

# Research on Method of Dynamic Stability Analysis for Slopes of Earth and Rockfill Dam Basing on the P-Z Model

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**Abstract:** According to the problems in dynamic stability analysis for slopes of earth and rockfill dam, the P-Z constitutive model, which is a kind of the multi-mechanism plastic model based on generalized plasticity, is introduced in the paper. Strength reduction factors of P-Z model are derived and verified, and based on them a new kind of method of dam slopes dynamic stability is put forward. For the method, the dynamic stability of dam slopes is judged by dynamic displacement time history and post-earthquake permanent displacement. The results show that local instability of dam slopes and variation features of dynamic response are obtained by the method, which is more reasonable.

**Keywords:** dynamic stability analysis; local instability; P-Z constitutive model; strength reduction FEM

## 1 INTRODUCTION

Currently, the common analysis methods of dynamic stability for earth and rockfill dam slopes are the quasi-static method and the Newmark-slider method. Although these methods are simple and easy to calculate, there are many assumptions and the calculation results have a larger arbitrariness. The finite element method was used in static stability analysis of slope in the 1970s. It is a more rational analysis method, which considers the balance of forces and moments of the whole slope and the constructive relation of slope soil.

However, in recent years the finite element method has been applied to slope dynamic stability. Liu Hanlong et al. [1] proposed a method that used average safety factor to evaluate the stability of slope based on the relation curve of safety factor and time. Li Yushu et al. [2] drew the time history curve of dynamic safety factor on the most dangerous sliding surface with the same method, and used the average dynamic safety factor method and permanent deformation of earthquake to evaluate the earthquake dynamic stability of slope. Ma Fangfang [3] proposed two-time history analysis methods of dynamic stability about high earth and rockfill dam. Recently, there are many other researchers who have done a lot of useful researches [4-11]. These studies on the dynamic stability of slope are very significant attempts, but in the studies the dynamic safety factor is defined as the ratio of shear force and shear stress of a moment, when the unit is not necessarily in the limit equilibrium state, which is inconsistent with premise (limit equilibrium state) defined by conventional safety factor. Therefore, based on the static slope stability analysis and taking into consideration the selection of constitutive relationship of slope material, dynamic artificial boundary and instability criterion of finite element strength reduction, a new dynamic stability analysis method of slope is proposed.

## 2 FINITE ELEMENT STRENGTH REDUCTION PARAMETERS OF P-Z MODEL

The P-Z model [12-15] which is based on the generalized plastic mechanics theory was proposed by

Pastor and Zienkiewicz. The model can describe the static and dynamic characteristics of soil in the condition of cyclic loading or uniaxial loading, drainage or not. Besides, in deriving plastic deformation, it is not necessary to firstly define the yield surface and plastic potential surface, but to deduce the yield surface and the plastic potential surface through loading direction vector and loading and unloading direction vector of plastic potential. There are 12 parameters about P-Z constitutive model, which are  $M_g$ ,  $M_f$ ,  $\alpha_g$ ,  $\alpha_f$ ,  $\beta_0$ ,  $\beta_1$ ,  $H_0$ ,  $\gamma_{DM}$ ,  $H_{u0}$ ,  $\gamma_u$ ,  $K_{evo}$  and  $K_{eso}$ , of which  $\gamma_{DM}$ ,  $H_{u0}$  and  $\gamma_u$  are related to dynamic analysis. The parameters can be determined by conventional three axis test or experience. The authors have recently studied the wetting deformation and dynamic characteristics of earth-rockfill Dam based on P-Z Model, and the reasonable results are obtained [16-17] (Intro parameters and test verification, see refs [18-19]).

### 2.1 Selection of finite element strength reduction parameter of P-Z model

Plastic potential surface equation of P-Z model:

$$G = q - M_g p \left( 1 + \frac{1}{\alpha_g} \right) \left[ 1 - \left( \frac{p}{p_g} \right)^{\alpha_g} \right] = 0 \quad (1)$$

Yield surface equation of P-Z model:

$$F = q - M_f p \left( 1 + \frac{1}{\alpha_f} \right) \left[ 1 - \left( \frac{p}{p_f} \right)^{\alpha_f} \right] = 0 \quad (2)$$

where  $q$  is deviatoric stress;  $p$  is spherical stress;  $p_g$  and  $p_f$  are the constants of spherical stress;  $M_g$  is the slope of critical state line; the value of  $M_f$  is related to  $M_g$ ;  $\alpha_g$  and  $\alpha_f$  are the material parameters which implicitly reflect the Poisson's ratio and are related to strain and stress state.

To the associated flow rule, there is

$$\begin{cases} \alpha_f = \alpha_g = mc + n \\ M_f = M_g = \frac{6 \sin \varphi}{3 - \sin \varphi \sin 3\theta} \end{cases} \quad (3)$$

And to the non-associated flow rule, there is

$$\begin{cases} \alpha_f = \alpha_g = mc + n \\ M_f = D_r M_g = D_r \frac{6 \sin \varphi}{3 - \sin \varphi \sin 3\theta} \end{cases} \quad (4)$$

where  $D_r$  is the relative density of material;  $c$ ,  $\varphi$  are the soil cohesion and internal friction angle;  $\theta$  is the Lode angle;  $m$ ,  $n$  are constants. The calculation of  $\alpha_g$  and  $\alpha_f$  is complex, and linear relationship with cohesion is assumed, and the values are generally 0.3 to 0.5 according to the experimental results.

### 2.2 Case 1

A rigid block which is 5 m wide is covered on the left top of the vertical soil slope, ignoring deadweight. Concentrated load is applied vertically downward on the centre of block, the value of which is 200 kN. The right and bottom of slope are imposed constraints, as shown in Fig. 1. The finite element mesh is shown in Fig. 2, and there are a total of 700 units and 756 nodes. The cohesion and friction angle of slope soil are:  $c=35$  kPa and  $\varphi=31^\circ$ , and the corresponding parameters of P-Z model are shown in Tab. 1.

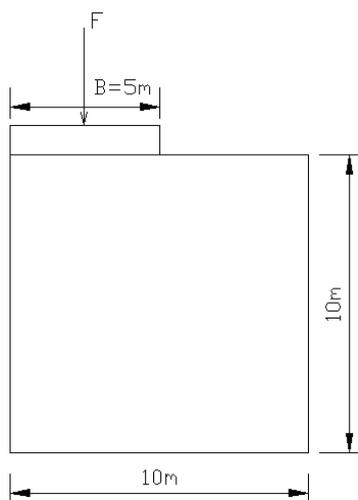


Figure 1 The size of model

Table 1 Parameters of P-Z model

Parameters	$M_g$	$M_f$	$\alpha_g$	$\alpha_f$
Value	1.42	0.64	0.45	0.45
Parameters	$\beta_0$	$\beta_1$	$H_0$	$\gamma_{DM}$
Value	4.2	0.2	380.0	1.5
Parameters	$H_{u0}$	$\gamma_u$	$K_{evo}$	$K_{eso}$
Value	5320	3.0	81325KPa	63980KPa

The angle between theoretical failure surface and the horizontal direction is  $52.4^\circ$ , and the safety factor is:  $F=Bc/F(ctan\alpha+tan\alpha)+ctan\alpha \cdot tan\theta=2.273$ , and putting the calculation parameters in it can get  $K=2.273$ . Based on the given reduction parameters of P-Z model, the static safety factor of model is 2.267 with the finite element strength reduction method. Comparing the calculated solution with the theoretical solution, the differences are only 4.4 %. The plastic zone distribution is shown in Fig. 3, and the diagram of displacement vector is shown in Fig. 4.

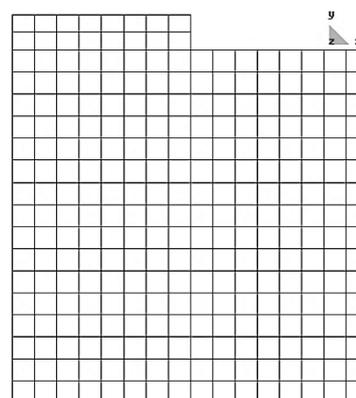


Figure 2 The finite element of model

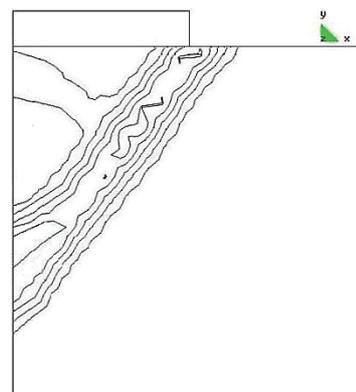


Figure 3 The plastic zone distribution

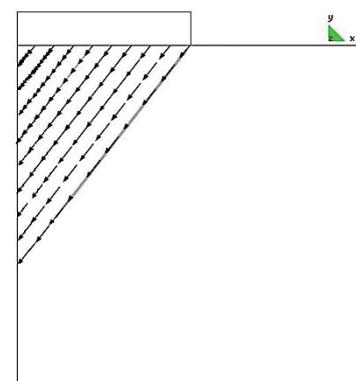


Figure 4 The diagram of displacement vector

## 3 THE METHOD OF STATIC AND DYNAMIC SLOPE STABILITY BASED ON P-Z MODEL

### 3.1 Basic Principle

The earthquake process is short, so it is assumed that the material parameters do not change in the dynamic stability analysis during the earthquake period on the basis of which a new method of dynamic stability analysis is proposed, and the basic ideas are: (1) Calculating a series of shear strength reduction factors of slope stability in static condition by nonlinear finite element mixing solution, then carrying out static and dynamic analyses with every reduction factor of material strength parameters, and the dynamic displacement time history and post-earthquake permanent displacement corresponding to every reduction factor are got. (2) Establishing the relation between post-earthquake permanent displacement of the feature point and shear strength reduction factor. (3) Taking the mutation of post-

earthquake permanent displacement as the criteria of slope failure, the strength reduction factor corresponding

to the displacement mutation of feature points is the dynamic stability safety factor of the slope.

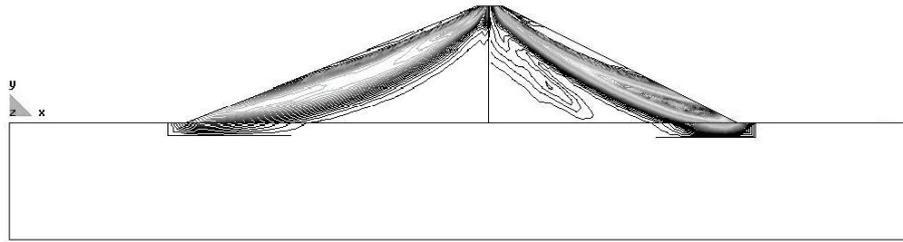


Figure 5 Distribution diagram of plastic zone

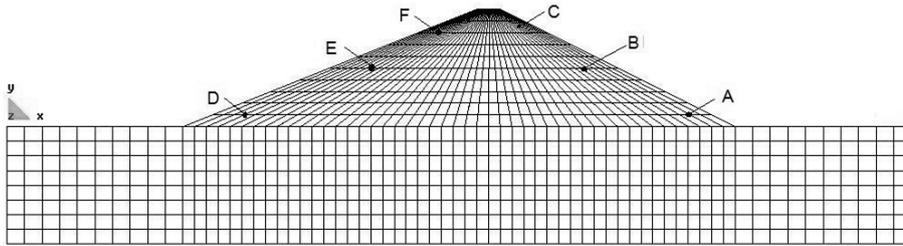


Figure 6 The finite element mesh and location of feature points

3.2 Case 2

The dam is 20 m height, and the upstream and downstream slope ratios are 1:2.5 and 1:2.0. The parameters are shown in Tab. 1.

The static safety factors of upstream and downstream slope are got by nonlinear finite element solution [20]. They are 1.818 and 1.515, and reduction factors of each iteration step are 1.0, 1.087, 1.190, 1.250, 1.316, 1.389, 1.471, 1.515, 1.667, 1.724, 1.818. Distribution diagram of plastic zone is shown in Fig. 5.

Based on the idea of slope dynamic safety evaluation before, the static and dynamic analysis of slope is implemented by the selection of all finite element strength reduction factors, which are 1.0, 1.087, 1.190, 1.250, 1.316, 1.389, 1.471, 1.515, 1.667, 1.724. Three feature points in the plastic transfixion zone on each side of upstream slope and downstream slope are selected to calculate their dynamic displacement time history and post-earthquake permanent displacement, and the feature points are shown in Fig. 6. In the dynamic analysis, the truncation boundary of dam foundation is simulated as viscoelastic artificial boundary, and the seismic wave figures of acceleration, velocity and displacement time history in X direction are shown in Fig. 7.

Results of relationship between post-earthquake permanent displacement of feature points and the reduction factors are shown in Tab. 2.

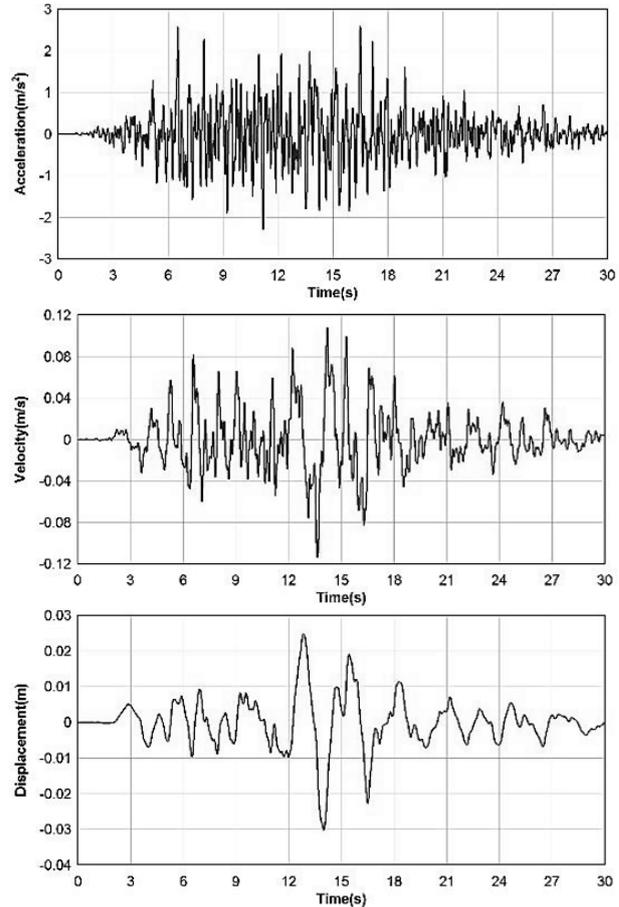
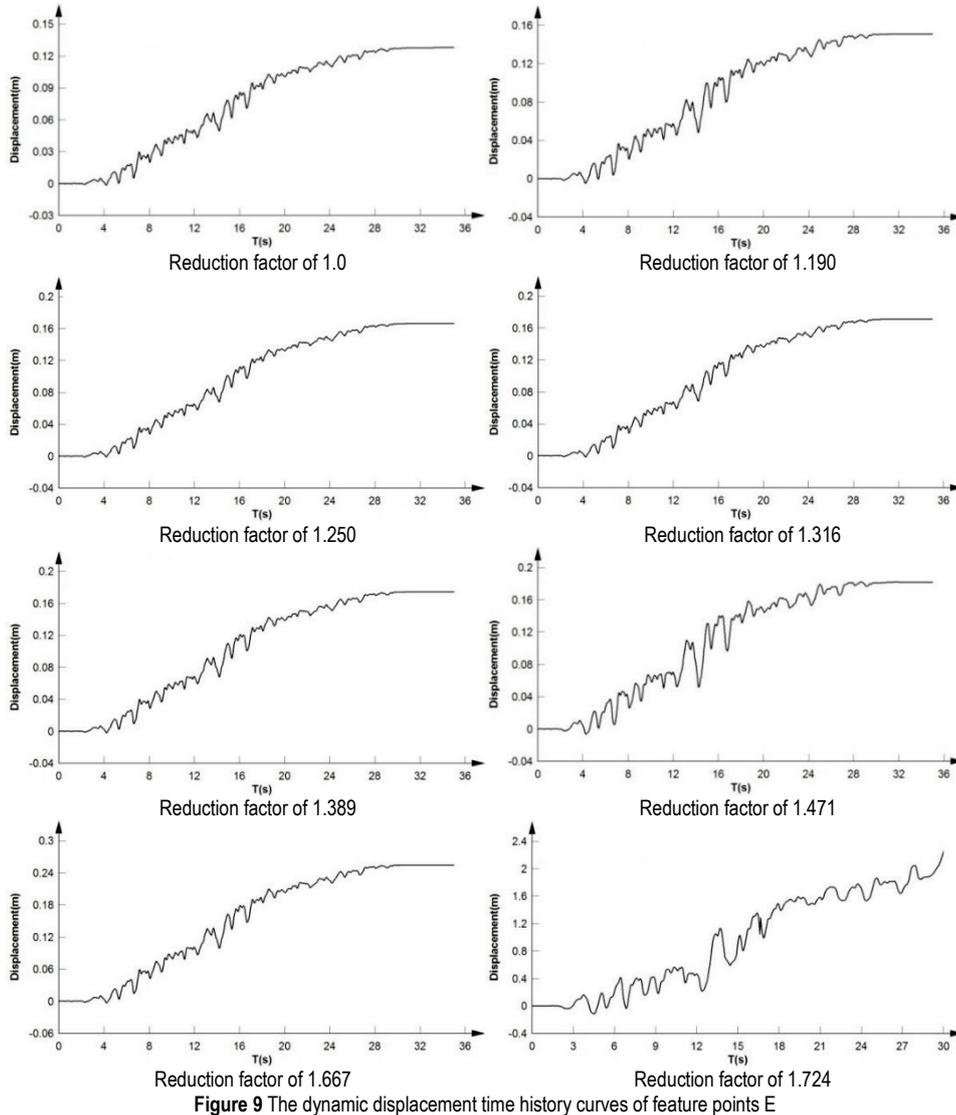
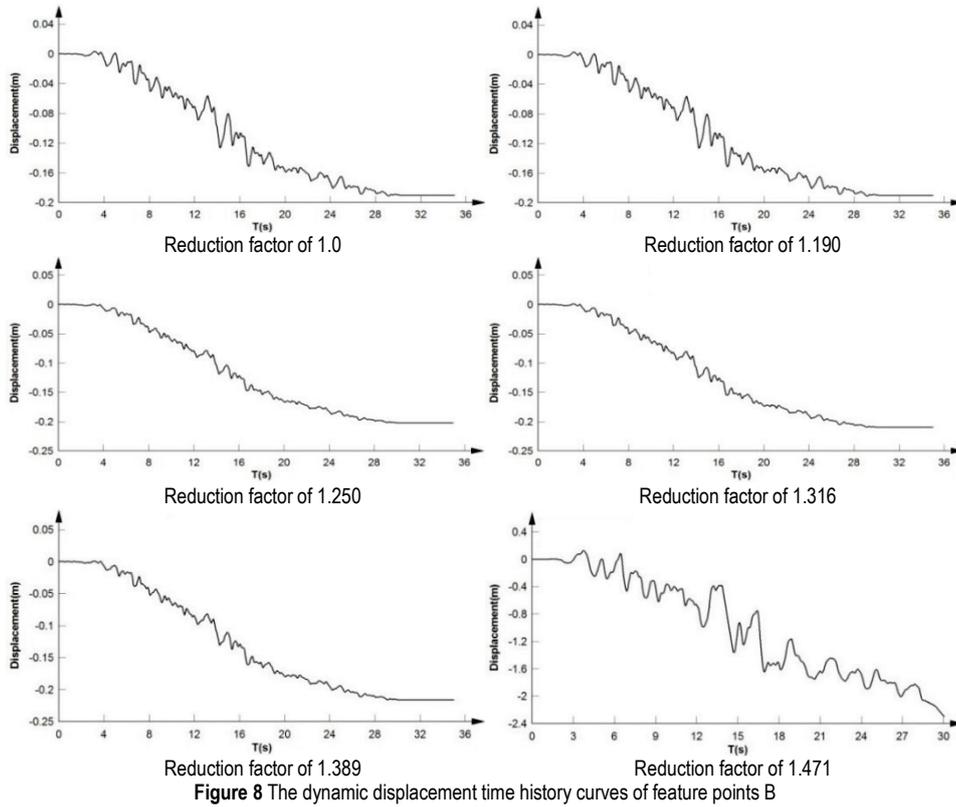


Figure 7 Figures of acceleration, velocity and displacement time history

Table 2 Relationship between post-earthquake permanent displacement and the reduction factors

The reduction factors		1.0	1.19	1.25	1.316	1.389	1.471	1.667	1.724
The post-earthquake permanent displacement of feature points (m)	A	-0.080	-0.086	-0.088	-0.091	-0.095	-0.102	-0.110	-0.122
	B	-0.161	-0.190	-0.202	-0.210	-0.226	-2.217	/	/
	C	-0.153	-0.181	-0.195	-0.198	-0.200	-0.214	-0.221	-0.230
	D	0.071	0.080	0.086	0.090	0.092	0.095	0.109	0.138
	E	0.128	0.151	0.166	0.171	0.174	0.182	0.255	2.259
	F	0.127	0.148	0.155	0.157	0.159	0.163	0.180	0.208



In Tab. 2, post-earthquake permanent displacement of feature point increases with the increase of reduction factor. The permanent displacement of point B is bigger than that of points A and C on downstream slope, and mutates in the reduction factor of 1.471. The displacement of E is bigger than the displacements of D and F on upstream slope, and mutates in the reduction factor of 1.724. The dynamic displacement time history curves of feature points B and E in each reduction factor are shown in Fig. 8 and Fig. 9.

It is known from Fig. 8 and 9 that the displacement time history of each point is constantly accumulating and increasing for the adoption of the P-Z elastic-plastic model, and the post-earthquake permanent displacement of each point increases with the increasing of reduction factor. The curves between post-earthquake permanent displacement of feature points B, E and the safety factor which are established according to Tab. 2 and Fig. 8 and 9 are shown in Fig. 10.

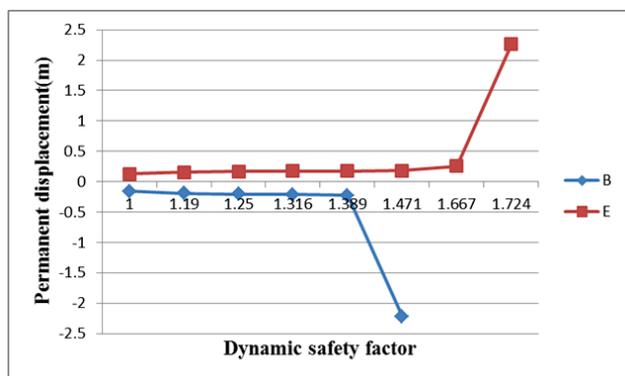


Figure 10 The curves between post-earthquake permanent displacement and the safety factor

It is known from figure 10 that the post-earthquake permanent displacement of point B mutates after the reduction factor of 1.389 under the dynamic load, which is the dynamic safety factor of the downstream slope. The post-earthquake permanent displacement of point E mutates after the reduction factor of 1.667, which is the dynamic safety factor of the upstream slope. For permanent movement of points A, C, D and F in the post-quake there are no immediate changes, but it can be considered that the dam locally loses stability.

### 3.3 Comparisons of Several Analysis Methods of Slope Dynamic Stability

Tab. 3 shows the dynamic safety factors of slope calculated by the proposed method, the pseudo-static method and the dynamic time-history method. Fig. 11 shows the dynamic safety factor curve by the time history method. Fig. 11 shows the dynamic safety factor curve of upstream and downstream slope.

Table 3 The dynamic safety factors calculated by different methods

	The proposed method	The pseudo-static method	The dynamic time-history method
Upstream slope	1.667	1.453	1.20
Downstream slope	1.389	1.211	0.833

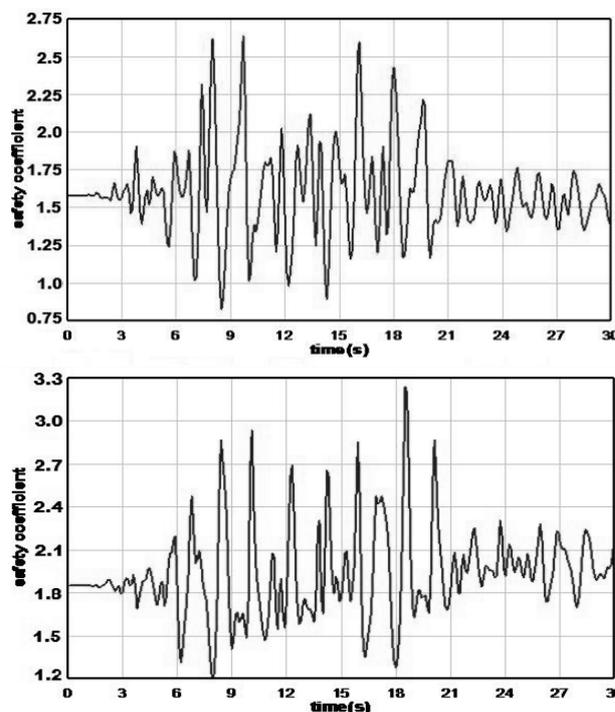


Figure 11 The dynamic safety factor curve of upstream and downstream slope

It is known from the table and figures above that the dynamic safety factor calculated by quasi-static method is less than that in the proposed model, and only it can judge the stability of slope in its entirety. The dynamic safety factor at each moment is obtained by time history analysis method, but there are no clear criteria about how to judge the stability of slope in the case the safety factor is less than 1. Besides, the dynamic safety factor is calculated by the ratio of stabilizing force and sliding force at a moment, when the slope is not always in the limit equilibrium state, which is unreasonable. Comparing with traditional methods, the local slope instability can be judged and the change of permanent displacement and dynamic displacement can be got by the proposed method, which is more reasonable.

## 4 CONCLUSION

The dynamic stability of earth and rockfill dam slope is worrying and complicated, so the definition of dynamic safety factor, the constitutive model, the dynamic boundary and other factors need to be considered. The P-Z constitutive model is introduced in the paper, and its strength reduction parameters are derived. Then a new method of dynamic stability analysis is put forward which judges the stability of slope by dynamic displacement time history and post-earthquake permanent displacement. By comparing the results of the proposed method to those of quasi-static method and dynamic safety factor time history method, it is known that the local slope instability can be judged and the change of permanent displacement and dynamic displacement can be got by the proposed method, which is more reasonable.

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