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ABSTRACT

Stock markets play an important role in spurring economic growth and development through diversification opportunities. However, diversification cannot be truly achieved if we continue to ignore additional dimensions of risk, namely skewness and kurtosis. This study incorporates higher moments of risk to form a mean-variance-skewness-kurtosis based framework for portfolio optimisation. Inclusion of higher moments in optimisation framework acknowledges the risk of asymmetric returns and fat-tail risk and can help investors in formulating optimal portfolios of stocks which can be significantly divergent from the ones they obtain through the Markowitz mean-variance optimisation. Our results confirm the presence of tradeoff between returns and additional dimensions of risk in Pakistan Stock Exchange (PSX) and strongly suggest including them in the optimisation framework to avoid sub-optimal decisions and to curtail exposure towards higher moments of risks.

1. Introduction

The role of stock markets in spurring economic growth and development is almost undeniable. Stock markets have a tendency to affect economic activities of a country by providing several functions, such as secondary trading of equity securities that helps businesses and industries raise capital because of the liquidity created in these markets. Stock markets also provide investors with the means to invest effectively and efficiently thanks to the diversification they can have by investing in different securities, asset classes or categories. Diversification also helps investors in improving asset allocation decision which ultimately leads to high long-term economic growth for both the country and the investors (Levine & Zervos, 1996).

One of the key features of diversification is to create a balance between the risk and rewards of different investments for which the gold standard is the modern portfolio theory (MPT), introduced by Markowitz (1952). The MPT provides a mean-variance framework to minimise risk for a given level of expected return, or to maximise expected returns for a given level of risk. Over the years, the Markowitz mean-variance framework has been widely accepted for portfolio optimisation and is a vital practical tool for practitioners. Nonetheless, one of the major loopholes in this framework is the assumption of normally distributed asset returns, which implies that expected value and variance should accurately represent the returns distribution. This assumption makes the entire process of optimisation prone to severe underestimation of investment risk, as it conveniently assumes the second moment (variance) is capable to proxy the risk of any investment in its entirety. However, several studies in empirical finance confirm that asset returns distributions are characterised by negative skewness and excess kurtosis, and so the assumption for a normal distribution is continuously being violated (Aggarwal, Rao, & Hiraki, 1989; Beedles, 1986; Lux & Marchesi, 2000). If portfolio returns are negatively skewed, the probability of getting negative returns is higher than the positive returns and vice versa. Similarly, the presence of excess kurtosis indicates the higher probability of extreme events also known as fat-tail risk. The presence of these two phenomena gives rise to a need for the inclusion of higher moments of risk to describe portfolio behaviour, otherwise the portfolio optimisation based on mean-variance under the normality assumption may lead us to underestimate the true investment risk and might end up with an inefficient portfolio rather than the efficient one.

In the presence of skewness and kurtosis, the major hurdle in the expansion of a mean-variance framework to include higher moments of risk is the difficulty in finding a tradeoff among the four objectives. This creates a non-convex and non-smooth multi-objective problem and due to this, many studies in portfolio selection mainly focus on the first three moments and often neglect kurtosis. Additionally, most of the models only consider the distribution of asset return and ignore investor's risk preferences and trading strategies.

In this study we use a polynomial goal programming (PGP) approach with a multi-objective method developed by Lai, Yu, and Wang (2006) to deal with this very issue. The multi-objective function is capable to incorporate investor preferences while finding an optimal solution based on multiple criteria. We incorporate the investor's weighted preferences for skewness and kurtosis in addition to the conventional criteria of expected returns and variance at Pakistan Stock Exchange (PSX) and found that additional dimensions of risk play an important role in determining the yields of optimised portfolios. Our results suggest that investors with the intention to minimise these additional dimensions of risk would probably have to settle for lower expected returns.

The remainder of the article is organised as follows. Section 2 includes a brief review of the literature. Section 3 describes the data and methodology used for this study. Sections 4 and 5 include the empirical results and conclusion, respectively.

2. Literature review

A portfolio is a suitable combination of securities that individual investors and institutions possess with the objective of earning a profit. In the financial world, portfolio optimisation is plagued with several problems. Investors want an optimal combination of securities that maximises returns at the minimum level of risk. The MPT solves this problem through the

diversification of investments (Aracioglu, Demircan, & Soyuer, 2011; Markowitz, 1952; Singh, Sahu, & Bharadwaj, 2010).

Markowitz (1952) proposed a mean-variance framework for portfolio optimisation which focused on the first two moments of return distributions. The primary assumption was that asset returns are normally distributed, or that the utility function is quadratic and only depends on the first two moments.

In a serious contradiction to the assumptions of MPT, a number of studies conducted in different markets suggest that financial returns are non-normal. Beedles (1979) concludes that returns are significantly positively skewed and uneven with respect to the Australian market. Aggarwal et al. (1989) find skewness and kurtosis in returns for the Japanese equity market. Lux and Marchesi (2000) confirm lepto-kurtosis in financial time series and also confirm that volatility clustering and kurtosis have a positive relationship. Tseng and Li (2012) analyse the presence of volatility clustering in financial time series and also find that high volatiltiy clustering increases kurtosis risk and asymmetry. They find that an increase in volatiltiy also increases kurtosis risk and skewness risk.

The above findings on the presence of non-normality, skewness and kurtosis make it necessary to incorporate higher order moments for optimal portfolio construction and selection. Kane (1982), Konno, Shirakawa, and Yamazaki (1993), Lai (1991) and Simonson (1972) emphasise the importance of incorporating skewness of returns, and suggest that the third moment of risk can improve mean-variance portfolio efficiency and portfolio selection and optimisation. In recent years, a lot of attention has been directed towards kurtosis due to its integral role in portfolio selection. Beardsley, Field, and Xiao (2012), Guidolin and Timmermann (2008), Hong, Tu, and Zhou (2007), Jarrow and Zhao (2006), Mitton and Vorkink (2007), Martellini (2008), Li, Qin, and Kar (2010), Liu, Liu, and Wang (2013) and Wilcox and Fabozzi (2009) have emphasised the importance of kurtosis and its inclusion in portfolio optimisation and selection.

Incorporation of higher order moments turns portfolio selection into a non-convex and non-smooth optimisation problem that can be characterised by multiple conflicting and competing objective functions such as maximising expected return and skewness, and minimising variance and kurtosis, respectively. In the past, studies used the mean-variance model to reformulate and simplify the solution of quadratic programming. However, finance literature is now enriched with various methods to solve this complex task (Smimou & Thulasiram, 2010). Although there are approaches like shortage function developed by Briec, Kerstens, and Jokung (2007) that uses non-parametric efficiency measurements or the evolutionary algorithm approach in an experimental setting by Chiam, Tan, and Al Mamum (2008), but in line with Lai et al. (2006) and Mhiri and Prigent (2010), we decided to use PGP with a multi-objective approach, to incorporate higher moments of risks while selecting an optimal portfolio. PGP, according to the existing literature is considered as highly effective and efficient to solve such problems with multiple conflicting risk preferences.

Tayi and Leonard (1988) were the first to introduce PGP, followed by Lai (1991) who applied the multi-objective method by incorporating skewness into portfolio selection with investors' preferences. PGP is a method flexible enough to incorporate investor preferences which they might have for the higher moments of returns. The superiority of PGP is that it allows simultaneous optimisation with reference to variance, skewness and kurtosis without explicitly specifying a utility function, however, it still requires that an investor's preferences towards higher moments of risk should explicitly be defined. Chunhachinda, Dandapani,

Hamid, and Prakash (1997); Sun and Yan (2003); Canela and Collazo (2007); Hafner and Wallmeier (2008) provide at length the superiority and practical efficiency of PGP over other approaches. Further improvements in PGP are brought by Prakash, Chang, and Pactwa (2003) who use the multi-objective method for construction of an optimal portfolio. Lai et al. (2006) also augmented the dimension of portfolio selection from mean-variance-skewness to mean-variance-skewness-kurtosis using the multi-objective method. Similarly, Mhiri and Prigent (2010) incorporated higher moments of skewness and kurtosis and also Davies, Kat, and Lu (2009), who focused on selection of efficient funds from hedge funds.

3. Data and methodology

In this study, we choose eight stocks listed at the PSX. PSX, formerly known as Karachi Stock Exchange, is the representative stock market of Pakistan. According to the Bloomberg, Morgan Stanley Capital International (MSCI) has officially stated that the market index would be reclassified to emerging market status from its present status of frontier market in May 2017. Although the PSX is comprises of 33 sectors with 578 listed companies, its performance is usually evaluated by *KSE-100 index* recently been declared as best performing Asian market index after yielding 14% returns in 2016. The market capitalisation of PSX is about PKR 8.4 trillion (80 billion US$) and it is expected that the recent performance and the inclusion in MSCI emerging market index would bring a huge demand for the securities, listed on PSX, by the investors seeking healthy returns and therefore this is the right time to analyse in-depth the structure of this market to identify the impact of any ignored risk that might lead new investors towards sub optimal decisions.

The semi strong inefficiency of PSX has been reported by several researchers. Nazir et al. (2010) and Asghar et al. (2011) have reported strong predictive power of dividend in subsequent returns and the volatility of stocks listed in PSX. This is one of the reasons most of the investors in PSX favours dividend yielding stocks. Another reason is the regular cash flows associated with these stocks taken positively by the investors. Keeping in view the above market sentiments, we select two stocks on the basis of highest dividend yield over the last 52 weeks (GASF and AGTL), top two over the last 52 weeks with highest average volume traded and dividend yield (GHNL and HCAR), top two with highest dividend yield in cement sector (FCCL and MLCF), one from the food sector based on highest dividend yield (CLOV) and one from commercial banks based on average volume traded and highest dividend yield (NBP). Additional attention has been paid while selecting these stocks to the sectors preferred by investors and are attracting significant proportion of investment, so that they should not be left out. Monthly closing prices of the selected stocks for the last 10 years, from May 2006 to April 2016, have been used to estimate monthly returns by taking natural log of price ratios, in line with Biglova, Jašić, Rachev, and Fabozzi (2004). The data has been taken from the data base of Standard Capital (a local brokerage house) that maintains financial data of companies listed in PSX for analysis purpose. The preference has been given to monthly returns over daily returns due to several factors. First of all, monthly returns are free from the noise factor which is prominent in daily returns and has a potential to distort real inferences about the investment strategy by increasing the volatility many folds. Secondly, the usage of monthly returns makes the results of this study more comparable to the performance of several mutual funds that usually report their performance on monthly basis.

$$R_{it} = \log(P_{it}/P_{it-1})$$

These calculated returns have been used in the optimisation process, which is explained below.

3.1. Optimisation and incorporation of higher moments

The first two equations represent the portfolio's expected return and variance which are also the first two moments of the returns distribution and are the essential components of optimisation process;

$$\text{Mean} = R(x) = X^T \overline{R} = \sum_{i=1}^{n} w_i R_i \tag{1}$$

$$\text{Variance} = V(x) = \sum_{i=1}^{n} \sum_{i=1}^{n} w_i w_j \sigma_{ij} (i \neq j) \tag{2}$$

Equation 3 represents portfolio skewness which is the weighted sum of individual coefficients of skewness and co-skewness among returns. Similarly, equation 4 defines portfolio kurtosis which is the weighted sum of individual coefficient of kurtosis and co-kurtosis among equity returns.

$$\text{Skewness} = S(x) = E[R - \overline{R}]^3 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} w_i w_j w_k s_{ijk} (i \neq j) \tag{3}$$

$$\text{Kurtosis} = K(x) = E\left[R - \overline{R}\right]^4 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} w_i w_j w_k w_l k_{ijkl} (i \neq j) \tag{4}$$

The notation for variance-covariance matrix used in equation 2 is as follows:

$$\sigma_{ij} = E\left[\left(R_i - \overline{R}_i\right)\left(R_j - \overline{R}_j\right)\right] \tag{5}$$

For the calculation of Skewness and co-skewness matrix we use the following structure where S_{iii} is the skewness coefficient for an individual stock, S_{iij} is the coefficient of co-skewness for two stocks portfolio, while for a three stocks portfolio the coefficient of co-skewness is S_{ijk}:

$$S_{iii} = E\left[\left(R_i - \overline{R}_i\right)^3\right], \tag{6}$$

$$S_{iij} = E\left[\left(R_i - \overline{R}_i\right)^2 \left(R_j - \overline{R}_j\right)\right], \tag{7}$$

$$S_{ijk} = E\left[\left(R_i - \overline{R}_i\right)\left(R_j - \overline{R}_j\right)\left(R_k - \overline{R}_k\right)\right] \tag{8}$$

Similarly, Kurtosis – co-kurtosis coefficient are k_{iiii}, k_{iiij}, k_{ijjj} and k_{ijkl} and are defined below:

$$k_{iiii} = E\left[\left(R_i - \overline{R}_i\right)^4\right], \tag{9}$$

$$k_{iijj} = E\left[\left(R_i - \overline{R}_i\right)^2 \left(R_j - \overline{R}_j\right)^2\right], \tag{10}$$

$$k_{ijjj} = E\left[\left(R_i - \overline{R}_i\right)\left(R_j - \overline{R}_j\right)^3\right], \tag{11}$$

$$k_{ijkl} = E\left[\left(R_i - \overline{R}_i\right)\left(R_j - \overline{R}_j\right)\left(R_k - \overline{R}_k\right)\left(R_l - \overline{R}_l\right)\right] \tag{12}$$

Finally, to consolidate multiple objectives, we use the PGP approach used by (Aracioglu et al., 2011; Lai et al., 2006; Mhiri & Prigent, 2010). With the intention to maximise portfolio return and skewness and minimise variance and kurtosis:

$$Mean = R * (x) = X^T \overline{R} (\text{Maximise}) \tag{13}$$

$$Variance = V * (x) = X^T V X (\text{Minimise}) \tag{14}$$

$$Skewness = S * (x) = E(X^T \left(R - \overline{R}\right))^3 (\text{Maximize}) \tag{15}$$

$$Kurtosis = K * (x) = E\left(X^T \left(R - \overline{R}\right)\right)^4 (\text{Minimize}) \tag{16}$$

Each Solution conditions with $X^T I = 1$, $X \geq 5\%$ and $X \leq 50\%$[1]

R*, V*, S* and K* are the optimised portfolio parameters for four moments, also called as aspired values (Lai et al., 2006). R is the returns distribution with the expected value of $\overline{R}$. X^T in equations 13 to 16 is the transpose of weight vector, where the weights represent the proportion of funds allocated to different assets included in the portfolio.

To model and incorporate various investor's preferences towards mean, variance, skewness and kurtosis, we use four parameters $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ respectively. By using lambda parameters, we turn the multi-objective portfolio selection into single-objective (Equation 17) for various investor's preferences, which is a general way to solve the multi-objective problem. This is primarily because multi-objective problems can be solved in two steps: (1) non-dominated solution, independent from investors' preferences; and (2) selection of most suitable solution among the given solutions according to risk preferences which included higher moments solutions:

$$Z = \left|\frac{d_1}{R^*}\right| \lambda_1 + \left|\frac{d_2}{V^*}\right| \lambda_2 + \left|\frac{d_1}{S^*}\right| \lambda_3 + \left|\frac{d_1}{K^*}\right| \lambda_4 \tag{17}$$

The goal variables in the above objective function are d_1, d_2, d_3 and d_4 which are the estimates of the deviation of actual moments (under achievement) with their respective aspired values.

This study uses Visual Basic for Applications (VBA) coding in excel to compute Variance-Covariance, skewness- CoSkewness and Kurtosis-CoKurtosis matrices. We then used solver function in Excel to optimise Z (the objective function) to estimate weights associated with each optimal portfolio given the investor's preferences (λ_1, λ_2, λ_3, λ_4).

4. Empirical results

A curious reader must seek a justification of such a huge emphasis on usually ignored dimensions of risk. Table 1 below provides the answer of it. We report in Table 1 the descriptive statistics of monthly USD and PKR returns earned by KSE-100 index during the last 10 years. Annualised returns are although very impressive, about 11.71% (USD) and 17.03% (PKR) but the cost to yield those returns in terms of risk is enormous. Not only that the annualised volatility is very high, about 30% in USD and 27.6% in PKR, the values of kurtosis and skewness also reveal the true risk embedded in the market. Kurtosis is 2.5–3 times greater than what it should have been if the returns were normally distributed, indicating a leptokurtic distribution with the significantly higher probability of extreme events. Moreover, negative skewness, both in terms of USD and PKR, is making this leptokurtic phenomenon worse implying higher chances of having extreme losses (fat-tail risk).

Table 2 shows the descriptive statistics of each selected stock at PSX (KSE100- index). The last two columns provide a ranking of these stocks based on coefficient of variation, a relative measure of riskiness. It seems that CLOV is at the top (worst) with the highest risk per unit return followed by GASF and NBP while AGTL is at the bottom (best) with the lowest risk per unit of return. This is one way of looking things and the similar sort of ranking based on standardised skewness and kurtosis have been provided in the last two columns of Table 2 showing significantly different ranks compared to the ranking based on Coefficient of Variation (CV). Such discrepancies are the true motivation for the inclusion of higher moments of risk in the process of portfolio optimisation.

As a first step in applying the PGP approach, we calculate the individual aspired levels by solving equation 13, 14, 15 and 16 separately and the results can be seen in Table 3. The upper panel of Table 3 shows the weights allocated to eight stocks while creating a portfolio providing optimal values of selected moment of mean, variance, skewness and kurtosis, all reported in lower panel.

Table 1. Descriptive statistics of KSE-100 returns (May 2006–April 2016).

	KSE 100 Returns (USD)	KSE 100 Returns (PKR)
Mean	0.009759696	0.014194215
Standard Error	0.007751045	0.00728352
Median	0.022121096	0.023220758
Mode	#N/A	#N/A
Standard Deviation	0.084908445	0.079786963
Sample Variance	0.007209444	0.00636596
Kurtosis	8.129780115	10.46559779
Skewness	−1.473962619	−1.670225169
Range	0.709188718	0.706450316
Minimum	−0.466480632	−0.461051644
Maximum	0.242708086	0.245398671
Sum	1.171163576	1.703305779
Count	120	120

Source: Authors' Estimations and Calculations.

Table 2. Ranking of selected stocks based on coefficient of variation.

Stocks	Mean	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	C.V*	Ranking 1**	Ranking 2***	Ranking 3****
GASF	0.00209	0.13877	−1.865	10.3474	−0.01228	66.3971	2	1	1
AGTL	0.01108	0.10782	−1.1488	7.62417	0.067568	9.73105	8	3	4
HCAR	0.01533	0.1492	0.02449	0.88923	0.077314	9.73255	7	6	8
GHNL	0.01246	0.18264	−0.146	2.09682	0.047444	14.6581	6	4	6
FCCL	0.00636	0.10913	−0.0091	0.70964	0.023506	17.1588	5	5	7
MLCF	0.0078	0.15185	0.63549	1.45833	0.026376	19.4679	4	7	5
NBP	0.00222	0.12483	−1.4163	6.60074	−0.01262	56.2297	3	2	2
CLOV	0.00127	0.16178	0.3604	1.6982	−0.01561	127.386	1	8	3

*Coefficient of variation (C.V) = Standard deviation / Mean; **Ranking based on Coefficient of Variation; ***Ranking based on Standardised Skewness, reversed to get the highest value for most negatively skewed stock; ****Ranking based on Standardised Kurtosis.
Source: Authors' Estimations and Calculations.

Table 3. Individual aspired levels of higher moments with weights.

Aspired Levels of Higher Moments				
Stocks	Mean	Variance	Skewness	Kurtosis
GASF	5.00	5.00	5.00	23.93
AGTL	5.00	32.95	5.00	11.53
HCAR	50.00	5.00	6.10	5.00
GHNL	20.00	5.00	18.90	5.00
FCCL	5.00	23.85	5.00	5.00
MLCF	5.00	5.00	5.00	12.42
NBP	5.00	5.04	5.00	11.18
CLOV	5.00	18.16	50.00	25.94
Aspired Levels	0.01149	0.00657	0.4475048	3.03

Source: Authors' Estimations and Calculations.

A maximum mean return an investor can get by solving equation 15 is 0.01149, and to get this level of return they must invest almost 50% of the funds in HCAR, 20% in GHNL and 5% each in the remaining six stocks. The minimum level of risk an investor can enjoy is 0.00657, obtained by solving equation 16. Equation 17 gives us the maximum value of skewness investor can have in a portfolio based on these stocks. And similarly, equation 18 solves for the minimum level of kurtosis (fat-tail risk) an investor has to deal with. It is important to note that the aspired levels for each moment have been calculated in a complete isolation and are of little use for an investor whose decision is based on multiple moments simultaneously with differing preferences towards each moment.

To incorporate simultaneous and varying preferences towards different criteria we estimate 11 portfolios with different preferences and the results are reported in Table 4.

Table 4 shows the Investors' preferences $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ which are importance given by the investor to mean, variance, skewness and kurtosis, respectively. Higher value designates higher importance to selected moments while estimating an optimal portfolio. Portfolio 1 is a benchmark mean-variance portfolio and used to make comparison with the other portfolios with multiple objectives. Portfolios 2, 4 and 5 are based on the preference structure of (3, 1, 1, 0), (3, 1, 2, 1) and (3, 1, 3, 1) which reflects strong linking of investors towards maximisation of returns. While in portfolios 3, 6, 7, 9 and 10 with preference structure of (1, 3, 1, 1), (1, 1, 1, 3), (1, 3, 1, 3), (1,1,3,0) and (1,1,0,3) the investor seems keen in optimising different dimensions of risk (variance, skewness and kurtosis).

Table 4. Asset allocation with weights and different preferences for optimal portfolios.

Portfolios	1	2	3	4	5	6	7	8	9	10	11
λ_1	1	3	1	3	3	1	1	3	1	1	1
λ_2	1	1	3	1	1	1	3	1	1	1	1
λ_3	0	1	1	2	3	1	1	2	3	0	1
λ_4	0	0	1	1	1	3	3	3	0	3	1
GASF	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050
AGTL	0.432	0.140	0.050	0.050	0.050	0.062	0.059	0.064	0.176	0.050	0.055
HCAR	0.094	0.075	0.090	0.096	0.099	0.050	0.050	0.050	0.099	0.070	0.115
GHNL	0.059	0.139	0.050	0.050	0.050	0.052	0.050	0.061	0.150	0.070	0.055
FCCL	0.159	0.050	0.050	0.050	0.050	0.083	0.050	0.135	0.050	0.399	0.050
MLCF	0.050	0.050	0.275	0.274	0.276	0.224	0.254	0.182	0.050	0.050	0.274
NBP	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050
CLOV	0.107	0.445	0.385	0.379	0.375	0.429	0.437	0.408	0.375	0.260	0.351
Mean	0.0085	0.006	0.01	0.006	0.006	0.005	0.005	0.005	0.007	0.006	0.006
Variance	0.007	0.01	0.01	0.01	0.01	0.01	0.01	0.009	0.009	0.008	0.01
Skewness	−1.171	0.374	0.26	0.249	0.242	0.321	0.34	0.277	0.255	−0.123	0.194
Kurtosis	5.0484	1.038	0.41	0.408	0.405	0.507	0.503	0.503	1.045	0.511	0.405
Sharpe Ratio	0.056	0.022	0.062	0.022	0.022	0.012	0.012	0.013	0.034	0.025	0.022

Source: Authors' Estimations and Calculations.

Table 5. Change of moments with investor's preferences.

Portfolios	2	2A	9	9A	10	10A
λ_1	3	3	1	2	1	2
λ_2	1	1	1	1	1	1
λ_3	1	2	3	3	0	0
λ_4	0	0	0	0	3	3
GASF	0.050	0.050	0.050	0.050	0.050	0.050
AGTL	0.140	0.161	0.176	0.170	0.050	0.253
HCAR	0.075	0.050	0.099	0.114	0.070	0.112
GHNL	0.139	0.115	0.150	0.181	0.070	0.137
FCCL	0.050	0.054	0.050	0.050	0.399	0.083
MLCF	0.050	0.050	0.050	0.050	0.050	0.050
NBP	0.050	0.050	0.050	0.050	0.050	0.050
CLOV	0.445	0.471	0.375	0.335	0.260	0.266
Mean	0.00573	0.00532	0.00653	0.00703	0.00577	0.00748
Variance	0.00994	0.00994	0.00907	0.00907	0.00779	0.00779
Skewness	0.37438	0.38101	0.25542	0.20986	−0.1234	−0.1234
Kurtosis	1.03784	1.05	1.04525	1.04525	0.51056	1.48703
Sharpe Ratio	0.019	0.015	0.029	0.034	0.022	0.042

Source: Authors' Estimations and Calculations.

Portfolio 8, however, with the preference structure (3,1,2,3) seems to achieve an ambitious objective with higher importance to several competing criteria, like maximisation of returns and minimisation of kurtosis and skewness.

To make this tradeoff more visible and comprehendible, we pick three portfolios, 2, 9 and 10 from Table 4 and create their sub-portfolios 2A, 9A and 10A by changing preferences for only one moment and leaving rest of the preferences constant.

Table 5 shows the results of six portfolios explained above. In portfolio 2A we leave preferences for expected return and variance similar to portfolio 2 and a higher preference is assigned to the maximisation of skewness. Results show that it helps in increasing portfolio skewness (desirable phenomenon) but it also increases portfolio kurtosis (undesirable phenomenon) showing a tradeoff even in this simplified situation. In portfolio 9A where

we leave preferences for variance and kurtosis similar to portfolio 9, we see that the higher preference to maximisation of returns yields us higher expected return, but at the cost of accepting lower positive skewness (undesirable phenomenon). Finally, when we try to change the preference parameter of portfolio 10 towards returns by holding preferences for variance and skewness constant in portfolio 10A, results show that the returns could be increased by almost 50% of the base value but at the cost of increasing fat-tail risk (kurtosis) to almost three times to its base value.

These results clearly indicate that the additional dimension of risk, namely skewness and kurtosis, plays an important role in determining the yields of optimised portfolios. And the investors with the intentions to minimise these additional dimensions of risk would probably have to settle for lower expected returns.

The above results are important in a way that they challenge the entire paradigm of Mean-Variance optimisation where variances are considered sufficient for the estimation of risk and no consideration is given to the additional dimensions of risks. To make the tradeoff between returns and the additional dimensions of risk more visible in an old-fashioned way, we take an indirect way pursuing Mean-Skewness and Mean-Kurtosis optimisation.

We first construct seven portfolios by optimising returns for different levels of risk through mean variance optimisation (Table 6) to obtain the baseline Markowitz Mean-Variance efficient frontier. We then hold the level of variance constant for each portfolio and optimise them for the third and fourth moment of risk (Table 7 and 8). Results show while optimising for higher moments of risk, the expected return goes down substantially, especially in the case of Mean-skewness case. This reduction in expected returns further substantiates the results we obtained in previous section and also indicates that the optimal portfolios in PSX are overpriced and the higher returns generated by them coming at the cost of additional risk embedded in skewness and kurtosis.

Figure 1 below shows the efficient frontiers based on Mean-Variance, Mean-Skewness and Mean-Kurtosis optimisation. Risk is on x-axis while y-axis shows respective returns. A clear downward shift in Mean-Skewness and Mean-Kurtosis efficient frontiers is observable compared to the base case of Mean-Variance efficient frontier. These results further confirm our apprehensions about the severe overpricing of emerging markets like PSX. Under the conventional framework of Mean-Variance optimisation, markets like PSX seem very attractive offering a substantially high reward to risk ratio visible thorough a higher efficient

Table 6. Benchmark mean-variance optimisation.

Portfolios	1	2	3	4	5	6	7
GASF	0.050	0.050	0.050	0.050	0.050	0.050	0.050
AGTL	0.431	0.450	0.466	0.415	0.358	0.314	0.277
HCAR	0.090	0.137	0.174	0.255	0.323	0.376	0.420
GHNL	0.057	0.074	0.087	0.080	0.069	0.060	0.053
FCCL	0.163	0.112	0.071	0.050	0.050	0.050	0.050
MLCF	0.050	0.050	0.050	0.050	0.050	0.050	0.050
NBP	0.050	0.050	0.050	0.050	0.050	0.050	0.050
CLOV	0.109	0.077	0.052	0.050	0.050	0.050	0.050
Mean	0.008441	0.009213	0.009831	0.010385	0.010788	0.011	0.011249
Variance	0.007001	0.007501	0.008001	0.008501	0.009001	0.009501	0.010001
SD	0.083672	0.086608	0.089448	0.092201	0.094874	0.097473	0.100005
Skewness	−1.17211	−1.14003	−1.08357	−0.91458	−0.77805	−0.67896	−0.59948
Kurtosis	5.03965	5.100087	5.007513	3.979445	3.125349	2.576206	2.185254
Sharpe Ratio	0.056	0.063	0.067	0.071	0.074	0.074	0.075

Source: Authors' Estimations and Calculations.

Table 7. Mean-skewness optimisation.

Portfolios	1	2	3	4	5	6	7
GASF	0.050	0.050	0.050	0.050	0.050	0.050	0.050
AGTL	0.216	0.238	0.239	0.207	0.161	0.122	0.086
HCAR	0.050	0.064	0.111	0.155	0.194	0.231	0.263
GHNL	0.050	0.119	0.141	0.164	0.182	0.195	0.203
FCCL	0.245	0.142	0.073	0.050	0.050	0.050	0.050
MLCF	0.050	0.050	0.050	0.050	0.050	0.050	0.050
NBP	0.050	0.050	0.050	0.050	0.050	0.050	0.050
CLOV	0.290	0.287	0.286	0.274	0.262	0.252	0.248
Mean	0.006102	0.006765	0.007311	0.007781	0.008086	0.008354	0.008546
Variance	0.007001	0.007502	0.008002	0.008502	0.009002	0.009501	0.010002
SD	0.083672	0.086614	0.089454	0.092206	0.094879	0.097475	0.10001
Skewness	−0.23494	−0.10335	−0.03296	0.02016	0.056399	0.076255	0.095811
Kurtosis	1.312269	1.344273	1.339685	1.205788	1.049371	0.942245	0.86471
Sharpe Ratio	0.028	0.034	0.039	0.043	0.045	0.047	0.048

Source: Authors' Estimations and Calculations.

Table 8. Mean-kurtosis optimisation.

| Portfolios	1	2	3	4	5	6	7
GASF	0.050	0.050	0.050	0.050	0.056	0.071	0.156
AGTL	0.323	0.340	0.361	0.368	0.334	0.294	0.161
HCAR	0.111	0.163	0.205	0.272	0.328	0.377	0.391
GHNL	0.078	0.100	0.118	0.086	0.067	0.051	0.050
FCCL	0.165	0.115	0.074	0.058	0.053	0.051	0.050
MLCF	0.064	0.063	0.050	0.050	0.052	0.051	0.050
NBP	0.050	0.050	0.050	0.050	0.056	0.055	0.092
CLOV	0.160	0.119	0.092	0.066	0.055	0.051	0.050
Mean	0.008005	0.008761	0.009421	0.009897	0.010263	0.01048	0.01058
Variance	0.007001	0.007501	0.008001	0.008501	0.009001	0.009501	0.010001
SD	0.083672	0.086608	0.089448	0.092201	0.094874	0.097473	0.100005
Skewness	−0.76657	−0.7843	−0.75903	−0.80426	−0.78266	−0.74256	−0.85016
Kurtosis	3	3.149999	3.209999	3.319999	3.049999	2.749999	2.869999
Sharpe Ratio	0.050	0.057	0.063	0.066	0.068	0.069	0.068

Source: Authors' Estimations and Calculations.
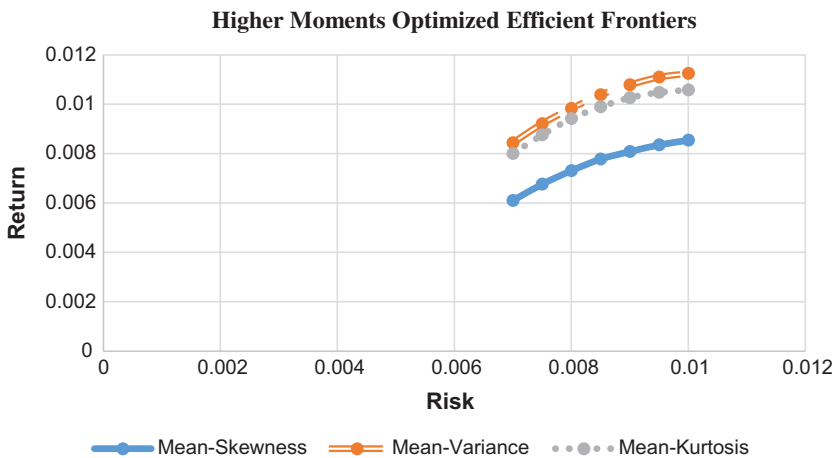
Figure 1. Efficient portfolios with mean-variance, skewness and kurtosis optimisation. Source: Authors' Estimations and Calculations

frontier. However, the extra ordinarily high returns are coming at a cost of being highly exposed to the negative skewness and high value of kurtosis, the kind of risks most of the investors are unaware of. These results strongly suggest the inclusion of higher moments in the process of optimisation to gauge the reward to risk ratio properly which could lead investors towards a truly optimised portfolio and save economy from the unintended losses in case of volatile periods.

5. Conclusion

The assumption of normality of stock returns which is usually considered as the cornerstone for mean variance optimisation is no longer a valid phenomenon. This assumption makes the entire process of portfolio optimisation prone to severe underestimation of investment risk as it conveniently assumes the second moment (variance) is capable to proxy the risk of any investment in its entirety.

Researchers from the various backgrounds, however, confirm the presence of negative skewness and excess kurtosis in diverse set of financial assets. The presence of these two phenomena gives rise to a need for the inclusion of higher moments of risk to describe portfolio behaviour otherwise the portfolio optimisation based on mean-variance under the normality assumption may lead us to underestimate the investment risk and might end up with an inefficient portfolio rather than the efficient one.

This study adopts the PGP approach to include higher moments of returns distribution in portfolio optimisation process on the data of portfolio of eight stocks listed on the PSX. The study confirms the presence of negative skewness and excess kurtosis both in the index and the portfolio and concludes in the presence of skewness and kurtosis risk, Markowitz Mean-Variance portfolio optimisation at the PSX may not provide a workable solution for the investors with preferences towards additional dimensions of risk.

Our results are broadly aligned with the studies confirming the presence of skewness and kurtosis risk in different markets such as Aggarwal et al. (1989), Aracioglu et al. (2011), Beedles (1986), Canela and Collazo (2007) and others, and clearly show strong tradeoffs between returns and additional dimensions of risks (skewness and kurtosis) which was traditionally assumed to be present only between returns and variance. The investors aware of these new dimensions of risks would have to accept lower returns if they chose to optimise these risks (skewness and kurtosis) in addition to the risk captured through the variance. This means that the observed efficient frontier based only on Mean-Variance optimisation is not a true efficient frontier and could lead investors towards sub-optimal decisions due to the misallocation of resources. Ignoring the risk embedded in skewness and kurtosis of returns distribution would lead the overpricing of the market and would expose investors and economy towards unintended and uninformed risks affecting overall growth and development.

Note

1. The conditions, $X \geq 5\%$ and $X \leq 50\%$, restrict the funds allocated to each security to move within this threshold to ensure diversification and avoid short selling.

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Appendix A.

Variance Covariance Matrix: $\sigma_{ij} = E\left[\left(R_i - \bar{R}_i\right)\left(R_j - \bar{R}_j\right)\right]$

	GASF	AGTL	HCAR	GHNL	FCCL	MLCF	NBP	CLOV
GASF	0.019256	0.006859	0.007330	0.008915	0.006342	0.008996	0.009787	0.005618
AGTL	0.006859	0.011624	0.006532	0.003829	0.003623	0.004250	0.006766	0.000923
HCAR	0.007330	0.006532	0.022260	0.010935	0.008492	0.010099	0.007388	0.004676
GHNL	0.008915	0.003829	0.010935	0.033355	0.007389	0.011293	0.002535	0.008206
FCCL	0.006342	0.003623	0.008492	0.007389	0.011910	0.012222	0.007653	0.000745
MLCF	0.008996	0.004250	0.010099	0.011293	0.012222	0.023057	0.007874	0.001935
NBP	0.009787	0.006766	0.007388	0.002535	0.007653	0.007874	0.015583	0.000270
CLOV	0.005618	0.000923	0.004676	0.008206	0.000745	0.001935	0.000270	0.026171

Correlation matrix

	GASF	AGTL	HCAR	GHNL	FCCL	MLCF	NBP	CLOV
GASF	1.0000	0.4585	0.3541	0.3518	0.4188	0.4270	0.5650	0.2503
AGTL	0.4585	1.0000	0.4061	0.1945	0.3080	0.2596	0.5027	0.0529
HCAR	0.3541	0.4061	1.0000	0.4013	0.5215	0.4458	0.3967	0.1938
GHNL	0.3518	0.1945	0.4013	1.0000	0.3707	0.4072	0.1112	0.2777
FCCL	0.4188	0.3080	0.5215	0.3707	1.0000	0.7376	0.5618	0.0422
MLCF	0.4270	0.2596	0.4458	0.4072	0.7376	1.0000	0.4154	0.0788
NBP	0.5650	0.5027	0.3967	0.1112	0.5618	0.4154	1.0000	0.0134
CLOV	0.2503	0.0529	0.1938	0.2777	0.0422	0.0788	0.0134	1.0000

Source: Authors' estimations and calculations.

Appendix B.

Skewness-CoSkewness Matrix: $S_{ijk} = E\left[(R_i - \bar{R}_i)(R_j - \bar{R}_j)(R_k - \bar{R}_k)\right]$

	GASF	AGTL	HCAR	GHNL	FCCL	MLCF	NBP	CLOV
GASF	−0.004983691	−0.003505212	−0.001704645	−0.000644031	−0.001487949	−0.00182069	−0.003390881	0.00034556
AGTL	−0.003505212	−0.002891854	−0.001512545	−0.000558871	−0.00127145	−0.001512562	−0.002936698	0.000415245
HCAR	−0.001704645	−0.001512545	−0.000828937	−0.00046487	−0.000916203	−0.000986817	−0.001705671	0.000353098
GHNL	−0.000644031	−0.000558871	−0.00046487	−0.000199563	−0.000205364	−0.000194249	−0.00070773	0.000597538
FCCL	−0.001487949	−0.00127145	−0.000916203	−0.000205364	−0.000507861	−0.000373864	−0.001418391	3.09699E-05
MLCF	−0.00182069	−0.001512562	−0.000986817	−0.000194249	−0.000373864	8.88921E-05	−0.001605041	5.40249E-06
NBP	−0.003390881	−0.002936698	−0.001705671	−0.00070773	−0.001418391	−0.001605041	−0.003558712	−0.000259518
CLOV	0.00034556	0.000415245	0.000353098	0.000597538	3.09699E-05	5.40249E-06	−0.000259518	0.000659418
GASF	−0.003505212	−0.002891854	−0.001512545	−0.000558871	−0.00127145	−0.001512562	−0.002936698	0.000415245
AGTL	−0.002891854	−0.001439704	−0.000647317	−0.000271042	−0.000894348	−0.001068192	−0.002095172	−0.000418702
HCAR	−0.001512545	−0.000647317	−0.000433241	−6.03675E-05	−0.000625142	−0.000600847	−0.001164903	−0.000136594
GHNL	−0.000558871	−0.000271042	−6.03675E-05	0.000708737	−0.000235172	−5.49395E-06	−0.000693941	0.000373498
FCCL	−0.00127145	−0.000894348	−0.000625142	−0.000235172	−0.000667961	−0.000705372	−0.001216064	3.13586E-05
MLCF	−0.001512562	−0.001068192	−0.000600847	−5.49395E-06	−0.000705372	−0.000633413	−0.001400708	4.94974E-05
NBP	−0.002936698	−0.002095172	−0.001164903	−0.000693941	−0.001216064	−0.001400708	−0.002600544	−4.22153E-05
CLOV	0.000415245	−0.000418702	−0.000136594	0.000373498	3.13586E-05	4.94974E-05	−4.22153E-05	0.000678456
GASF	−0.001704645	−0.001512545	−0.000828937	−0.00046487	−0.000916203	−0.000986817	−0.001705671	0.000353098
AGTL	−0.001512545	−0.000647317	−0.000433241	−6.03675E-05	−0.000625142	−0.000600847	−0.001164903	−0.000136594
HCAR	−0.000828937	−0.000433241	8.13356E-05	0.000565729	−0.000988111	−0.001157145	−0.001374891	0.000283827
GHNL	−0.00046487	−6.03675E-05	0.000565729	7.1505E-05	−0.00038818	−0.00056862	−0.000629394	0.000680797
FCCL	−0.000916203	−0.000625142	−0.000988111	−0.00038818	−0.000776411	−0.000720866	−0.001087881	−5.03381E-05
MLCF	−0.000986817	−0.000600847	−0.001157145	−0.00056862	−0.000720866	−0.000407725	−0.000975423	−0.00018189
NBP	−0.001705671	−0.001164903	−0.001374891	−0.000629394	−0.001087881	−0.000975423	−0.001869166	−0.000372775
CLOV	0.000353098	−0.000136594	0.000283827	0.000680797	−5.03381E-05	−0.00018189	−0.000372775	0.001240994
GASF	−0.000644031	−0.000558871	−0.00046487	−0.000199563	−0.000205364	−0.000194249	−0.00070773	0.000597538
AGTL	−0.000558871	−0.000271042	−6.03675E-05	0.000708737	−0.000235172	−5.49395E-06	−0.000693941	0.000373498
HCAR	−0.00046487	−6.03675E-05	0.000565729	7.1505E-05	−0.00038818	−0.00056862	−0.000629394	0.000680797
GHNL	−0.000199563	0.000708737	7.1505E-05	−0.000889129	−0.000159521	−0.000575656	0.000249291	0.001610911
FCCL	−0.000205364	−0.000235172	−0.00038818	−0.000159521	−0.000192082	−4.89123E-07	−0.00081033	0.000133314
MLCF	−0.000194249	−5.49395E-06	−0.00056862	−0.000575656	−4.89123E-07	0.000412455	−0.000525213	2.75507E-05
NBP	−0.00070773	−0.000693941	−0.000629394	0.000249291	−0.00081033	−0.000525213	−0.001716887	−4.67617E-05
CLOV	0.000597538	0.000373498	0.000680797	0.001610911	0.000133314	2.75507E-05	−4.67617E-05	0.000915723

	GASF	AGTL	HCAR	GHNL	FCCL	MLCF	NBP	CLOV
GASF	-0.001487949	-0.00127145	-0.000916203	-0.000205364	-0.000507861	-0.000373864	-0.001418391	3.09699E-05
AGTL	-0.00127145	-0.000894348	-0.000625142	-0.000235172	-0.000667961	-0.000705372	-0.001216064	3.13586E-05
HCAR	-0.000916203	-0.000625142	-0.000988111	-0.00038818	-0.000776411	-0.000720866	-0.001087881	-5.03381E-05
GHNL	-0.000205364	-0.000235172	-0.00038818	-0.000159521	-0.000192082	-4.89123E-07	-0.00081033	0.000133314
FCCL	-0.000507861	-0.000667961	-0.000776411	-0.000192082	-1.18177E-05	8.47605E-06	-0.000677752	-0.000219614
MLCF	-0.000373864	-0.000705372	-0.000720866	-4.89123E-07	8.47605E-06	0.00055086	-0.000669756	-3.73328E-05
NBP	-0.001418391	-0.001216064	-0.001087881	-0.00081033	-0.000677752	-0.000669756	-0.001394176	-0.000425399
CLOV	3.09699E-05	3.13586E-05	-5.03381E-05	0.000133314	-0.000219614	-3.73328E-05	-0.000425399	0.000690419

	GASF	AGTL	HCAR	GHNL	FCCL	MLCF	NBP	CLOV
GASF	-0.00182069	-0.001512562	-0.000986817	-0.000194249	-0.000373864	8.88921E-05	-0.001605041	5.40249E-06
AGTL	-0.001512562	-0.001068192	-0.000600847	-5.49395E-06	-0.000705372	-0.000633413	-0.001400708	4.94974E-05
HCAR	-0.000986817	-0.000600847	-0.001157145	-0.00056862	-0.000720866	-0.000407725	-0.000975423	-0.00018189
GHNL	-0.000194249	-5.49395E-06	-0.00056862	-0.000575656	-4.89123E-07	0.000412455	-0.000525213	2.75507E-05
FCCL	-0.000373864	-0.000705372	-0.000720866	-4.89123E-07	8.47605E-06	0.00055086	-0.000669756	-3.73328E-05
MLCF	8.88921E-05	-0.000633413	-0.000407725	0.000412455	0.00055086	0.002224922	-0.000448523	0.000181321
NBP	-0.001605041	-0.001400708	-0.000975423	-0.000525213	-0.000669756	-0.000448523	-0.001634817	-0.000312357
CLOV	5.40249E-06	4.94974E-05	-0.00018189	2.75507E-05	-3.73328E-05	0.000181321	-0.000312357	0.000831577

	GASF	AGTL	HCAR	GHNL	FCCL	MLCF	NBP	CLOV
GASF	-0.003890881	-0.002936698	-0.001705671	-0.00070773	-0.001418391	-0.001605041	-0.003558712	-0.000259518
AGTL	-0.002936698	-0.002095172	-0.001164903	-0.000693941	-0.001216064	-0.001400708	-0.002600544	-4.21153E-05
HCAR	-0.001705671	-0.001164903	-0.001374891	-0.000629394	-0.001087881	-0.000975423	-0.001869166	-0.000372775
GHNL	-0.00070773	-0.000693941	-0.000629394	0.000249291	-0.00081033	-0.000525213	-0.001716887	-4.67617E-05
FCCL	-0.001418391	-0.001216064	-0.001087881	-0.00081033	-0.000677752	-0.000669756	-0.001394176	-0.000425399
MLCF	-0.001605041	-0.001400708	-0.000975423	-0.000525213	-0.000669756	-0.000448523	-0.001634817	-0.000312357
NBP	-0.003558712	-0.002600544	-0.001869166	-0.001716887	-0.001394176	-0.001634817	-0.00275519	-0.0008858
CLOV	-0.000259518	-4.21153E-05	-0.000372775	-4.67617E-05	-0.000425399	-0.000312357	-0.0008858	0.000625566

	GASF	AGTL	HCAR	GHNL	FCCL	MLCF	NBP	CLOV
GASF	0.00034556	0.000415245	0.000353098	0.000597538	3.09699E-05	5.40249E-06	-0.000259518	0.000659418
AGTL	0.000415245	-0.000418702	-0.000136594	0.000373498	3.13586E-05	4.94974E-05	-4.21153E-05	0.000678456
HCAR	0.000353098	-0.000136594	0.000283827	0.000680797	-5.03381E-05	-0.00018189	-0.000372775	0.001240994
GHNL	0.000597538	0.000373498	0.000680797	0.001610911	0.000133314	2.75507E-05	-4.67617E-05	0.000915723
FCCL	3.09699E-05	3.13586E-05	-5.03381E-05	0.000133314	-0.000219614	-3.73328E-05	-0.000425399	0.000690419
MLCF	5.40249E-06	4.94974E-05	-0.00018189	2.75507E-05	-3.73328E-05	0.000181321	-0.000312357	0.000831577
NBP	-0.000259518	-4.21153E-05	-0.000372775	-4.67617E-05	-0.000425399	-0.000312357	-0.0008858	0.000625566
CLOV	0.000659418	0.000678456	0.001240994	0.000915723	0.000690419	0.000831577	0.000625566	0.0015259

Source: Authors' Estimations and Calculations.