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# A new proposal for efficiency quantification of capital markets in the context of complex non-linear dynamics and chaos

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## ABSTRACT

The main purpose of this paper is efficiency analysis as well as its quantification in the case of emerging capital markets, by building a new measure of market efficiency. The basic assumption of such markets is the lack of correlation between returns, and therefore the existence of low entropy, the lack of randomness, manifestation of fractality and long-term memory, integrated into a single measure, will indicate the distancing from the state of efficient market. This paper proposes five different estimates (for informational entropy, run test, Hurst exponent, long-term correlation coefficient and fractal dimension) to construct a new measure of market efficiency based on a deviation from the ideal state (expressed by the efficient market). The Capital Market Efficiency Exponent is estimated for nine emerging capital markets and, for comparison, for three developed capital markets, at different stages of development over a 16-year time span.

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## 1. Introduction

Testing the efficiency of different capital markets in various regions is, undoubtedly, a popular topic in financial journals. However, the hypothesis of market efficiency is normally either rejected or not and markets are ranked quite infrequently. In addition, the researchers significantly concentrate on a single method and comment on the results. Also, the whole idea of testing or measuring capital market efficiency has been managing the joint-hypothesis issue (i.e., when we reject the efficiency of a particular market, it might be caused by a wrong assumption of the market's behaviour) since its beginnings. This issue was also touched on by Eugene Fama (1965) himself.

The Efficient Market Hypothesis (EMH) developed by Fama (1965), with its subsequent developments, is a large research area, since the question is to what extent and from what perspective this theory is realistic. From the perspective of tests' applicability to these markets, the greatest challenges of EMH concern the following elements:

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- despite the simplicity of the assumptions underlying the EMH, it is surprisingly difficult to test and requires considerable attention when it is applied and when the empirical tests are processed;
- tests only occasionally showed evidence for the existence of informational efficiency (Borgesa, 2010; Chen & Jarrett, 2011; Ito, Noda, & Wada, 2016; Urquhart, 2014);
- the limits that exist related to statistical and mathematical models used to test EMH (Malkiel, 2003);
- the existence of complex non-linear dynamics and chaos in capital markets (Cajueiro & Tabak, 2005; Granero, Segovia, & Pérez, 2008; Oprean, Tănăsescu, & Brătian, 2014; Oprean & Tănăsescu, 2014; Reboredo, Rivera-Castro, Miranda, & García-Rubio, 2013).

We chose to apply this study mainly in the case of emerging countries because we have identified a number of problems and difficulties that can arise when applying the tests of efficiency, problems that reside in the fact that conventional classic efficiency tests were designed to be implemented only in the case of developed capital markets. Additional volatility, which means a higher memory, can be ascribed to typical weaknesses of emerging markets: lack of investors, dependence of foreign interest (Kitt, 2003), non-linearity of assets prices (Ahmed, Rosser, & Uppal, 2010; Hasanov & Omay, 2008; Lim & Brooks, 2009) or thin trading (Antoniou, Ergul, Holmes, & Priestley, 1997; Lilam, 2003; Loc, Lanjouw, & Lensink, 2010; Plesoianu & Todea, 2012). These characteristics of emerging markets may lead to erroneous results when testing the efficient emerging market hypothesis.

To overcome all these problems, this paper proposes five different estimates (one estimate of informational entropy, one estimate of run test as a random test, one estimate of the Hurst exponent, one estimate for the long-term correlation coefficient and one estimate for fractal dimension) to construct a new measure of market efficiency – the Capital Market Efficiency Exponent (CMEE), in order to control the possible correlations and complexity using a single measure.

Another reason to apply the study on emerging markets is the fact that most studies so far have focused on stochastic long-memory behaviour of stock returns in developed capital markets. By contrast, the question of long memory in smaller markets has received little attention. Outside the world's developed economies, many emerging capital markets in recent years have attracted a great deal of attention from investors and investment funds seeking to diversify further their assets.

In this paper, we try to bypass the problem by defining the efficient market as a martingale, because it is less restrictive. Samuelson (1965) provided the first formal economic argument for 'efficient markets' and focused on the concept of a martingale, rather than a random walk (as in Fama, 1965). His contribution is perfectly summarised by the title of his article: 'Proof that properly anticipated prices fluctuate randomly'.

Taking into account this definition, we assume that the returns of a financial asset are serially uncorrelated and with finite variance for the efficient market circumstance. Such a simple definition permits us to utilise different measures of market efficiency, which are described in this article. We utilise the fact that the measures are bounded and subsequently can be utilised to develop an informative norm representing the said deviation from the ideal state. In the end, we propose the CMEE, which takes these statistics into consideration and helps to rank different assets according to their efficiency while using different dynamic properties of the time series in this study.

This paper is structured as follows. Section 2 outlines the context of the paper, respectively some views regarding the classical EMH and the implications of complexity and chaos on capital markets. Section 3 contains the literature review. Section 4 discusses the theoretical framework and methodology of the paper. Section 5 presents and validates the data and the empirical results of the study. Conclusions are given in Section 6.

## **2. The context: from classical EMH to the implications of complexity and chaos on capital markets**

EMH, outlined by Eugene Fama (1965, 1970), assumes that capital markets process the economic information in a highly sensitively manner and react quickly to adjust the prices of financial assets. The possibility to predict future returns, starting from the returns noticed in the past, is naturally an interesting point in finance, both from the academic point of view and from a practical standpoint.

In finances great interest is shown in the role of chaos in this area, due to the multitude of information and the interest in detecting predictable patterns. Tests have shown that asset prices on the market, although unpredictable, show a certain trend. The capital market is accepted as a self-similar system in the sense that the components are similar or even identical to the whole. In this regard, the association of capital market with a fractal seems appropriate because if one examines the graphs in stock prices over a period, one may see that they have a similar structure. However, like the fractal, the capital market has a sensitive dependence on initial conditions, which makes the dynamic market systems so difficult to predict. It is argued that although a system can be unpredictable in the short term, it can become deterministic in the long term.

The Fractal Market Hypothesis (Peters, 1994, 1996) is a replica of efficient markets, but the focus lies not on the market efficiency, but on its stability. Thus, the daily randomness of the market and the anomalies occurring on the market are taken into consideration, and it is suggested that the apparent random evolution of the rate can be decomposed into several periods of evolution, similar to each other, the original series thus becoming mouldable.

Regarding entropy, it is a measure both for the uncertainty and for the complexity of a system, with many applications in physics, in information theory, in biology, and in economics (complexity of a system). In other words, high levels of entropy are produced in situations of deep uncertainty and low levels of entropy are associated with a low-uncertainty situation.

## **3. Literature review**

According to Fama's (1970) early work, the prices on an efficient capital market should follow a random walk and always fully reflect all available and relevant information. From this early work, an entire literature appeared on testing the efficiency of emerging and developed capital markets.

Using traditional statistical tests, such as autocorrelation tests, most of the EMH tests were based on linear prediction of future changes in stock prices (Brătian & Opreana, 2010; Oprean, 2012). The weak form of market efficiency is disregarded when the return series display long-range dependence. In view of previous information one can make forecasts for future returns. According to Hurst's methodology (and co-authors) (Hurst, Black, & Simaika, 1965), the discovery of long memory should be heuristically possible by evaluating

its intensity, namely the value of parameter  $H$ , which fluctuates somewhere around 0 and 1. The  $H$  value equivalent to 0.5 means non-appearance of long memory.

Among the first studies on long-range dependence are Mandelbrot's (1972), who applied range/standard deviation (R/S) analysis to returns, and Greene and Fielitz's (1977), who claimed that many daily stock return series are characterised by long-term dependence. Long-term memory of assets returns was investigated and demonstrated in many recent works (Adel & Lamia, 2012; Ahamed, Kalita, & Tiwari, 2015; Anoruo & Gil-Alana, 2011; Cajueiro & Tabak, 2004, 2005; Hull & McGroarty, 2014; Kale & Butar, 2011; Kristoufek & Vosvrda, 2013, 2014a, 2014b; Lilam, 2003; Necula & Radu, 2012; Oprean & Tănăsescu, 2014; Oprean et al., 2014; Pele & Tepus, 2011; Plesoianu & Todea, 2012; Risso, 2008; Tan, Chin, & Galagedera, 2014; and many others). Moreover, there are some authors who did not find any significant evidence for long-term memory in common stock returns (Grau-Carles, 2005; Oh, Um, & Kim, 2006).

Previous studies on efficiency have also applied Shannon's entropy (Adel & Lamia, 2012; Mensi, 2012, etc.), some efficiency rankings have been presented (for example, Mensi, 2012, ranks efficiency for 26 emerging markets applying Shannon's entropy; and Baciu, 2014, ranks efficiency for 20 European markets) and also a similar efficiency index has been already advanced and applied by Kristoufek and Vosdra (2013, 2014a, 2014b).

#### 4. Theoretical framework and methodology of constructing the CMEE

The methodology that is used in this paper for constructing a mathematical model, namely the CMEE, consists of the following steps (Bucur, 2014).

Step 1. *Setting the goal of mathematical modelling.*

In this paper, the aim of mathematical modelling is to create an indicator to measure market efficiency. The efficiency measure is taken as a Euclidean distance from an ideal efficient market situation. For the construction of capital market efficiency measure  $EI$ , in this paper the procedure of Kristoufek and Vosvrda (2013) is followed, which is equivalent to the Chebyshev measure, and is easy to apply and interpret by experts in financial or economic fields, by managers, etc.

Step 2. *Setting input and output variables in the mathematical model.*

In this paper, for the construction of CMEE, five partial indicators are used as input variables.

(1) Shannon entropy.

The link between a certain level of complexity and EMH is quite clear: if one takes into consideration a (weak form) efficient market, then equity prices follow the random-walk pattern, i.e., the range of returns is a white noise process. In quantitative terms, such a white noise process has the greatest level of complexity; conversely, if the EMH is not accepted, then prices are not random-walk processes and, consequently, the complexity of the market will be lower. Taking into consideration the explanations above, informational entropy can be considered as a measure for informational efficiency of capital markets (Oprean, 2016). To measure the informational efficiency of capital markets, the *Shannon entropy* is used in this article (Shannon, 1948).

To measure efficiency one can follow two steps. First, the returns are represented in order to be less demanding to identify their changes over the analysed period. Second, the

Shannon entropy is applied and computed to measure the amount of information contained in the returns series. Using the symbolic time series analysis (Daw et al., 2003), it is possible to get more information from an ordinary time series, as the ordinary information with very different values is transformed into a series symbolised by just a few distinct values.

Taking into account that there is no formal method by which time series symbols are characterised, for this paper it was very helpful to find combinations of the positive and negative values of the returns series. Therefore, the negative returns were denoted by (0) and positive ones by (1). In fact, for this situation, the procedure is a sequence of Bernoulli trials ((0) and (1)). In this way, an  $N$ -size time series was defined as  $(R_1, R_2, R_3, \dots, R_N)$ ,  $R_t$  being the security returns at time  $t$ , where  $t = 1, 2, 3, \dots, N$ . The returns were measured by using the first difference of logarithmic values of daily prices.

Then, the series of returns  $(R_1, R_2, R_3, \dots, R_N)$  was transformed into a symbolic series  $(s_1, s_2, s_3, \dots, s_N)$  in accordance with the following relations:

$$\begin{cases} \text{if } R_t < 0, & s_t = 0 \\ \text{if } R_t \geq 0, & s_t = 1 \end{cases} \quad (1)$$

As a measure of informational efficiency, the Shannon entropy ( $H_S$ ) is implemented, which can take a maximum value of 1 when the process is completely random and a minimum value of 0 when alluding to a sure event. The theoretical expression of  $H_S$  for two events,  $p$  (the probability of having negative returns) and  $1 - p$  (the probability of having positive returns) is given as (Risso, 2008):

$$H_S = -[p * \log_2 p + (1 - p) * \log_2(1 - p)] \quad (2)$$

The maximum value ( $H_S = 1$ ) is obtained for  $p = 1/2$  (maximum uncertainty) and the minimum is obtained when one of the events is a sure event ( $p = 0$  or  $p = 1$ ). In this case, it means that the market is not informationally efficient and the Shannon entropy value ( $H_S$ ) will be less than 1.

## (2) Run test.

The run test is a non-parametric statistical test checking a randomness hypothesis for a two-valued data sequence and it is used in this paper to discover whether the indices exhibit random behaviour or whether they follow a specific growth pattern. Under the null hypothesis of independent successive results, the aggregate number of series is distributed with the following mean and standard variance (Islam et al., 2007):

$$E(R) = \frac{n + 2n_A n_B}{n} \quad (3)$$

$$\sigma_R = \sqrt{\frac{2n_A n_B (2n_A n_B - n)}{n^2 (n - 1)}} \quad (4)$$

where  $n$  represents the total number of observations,  $n_A$  stands for the number of observations within the first cycle, and  $n_B$  stands for the number of the second cycle.  $R$  is used to designate the observed number of runs.

The test for the run dependence should be performed by comparing the total number of effective runs with the expected number.  $H_0: E(\text{sequences}) = E(R)$  represents the null hypothesis and a random hypothesis is checked for a two-valued data sequence. The value expressed by  $H_0$  shows that the changes that follow are not dependent and they are in random walk. In the case of a high number of observations, its distribution becomes practically equivalent to the normal distribution. Along these lines the normal distribution  $Z$  might be used to implement a run test. The following formula is the formula used for the standard score:

$$Z = \frac{R - E(R)}{\sigma_R} \quad (5)$$

In the event that the  $Z$  value fundamentally contrasts from the critical value (upper tail), one can dismiss the null hypothesis and infer that the tested index might be anticipated. For this situation, the capital market will not meet the low weak form of market efficiency.

### (3) Hurst exponent.

R/S analysis in economics was introduced by Mandelbrot (1972), who argued that this methodology was superior to autocorrelation, variance analysis and spectral analysis and it can identify the existence of long-term linear dependence in the stock market returns.

In the following, the fact that three different values of Hurst exponent can express three different types of time series is examined.

In the case of  $H = 0.5$ , the series being no longer correlated, the events become perfectly random. It is standard Brownian motion. Fractional Brownian motion is a well-known stochastic process where the second-order moments of the increments scale as (Kantelhardt et al., 2002; Nualart, 2006):

$$E\left\{ [Y(t_i) - Y(t_{i-1})]^2 \right\} \sim |t_i - t_{i-1}|^{2H} \quad (6)$$

with  $H \in [0, 1]$  (the symbol ' $\sim$ ' is used so that they have the same finite joint distribution functions) (Sánchez et al., 2015). The Brownian motion represents the particular case where  $H = 0.5$ .

In the case of  $0 \leq H < 0.5$ , the series is negatively correlated. This type of dynamic series represents the *antipersistent* fractal Brownian motion (a characteristic of self-adjustment systems), therefore any increase in any of the series terms will entail its decrease at the next moment in time.

In the next case, if  $0.5 < H \leq 1$ , this signifies that the series is positively correlated and a persistent dynamics is thus encountered, that is, if a value increased (decreased) previously, it will very probably continue increasing (decreasing). As one can see, the intensity of persistent behaviour depends on the value of  $H$ .

The relationship between long-term memory and potential efficiency has been discussed in numerous studies (Adel & Lamia, 2012; Pele & Tepus, 2011; Risso, 2008; Taback & Cajuiero, 2007, 2009; and many others), and in many cases the less-developed markets were characterised by signs of long-term memory. However, it is also necessary to mention the criticism of using the Hurst exponent, as there are authors who argue that such an exponent different from 0.5 does not necessarily mean such long time correlations as those found in fractional Brownian motion (Bassler et al., 2006; McCauley et al., 2007). So, for financial



time series, even an uncorrelated series can yield  $H$  different from 0.5 due to several reasons, such as heteroscedasticity, short-term memory and fat tails. The effects of these are discussed in various papers, e.g., Barunik et al. (2012), Barunik and Kristoufek (2010) and Kristoufek (2012). Similarly, Plesoianu and Todea (2012) confirm that thin trading in emerging markets leads to spurious correlations and an overestimation bias of the Hurst exponent.

It must be noted that Hurst exponent estimators are usually biased by the presence of short-term memory in the underlying process (Barunik et al., 2012); Kristoufek, 2012). However, this is not an issue for the proposed efficiency measure because when short-term memory (which is of course a form of inefficiency as well) biases the Hurst exponent estimate, it is in turn reflected in the efficiency measure.

This study applies the R/S analysis (using rolling window applied on Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) residuals) for the Hurst exponent estimation. A dynamic measurement of this exponent is chosen, taking into account that market efficiency (predictability) appears to evolve over time (Cajueiro & Tabak, 2005). A 'rolling sample' approach of  $N = 100$  observations is used to evaluate the Hurst exponent and to check for the presence of long-term linear dependence in stock market volatility. Since this approach makes available several Hurst exponents for each stock market and it is impossible to compare all of them, statistical inference with the means, medians and other statistical measures of these Hurst exponents is made in order to compare their market inefficiency degree.

When using the R/S method, i.e., range/standard deviation, an initial dynamic series is required that stands for the evolution of a natural phenomenon or process. Starting from this point one can determine the next dimension (given that the series is divided into subperiods of length  $t$ ):

$$X_{t,N} = \sum_{i=1}^t (x_i - m_N) \quad (7)$$

where  $X_{t,N}$  represents the cumulative deviation for  $N$  periods,  $x_i$  is the component  $i$  of the dynamic series, and  $m_N$  is the average of the values  $x_i$  on  $N$  periods.  $N$  can be equal to the total number of observations and  $t \leq N$ .

Subsequently, the size of the interval  $R$  (*range*) is determined, as the difference between the maximum and the minimum level reached by the dimensions of  $X_{t,N}$ :

$$R = \text{Max}(X_{t,N}) - \text{Min}(X_{t,N}) \quad (8)$$

In order to compare different dynamic series, Hurst divides the size  $R$  to the standard deviation of initial observations, namely:

$$S = \sqrt{\frac{\sum_{i=1}^N (x_i - m_N)^2}{N}} \quad (9)$$

Then the ratio is determined:

$$R/S = \frac{\text{Max}(X_{t,N}) - \text{Min}(X_{t,N})}{\sqrt{\frac{\sum_{i=1}^N (x_i - m_N)^2}{N}}} \quad (10)$$



Then, by regression the Hurst coefficient is determined:

$$\log\left(\frac{R}{S}\right) = \log(a * N^H) = \log(a) + H * \log(N) \quad (11)$$

Andrew W. Lo (1991) has discovered that the R/S statistic is sensitive to short-range dependence in financial time series. In order to avoid any short-range dependence in mean and variance of the stock returns, the rolling R/S statistic (a rolling window of  $N = 100$  observations) has to be implemented on the adjusted time series. In line with previous studies, including Cajueiro and Tabak (2004, 2005) and others, in this article the GARCH (1, 1) model is used to filter the stock return time series. Consequently, the R/S statistic was implemented on the standardised residuals. The GARCH (1, 1) residual can be used to eliminate or reduce linear dependence degree, that is, autocorrelation, because linear dependence will deviate from Hurst exponent  $H$  or easily lead to the first kind of error. In the literature this process is usually called the pre-white noise treatment or trend expunction method.

#### (4) Long-term correlation coefficient.

In order to measure the correlation between recent and previous observations (also called long-term memory character), the *correlation coefficient* is used, evinced throughout the time range subject to analysis. This coefficient for measuring the correlation (between increases occurring at future moments to increases at previous time),  $C$ , has the following significance in correlation with the Hurst exponent:

$$C = 2^{(2H-1)} - 1 \quad (12)$$

If the Hurst exponent equals 0.5 the series is no longer correlated, and the events are perfectly random ( $C = 0$ ). It is standard Brownian motion. In other words, the present cannot influence the future.

#### (5) Fractal dimension.

The relation between *fractal dimension*  $D$  (used to identify whether the system is random) and Hurst exponent is:  $D = 1/H$  (Mandelbrot, 1982). The random walk has a fractal dimension of  $D = 1/0.5 = 2$ .

To sum up, the input variables in the mathematical model are:  $M_i$  is the  $i$ th measure of efficiency (Shannon entropy  $H_s$ , run test  $Z$ , Hurst exponent  $H$ , correlation coefficient  $C$  and the fractal dimension  $D$ , in our case),  $\hat{M}_i$  is an estimate of the  $i$ th measure,  $M_i^*$  is an expected value of the  $i$ th measure for the efficient market and  $R_i$  is a range of the  $i$ th measure. As ranges of different measures may vary, they are standardised so that the range is equal to one, implying a unit cube as a resulting space. Therefore, the CMEE is defined on a unit  $n$ -dimensional cube with the efficient market in the centre, i.e.,  $CMEE = 0$  for the efficient market.

The output variable is the indicator  $CMEE$  for market efficiency.

Step 3. *Setting the interdependence relationships between variables and, respectively, the mathematical model itself.*

The mathematical model of CMEE is (after the procedure proposed by Kristoufek & Vosvrda, 2013):

$$CMEE = \sqrt{\sum_{i=1}^n \left( \frac{\hat{M}_i - M_i^*}{R_i} \right)^2} \quad (13)$$

In this particular case, the capital market efficiency measure *CMEE* is based on five estimates (one estimate of informational entropy, one estimate of run test, one estimate of Hurst exponent, one estimate for correlation coefficient and one estimate for the fractal dimension). For our specific case, *CMEE* is defined as:

$$CMEE = \left[ (\hat{H}_s - 1)^2 + (\hat{Z}_s - 0)^2 + (\hat{H}_{R/S} - 0.5)^2 + (\hat{C} - 0)^2 + (\hat{D} - 2)^2 \right]^{1/2} \quad (14)$$

where  $H_s$  is estimated informational entropy,  $Z$  is estimated value of run test,  $H_{R/S}$  is estimated Hurst exponent based on R/S methodology,  $C$  is estimated correlation coefficient and  $D$  is estimated fractal dimension.

Step 4. *Interpretation of mathematical model results on a scale.*

The result of the mathematical model, the output variable *CMEE*, can be interpreted according to Table 1.

## 5. Data and empirical results

The data used in this article represent daily stock market quotes of the most traded indices in the nine emergent capital market countries, respectively, five European Union (EU) countries and the four BRIC countries (Brazil, Russia, India and China; South Africa was not included in the study because its main exchange, JSE Limited, demutualised and listed on its own exchange in 2005). In Table 2 there is a short presentation of these indices.

In this paper, we also investigated the relationship between capital market efficiency and economic development level using the data sample from the emerging markets in the study. According to some authors (Hull & McGroarty, 2014) economic development levels and stock market efficiency are closely related. In order to verify this, we use the FTSE (2015) classification of advanced, secondary or frontier emerging markets. FTSE formally reviews

**Table 1.** Interpretation of the results.

Value of the indicator <i>CMEE</i>	Interpretation
$0 \leq CMEE \leq 1$	The market has a high efficiency
$1 \leq CMEE \leq 3$	The market has a medium efficiency
$3 \leq CMEE$	The market has a low efficiency

Source: Own interpretation.

**Table 2.** The presentation of indices series.

No.	Index	Symbol	Country	FTSE 2015 classification <sup>a</sup>
1.	Prague Stock Exchange Index	PX	Czech Republic	Advanced
2.	Bucharest Stock Exchange Trading Index	BET	Romania	Frontier
3.	Zagreb Stock Exchange Equity Index	CROBEX	Croatia	Frontier
4.	Budapest Stock Exchange Stock Index	BUX	Hungary	Advanced
5.	Athen Index Compos	GD.AT	Greece	Advanced <sup>b</sup>
6.	Sao Paulo Bovespa Index	IBOVESPA	Brazil	Advanced
7.	'Russia Trading System' Index	RTSI	Russia	Secondary
8.	Bombay Stock Exchange Index	SENSEX	India	Secondary
9.	Shanghai Composite Index	SSEC	China	Secondary

<sup>a</sup>FTSE 2015 classification of emerging countries.

<sup>b</sup>Although in FTSE 2015 Greece is considered a developed country, following this classification it will be reclassified from developed to advanced emerging country.

Source: Own processing, using FTSE 2015 classification.

country classification within its global equity indices on an annual basis each September using a comprehensive, transparent and consistent methodology. The main criteria for FTSE (2015) classification are: market and regulatory environment, custody and settlement, dealing landscape, derivatives, and size of market. In this study, the countries that meet the ‘advanced’ criteria are Brazil, Czech Republic, Hungary and Greece. The ‘secondary’ emerging markets are China, India and Russia, while Romania and Croatia are considered to be frontier countries (see Table 2).

In order to allow more interesting comparisons and an enhanced understanding of stock market efficiency, we applied this study also in the case of developed capital markets, namely in the case of German, French and Spanish capital markets. Information has basically been gathered from the stock markets’ own sites. They are registered daily values over a 16-year time range, January 2000–January 2016. The information comprises an average of 4000 daily values of every index price.

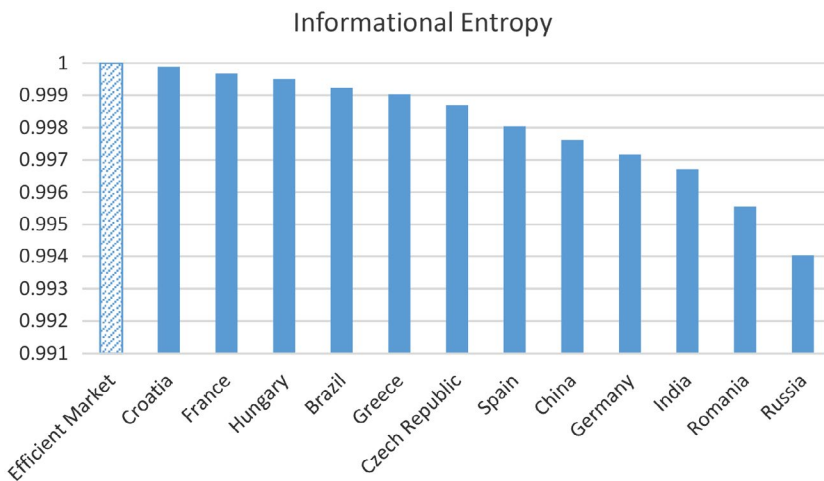
### 5.1. Applying information entropy to emerging capital markets

Using the methodology outlined earlier to determine the Shannon entropy, these emerging countries are rated on the basis of the entropy value obtained, as shown in Figure 1.

The results obtained show that, in general, all nine emerging countries and the three developed markets are characterised by weak informational efficiency. However, the most efficient capital market in terms of information seems to be the Croatian market, followed by the French and Hungarian markets. The least effective market in terms of information, using the Shannon entropy as an indicator of appreciation, is the Russian capital market, which has the lowest entropy value.

### 5.2. Run test analysis

The results of the basic parameters for the run test applied to indices time series are given in Table 3. It may be concluded that the Romanian capital market does not meet the



**Figure 1.** Informational entropy. Source: According to the authors’ calculations.

**Table 3.** The run test for the emerging capital markets indices.

	dlog_SSEC	dlog_BET	dlog_SEN-SEX	dlog_BOVESPA	dlog_BUX	dlog_PX	dlog_CROBEX	dlog_RTSI	dlog_GDAT	dlog_DAX	dlog_CAC40	dlog_IBEX35
Test value <sup>a</sup>	.0003	.00019654	.00042031	.00011914	.00014592	.00022981	.00003471	.00034749	.00000000	.00032405	.00013944	.00030261
Cases < test value	1937	2000	1981	1984	1997	2007	1996	2042	1996	2037	2045	2029
Cases ≥ test value	1938	2000	1981	1985	1998	2007	1996	2043	2148	2038	2045	2029
Total cases	3875	4000	3962	3969	3995	4014	3992	4085	4144	4075	4090	4058
Number of runs	1898	1807	1835	2006	2001	1959	1942	1930	1871	2111	2155	2061
E(R)	1938	2001	1982	1988	1998	2008	1997	2043	2070	2038	2046	2030
Z <sup>a</sup>	-1.301	-6.136	-4.671	0.651	0.079	-1.547	-1.741	-3.552	-6.198	2.272	3.409	0.973
Asymp. Sig. (2-tailed) <sup>a</sup>	.193	.000	.000	.515	.937	.122	.082	.000	.000	.023	.001	.330
Z <sub>α</sub> = 0.05	±1.96	±1.96	±1.96	±1.96	±1.96	±1.96	±1.96	±1.96	±1.96	±1.96	±1.96	±1.96
Hypothesis	H0	H1	H1	H0	H0	H0	H0	H1	H1	H1	H1	H0

<sup>a</sup>Median.

Source: Own calculation.

quality of efficiency. It is not efficient in light of the fact that the stock price changes can be forecast. Additionally, India, Russia, Greece, Germany and France (at a critical-level median) present an unexpected number of sequences lower than the total number, subsequently they clearly reject the hypothesis of random walk. By results, in this period, all the other capital markets: China, Brazil, Hungary, The Czech Republic, Croatia and Spain, are weak-formed efficient.

### 5.3. Hurst exponent analysis

In order to test the presence of long-term linear dependence in stock market volatility over time, the Hurst exponent is calculated from a rolling window of  $N = 100$  observations to process the extent of inefficiency. The GARCH (1, 1) model is employed in order to filter the stock return time series. In Tables 4(a), 4(b) and 4(c) the descriptive statistics for these Hurst exponents are shown for each of the 12 indices.

From these tables, one can see that the mean estimations of the Hurst exponent range from 0.756 (France) to 0.835 (Romania), while the median values of the Hurst exponent range from 0.762 (France) to 0.834 (Romania). It is also pointed out that the Hurst exponents are constantly above 0.5, demonstrating the presence of long-range dependence on stock market returns. The highest standard deviation of the Hurst exponent is attributed to Croatia's market, whereas the lowest standard deviation is a feature of the French Hurst exponent. Besides, the skewness and kurtosis statistics reveal that the Hurst exponents are not normally distributed. Almost all markets evince significant deviations from normality.

In order to ensure that the variation of the Hurst statistics over time is not due to noise, this preliminary finding is supported by testing normality. In this paper, in order to test the normality of Hurst exponent series, the Kolmogorov–Smirnov test (statistics  $D$ ) and Shapiro–Wilk test (statistic  $W$ ) are used initially (shown in Table 5).

To interpret the results, the following decision rule is used: if  $p$  (Sig) is greater than 0.05, then the hypothesis of normality of distribution is accepted for large samples and the conclusion is considered uncertain in the case of small samples (Wiener et al., 2003). The

**Table 4a.** Descriptive statistics for Hurst exponent (applied on GARCH residuals, rolling window of 100 observations) – BRIC capital markets.

	H_100_IBOVESPA	H_100_RTSI	H_100_SENSEX	H_100_SSEC
Mean	0.775683	0.822836	0.808687	0.789603
Median	0.771337	0.817734	0.802141	0.788871
Maximum	0.992314	1.072319	1.106656	1.059526
Minimum	0.537326	0.589692	0.603418	0.514873
Std dev.	0.088160	0.088444	0.092221	0.078267
Skewness	-0.093591	0.103278	0.427140	0.051299
Kurtosis	2.326232	2.502891	3.152923	3.005744
Jarque–Bera	76.87499	45.55618	118.4063	1.660017
Probability	0.000000	0.000000	0.000000	0.436045
Sum	2926.651	3104.559	3051.176	2979.171
Sum sq. dev.	29.31672	29.50580	32.07994	23.10608
Observations	3773	3773	3773	3773

Source: Own calculation using EViews software.

**Table 4b.** Descriptive statistics for Hurst exponent (applied on GARCH residuals, rolling window of 100 observations) – European emergent capital markets.

	H_100_BET	H_100_BUX	H_100_CROBEX	H_100_GD_AT	H_100_PX
Mean	0.835119	0.774266	0.828723	0.805050	0.787062
Median	0.834198	0.773101	0.820586	0.801254	0.791639
Maximum	1.112039	0.984407	1.109398	1.053295	1.018255
Minimum	0.545471	0.547750	0.547858	0.555558	0.598706
Std dev.	0.090289	0.089093	0.104438	0.095716	0.094608
Skewness	-0.018257	-0.032147	0.107946	0.110366	0.095483
Kurtosis	3.079950	2.589741	2.643235	2.595996	2.005094
Jarque–Bera Probability	1.214465 0.544857	27.10994 0.000001	27.33707 0.000001	33.31896 0.000000	161.3440 0.000000
Sum	3150.903	2921.305	3126.770	3037.452	2969.585
Sum sq. dev.	30.75005	29.94022	41.14205	34.55761	33.76191
Observations	3773	3773	3773	3773	3773

Source: Own calculation using EViews software.

**Table 4c.** Descriptive statistics for Hurst exponent (applied on GARCH residuals, rolling window of 100 observations) – European developed capital markets.

	H_100_CAC40	H_100_DAX	H_100_IBEX35
Mean	0.755898	0.768859	0.776617
Median	0.761588	0.764018	0.781996
Maximum	0.973822	0.998924	0.989894
Minimum	0.513911	0.536479	0.553054
Std dev.	0.078235	0.084590	0.084155
Skewness	-0.172618	0.174127	-0.134126
Kurtosis	2.912002	2.738319	2.723426
Jarque–Bera Probability	20.93321 0.000028	31.29423 0.000000	24.48224 0.000005
Sum	2991.842	3043.146	3073.849
Sum sq. dev.	24.21967	28.31422	28.02353
Observations	3958	3958	3958

Source: Own calculation using EViews software.

**Table 5.** Tests of normality for the series of Hurst exponents.

	Kolmogorov–Smirnov <sup>a</sup>			Shapiro–Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
H_IBOVESPA	.049	3773	.000	.985	3773	.000
H_SSEC	.015	3773	.062	.998	3773	.000
H_SENSEX	.045	3773	.000	.986	3773	.000
H_RTSI	.053	3773	.000	.989	3773	.000
H_PX	.080	3773	.000	.971	3773	.000
H_GD_AT	.032	3773	.000	.994	3773	.000
H_CROBEX	.038	3773	.000	.994	3773	.000
H_BUX	.031	3773	.000	.993	3773	.000
H_BET	.022	3773	.000	.998	3773	.000
H_DAX	.038	3958	.000	.994	3958	.000
H_CAC40	.044	3958	.000	.995	3958	.000
H_IBEX35	.041	3958	.000	.993	3958	.000

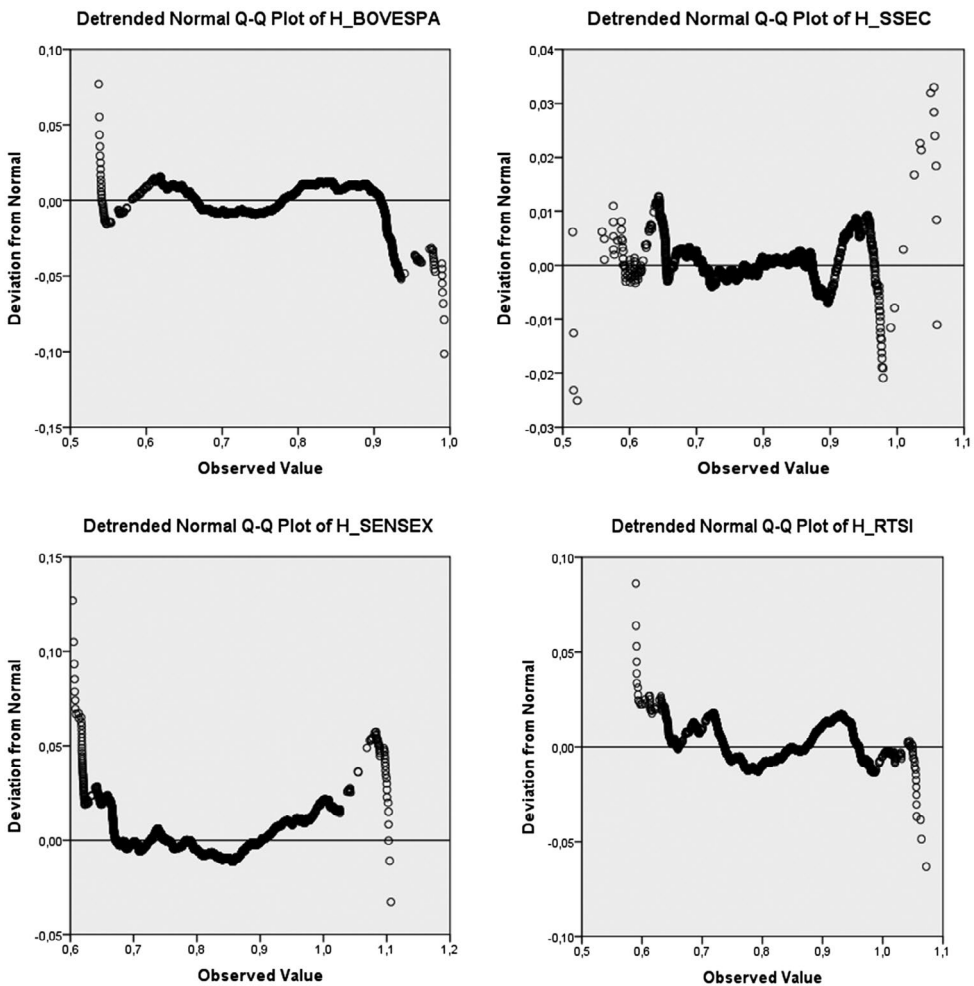
<sup>a</sup>Lilliefors significance correction.

Source: Own calculation, using SPSS software.

values obtained in Table 5 show that the distribution of the values of Hurst exponent series does not follow the normal distribution.

Another test of normality is also performed through the detrended normal QQ-plot, which charts the observed values against the normal distribution in our case. The results of this normality test are reported in Figure 2 (a–c). The plots show some deviation from the normal distribution, particularly at the lower and higher ends. Our overall conclusion is that this distribution of these Hurst exponents is not normal and therefore one should analyse the medians of the Hurst exponents in order to compare these indices.

In Figure 3, the distancing of each individual capital market from market efficiency is presented graphically (in the figure, the centre point corresponds to a value of 0.5 for the Hurst exponent). Subsequent to the situation presented in this figure, according



**Figure 2.** (a) Detrended normal QQ-plot for Hurst exponents in BRIC capital markets. (b) Detrended normal QQ-plot for Hurst exponents in European emergent capital markets. (c) Detrended normal QQ-plot for Hurst exponents in European developed capital markets. Source: According to the authors' calculations.



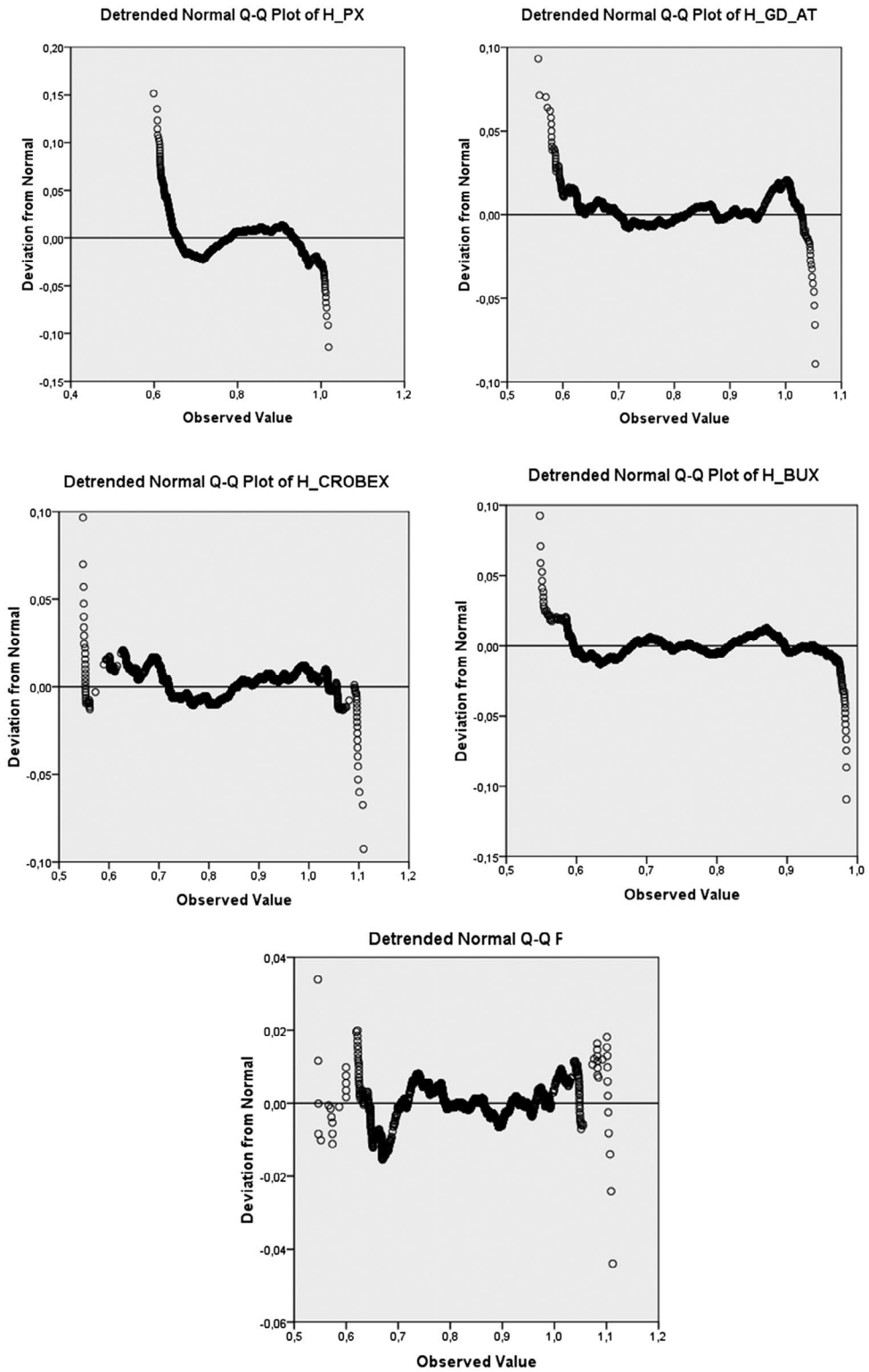
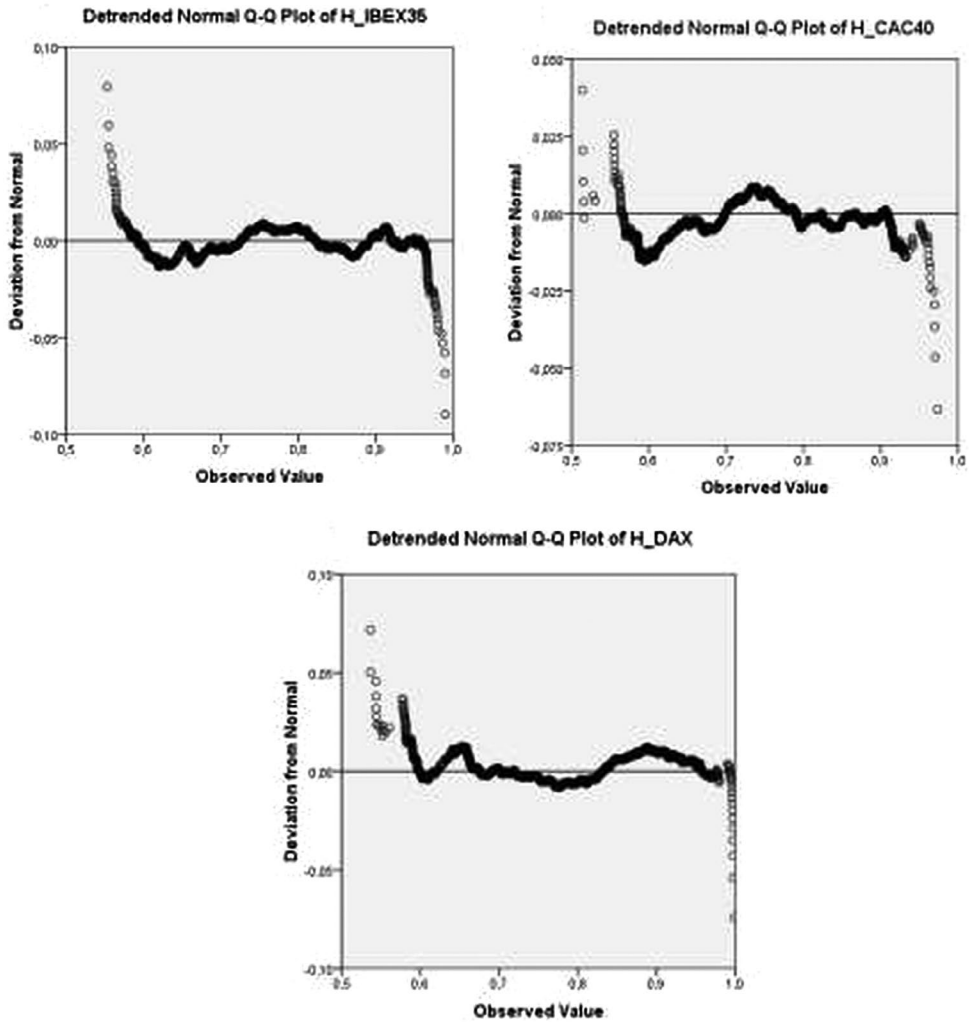


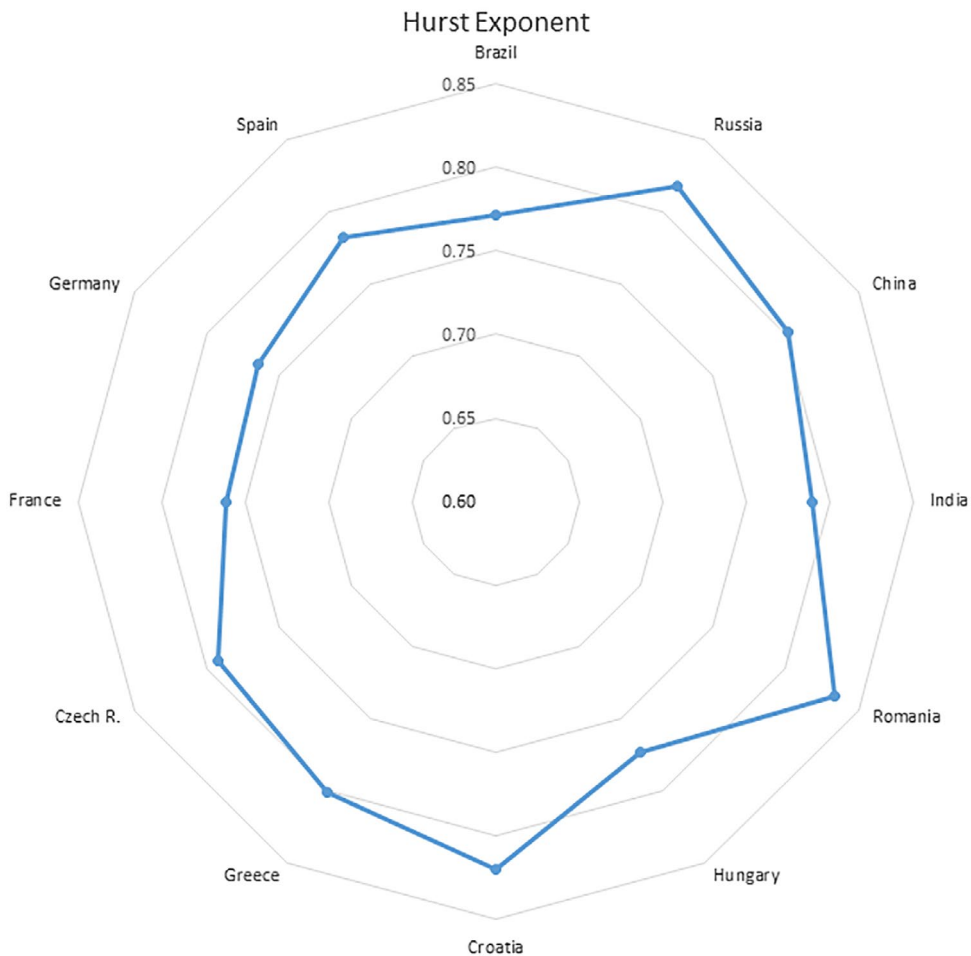
Figure 2. (Continued).



**Figure 2.** (Continued).

to the median value of the Hurst exponent, one may notice, as an overall trend, that all rate series are persistent, since the value of the Hurst exponent ranges between 0.5 and 1.

As mentioned earlier, the Hurst exponents' behaviour exhibits large deviation from the Gaussian distribution. The non-parametric tests could be used (non-parametric tests do not make assumptions about a specific distribution) for equality of medians for the different sample markets in order to test whether these rankings are meaningful. The results are summarised in Table 6 and they suggest significance of the difference between the median values of the Hurst exponents among all the selected non-parametric tests at the 1% significance level, and consequently our ranking is meaningful.



**Figure 3.** The situation of capital markets according to Hurst exponent values. Source: According to the authors' calculations.  
Note: R/S analysis: rolling sample, GARCH residuals.

#### **5.4. Long-term correlation coefficient and fractal dimension**

In Table 7 the long-term correlation coefficient and the fractal dimension are computed for all the indices and these stock markets are ranked according to these indicators, using medians for the calculated Hurst exponents.

Subsequent to the classification displayed in Table 7, one may see that the developed capital markets in France and Germany, followed by the emerging countries Brazil and Hungary are closest to the random walk, a characteristic of EMH, accordingly it can be said that they are the less inefficient stock markets. On the other hand, the capital markets in Romania, Croatia and Russia prove to be furthest from the random walk, accurately observing the fractal Brownian motion, and thus being the most inefficient markets. These outcomes reinforce our previous findings and suggest that this fractal exponent may be useful in surveying the stage of stock market inefficiency.

**Table 6.** Test for equality of medians between series.

Method	df	Value	Probability
Med. chi-square	8	812.9377	0.0000
Adj. med. chi-square	8	810.8076	0.0000
Kruskal–Wallis	8	1437.469	0.0000
Kruskal–Wallis (tie-adj.)	8	1437.469	0.0000
van der Waerden	8	1635.635	0.0000

Source: Own calculation, using EViews software.

**Table 7.** Indices classification according to the value of Hurst exponent, correlation coefficient and fractal dimension.

	Hurst median	Correlation coefficient ( $C_n$ )	Fractal dimension ( $D$ )	Rank
BET (Romania)	0.834198	0.589304983	1.198756171	12
IBOVESPA (Brazil)	0.771337	0.45667	1.29645	3
BUX (Hungary)	0.773101	0.460236	1.293492	4
CROBEX (Croatia)	0.820586	0.559596	1.218641	11
GD.AT (Greece)	0.801254	0.518354	1.248044	8
PX (Czech Republic)	0.791639	0.49825	1.263202	7
RTSI (Russia)	0.817734	0.553442	1.222892	10
SENSEX (India)	0.802141	0.520222	1.246664	9
SSEC (China)	0.788871	0.492511	1.267634	6
CAC40 (France)	0.761588	0.43711549	1.313046	1
DAX (Germany)	0.764018	0.441965	1.30887	2
IBEX35 (Spain)	0.781996	0.478354	1.278779	5

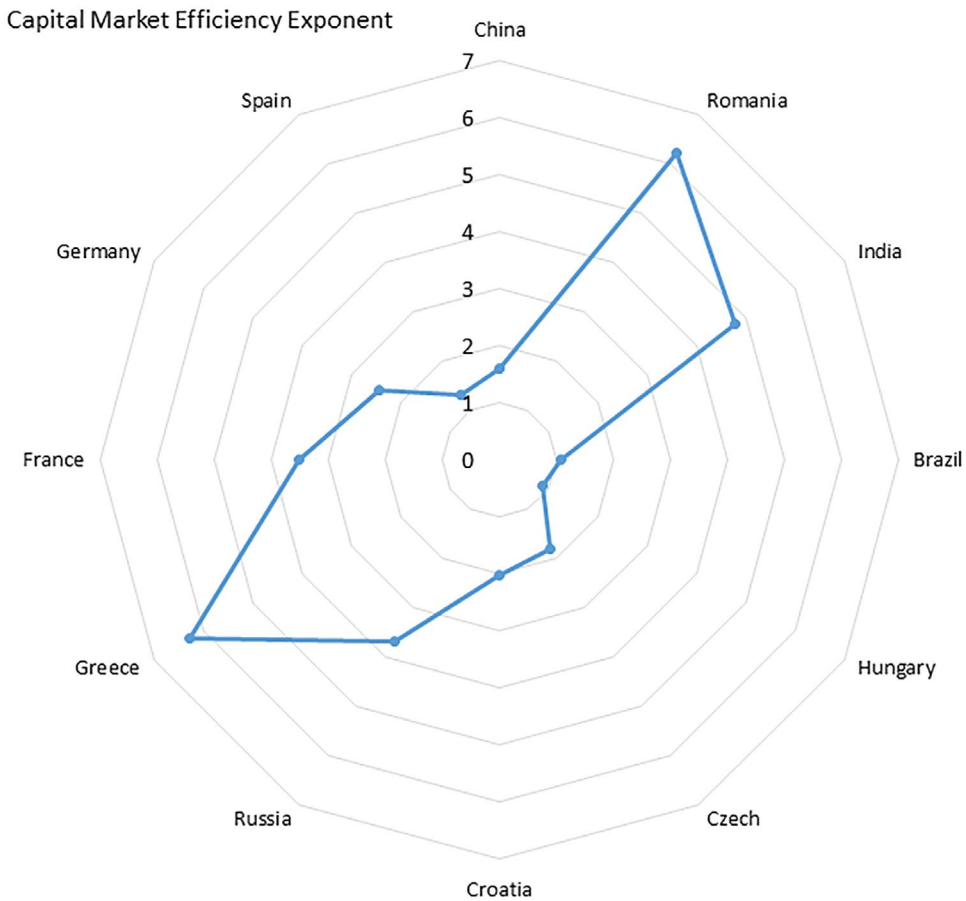
Source: Own calculation.

### 5.5. Capital Market Efficiency Exponent

The efficiency of the nine stock market indices is analysed. The results for *CMEE* are summarised in Figure 4, which graphically shows the distancing of each individual capital market from market efficiency (in the figure, the centre point corresponds to a value of zero for *CMEE*). The most efficient markets turn out to be the Hungarian, Brazilian and Spanish capital markets and the least efficient are the Greek and Romanian markets. According to the scale proposed in Section 4 for the interpretation of the mathematical model, the only capital market characterised by high efficiency is the Hungarian market, while the Russian, Indian, Romanian and Greek capital markets are the least efficient ones. All the other markets are characterised by medium efficiency.

In conclusion, the developed capital markets have an average *CMEE* of 2.42, the advanced emerging markets' return series have an average *CMEE* of 2.51, while the secondary emerging markets' return series have an average *CMEE* of 3.35. By contrast, the frontier markets have an average *CMEE* of 4.12, which suggests that the least efficient are these frontier markets. In conclusion, we found that, indeed, economic development levels and stock market efficiency are closely related, the developed capital markets being the most efficient compared with the advanced emerging markets, secondary and frontier emerging countries selected and analysed in this paper.

Following the results, the developed capital markets are more efficient compared with the emerging BRIC capital markets, with an average *CMEE* of 2.785, and the emerging European countries selected and analysed in this paper (with an average *CMEE* of 3.442).



**Figure 4.** Capital Market Efficiency Exponent. Source: According to the authors' calculations.

Empirical evidence of emerging markets has opened opportunities for portfolio investors to exploit the predictability in these markets, and these results confirm other results in the existing literature (Chong et al., 2010; Karamchandani et al., 2014).

## 6. Conclusions

The topic of EMH legitimacy in capital markets has been investigated in much research. The limitations posed on many widely used estimating tools due to the impact of non-linear dynamics in stock markets can be overcome by replacing or supplementing those tools with the ones of econophysics, such as the Hurst exponent or Shannon entropy.

In this paper, a measure of market efficiency has been developed based on deviation from the perfect state (expressed by the efficient market). For this measure of market efficiency five different estimates have been proposed (one estimate of informational entropy, one estimate of run test, one estimate of Hurst exponent, one estimate for correlation coefficient and one estimate for fractal dimension).

*Entropy* is seen as a measure of informational efficiency of capital markets. Likewise, in order to find out whether the indices exhibit random behaviour or whether they follow a specific growth pattern, the *run test* is used. This analysis applied the R/S analysis method with rolling window applied on GARCH residuals to estimate the Hurst exponent  $H$ . In order to measure the correlation between recent and previous observations (also called long-term memory character), it used the *correlation coefficient* evinced throughout the time range subject to analysis. It is also used the relation between *fractal dimension*  $D$  (used to identify whether the system is random) and Hurst exponent. To sum up, the tests performed in this paper indicate that the returns of these emerging stock markets are series submitting to fractal distribution. Fractal market means that there is memory, enhancement and continuity in the variation of the stock market. The changes in asset prices increase and continue on the basis of the previous state.

The final obtained results show that the most efficient market, with the lowest value of *CMEE*, is the Hungarian capital market, and the least efficient, with the highest value of *CMEE*, is the Greek capital market.

In order to allow more interesting comparisons and an enhanced understanding of stock market efficiency, we also applied this study in the case of developed capital markets, namely the German, French and Spanish capital markets. The results obtained in our study, like the results from most studies already undertaken in this regard (for example, Baciu, 2014; Hull & McGroarty, 2014; Kitt, 2003; Podobnik et al., 2006; Wright, 2001, show that even though the series of developed markets usually possess only short or no memory, emerging markets exhibit a different behaviour.

Then the question arises: why do emerging markets act unlike developed markets? This phenomenon deserves further study; a possible clarification is the following. In stock markets, short-term moves are described by the psychology of crowd behaviour. In developed markets, investors have seen drops and rises of indices and they behave more rationally: when the index is down, they are entering the market to buy; when the index is up, they are selling. This adjusts the market, yet makes short-term randomness in prices, which leads to lower Hurst exponents. In emerging markets, local investors are not experienced with financial markets and they behave more unpredictably. They can buy when prices are rising and create bubbles, or they can sell when prices are dropping, and create crashes.

Also, the absence of long-memory dynamics can be rationalised for developed stock markets on the grounds that they are informationally efficient. Prices tend to reflect all publicly available information and any new one is fully arbitrated away. Thin markets, on the other hand, are characterised by various institutional rigidities that perpetuate informational inefficiency and asymmetry (Limam, 2003).

Regarding this study, a possible improvement would be in terms of enlarging the sample of companies in the study to obtain more representative results. This research aims in the future to cover indices from North and Latin America, Western Europe, Asia, Oceania and Africa in order to gain in depth and relevance with the inclusion of emerging markets from several continents.

For investors, the implications of trading on BRIC emerging markets, more efficient compared with the European ones analysed in this study, refer to:

- (a) The more efficient the market, the more random and unpredictable the price evolution is on such a market. This is the result of the fact that agents on the market try

to take advantage of the available information, and quickly using this information, they will be quickly incorporated into prices, thus eliminating the opportunities to gain from arbitrage.

- (b) In an efficient market, equity research and valuation would be a costly task giving no advantages, for the chances of finding an undervalued stock ought to be arbitrary.
- (c) In an efficient market, a strategy of randomly diversifying across stocks or indexing to the market, conveying practically zero data cost and insignificant execution expenses, would be better than any other technique that created larger data and execution costs.
- (d) In an efficient market, a strategy of minimising trading, i.e., creating a portfolio and not trading unless cash was needed, would be better than a strategy that required regular trading.

Obviously, CMEE is more complex and difficult than tests developed to test Fama's EMH. Additionally, some early tests have been improved and new tests have been advanced thanks to the advances in financial econometrics. The new CMEE does not include in its full model additional public and private information. Fama's semi-strong hypothesis considers explicitly fundamental analysis which overcomes the limitations of price analysis only of the weak hypothesis. In short, CMEE is only a complex form to test information derived from stock prices. Considering the complex volume of data and complex quantitative methods integrated into the CMEE, this type of model can easily be applied by large traders of mature markets in order to produce solid results and avoid spurious evidence. However, in developing countries there are many limitations concerning advanced financial training, availability and reliability of financial data, and access to sophisticated software, so applications of CMEE would be limited.

The financial literature is rich in tests proving the presence or not of informational efficiency; fortunately, many studies have also proved the presence of anomalies. These studies have become important foundations to further advances in theory and research in finance. Indeed, frontier econometric, statistical and mathematical methods to test EMH and other stock market issues are currently being applied in all financial research.

## Disclosure statement

No potential conflict of interest was reported by the authors.

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