with its spread (behaviour). If all noise is random (Gaussian), the Kalman filter minimises the mean square error of the estimated parameters, in order to generate the optimal estimator (Kleeman, 1995).

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2 It is a recursive process in the sense that new measurements can be processed as they arrive.
The UC approach, written in a state space form, represents the structure of the Kalman filter. A Kalman filter is implemented conditional on the initial parameter values \((\mu_g, \phi_1, \phi_2, \sigma^2_\epsilon, \text{and } \sigma^2_\nu)\), which ‘design’ the Kalman filter on modelling the desired components of our model. The initial parameter values need to be assumed or calibrated before implementing the Kalman filter. The parameters of the UC model can then be estimated using the maximum likelihood (ML) function. The criteria for determining the value of the initial parameters will be such that they will minimize the difference of the mean squared error \(E\{ |y_t - \hat{y}_t|^2\}\).

In the case of discrete linear data modelling, the filter is always designed as an algorithm; in our case, measurement and transition equations algorithms. Algorithms serve as a guide to predict the unobserved components, where entering a set of observed and otherwise assumed information \((y_t, \mu_g, \phi_1, \phi_2, \sigma^2_\epsilon, \text{and } \sigma^2_\nu)\) in the system will generate an outcome \((g_t \text{ and } c_t)\). These parameters, together with other observed variables in the case of multivariate UC model, feed in the system and basically design (‘train’) the Kalman filter in how to filter or predict the future values of the unobserved variables.

The lagged terms in the transition equation (see Appendix 1) indicate that the transition algorithm in the Kalman filter will be a recursive process. A recursive process in the discrete data modelling means that each additional observation entered into the algorithm will predict each future value (one-step ahead predicted errors) based on past values at time \(t\), then the following predicted values \((t + 1, t + 2, \ldots t + n)\) will be updated (estimated) based on all past values plus the new estimated predicted value and so on. The recursive process runs into every term in the measurement and transition equation, similarly for trend, for cycle and for each parameter in succession.\(^3\) After much iteration, the algorithm will eventually converge into an optimal estimator. Estimating the unobserved components one by one at each point in time, ‘allows the data to speak’, meaning that the algorithm is able to identify itself the structural breaks in the sample, without having to impose external restrictions.

The estimation procedure using the Kalman filter is as follows. The first step is to write the model in a state space form, as denoted in equations A3-A12 (Appendix 1) Second, it requires setting the initial parameter values of the model \((\mu_g, \phi_1 \text{ and } \phi_2)\), which in our case, are time invariant and characterize the optimal linear state of estimation, i.e. define the model as filtering or prediction. Also, the filter requires setting the starting values of the unobserved variables \((g_t, c_t)\) and the variances of the unobserved variables and their co-variance matrix \(\text{cov}(\sigma^2_\epsilon, \sigma^2_\nu)\) which in effect will define the initial state of the model. The starting values of the unobserved components \((g_t, c_t)\) and other parameters are generated from

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\(^3\) Welch and Bishop (2006) explain that the recursive nature of the Kalman filter represents one of its most appealing features, since it makes the practical implementations more feasible, compared to other filters (e.g. Wiener filter) which are designed to operate on all of the data directly for each estimate.
other methods, such as OLS, calibrated or assumed, thus are not generated by the filter itself. In the third step, the transition algorithm (Appendix 1, eq. A7) generates the current state variables, conditional on the initial parameter values. These are the predicted values of the unobserved variables (potential output and output gap), which still contain some noise (error) and uncertainties in them. This stage is also known as the prediction step or a priori estimate for time \( t \).

In the fourth step, also called the updating (correcting) step, the ‘cleaning’ process continues, where the predicted values of unobserved variables are cast into the measurement algorithm (Appendix 1, eq. A1) to generate the predicted values for the observed variables. The predicting-correcting process in the Kalman filter represents the feedback control process.

After each predicted and updated measurement pair, the process is repeated with the previous estimates used to project or predict new estimates, until the parameter estimates converge (Commandeur and Koopman, 2007). The difference between the observed actual output and the predicted output \( E\{|y_t - \hat{y}_t|\} \) is called the measurement innovation or the residual, which quantifies the lack of the accuracy of the estimated unobserved estimates in predicting the observed values at time \( t \) (Welch and Bishop, 2006). Otherwise, following Hamilton (1994) the measurement innovation looks like the following:

\[
E\{|y_t - \hat{y}_t|\} = \min\{(\sigma^2_\epsilon - \overline{\sigma}^2_\epsilon) + \min (\sigma^2_\nu - \overline{\sigma}^2_\nu)\}
\]

(4)

Simultaneously, the Kalman filter also calculates the prediction error and the variance terms \((\sigma^2_\epsilon \text{ and } \sigma^2_\nu)\). If the measurement noise vector components \((\epsilon_i \text{ and } \nu_i)\) are uncorrelated, then the state update can be carried out one measurement at a time. Here, in a recursive manner, the Kalman algorithm updates the predicted values by giving a higher weighted average to the estimates with higher certainty and vice versa (as measured by \( q \)). The unobserved variables will be estimated using a ML function “….which reflects how likely would have been to have observed the data [on potential output and output gap] if the initial parameter values were the true values…” (Hamilton, 1994, p. 3055).

The value of the ML function is maximised by minimizing the prediction errors \((\epsilon_i \text{ and } \nu_i)\) and their variances \((\sigma^2_\epsilon \text{ and } \sigma^2_\nu)\). If the likelihood function is maximised, conditional on initial parameter values and initial values of \( g_t \) and \( c_t \):

\[
\log (y_t, y_{t-1}, \ldots, y_1| X_t, X_{t-1}, \ldots, X_1; \mu_g, \phi_1, \phi_2, \sigma^2_e, \sigma^2_\nu)
\]

(5)

then the estimated unobserved \((g_t \text{ and } c_t)\) components and the parameters \((\mu_g, \phi_1, \phi_2, \sigma^2_e \text{ and } \sigma^2_\nu)\) are ML estimates; if not, then the algorithm will continue a set of iterations and recursive processes by iterating on measurement and transition equations, using new initial parameter values, until it generates the optimal error estimates.

Overall, the UC method written in the state space notation has several advantages relative to other methods. First, the UC method written in state
space notation offers enormous flexibility when dealing with irregular data, such as missing observations and observations with mixed frequencies such as monthly and quarterly (Koopman and Ooms, 2002; Harvey, 2005). Second, while other methods, such as the Hodrick-Prescott filter, require the researcher to arbitrarily decide on the smoothness component of the cycle $\lambda$, an important advantage of UC is that it does not require identifying the break points prior to estimation, since it identifies them through the recursive process. Third, the UC method does not require other unobservable variables to estimate potential output, such as the natural rate of unemployment. What is important in the case of policymaking is that the decomposition made through the UC model is usually made based on the past and current data, so that the computation of potential output and output gap can be made in real time, and thus continuously updated (Commandeur and Koopman, 2007). Last, the most important feature of the UC model is that through its recursive process, accounts for the several structural changes that may characterise data series from TEs. Regarding the disadvantages, the UC method requires considerable programming and is sensitive to the initial set of parameter values (Cerra and Saxena, 2000).

3. DATA

The estimation of potential output and the output gap will be undertaken for three country examples, namely the Czech Republic, Estonia and Kosovo. Since each country has its specific economic characteristics, like their long-run potential output, fiscal stance, regulatory framework etc., it makes more economic sense to undertake a country by country business cycle analysis rather than pooling the data for all the countries together.

The data used for all the countries is quarterly. The time span of the data is from Q1 2002 to Q4 2013 for Kosovo and for the Czech Republic and Estonia the time series span from Q1 1998 to Q4 2013. However, most empirical research on the TEs is characterised by a short time-span of data, particularly in business cycle analysis (see for example, Benes and N’Diaye (2004), Tsalinksi (2007), Sramkova et al. (2010), etc.).

The nominal GDP data will be adjusted with the GDP deflator, to extract the real GDP data for all three countries. Apart from changes in Consumer Price Index (CPI), following Kuttner’s (1994) model extensions to augment the Phillips curve equation, other explanatory variables that proxy for inflation, such as changes in food ($\Delta FP$) and oil prices ($\Delta OP$) will be used in

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4 The Statistical Agency of Kosovo provides only yearly data for the GDP series for Kosovo, hence a disaggregated data from yearly into quarterly will be used. The disaggregation process from yearly GDP into quarterly GDP was conducted from the economists in the Central Bank of the Republic of Kosovo. The methodology used to disaggregate yearly into quarterly data was mainly based on the weighted averages of the historical data of all the sectors of economy, e.g. external and financial sectors.
the multivariate UC model (see also Gerlach and Smets, 1997 and Ogunc and Ece, 2004). Oil prices are proxied by the average crude oil prices. Since this series was expressed in U.S. dollars per barrel we multiplied the series with the euro per dollar exchange rate to obtain the series expressed in euros. The food prices are taken from the international food price index FAO. The ΔOP and ΔFP are the same for the three countries. All the changes will be transformed into the first differences of the log of the respective variables in order to obtain the percentage changes for the period. The base year of the CPI in Kosovo is 2002, whereas in the Czech Republic and Estonia it is 2000. The inflation variables are annualised. The data for Kosovo are taken from the Statistical Agency of Kosovo (SAK), IMF World Economic Outlook (2015) and calculations from the Central Bank of the Republic of Kosovo. For the Czech Republic and Estonia the data source is EUROSTAT. In addition, since quarterly data are being used, all the data is this paper are seasonally adjusted.

4. RESULTS

The initial parameter values, for Czech Republic in Table 1 and the other two countries were set up in the following way: the drift term $\mu_g$ represents the average quarterly growth rate of the actual output to approximate for the potential output growth, the autoregressive coefficients $\phi_1$ and $\phi_2$ were calibrated based on theoretical guidance (1.4 and -0.5, respectively, see for example Benes and Ndiaye, 2004). The variance parameters were obtained by initially HP filtering the GDP series into the trend and cycle; then regressing them separately into an OLS regression and then using these error variances as initial parameters for the $\sigma^2_\varepsilon$ and $\sigma^2_\eta$. Last, the initial value for the dummy coefficient $\lambda$, representing the structural break during the GFC was obtained by including an impulse dummy in the trend equation (eq. 2) and estimating it in an OLS regression.

In addition, the initial parameter estimated for the structural equation of the inflation versus output gap relationship (eq. 6 to 8), is constructed as an ARIMA process, following the first two stages of the Box-Jenkins (1970) procedure: identification and estimation with diagnostic checking. Firstly, a pure autoregressive model of order one was tested, by adding only one autoregressive log-CPI variables into the equation, then following Kuttner (1994) we added one by one other independent variables (Table 1).
Table 1

<table>
<thead>
<tr>
<th>Parameter estimates</th>
<th>Diagnostics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_\pi$</td>
<td>$a_1$</td>
</tr>
<tr>
<td>-0.344 (0.787)</td>
<td>-0.460 (0.151)</td>
</tr>
<tr>
<td>-0.045 (0.070)</td>
<td>1.08*** (0.036)</td>
</tr>
<tr>
<td>-0.358 (0.555)</td>
<td>0.408** (0.167)</td>
</tr>
<tr>
<td>-0.542 (0.569)</td>
<td>0.377** (0.171)</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses.

*** Significant at 1% level of significance.
** Significant at 5% level of significance.
* Significant at 10% level of significance.

The information criteria in Table 1 suggest that the equation 2 is the best specification model; hence the inflation equation takes the following form:

$$\pi_{CZ}^t = a_1\pi_{t-1} + b_1c_t + b_2c_{t-1} + d_1\Delta FP + e_t \sim iid(0, \sigma^2)$$  \hspace{1cm} (6)

Based on the initial parameter estimates inserted into the Kalman filter, the estimated results via the ML function for the Czech Republic are presented in Table 2 below.

One of the most important relations in the UC multivariate model is the relation of the output gap and inflation. The relation between output gap and the inflation is negative in the case of the Czech Republic, which is in line with New Keynesian Phillips curve theory, which implies that the output gap, the deviation of the actual output from its natural level due to nominal rigidities, drives the dynamics of inflation relative to expected inflation and lagged inflation (Zhang and Murasawa, 2011). The current and previous output gap to GDP ratios ($b_1$ and $b_2$) suggests that a negative output gap is an indication of lower inflationary pressures. However, the output gap coefficient was insignificant. Inflation inertia is highly significant with a relatively large magnitude, confirming that the previous period inflation rate tends to also persist in the current period.
From the estimated model presented in Table 2, the estimated output gap generated from the Kalman filter is presented in Figure 1 below:

![Figure 1. The output gap in Czech Republic](image)

As can be observed in Figure 1, at the very beginning of our sample (1998) the output gap of the Czech Republic was negative (below potential output), coinciding with a period after the currency crisis, and followed by a period of negative growth rate of the actual output, until the beginning of the 2000s. Based on the estimated output gap, the expansion of the economy returned in the last quarter of 2004, which represents a period of high inflows of foreign investments and an export boom. The late impact of the GFC was reflected in the Czech economy only in the third quarter of 2009, putting
the positive output gap at a fairly low base (around 0.5 percent of GDP) until the recession starts in 2011 onwards.

It is worth noting that this crisis did not have a permanent shock on future potential output (Figure 1). The beginning of a positive cycle does not start until Q1 2005 and the positive output gap is larger, reaching up to 9.8 percent of GDP. The negative output gap during late 1990s and the positive output gap largely coincide with IMF’s estimations for the Czech Republic (IMF, 2012). The significant structural break dummy d2008 controlling for the impact of GFC, is important, since it suggests that the potential output suffers a supply-side shock in the last quarter of 2008. A second important point to emphasise is the business cycle turning point in the same period, which is in not in line with the main assumptions of the model that suggest that shocks affecting potential output and the output gap should not be correlated. After 2009, the negative output gap is consistent with the slow and even negative growth rates during this period.

From orthodox economic theory, another important link in this model is the relation between the inflation rate and output gap. Inflation inertia is highly significant with a relatively large magnitude, confirming that the previous period inflation rate tends to also persist in the current period.

In Estonia, the ARIMA procedure generated the following results (Table 3).

Table 3

<table>
<thead>
<tr>
<th>Parameter Estimates</th>
<th>Diagnostics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$, $a_1$, $a_2$, $a_3$, $d_1$, $d_2$, $d_3$</td>
<td>Akaike i.c.</td>
</tr>
<tr>
<td>1</td>
<td>0.014***, 0.384***, 0.181, -0.286*</td>
</tr>
<tr>
<td>(0.005) (0.129) (0.129) (0.154)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.013***, 0.385***, 0.217*, -0.242</td>
</tr>
<tr>
<td>(0.005) (0.124) (0.129) (0.154) (0.0001)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.019***, 0.109, 0.432***, -0.616**,-0.0007</td>
</tr>
<tr>
<td>(0.005) (0.113) (0.128) (0.099) (0.0001) (0.0001)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-0.003, 0.203**, 0.062, 0.754***, 0.0001*, 0.005**,**</td>
</tr>
<tr>
<td>(0.003) (0.089) (0.134) (0.117) (0.0002) (0.0001)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses.

*** Significant at 1% level of significance.
** Significant at 5% level of significance.
* Significant at 10% level of significance.

Given that the information criteria suggest that the second equation is the best specification model, the inflation equation for Estonia is as follows:

$$\pi_{EE}^t = \mu + a(3)\pi_{t-1} + b_1c_t + b_2c_{t-1} + d_1\Delta F_t + d_2\Delta OP_t + d_3\Delta GDP(t-1) + e_t \sim iid(0, \sigma^2)$$

(7)
In the following table the estimated parameters of the multivariate UC model will be presented.

The starting and estimated multivariate UC model coefficients, Estonia

<table>
<thead>
<tr>
<th>Starting parameter values</th>
<th>Estimated coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_g$</td>
<td>1.6</td>
</tr>
<tr>
<td>$\sigma^2_r$</td>
<td>0.226</td>
</tr>
<tr>
<td>$\sigma^2_v$</td>
<td>0.245</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>-0.6</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>-0.326</td>
</tr>
<tr>
<td>$\mu_\pi$</td>
<td>0.012</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.104</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.008</td>
</tr>
<tr>
<td>$b_1$</td>
<td>-0.006</td>
</tr>
<tr>
<td>$b_2$</td>
<td>-0.006</td>
</tr>
<tr>
<td>$d_1$</td>
<td>0.0003</td>
</tr>
<tr>
<td>$\sigma^2_\pi$</td>
<td>0.013</td>
</tr>
</tbody>
</table>

A one percentage point (pp) increase in the inflation rate of the previous period, on average, will increase the inflation rate in the current period by 0.012 pp, ceteris paribus. The second lag is insignificant, meaning that probably expectations do not play a significant role and the market has a short-term memory. From orthodox economic theory, another important link in this model is the relation between the inflation rate and output gap. The output gap of the current period is highly significant indicating that, on average, a one pp increase in the output gap ratio to GDP in the current period will decrease the inflation rate by 0.006 pp, keeping other things constant. Moreover, the previous period’s output gap ratio is also highly significant; suggesting that a one pp increase in the previous periods of output gap ratio will also decrease the inflation in the current period by 0.006 pp. However, even though the output gap is significant, the economic significance is clearly less so.

From the estimated model presented in Table 4, the estimated output gap generated from the Kalman filter is presented in Figure 2 below:
Figure 2. The output gap in Estonia

In Estonia, the largest output gap stands at +6.3 percent of GDP in Q1 2007, which coincides with the highest peak of the overheating period, and the lowest output gap of -5.3 percent of GDP in Q1 2010 (Figure 2). The robust and fast catching-up period (from Q4 1998 to Q3 2008) resulted in a prolonged overheating period, which slows down only in the last quarter of 2008. The beginning of 2009 marks a turning point in the business cycle, when the deep recession starts due to the crisis. Since the crisis resulted in a negative output gap, whereas its impact on potential output was only transitory s the trend path soon gets back to the previous one, it may be concluded that shocks affecting potential output and output gap are uncorrelated for the case of Estonia, which is in line with the main assumption of the UC model.

In Kosovo, the ARIMA tests generated the following results in Table 5 below:

Table 5

<table>
<thead>
<tr>
<th>Parameter estimates</th>
<th>Diagnostics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_n$ $a_1$ $a_2$ $d_1$ $d_2$ $d_3$</td>
<td>Akaike</td>
</tr>
<tr>
<td>1</td>
<td>0.106</td>
</tr>
<tr>
<td></td>
<td>(0.721)</td>
</tr>
<tr>
<td>2</td>
<td>0.964</td>
</tr>
<tr>
<td></td>
<td>(0.283)</td>
</tr>
<tr>
<td>3</td>
<td>0.081</td>
</tr>
<tr>
<td></td>
<td>(0.287)</td>
</tr>
<tr>
<td>4</td>
<td>0.100</td>
</tr>
<tr>
<td></td>
<td>(0.298)</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses.

*** Significant at 1% level of significance.
** Significant at 5% level of significance.
* Significant at 10% level of significance.
Thus the selection criteria suggest that the best specification model in equation 2, thus the inflation

$$\pi_{KS}^t = \mu + a(L)\pi_{t-1} + b_1 c_t + b_2 c_{t-1} + d_1 \Delta FP + d_2 \Delta OP + d_3 \Delta GDP_{t-1} + e \sim \text{iid}(0, \sigma^2)$$

(8)

In the following table the estimated parameters of the multivariate UC model will be presented.

Table 6.

The starting and estimated parameters for multivariate model, Kosovo

<table>
<thead>
<tr>
<th>Starting parameter values</th>
<th>Estimated coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma^2_e$</td>
<td>0.2</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma^2_v$</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>-0.5</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.174</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_2$</td>
<td>-0.145</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.039</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_2$</td>
<td>-0.058</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_1$</td>
<td>0.519</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma^2_{\pi}$</td>
<td>0.795</td>
</tr>
</tbody>
</table>

Output equation

H0: No serial correlation: Q(10) = 0.183

Inflation equation

H0: No serial correlation: Q(10) = 0.814

H0: Normality in the residuals: Jarque-Berra = 0.072

Note: Standard errors in parentheses. The log-likelihood of the function is 148.28. The model converged after 46 iterations.

*** Significant at 1% level of significance.

** Significant at 5% level of significance.

* Significant at 10% level of significance.

The relationship between the output gap and inflation rate turned out as insignificant in Kosovo. An implication of this result may be that when the economy is
performing below the potential, it is easier to activate the working force at the same price level. Meanwhile, if the aggregate demand increases, the need for extending productivity increases too. However, the employees may also require higher wages. This in turn may increase prices. However, the insignificant relationship between output gap and the inflation rate may also reflect the specific labour market in Kosovo, where even though the unemployment rate is considerably high, the reservation wage is also considered to be relatively high (Kastrati, 2014), whereas wages are not necessarily flexible in short-run. Therefore, the impact of the output gap in the inflation rate may be insignificant. Additionally, nevertheless the presence of a positive or negative output gap, Kosovo has a very limited scope of monetary policy at hand to influence the price levels.

From the estimated model presented in Table 6, the estimated output gap generated from the Kalman filter is presented in Figure 3 below:

![Figure 3. The output gap in Kosovo](image)

In Kosovo, the period between Q12002 to Q32008 represents a period where the economy was performing below its potential (Figure 3). After around ten years performing below its potential, around 2009 for the first time the economy reaches a temporary equilibrium state, followed by a short (+07 percent of GDP) overheating period. In 2011 the business cycle once again turns negative. Throughout the period under examination, the business cycle in Kosovo appears to closely mimic the fiscal policy developments.

5. **CONCLUSION**

Given that the potential output and the corresponding output gap concepts are mainly discussed in the light of mainstream theories, this paper discusses the relevance of the output gap in the transition context, with special reference to the Czech Republic, Estonia and Kosovo. Given the macroeconomic
imbalances and structural changes characterizing many TEs, such as relatively sluggish growth and chronic balance of payments deficits together with structural deficiencies, led us to believe that TEs have continuously been operating in the presence of relatively large underutilized resources. Furthermore, the skill mismatch, together with relatively high reservation wage, obsolete capital and other market rigidities meant that the NAIRU was extremely high in most TEs, whereas the actual unemployment rate could be much lower. Given that the high NAIRU and obsolete capital do not directly enter into the calculation of the potential output, the findings of the paper led to the conclusion that the potential output in these countries was low relative to the factors of production apparently available, hence the output gap was relatively small, which is reflected in the output gaps estimated and reported in this paper for the Czech Republic, Estonia and Kosovo. Therefore, the output gap in TEs may to a large extent reflect the structural changes and to a smaller extent the cyclical component.

In order to reflect persistent underutilised resources i.e. negative output gap, as well as several structural breaks, the UC approach operationalized via the Kalman filter was employed as the appropriate method of estimation for TEs. A typical case was the business cycle in Kosovo where for most of the time our observed results indicated that the actual output was below its potential, which reflects the economic characteristics of this country. On the other hand, in more developed TEs, where major structural changes have been largely completed, such as the Czech Republic, the differences with the developed countries have become smaller, thus the behaviour of the output gap reflected a more symmetric (developed country-wise) business cycle. Along with presenting new estimates of the output gap for Czech Republic, Estonia and Kosovo, another novelty of this study was the intuitive explanation of the technicalities underpinning the Kalman filter method for economists.

Another important finding suggests that the GFC shock had a significant supply-side shock but with a temporary effect on potential output of the Czech Republic during the following time period modelled. After 2009, the negative output gap is consistent with the slow and even negative growth rates during this period.

In Estonia, the GFC effect resulted in a relatively significant negative output gap, however its impact on potential output was again only temporary, as the trend path soon returned to the previous one. Due to relatively low external exposure and domestically funded banking system, the GFC caused no recession in Kosovo, but rather slowed the pace of growth mainly via the external sector channels and the uncertainties perceived by the banking sector. Finally, the negative relationship between inflation and output gap was informative in the case of Czech Republic and Estonia, because it suggested a presence of inflation inertia in these countries, even though the economic significance was relatively small. In Kosovo, structural issues like relatively high unemployment rate and reservation wage and inflexible wages in short-run, led to the conclusion that the impact of the output gap in the inflation rate may be insignificant.
REFERENCES


APPENDIX 1

Following Watson (1986) model, we first present signal and state variables as equations, and then transform them into a state space form of vectors and matrices, in a constant-parameter linear state space model (Watson model):

The measurement (observation) equation:

\[ Y_t = ZX_t + DD_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2_{\varepsilon}) \]  

(A1)

where \( Y_t \) is the (1 x 1) vector of the observed variables, \( X_t \) is the (3 x 1) vector of the unobserved variables, \( Z \) is the (1 x 3) matrix of the coefficients of unobserved variables, and in the case where other exogenous variables are included (e.g. inflation in the Phillips curve or unemployment rate in NAIRU, in multivariate representations of the UC model), \( D \) will stand for the matrix of the coefficients of the exogenous variables and \( D_t \) is the vector of exogenous variables. In the univariate representation of the state space model, the exogenous variables are missing since we extract the unobserved potential output and output gap by using the actual output as the only known information. In the univariate form of the measurement equation, we do not include exogenous variables (e.g. inflation rate, unemployment rate, etc.), so this term is equal to zero, and the restricted Watson (1986) form will be:

\[ Y_t = ZX_t \]  

(A2)

As mentioned earlier, in the UC model, the framework should be written in a state space form, that is, as a system of vectors and matrices. In the state space form, the expression in (2.12) can equivalently be written as:

\[ Y_t = [y_t], \quad Z = [1 \ 1 \ 0], \quad X_t = \begin{bmatrix} g_t \\ c_t \\ c_{t-1} \end{bmatrix} \]  

(A3)

(A4)

(A5)

Writing these expressions as a system of vectors and matrices will give us the explicit form of the measurement equation:

\[ [y_t] = [1 \ 1 \ 0] \begin{bmatrix} g_t \\ c_t \\ c_{t-1} \end{bmatrix} = g_t + c_t \]  

(A6)

The measurement equation relates the observation \( y_t \) to the state vector \( X_t \) through the signal \( ZX_t \). Here the algorithm of the Kalman filter does the prediction of the unobserved components.

The second equation is the transition equation (state) of the UC model:

\[ X_t = BX_{t-1} + AZ_t + v_t; \quad v_t \sim N(0, \sigma^2_v) \]  

(A7)

5 In the univariate approaches, unobserved components, such as the trend and the cycle, are filtered out of some single series (e.g. actual output). Conversely, in multivariate approaches, this filtering is accomplished conditional on values of other variables, which are treated as exogenous.
Where $B$ is the $(4 \times 4)$ matrix of coefficients of unobserved variables, $A$ is the matrix of coefficients of exogenous variables and the $Z_t$ is the vector of exogenous variables.

In the state space form, again, since we are presenting the univariate framework with no exogenous variables, the term $AZ_t$ is omitted. So, presenting the transition equation in the state space form, where:

\[
X_t = \begin{bmatrix} g_t \\ 1 \\ c_t \\ c_{t-1} \end{bmatrix}, \quad (A8)
\]

\[
B = \begin{bmatrix} 1 & \mu_g & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \phi_1 & \phi_2 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad (A9)
\]

\[
X_{t-1} = \begin{bmatrix} g_{t-1} \\ 1 \\ c_{t-1} \\ c_{t-2} \end{bmatrix}, \quad \text{and} \quad (A10)
\]

\[
\epsilon_t \text{ and } \nu_t = \begin{bmatrix} \epsilon_t \\ 0 \\ \nu_t \\ 0 \end{bmatrix}. \quad (A11)
\]

Here, the lagged signal $(4 \times 1)$ $X_{t-1}$ indicates that the transition algorithm is a recursive process. The combinations of all these vectors and matrices will give us the explicit state space form of the transition equation:

\[
\begin{bmatrix} g_t \\ 1 \\ c_t \\ c_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & \mu_g & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \phi_1 & \phi_2 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} g_{t-1} \\ 1 \\ c_{t-1} \\ c_{t-2} \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ 0 \\ \nu_t \\ 0 \end{bmatrix}. \quad (A12)
\]

Generating:

\[
g_t = g_{t-1} + \mu_g + \epsilon_t \quad \text{(random walk with drift)} \quad (A13)
\]

\[
c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \nu_t \quad \text{(an AR(2) process)} \quad (A14)
\]
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PROIZVODNI JAZ U TRANZICIJSKIM EKONOMIJAMA PRIMJENOM METODE NEVIDLJIVE KOMPONENTE: SLUČAJ ČEŠKE, ESTONIJE I KOSOVA

Sažetak
U radu se istražuje koncept i procjena proizvodnog jaza u tranzicijskim ekonomijama, s posebnim osvrtom na Češku, Estoniju i Kosovo. Motivacija za istraživanje ovog fenomena leži u makroekonomskim neravnotežama koje karakteriziraju brojne ekonomije u tranziciji, poput relativno slabog rasta, kroničnih deficita u platnoj bilanci i strukturnih nedostataka, a koje funkcioniraju s nedovoljno upotrijebljenim, ali relativno velikim sredstvima. Budući da se konceptima potencijalne proizvodnje i odgovarajućem proizvodnom jazu većinom raspravlja u svjetlu glavnih teorija, novina je ovog rada ispitivanje relevantnosti proizvodnog jaza u kontekstu tranzicije. Kako bi se prikazala dugotrajno neupotrijebljena sredstva, kao i nekoliko strukturnih prekida, koristili smo se modelom nevidljive komponente primijenjenim na temelju Kalmanova filtra, odgovarajuće metode procjene za ekonomije u tranziciji. Još je jedna novost ovog istraživanja tekstualno objašnjenje tehničkih podloga Kalmanova filtriranja. Uzrokujući pad proizvodnje ispod potencijala, globalna ekonomska kriza imala je značajan, ali prolazan utjecaj na Češku i Estoniju. Zbog relativno slabe vanjske izloženosti i domaćeg financiranja bankarskog sustava, globalna ekonomska...

* Stavovi izraženi u ovom radu isključivo su stavovi autora. Zahvaljujemo profesoru Nicku Adnettu i sudionicima konferencije SINCERE, posebno Zoranu Aralicu na korisnim komentarima.
Križa nije izazvala recesiju na Kosovu, već je usporila tempo rasta kanalima vanjskog sektora i neizvjesnosti koju percipira bankarski sektor. Naposljetku, negativni odnos inflacije i proizvodnog jaza bio je informativan za Češku i Estoniju jer je ukazao na inertnost inflacije u tim zemljama, a utjecaj proizvodnog jaza na stopu inflacije na Kosovu pokazao se neznatnim.

**Ključne riječi:** proizvodni jaz, model nevidljive komponente, Kalmanov filter, ekonomije u tranziciji.

**JEL klasifikacija:** E32, C32, P2