## The Bonding of $\mathrm{P}_{4}$ to $\mathrm{d}^{\mathbf{8}}-\mathrm{ML}_{3}$ Complexes

Sung-Kwon Kang, Thomas A. Albright*1a, and Jerome Silvestre ${ }^{1 \mathrm{~b}}$<br>Department of Chemistry, University of Houston, Houston, Texas 77004, USA<br>Received February 3, 1984

Extended Hückel calculations were carried out on $\eta^{1}, \eta^{2}$, and $\eta^{3}$ complexes of $\mathrm{P}_{4}$ to $\mathrm{Rh}\left(\mathrm{PH}_{3}\right)_{2} \mathrm{Cl}$. The $\eta^{1}$-square planar and an $\eta^{2}$ complex with $C_{2 v}$ symmetry are the most stable. Geometrical optimizations and a detailed account of the bonding in each have been carried out. $d^{10} \eta^{1}$-tetrahedral complexes of $\mathrm{P}_{4}$ are expected to be quite stable. The best candidate for an $\eta^{3}$ mode of bonding is the trimer $\mathrm{Fe}_{3}(\mathrm{CO})_{9}$. Alternative complexes at $\eta^{3}$ include a $d^{6}-\mathrm{ML}_{3}$ and $d^{4}-\mathrm{ML}_{4}$ species, however reactivity problems in the former and steric constraints in the latter may prohibit their isolation.

There have been a number of compounds recently prepared and structurally determined of the $\eta^{3}-\mathrm{P}_{3}-\mathrm{ML}_{3}$ and $\eta^{3}-\mathrm{P}_{3}-\mathrm{M}_{2} \mathrm{~L}_{6}$ type. ${ }^{2,3}$ These are usually made by reaction of white phosphorus with a labile transition metal complex. Several other $\mathrm{P}_{n}$ »clusters« coordinated to a transition metal are known, however, many are not well categorized at the present time. Intact Ni and Pd complexes of $\mathrm{P}_{4}$ have been prepared using the tris (2-diphenylphosphinoethyl)amine chelate. ${ }^{2 \mathrm{~b}, 5}$ The structure ${ }^{5}$ of the Ni complex shows it to be an $\eta^{1}$ complex, as shown in $\mathbf{1}$. What initialy caught our attention and



I
2
provided the impetus for this work was the report ${ }^{6}$ that $d^{8}-\mathrm{M}\left(\mathrm{PR}_{3}\right)_{2} \mathrm{X}$ complexes of $\mathrm{P}_{4}$ could be isolated where $\mathrm{M}=\mathrm{Rh}$, Ir; $\mathrm{X}=\mathrm{Cl}, \mathrm{Br}$, I. Most importantly, the ${ }^{31} \mathrm{P}$ spectrum of $\mathrm{Rh}\left(\mathrm{P}_{4}\right)\left(\mathrm{PPh}_{3}\right)_{2} \mathrm{Cl}$ was an $\mathrm{A}_{2} \mathrm{~B}_{2} \mathrm{M}_{2} \mathrm{X}$ spin system ${ }^{6 a}$ which implies either a static structure of $C_{2 v}$ symmetry or a rapidly equilibrating
one which averages to a structure of this symmetry. The spectra remain unchanged over a temperature range of $-70^{\circ} \mathrm{C}$ to $+40^{\circ} \mathrm{C}$, thus it was reasonble to postulate a novel $\eta^{2}$ type of coordination, as shown in 2. Very recently the structure of $\mathrm{Rh}\left(\mathrm{P}_{4}\right)\left(\mathrm{PPh}_{3}\right)_{2} \mathrm{Cl}$ was determined ${ }^{7}$ which indeed does show an $\eta^{2}$ coordination mode for the $\mathrm{P}_{4}$ unit, and preliminary extended Hückel calculations were carried out to examine the bonding in the complex. We were interested in what geometrical factors favored the formation of an $\eta^{2}$ geometry over a normal 16 electron, $\eta^{1}$-square planar isomer. Furthermore, we wondered whether or not an $\eta^{1}$-tetrahedral or $\eta^{3}$-piano stool were viable possibilities in this or other electron counts.

A series of extended Hückel calculations were carried out on $\operatorname{Rh}\left(\mathrm{P}_{4}\right)\left(\mathrm{PH}_{3}\right)_{2} \mathrm{Cl}$ for the isomers shown in 3-7. Here all P-P distances in $P_{4}$ were fixed at $2.21 \AA$; the value of the isolated molecule. ${ }^{8}$ The other pertinent geometrical

(0)

(37)
(53)
3
4
5

(41)

6

(81)
7
details and parameters are listed in the Appendix. The relative energies (in $\mathrm{kcal} / \mathrm{mol}$ ) are listed in parenthesis under each structure. Surprisingly the $\eta^{1}$-square planar structure, $\mathbf{3}$, is computed to be $37 \mathrm{kcal} / \mathrm{mol}$ more stable than the correct $\eta^{2}$ isomer, 4. Before we discuss the bonding and geometrical optimizations performed on 3 and 4, it is worthwhile to digress for a moment and derive the valence orbitals of the $\mathrm{P}_{4}$ fragment.

## The $P_{4}$ Fragment Orbitals

There have been several previous theoretical studies ${ }^{9}$ of $\mathrm{P}_{4}$. Perhaps the easiest way to construct the valence orbitals of this cluster starts by artifically partitioning the orbitals on each phosphorus atom into three types. Firstly, each atom contains one $s p$ hybrid which is pointed towards the center of the tetrahedron, as shown by 8 in Scheme I. These four radial orbitals form

## SCHEME I



9


10
tangential


11

lone pair


13
bonding ( $1 a_{1}$ ) and antibonding ( $3 t_{2}$ ) molecular orbitals, see 9 . Since the overlap is large between the radial hybrids, the $1 a_{1}$ and $3 t_{2}$ molecular orbitals are split by a large amount. Each phosphorus atom also uses two atomic p orbitals, 10, for tangential bonding. The overlap here is less than that for the radial types so the $1 t_{2}+e+t_{1}$ members of the tangential set are not seperated in energy as much as the radial set. Finally each phosphorus will have an $s p$ hybrid pointed away from the cluster, 12. These lone pairs will have an even smaller overlap with each other. Therefore, the $2 a_{1}+2 t_{2}$ set is split to a very small extent in 13. Notice that the radial, tangential, and lone pair functions each yield one $t_{2}$ set. Unfortunately there is some intermixing between them, but the level ordering and basic composition is retained in an extend Hückel calculation. With 20 valence electrons $2 t_{2}$ is the HOMO and $t_{1}$ is the LUMO.

The $t_{1}, 2 t_{2}$, e, and $2 a_{1}$ molecular orbitals will figure heavily into our discussion; they are presented in two ways in Figure 1. On the left side of Figure 1 these orbitals are displayed in manner convenient for interaction with a $\mathrm{ML}_{3}$ fragment via edge-bonding ( $\eta^{2}$ ). Intermixing of tangential character from the $1 t_{2}$ set into $2 t_{2}$ is evident. On the right side of Figure 1 the valence orbitals of $P_{4}$ are drawn for $\eta^{1}$ and $\eta^{3}$ bonding. Each member of $t_{1}, 2 t_{2}$, and $e$ have been individually labeled to facilitate our discussion. Different linear combinations have been taken for the $2 t_{2}$ set on the left and right sides of Figure 1. This will simply their interactions with the $\mathrm{ML}_{3}$ unit at the $\eta^{1}, \eta^{2}$, and $\eta^{3}$ geometries.

The reader should note that the low energy of the $t_{1}$ set and high energy of $e$ make these orbitals good electron acceptors and donors, respectively. The

$X_{1}$

$\mathrm{X}_{1}$

$X_{3}$

$X_{2}$


X

$\mathrm{X}_{2}$


$\mathrm{X}_{3}$

$\mathrm{X}_{2}$

$22_{2}$
$\#$
$\#$

e \#




$X_{1}$

Figure 1. Two perspectives of the important valence orbitals in $\mathrm{P}_{4}$.
lone pair combinations, $2 a_{1}$ and $2 t_{2}$, are pointed away from the vertices of the tetrahedron. Consequently their overlap with $\mathrm{ML}_{3}$ at $\eta^{2}$ (on the edge of the tetrahedron) or $\eta^{3}$ (on a face) is expected to be small.
$\eta^{1}$ and $\eta^{2}$ Bonding to a $C_{2 v}$ ML Fragment
Using an undistorted $\mathrm{P}_{4}$ geometry the $\eta^{1}$ and $\eta^{2}$ coordination modes to a square planar $\mathrm{Rh}\left(\mathrm{PH}_{3}\right)_{2} \mathrm{Cl}$ unit in 3 and 4 are computed to be more stable than the other conceivable geometries. Let us start our analysis with the $\eta^{1}$ geometry. The molecular orbitals are constructed for 3 in Figure 2. On the right side of this Figure are the orbitals of a $C_{2 v} M L_{3}$ fragment. ${ }^{10}$ At low energy are the four filled orbitals, $1 a_{1}+1 b_{2}+b_{1}+a_{2}$, associated with a square planar splitting pattern. The $1 b_{2}$ and $b_{1}$ orbitals are pushed up somewhat in energy from $a_{2}$ because of antibonding from the C1 ligand. At higher energy the $2 a_{1}$ fragment orbital is primarily the metal-ligand antibonding $x^{2}-y^{2}$ level. However, it is also hybridized by metal $s$ and $p$ character towards the missing fourth ligand in a square planar complex. ${ }^{10}$ Finally, at higher energy is an empty metal $p$ orbital, $2 b_{2}$. The $2 a_{1}$ and $\chi_{3}$ of $2 t_{2}$ lone pair levels interact strongly with the $2 a_{1}$ hybrid on $\mathrm{ML}_{3}$ and are stabilized. This forms the principal source of bonding in this complex.

We also find that there is some back donation of electron density from the filled $1 b_{2}$ and $b_{1}$ fragment orbitals of $\mathrm{ML}_{3}$ into the empty members of $t_{1}$ ( $\chi_{2}$ and $\chi_{3}$, respectively). It was computed that 0.16 and 0.14 electrons are transfered from $1 b_{2}$ and $b_{1}$, respectively, to the members of $t_{1}$. This means that the Rh-P distance of $\mathrm{P}_{4}$ should be shorter than normal. Furthermore, since $t_{1}$ is antibonding between the phosphorus atom coordinated to Rh and








$$
\mid
$$


$\left(X_{3}\right)$


$$
2 t_{2}
$$



$$
2 t_{2}
$$

Figure 2. An orbital interaction diagram for the $\eta^{1}$-square planar geometry.
the other three basal phosphorus atoms, these P-P distances should increase. These distortions do indeed occur for the $\eta^{1}$-tetrahedral complex, $\mathbf{1},{ }^{5}$ and, as we shall show later, for precisely the same reasons. An optimization of the Rh-P and P-P distances was carried out on the $\eta^{1}$-square planar complex to check the predictions we have made. Using the numbering system shown in 14, an independent optimization within a $C_{s}$ symmetry constraint yielded $P_{1}-P_{2}$ and $P_{1}-P_{3}$ distances of $2.31 \AA$ while those for $P_{2}-P_{3}\left(P_{2}-P_{4}\right)$ and


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$\mathrm{P}_{3}-\mathrm{P}_{4}$ were found to be $2.10 \AA$. The $\mathrm{Rh}-\mathrm{P}_{1}$ distance was found to be too short - $1.85 \AA$. But given the approximate nature of the extended Hückel
method these results are encouragingly in the right direction. Finally, the $\mathrm{H}_{3} \mathrm{P}-\mathrm{Rh}-\mathrm{PH}_{3}$ was also varied in the optimization. We find that it is slightly bent back $\left(170^{\circ}\right)$ away from the $\mathrm{P}_{4}$ unit for steric reasons.

The molecular orbitals for an $\eta^{2}$ complex using a $T_{d}$ geometry for $\mathrm{P}_{4}$ are constructed in Figure 3. Here there is essentially no overlap between $\chi_{3}$ of the $2 t_{2}$ set (see the left side of Figure 1) and $2 a_{1}$ on $\mathrm{P}_{4}$ with the $2 a_{1}$ fragment orbital of $\mathrm{ML}_{3}$. The latter fragment orbital is concentrated towards the bisector of the coordinated P-P bond while the lone pairs are directed away from it. Instead the $\chi_{2}$ component of the $e$ set on $P_{4}$ finds a good overlap with $2 a_{1}{ }^{11}$ and is stabilized to form the molecular orbital labeled $a_{1}$ in Figure 3. There are also $\pi$ interactions at work between the $\mathrm{P}_{4}$ and $\mathrm{ML}_{3}$ fragments. In particular the $1 b_{2}$ and $2 b_{2}$ orbitals of $\mathrm{ML}_{3}$ interact to a considerable extent with the $\chi_{1}$ components of $t_{1}$ and $2 t_{2}$. Three of the four resulting molecular orbitals are explicitly shown in Figure 3. The $1 b_{2}$ fragment orbital on $\mathrm{ML}_{3}$ overlaps with $\chi_{1}$ on $2 t_{2}$ and $t_{1}$ to approximately the same extent at this geometry; the magnitudes of the corresponding overlap integrals were 0.197 and 0.196 , respectively. Notice that both $1 b_{2}$ on $\mathrm{ML}_{3}$ and $\chi_{1}$ of $2 t_{1}$ are filled. Thus, the interaction between them is repulsive. The antibonding combination between the two (the molecular orbital labeled $2 b_{2}$ ) is kept at low energy only because of secondary mixings via $\chi_{1}$ on $t_{1}$ and $2 b_{2}$ on $\mathrm{ML}_{3}$. Therefore,


Figure 3. An orbital interaction diagram for the $\eta^{2}$ complex.
with an undistorted $\mathrm{P}_{4}$ ligand it should come as no surprise that the $\eta^{1}$ complex is computed to be $37 \mathrm{kcal} / \mathrm{mol}$ more stable than $\eta^{2}$. There is much more efficient $\sigma$ bonding at the $\eta^{1}$ geometry for overlap and energy gap reasons (the $2 t_{2}$ set is higher in energy and closer to $2 a_{1}$ than the $e$ set is). Furthermore, there is little, if any, $\pi$ stabilization in the $\eta^{2}$ species. Although no quantitive reliance should be placed on binding energies computed at this level, we find that the complex is not bound with respect to dissociation to $\mathrm{P}_{4}$ and $\mathrm{Rh}\left(\mathrm{PH}_{3}\right)_{2} \mathrm{Cl}$.

Careful inspection of Figure 3 shows that stabilization of the $\eta^{2}$-complex will be achieved by distorting the $P_{4}$ ligand so that the energy of $\chi_{2}$ in the $e$ set is raised, making it a better $\sigma$ donor to the empty $2 a_{1}$ level on $\mathrm{ML}_{3}$. More importantly this distortion must lower the energy of $\chi_{1}$ in the $t_{1}$ set so the backbonding from the filled $1 b_{2}$ orbital on $\mathrm{ML}_{3}$ is maximized. One way to achieve this is by elongating the $P_{1}-P_{2}$ bond (see the numbering system in 2 and 15). A Walsh diagram for this distortion is given in Figure 4. Here $\Theta$ is defined as the $P_{1}-P_{3}, P_{4}$ bisector- $P_{2}$ angle shown in 15. At the tetrahedral


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geometry $\Theta=70.52^{\circ}$. At larger angles the symmetry of the molecule is lowered to $C_{2 v}$. The orbitals on the far right of Figure 4 have been relabeled to reflect this fact. Referring back to the valence orbitals of $\mathrm{P}_{4}$ on the left side of Figure 1, nothing much happens to the orbitals of the $2 t_{2}$ set and $2 a_{1}$. Overlap between these hybrids within each component is small so they stay at relatively constant energy. The $\chi_{1}$ component of the $e$ set is stabilized as $\Theta$ increases. Antibonding between $P_{1}$ and $P_{2}$ is decreased while $\pi$ bonding between $P_{1}$ and $P_{2}$ to $P_{3}$ and $P_{4}$ is increased. The $\chi_{2}$ component of the $e$ set is destabilized greatly since bonding between $P_{1}$ and $P_{2}$ is diminished. Actually this orbital along with $\chi_{3}$ in the $2 t_{2}$ set undergo a weakly avoided crossing (both become $a_{1}$ in symmetry). The resultant HOMO is drawn on the right side of Figure 4. The energetic variation of the individual members in $t_{1}$ can be derived in a similar fashion. Most importantly the $\chi_{1}$ component which is strongly $\mathrm{P}_{1}-\mathrm{P}_{2}$ antibonding is tremendously stabilized as $\Theta$ increases. It becomes the LUMO of the molecule and its final form is also shown on the right of this Figure. For reference the energy of the two critical orbitals, $2 a_{1}$ and $1 b_{2}$, in the $\mathrm{Rh}\left(\mathrm{PH}_{3}\right)_{2} \mathrm{Cl}$ fragment are indicated on the far left side of the Figure. In conclusion, increasing $\Theta$ raises the energy of $3 a_{1}$ making it a better $\sigma$ donor to the empty $2 a_{1}$ fragment orbital on $\mathrm{ML}_{3}$ and the energy of $2 b_{2}$ is lowered making it a better $\pi$ acceptor to the filled $1 b_{2}$ orbital on ML $L_{3}$.


Figure 4. A Walsh diagram for elongating the $P_{1}-P_{2}$ bond (increasing $\Theta$-see 15) in $P_{4}$.
We optimized the $\mathrm{P}-\mathrm{P}$ bond lengths, $\mathrm{Rh}-\mathrm{P}$ bond lengths and $\mathrm{H}_{3} \mathrm{P}-\mathrm{Rh}-$ $-\mathrm{PH}_{3}$ angle within a $\mathrm{C}_{2 \mathrm{v}}$ symmetry constraint. Using the numbering system in 2 we find that in the optimized structure $P_{1}-P_{2}=3.12 \AA, P_{2}-P_{3}$ (etc.) $=$ $=2.24 \AA, \mathrm{P}_{3}-\mathrm{P}_{4}=2.01 \AA$, and $\mathrm{Rh}-\mathrm{P}_{1}=2.74 \AA$. The $\mathrm{H}_{3} \mathrm{P}-\mathrm{Rh}-\mathrm{PH}_{3}$ angle was computed to be $175^{\circ}$. Most importantly the optimized energy for the $\eta^{2}$ geometry is now $15 \mathrm{kcal} / \mathrm{mol}$ more stable than the optimized $\eta^{1}$ structure. Here $\Theta$ (defined in 15) was $102^{\circ}$. The agreement with the experimentally determined structure ${ }^{7}$ of 2 is marginal. The $P_{2}-P_{3}$ (etc.) bond lengths average to $2.21 \AA$. The $\mathrm{P}_{3}-\mathrm{P}_{4}$ bond is slightly shortened ( $2.19 \AA$ ) but not as much as in our optimized structure. The $\mathrm{Rh}-\mathrm{P}_{1}$ distance ( $2.29 \AA$ ) was much shorter than we have calculated it to be. Finally, the $\mathrm{P}_{1}-\mathrm{P}_{2}$ distance was significantly longer than the other $\mathrm{P}-\mathrm{P}$ distances, however, it is only $2.46 \AA\left(\Theta_{=}=79.6^{\circ}\right)$. Our calculations do show the expected features which maximize bonding of the metal to an edge of $\mathrm{P}_{4}$. Namely, there is increased bonding between the filled $3 a_{1}$ orbital of $\mathrm{P}_{4}$ (see Figure 4) and empty $2 a_{1}$ on $\mathrm{Rh}\left(\mathrm{PH}_{3}\right)_{2} \mathrm{Cl}$. This is evident from a plot of the bonding molecular orbital, labeled $a_{1}$ in Figure 3. In Figure 5a, this orbital is shown at the undistorted $T_{d}$ geometry for $P_{4}$. Most of the electron density remains in the $P_{4}$ region. Figure 5 c shows that same molecular orbital at the optimized geometry. It is clear that the electron density is more evenly divided between both fragments and is concentrated in the $\mathrm{Rh}-\mathrm{P}$ bonding region. Figures $5 b$ and $d$ illustrate the analogous trend for the filled molecular
orbital labeled $2 b_{2}$ in Figure 3. There is again much more efficient $\mathrm{Rh}-\mathrm{P}$ bonding at the optimized geometry in Figure 5d than that in Figure 5b. Our calculations overemphasize the distortion. This is particularly evident for the $2 b_{2}$ molecular orbital. As $\Theta$ increases this molecular orbital is considerably stabilized and falls between the $1 a_{1}+b_{1}+a_{2}$ metal-centered orbitals in Figure 3 . With the increasing involvement of the $\chi_{1}$ component of $t_{1}$ into the $2 b_{2}$ molecular orbital (as $\Theta$ increases), the $P_{3}-P_{4}$ distance is expected to contract. As shown in Figure 1 this orbital is bonding between $P_{3}$ and $P_{4}$ in a $\pi$-type fashion. This is the reason behind why our calculations for the optimized structure predict that it should be so short. Notice also in Figure 4 that at $\Theta \sim 90^{\circ}$ the $2 b_{2}$ LUMO of $P_{4}$ lies lower in energy than the $1 b_{2}$ fragment orbital of $\mathrm{Rh}\left(\mathrm{PH}_{3}\right)_{2} \mathrm{Cl}$. Consequently a large amount of electron density is transfered from the metal to the $P_{4}$ unit. Going from the undistorted $P_{4}$ geometry to the observed $x$-ray structure to the optimized one the occupation of the $\chi_{1}$ component in $t_{1}\left(2 b_{2}\right)$ for the complex increases from 0.488 to 0.838 to 1.605 electrons, respectively. This is also reflected in the charges calculated for Rh which increase from +0.396 to 0.713 to 1.294 along this series.

What we have been describing in electronic terms is the oxidative insertion of a 14 electron $\mathrm{Rh}\left(\mathrm{PH}_{3}\right)_{2} \mathrm{Cl}$ unit into a $\mathrm{P}-\mathrm{P}$ bond of the $\mathrm{P}_{4}$ cluster. The resulting product can be formulated in $\mathbf{1 6}$ as a 16 electron trigonal bipyramid.


Figure 5. Plots of the $a_{1}$ and $2 b_{2}$ molecular orbitals of the $\eta^{2}$ complex at an undistorted $\left(\Theta=70.52^{\circ}\right)$ geometry in a and $b$ and optimized $\left(\Theta=102^{\circ}\right)$ structure in c and d . The contours have the values of $0.025,0.05,0.075,0.10,0.15$, and 0.20 . The dashed and full lines refer to opposite signs of $\Psi$.


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This reaction type does indeed have precedent. $\operatorname{IrCl}(\mathrm{CO})\left(\mathrm{PPh}_{3}\right)$ is inserted into the $\mathrm{P}_{4} \mathrm{~S}_{3}$ cluster. ${ }^{4 \mathrm{f}}$ In this case a dimeric compound with an octahedral, 18 electron coordination around each Ir atom is formed. Significantly the P-P distance of the cleaved bond is opened from $2.24 \AA$ in the free ligand to $3.06 \AA$ in the complex. This is close to the value that we find for the optimized $\eta_{j}^{2}$ complex of $P_{4}$. There is clearly not much bonding between $P_{1}$ and $P_{2}$ in our optimized structure. One way to quantify this in an approximate manner is via the Mulliken overlap population. In the free $P_{4}$ ligand the $\mathrm{P}_{1}-\mathrm{P}_{2}$ overlap population is 0.585 . It is reduced slightly to 0.405 in the $\eta^{2}$ complex at a $T_{\mathrm{J}}$ geometry. At the experimentally determined structure it is much smaller (0.136) and at the optimized structure we find the overlap population to be negative ( -0.105 ) which is typical for nonbonded atoms. The situation here is not unlike that for the metal-olefin versus metallacyclopropane controversy. ${ }^{10 a, c}$ The two modes of bonding between $\mathrm{ML}_{3}$ and $\mathrm{P}_{4}$, namely forward donation of $P_{4}$ to $\mathrm{ML}_{3}$ of $a_{1}$ symmetry and backdonation from $\mathbb{M L}_{3}$ to $P_{4}$ of $b_{2}$ symmetry, are very close to the Dewar-Chatt-Duncanson model of metal-olefin bonding. It is difficult to determine, however, whether the compound is represented better as an $\eta^{2}$ complex to the $\mathrm{P}-\mathrm{P}$ bond (as drawn at the top of Figure 3) or the oxidative insertion product 16. $\mathrm{Rh}\left(\mathrm{P}_{4}\right)\left(\mathrm{PPh}_{3}\right)_{2} \mathrm{Cl}$ is perhaps closer to the $\eta^{2}$ formulation but our calculations at the experimental geometry do point to a substantial reduction of the $\mathrm{P}_{1}-\mathrm{P}_{2}$ bond order. ${ }^{12}$ What we should realize is that there ought to be a continuum of structures that lie between these two extremes. The substitution of $\pi$-donors around the metal should raise the energy of the $1 b_{2}$ fragment orbital making donation of electron density of $\chi_{1}$ in $t_{2}$ more efficient. This will push the ground state structure towards 16.

The careful reader will have realized that there is something peculiar about the formulation of especially $\mathbf{1 6}$ as a trigonal bipyramidal complex. This is a 16 electron compound, but the vast majority of pentacoordinate complexes have an 18 electron count. The normal splitting pattern for a $D_{3 \mathrm{~h}}$ trigonal bipyramid ${ }^{13}$ is given in $\mathbf{1 7}$. In a formal sense the $\chi_{1}$ component of $t_{2}$ becomes

doubly occupied so the $R h$ is $d^{6}$. That will half-fill the $e^{\prime}$ set in 17 and a high spin complex is anticipated. One empty molecular orbital in the $\eta^{2}$ complex is appreciably stabilized as $\Theta$ is increased. It is labeled $3 b_{2}$ in Figure 3 and its derivation is identical to the $x y$ component of $e^{\prime}$ in $\mathbf{1 7}$. However, it is energetically well-separated from the HOMO at the optimized structure. The reason behind this lies in the fact that the $\mathrm{P}_{1}-\mathrm{Rh}-\mathrm{P}_{2}$ angle is only $69.5^{\circ}$ at the optimized geometry. This is far away from the angle of $120^{\circ}$ between two equatorial ligands in 17. We shall not belabor the point here but this insures that the $x y$ component of $e^{\prime}$ lies at a higher energy than the $x^{2}-y^{2}$ component. Instead, we direct the reader to the analysis of distortions in six-coordinate complexes by Kubacek and Hoffmann ${ }^{14}$ where identical considerations have been given for low electron count complexes. This point of view does easily show why the alternative $\eta^{2}$-isomer, $\mathbf{5}$, is so much less stable than the others. First of all, the bite angle for even a widely opened $P_{4}$ ligand is quite small for a square pyramidal (5) molecule. Secondly, there is a very small HOMO--LUMO gap. The splitting pattern for a $C_{4 \mathrm{v}}$ square pyramid is given in 18. An orbital which is basically $z^{2}$ is the LUMO in our calculations of 5. Any distortion of the $P_{4}$ ligand does not have an effect on $z^{2}$. Therefore, this geometry is not thought to be a viable one for complexes of $P_{4}$.

In conclusion, the $\eta^{1}(3)$ and $\eta^{2}$ (4) coordination modes of $\mathrm{P}_{4}$ to a $d^{8}-\mathrm{MIL}_{3}$ unit can be made energetically quite close to one another when geometrical optimization of the $\mathrm{P}_{4}$ ligand is taken into account. An $\eta^{1}$ coordination geometry will be favored when the $x^{2}-y^{2}$ hybrid, $2 a_{1}$ in Figure 2 lies low in energy, close to $\chi_{3}$ of $2 t_{2}$ and $2 a_{1}$ on $\mathrm{P}_{4}$. Weak $\sigma$-donor ligands around the metal will accomplish this task. A high-lying $1 b_{2}$ orbital on the metal will favor interaction with the $\chi_{1}$ component of $t_{2}$. Therefore, the substitution of a strong $\pi$ donor trans to the $\mathrm{P}_{4}$ ligand favors the $\eta^{2}$ mode of bonding.

## $\eta^{1}$-Tetrahedral and $\eta^{3}$ Bonding

The orbitals of a pyramidal $\mathrm{ML}_{3}$ species have been discussed extensively elsewhere. ${ }^{10,15}$ Figure 6 presents an interaction diagram for a pyramidal, $C_{3 v}$, $M L_{3}$ fragment interacting with $\mathrm{P}_{4}$ at an $\eta^{1}$ geometry. On the right side of the Figure are displayed the valence orbitals of an arbitrary $C_{3 v} \mathrm{ML}_{3}$ fragment. At low energy are the three reminants of the octahedral $t_{2 g}$ set. They are labeled $1 a_{1}$ and $1 e$. At higher energy are three hybrid orbitals. The $2 e$ set is primarily metal $d$ with some $p$ character mixed into them so they are hybridized away from the three auxiliary ligands, L , towards the $\mathrm{P}_{4}$ unit. The $2 a_{1}$ fragment orbital contains basically metal $s$ and $p$ character and is hybridized in the same direction as the $2 e$ set. The $\chi_{3}$ component of $2 t_{2}$ and the $2 a_{1}$ orbital of $\mathrm{P}_{4}$ (see the right side of Figure 1) are stabilized by $1 a_{1}$ and $2 a_{1}$ on $\mathrm{ML}_{3} . \chi_{2}$ and $\chi_{3}$ on $2 t_{2}$ and the $e$ set of $\mathrm{P}_{4}$ are left nonbonding. Likewise, the $1 e$ set on $\mathrm{ML}_{3}$ will overlap with $\mathrm{P}_{4}$ in primarily a $\delta$ type fashion and so it remains nonbonding. The $1 a_{1}$ fragment orbital on $\mathrm{ML}_{43}$ is destabilized by $\chi_{3}$ of $2 t_{2}$ and $2 a_{1}$ from $\mathrm{P}_{4}$. What stops it from rising to excessively high energies is that the $2 a_{1}$ fragment orbital on $\mathrm{ML}_{3}$ mixes into this molecular orbital in a way which is bonding to $\chi_{3}$ components of $t_{1}$ on the $\mathrm{P}_{4}$ fragment. These two molecular orbitals along with that previously described for $1 a_{1}$ on $\mathrm{VIL}_{3}$ are very close to being degenerate. Therefore, the classic splitting pattern of three above two for a tetrahedral complex is established. A low spin $d^{8}$ complex (like $\mathrm{Rh}\left(\mathrm{PH}_{3}\right)_{2} \mathrm{Cl}$ ) will




Figure 6. An interaction diagram for an $\eta^{1}$-tetrahedral- $\mathrm{ML}_{3}$ complex.
not be stable at this geometry. The three molecular orbitals of the triply »degenerate« set will be occupied by only four electrons. A first order Jahn--Teller distortion relaxes the $\mathrm{ML}_{3}$ unit back to a square planar structure, thus 6 does not represent a minimium on the potential energy surface. With two additional electrons (i.e. a $d^{10}$ complex) these three molecular orbitals are totally occupied. As mentioned in the Introduction there exists one example of this type. The structure of $\mathbf{1}$ is quite close to what one expects for a tetrahedral complex. Notice in Figure 6 that with the $2 e$ set formally filled in such a $d^{10}$ complex there is appreciable backbonding to the $\chi_{1}$ and $\chi_{3}$ components of $t_{1}$ on $\mathrm{P}_{4}$. This backbonding should be reflected in a shorted $\mathbb{M}-\mathrm{P}$ distance and, since $\chi_{1}$ and $\chi_{3}$ of $t_{1}$ are antibonding between the coordinated phosphorus atom and the other three, these three P-P distances should be longer than the other three »basal« $\mathrm{P}-\mathrm{P}$ bond lengths. This is found in $1 .{ }^{5}$ The Ni-P distance is $\sim 0.2 \AA$ shorter than normal. The distances of the coordinated phosphorus atom to the other three are $2.20 \AA$ while the three basal P-P distances are $2.09 \AA$.

No categorized complex of the $\eta^{3}$ type exists. Much of the strategy for stabilizing this coordination mode can be extrapolated from the $\eta^{3}-\mathrm{P}_{3}-\mathrm{ML}_{n}$ system. ${ }^{2 a, 3}$ There is, however, one interesting twist that the $\mathrm{P}_{4}$ ligand presents. An obvious candidate for stabilizing an $\eta^{3}-\mathrm{P}_{4}$ ligand is the pyramidal, $\mathrm{C}_{3 \mathrm{v}}, \mathrm{ML}_{3}$ fragment. A simplified interaction diagram is shown in Figure 7. The $\chi_{3}$
component of $2 t_{2}$ and $2 a_{1}$ on $\mathrm{P}_{4}$ are stabilized somewhat by the $2 a_{1}$ orbital on $\mathrm{ML}_{3}$. Using a normal coordinate system where the $z$ axis is coincident with the three-fold rotation axis of $\mathrm{ML}_{3}$, the $1 a_{1}$ along with $1 e$ in $\mathrm{ML}_{3}$ remain nonbonding. The $2 e$ set of $M L_{3}$ overlaps to an approximately equal extent with $\chi_{1}$ and $\chi_{3}$ of $t_{1}$ and $e$. The overlap of $2 e$ to the $\chi_{1}$ and $\chi_{2}$ components of $2 t_{2}$ is much smaller. The resultant molecular orbital diagram in Figure 7 follows the typical splitting pattern of an octahedral complex with three metal centered orbitals below two, i.e. $t_{2 \mathrm{~g}}$ below $e_{\mathrm{g}}$. A $d^{6}$ complex should be stable. Our calculations on a $\mathrm{Rh}\left(\mathrm{PH}_{3}\right)_{2} \mathrm{Cl}^{2+}$ complex yield a binding energy of the $\mathrm{ML}_{3}$ fragment to $P_{4}$ that is somewhat on the small side ( $13 \mathrm{kcal} / \mathrm{mol}$ ), however, not much reliance should be put on this result since we have not optimized the geometry. What is somewhat worrisome is that the "eg" set is at relatively low energy because of the overlap of $2 e$ on $\mathrm{ML}_{3}$ to $\chi_{1}$ and $\chi_{3}$ in $t_{1}$. Consequently a rather small energy gap ( 1.1 eV ) between the HOMO and LUMO is found. Thus, while a $d^{6}$ complex of, for example, $\mathrm{Cr}(\mathrm{CO})_{3}$ may be isolated, it ought to be quite reactive.

Referring back to Figure 7, one can see that there is a large energy gap between the $2 e$ set on $\mathrm{ML}_{3}$ and $e$ of $\mathrm{P}_{4}$. If this can be made smaller, then more favorable bonding will result between $\mathrm{P}_{4}$ and the $\mathrm{ML}_{n}$ fragments. An $\mathrm{ML}_{n}$ fragment which encorporates this feature is the $C_{3 v} M L_{4}$ one. The valence


Figure 7. An orbital interaction diagram for an $\eta^{3}-\mathrm{P}_{4}-\mathrm{ML}_{3}$ complex.
orbitals of this fragment ${ }^{10}$ are shown in 19 . The $1 e$ set is basically $x z$ and $y z$ using the coordinate system in 19. It lies at much lower energy than the $2 e$ set in $\mathrm{ML}_{3}$. Consequently it will stabilize the $e$ set on $\mathrm{P}_{4}$ to a greater extent. The





$2 e$ set of $\mathrm{ML}_{4}$ because of its $\delta$ symmetry will not overlap to a significant extent with the orbitals of $P_{4}$. Therefore, the antibonding combination of $1 e$ with the $e$ set on $P_{4}$ should lie at a higher energy than the $2 e$ set of ML4. A $d^{4}$ complex is then expected to be stable. We do find this basic pattern on a model calculation of $\mathrm{P}_{4} \mathrm{Fe}(\mathrm{CO})_{4}{ }^{4+}$. However, we hasten to add that there are severe steric constraints at work here. With a Fe-P distance of $2.30 \AA$ and $\mathrm{P}-\mathrm{P}$ distances of $2.21 \AA$, the equatorial carbonyl groups are quite close to the basal phosphorus atoms ( $2.48 \AA$ ). Thus, nonbonded repulsions between the filled CO $\pi$ orbitals and filled orbitals on $P_{4}$ dominate and we find that the two fragments are not bound. One could bend the equatorial carbonyl groups away from the $P_{4}$ unit, however, this will serve to rehybridize the $1 e \operatorname{set}^{10 \mathrm{~b}}$ with a concomitant loss of overlap to the $e$ set on $\mathrm{P}_{4}$.

The best candidate that we have found for an $\eta^{3}-\mathrm{P}_{4}$ complex is not a mononuclear one, but a trinuclear transition metal species. An $\mathrm{Fe}_{3}(\mathrm{CO})_{9}$ fragment ${ }^{16}$ has three empty valence orbitals, as shown in $\mathbf{2 0}$, which are directed towards cluster. They are individually equivalent to the $a_{1}$ orbital in 19. Linear


20
combinations of these three generate orbitals of $a_{1}+e$ symmetry. They now match the symmetry properties and overlap well with the three members of the $2 t_{2}$ set of $\mathrm{P}_{4}$. A substantial HOMO-LUMO gap and healthy overlap population (0.826) between the two fragments suggest that this complex may be a viable possiblity. We look forward to experimental tests in this direction.

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## APPENDIX

All calculations were performed using the extended Hückel method ${ }^{17}$ using the modified Wolfsberg-Helmholz technique. ${ }^{18}$ The $H_{i j}$ 's for Rh were obtained from a charge iteration on $\mathrm{P}_{4} \mathrm{Rh}\left(\mathrm{PH}_{3}\right)_{2} \mathrm{Cl}$ at an $\eta^{1}$ geometry using a $T_{d}$ symmetry for $\mathrm{P}_{4}$. The orbital exponents for Rh were taken from Basch and Gray. ${ }^{19}$ The other parameters were taken from previous work. ${ }^{15}$ The values of the $H_{i j}$ 's and orbital exponents are listed in Table I. The $\mathrm{Rh}-\mathrm{Cl}, \mathrm{Rh}-\mathrm{P}\left(\mathrm{H}_{3}\right)$, and $\mathrm{P}-\mathrm{H}$ distances used were $2.34,2.30$, and $1.41 \AA$, respectively. All calculations employed $\mathrm{H}-\mathrm{P}-\mathrm{H}$ angles of $90^{\circ}$. The $\mathrm{Fe}-\mathrm{C}, \mathrm{C}-\mathrm{O}$, and $\mathrm{Fe}-\mathrm{Fe}$ distances used were $1.78,1.14$, and $2.53 \AA$, respectively. In $\mathrm{P}_{4} \mathrm{Fe}_{3}(\mathrm{CO})_{9}$ the $\mathrm{Fe}-\mathrm{P}$ distance were $2.41 \AA$.

TABLE I
Parameters Used in the Extended Hückel Calculations

| Orbital | $H_{i i} / \mathrm{eV}$ | $\zeta_{1}$ | $\zeta_{2}$ | $C_{1}{ }^{\text {a }}$ | $\mathrm{C}_{2}{ }^{\text {a }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Rh | $\begin{array}{ll} 4 d & -10.51 \\ 5 s & -6.63 \\ 5 p & -3.39 \end{array}$ | $\begin{aligned} & 4.29 \\ & 2.135 \\ & 2.10 \end{aligned}$ | 1.97 | 0.5807 | 0.5685 |
| Fe | $\begin{array}{ll} 3 d & -12.70 \\ 4 s & -9.17 \\ 4 p & -5.37 \end{array}$ | $\begin{aligned} & 5.35 \\ & 1.90 \\ & 1.90 \end{aligned}$ | 1.801 | 0.5366 | 0.6678 |
| P | $\begin{aligned} & 3 s-18.60 \\ & 3 p-14.00 \end{aligned}$ | $\begin{aligned} & 1.60 \\ & 1.60 \end{aligned}$ |  |  |  |
| Cl | $\begin{aligned} & 3 s-30.00 \\ & 3 p-15.00 \end{aligned}$ | $\begin{aligned} & 2.033 \\ & 2.033 \end{aligned}$ |  |  |  |
| C | $\begin{aligned} & 2 s-21.40 \\ & 2 p-11.40 \end{aligned}$ | $\begin{aligned} & 1.625 \\ & 1.625 \end{aligned}$ |  |  |  |
| O | $\begin{aligned} & 2 s-32.30 \\ & 2 p-14.80 \end{aligned}$ | $\begin{aligned} & 2.275 \\ & 2.275 \end{aligned}$ |  |  |  |
| H | 1s -13.60 | 1.30 |  |  |  |

a Contraction coefficients used in the double zeta expansion.

## REFERENCES

1. (a) Camille and Henry Dreyfus Teacher-Scholar, 1979-1984. Alfred P. Sloan Research Fellow, 1982-1984; (b) Present address, Department of Chemistry, Cornell University, Ithaca, NY 14853.
2. (a) For a review see M. Di Vaira and L. Sacconi, Angew. Chem. 94 (1982) 338; Angew. Chem. Int. Ed. Engl. 21 (1982) 330; (b) P. Dapporto, L. S acconi, P. Stoppioni, and F. Zanobini, Inorg. Chem. 20 (1981) 3834; (c) M. Di Vaira, L. Sacconi, and P. Stoppioni, J. Organomet. Chem. 250 (1983) 183; (d) M. Di Vaira, M. Peruzzini, and P. Stoppioni, Acta Crystallogr. Sect. C 39 (1983) 1210; (e) M. Di Vaira, G. A. Ghilardi, S. Midollini, and Sacconi, J. Amer. Chem. Soc. 100 (1978) 2550; (f) M. D. Vaira, S. Midollini, and L. Sacconi, ibid. 101 (1979) 1757; (g) C. Bianchini, M. Di Vaira, A. Meli, and L. Sacconi, ibid 103 (1981) 1448; (h) F. Cecconi, P. Dapporto, S. Midollini, and L. Sacconi, Inorg. Chem. 17 (1978) 3292; (i) C. A. Ghilardi, S. MI dollini, A. Orlandini, and L. Sacconi, ibid 19 (1930) 301; (j) C. Bianchini, M. Di Vaira, A. Meli, and L. Sacconi, ibid 20 (1981) 1169; (k) C. Bian-
chini, C. Mealli, A. Meli, and L. Sacconi, Inorg. Chim. Acta 37 (1979) L543.
3. For theoretical calculations see Ref. 2a and S. Shen-shu and R. Hoffmann, submitted for publication.
4. (a) G. Schmid and H. P. Kempry, Z. Anorg. Allg. Chem. 432 (1977) 160; (b) A. Vizi-Orotz, J. Organomet. Chem. 111 (1976) 61; (c) G. Fritz and R. Uhlmann, Z. Anorg. Allg. Chem. 465 (1980) 51; (d) W. Hönle and H. G. Von Schnering, ibid 465 (1980) 72; (e) A. W. Cordes, R. D. Joyner, R. D. Shores, and E. D. Dill, Inorg. Chem. 13 (1974) 32; (f) C. A. Ghilardi, S. Midollini, A. Orlandini, Angew. Chem. 95 (1983) 801; Angew. Chem. Int. Ed. Engl. 22 (1983) 790; (g) For theoretical computations on several $\mathrm{P}_{n}$ clusters see R. Gleiter, M. C. Böhm, M. Eckert-Maksić, W. Schäfer, M. Baudler, Y. Aktalay, G. Fritz, and K. - D. Hoppe, Chem. Ber. 116 (1983) 2972; R. Gleiter, M. C. Böhm, and M. Baudler, ibid 114 (1981) 1004; M. C. Böhm and R. Gleiter, Z. Naturforsch. Teil B 36 (1981) 498; W. W. Schoeller and T. Dabisch, J. Chem. Soc., Dalton Trans. (1983) 2411; W. W. Schoeller and C. Lerch, Inorg. Chem. 22 (1983) 2992.
5. P. Dapporto, S. Midollini, and L. Sacconi, Angew. Chem. 91 (1979) 510, Angew. Chem. Int. Ed. Engl. 18 (1979) 469.
6. (a) W. E. Lindsell, J. Chem. Soc. Chem. Commun. (1982) 1422; (b) A. P. Ginsberg, W. E. Lindsell, and W. E. Silverthorn, Trans. N. Y. Acad. Sci. 33 (1971) 303; (c) A. P. Ginsberg and W. E. Lindsell, J. Amer. Chem. Soc. 93 (1971) 2082.
7. W. E. Lindselll, K. J. Mc Cullough, and A. J. Welch, ibid 105 (1983) 4487.
8. L. E. Sutton, Chem. Soc. Spec. Pub. No. 11 (1958).
9. (a) R. R. Hart, M. B. Robin, and N. A. Kuebler, J. Chem. Phys. 42 (1965) 3631; C. R. Brundle, N. A. Kuebler, M. B. Robin, and H. B as ch, Inorg. Chem. 11 (1972) 20; (b) S. F. A. Kettle, Theor. Chim. Acta 4 (1966) 150; (c) R. M. Archibald and P. G. Perkins, Chem. Commun. (1970) 569; (d) M. F. Guest, I. H. Hillier, and V. R. Saunders, J. Chem. Soc. Faraday Trans. 268 (1972) 2070; (e) R. Osman, P. Coffey, and J. R. van W a zer, Inorg. Chem. 15 (1976) 287.
10. (a) T. A. Albright, R. Hoffmann, J. C. Thibeault, and D. L. Thorn, J. Amer. Chem. Soc. 101 (1979) 3801; (b) M. Elian and R. Hoffmann, Inorg. Chem. 14 (1974) 1058; (c) T. A. Albright, Tetrahedron 38 (1982) 1339; (d) Molecular Shapes, J. K. Burdett, New York, Wiley-Interscience, 1980, pp. 219-231.
11. For example, the overlap between $2 a_{1}$ on $\mathrm{ML}_{3}$ with $\chi_{3}$ in the $2 t_{2}$ set was 0.0196 while the between $2 \alpha_{1}$ and $\chi_{2}$ of the set was 0.3004 .
12. At this geometry the $\mathrm{P}_{4}$ unit is computed to be bound to the $\mathrm{Rh}\left(\mathrm{PH}_{3}\right)_{2} \mathrm{Cl}$ unit, however, it is still $25 \mathrm{kcal} / \mathrm{mol}$ higher in energy than the optimized $\eta^{1}$ structure.
13. A. R. Rossi and R. Hoffmann, Inorg. Chem. 14 (1975) 365.
14. P. Kubacek and R. Hoffmann, J. Amer. Chem. Soc. 103 (1981) 4320.
15. T. A. Albright, P. Hofmann, and R. Hoffmann, ibid 99 (1977) 7546.
16. B. E. R. Schilling and R. Hoffmann, ibid 101 (1979) 3456.
17. R. Hoffmann, J. Chem. Phys. 39 (1963) 1397; R. Hoffmann and W. N. Lipscomb, ibid 36 (1962) $2179 ; 37$ (1962) 2872.
18. J. H. Ammeter, H.-B. Bürgi, J. C. Thibeault, and R. Hoffmann, J. Amer. Chem. Soc. 100 (1978) 3686.
19. H. Basch and H. B. Gray, Theor. Chim. Acta 4 (1966) 367.

## SAZ̆ETAK

## Vezanje $\mathbf{P}_{4}$ na $d^{8}-\mathbf{M L}_{3}$ kompleksima

## Sung-Kwon Kang, Thomas A. Albright i Jerome Silvestre

Razmatrani su $\eta^{1}$, $\eta^{2}$ i $\eta^{3}$ kompleksi $\mathrm{P}_{4}$-grozda $\mathrm{s} \mathrm{Rh}\left(\mathrm{PH}_{3}\right)_{2} \mathrm{Cl}$ primjenom proširene Hückelove (EH) metode. Najstabilnije su strukture $\eta^{1}$-kvadratni i $\eta^{2}$-kompleks $C_{2 v}$-simetrije; $d^{10}-\eta^{1}$ tetraedarski kompleks također je prilično stabilan. Najboīji kandidat za tip vezanja $\eta^{3}$ jest trimer $\mathrm{Fe}_{3}(\mathrm{CO})_{9}$.

# Interaction of the Hydrogen Molecule with the Palladium Atom. A Force Theoretic Study 

Hiroshi Nakatsuji and Masahiko Hada<br>Division of Molecular Engineering, Graduate School of Engineering, Kyoto University, Kyoto 606, Japan

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As a model of chemisorption, we studied an interaction of the $\mathrm{H}_{2}$ molecule with the ${ }^{1} S\left(4 d^{10}\right)$ and ${ }^{3} D\left(4 d^{9} 5 s^{1}\right)$ states of the Pd atom. We calculated an accurate Hellmann-Feynman force acting on the $\mathrm{H}_{2}$ molecule by adding the first derivatives of the hydrogen basis set. The force and density origins of the interaction were clarified. The singlet $\operatorname{Pd}\left({ }^{1} S\right)-\mathrm{H}_{2}$ system, which is the ground state, is attractive but the triplet $\operatorname{Pd}\left({ }^{3} D\right)-\mathrm{H}_{2}$ system is repulsive. The side-on approach is a preferable path. For the $\operatorname{Pd}\left({ }^{1} S\right)-\mathrm{H}_{2}$ system, the $\mathrm{Pd}-\mathrm{H}$ bonds are gradually formed. The electron density is accumulated in the overlap region of the $\mathrm{H}_{2}$ molecule and the Pd atom and it pulls the $\mathrm{H}_{2}$ molecule onto the Pd atom. The bond of the $\mathrm{H}_{2}$ molecule is gradually weakened. The electrons are transferred from the bonding MO of the $\mathrm{H}_{2}$ molecule to the empty $5 s$ and $5 p_{z}$ AO's of the Pd atom. For the $\operatorname{Pd}\left({ }^{3} \mathrm{D}\right)-\mathrm{H}_{2}$ system, the density of the Pd atom tends to keep its spherical symmetry even when the $\mathrm{H}_{2}$ molecule approaches. The Pd atom in the $4 d^{9} 5 s^{1}$ configuration is more repulsive than that in the $4 d^{10}$ configuration. Therefore, the $\mathrm{H}_{2}$ molecule is repelled by the Pd atom.

## I. INTRODUCTION

Chemisorption of hydrogen molecules on transition metals is an important step for hydrogen storage and activation of molecular hydrogen followed by a variety of catalytic processes. ${ }^{1-4}$ Palladium shows, especially, a unique affinity for hydrogen. The »solubility« of hydrogen in palladium metal markedly exceeds that in the other group 8 metals. ${ }^{1}$ Molecular hydrogen can diffuse through metallic palladium at a higher rate than in other platinum-series metals. This process is very selective so that it is used for the purification of hydrogen gas from a mixture. Further, homogeneous palladium complexes also show a variety of catalytic reactions including some industrially important ones (e.g., Hoechst-Wacker reaction)..$^{2-4}$

The electronic structure and the bonding nature in the diatomics, PdH and PtH have been studied theoretically at various levels of approximation. ${ }^{3-8}$ For these diatomic hydrides, bonding properties such as bond energy, force constant, etc., are known experimentally for some lower electronic states ${ }^{9}$, so that they offer a good test for various levels of theoretical method. Further, these metal hydrides are thought to be a model product of the dissociative chemisorption of the hydrogen molecule on the metal surface.

Bagus and Björkman ${ }^{6}$ studied the bonding in NiH and PdH by an $a b$ initio SCF and CI methods. The bonding between Pd and H is primarily due to the $5 s$ electron in the metal. The $d$-electrons are localized on the metal and participate only slightly in the $\mathrm{Pd}-\mathrm{H}$ bond. Pacchioni et al. ${ }^{7}$ reported a multi--reference CI study on the PdH, PdC, and PdCO molecules based on the effective-core potential SCF MO's. They reported the importance of the choice of the Pd basis set and also of the effects of electron correlation. Their results showed that the effect of electron correlation on PdH is to deepen the potential energy curve. The equilibrium bond length and force constant calculated by the SCF method compare relatively well with experimental results, although for the dissociation energy the SCF result is only half of the experimental value. Wang and Pitzer ${ }^{8}$ also reported a similar effect of electron correlation on PtH . The correlation energy has little effect on the bond length and force constant of the ground state of PtH , but it does contribute significantly to the binding energy. Basch and Topiol ${ }^{5}$ and Wang and Pitzer ${ }^{8}$ further pointed out the importance of the relativistic effect on the electronic structure of PtH .

Bagatur'yants et al..$^{10}$ studied the approach of the hydrogen molecule to the Pd atom with a fixed $\mathrm{H}-\mathrm{H}$ length $(0.74 \AA)$. They carried out all electron SCF calculations with the minimal and extended basis sets. They showed that formation of the molecular complex is favorable energetically and analyzed the donor-acceptor interactions between $\mathrm{H}_{2}$ and Pd. They pointed out the important role of the outer $5 s$ and $5 p$ AO's of the Pd atom in the formation of the $\mathrm{Pd}-\mathrm{H}_{2}$ complex.

In this paper we study the interaction of the $\mathrm{H}_{2}$ molecule with the Pd atom, as a model of the electronic processes in the chemisorption of the hydrogen molecule. We use the effective-core potential (ECP) SCF method ${ }^{11}$ for the Pd atom. ${ }^{12}$ We include the relativistic effects through the ECP potential but neglect, in this paper, the effect of electron correlation. For the ground state of PdH and PtH , the effect of electron correlation was to deepen the potential minima without much affecting the equilibrium length and the force constant. ${ }^{7,8}$

The force concept based on the Hellmann-Feynman theorem gives a simple and intuitive method of studying chemical phenomena. ${ }^{13,14}$ The force acting on the nucleus $\mathrm{A}, \mathrm{F}_{\mathrm{A}}$ is determined by the electrostatic interaction of the nucleus A with the electron cloud and the other nuclei surrounding it.

$$
\begin{equation*}
F_{\mathrm{A}}=Z_{\mathrm{A}} \int r_{\mathrm{A} 1} / r_{\mathrm{A} 1}^{3} \varrho\left(r_{1}\right) \mathrm{d} r_{1}-Z_{\mathrm{A}} \sum_{\mathrm{B}(\neq \mathrm{A})}^{\Sigma} Z_{\mathrm{B}} R_{\mathrm{AB}} / R_{\mathrm{AB}}^{3} \tag{1}
\end{equation*}
$$

We used the force concept to obtain a deeper understanding of the nature of the interaction.

Further, an accurate and reliable calculation of the Hellmann-Feynman force has been realized recently. ${ }^{15,16}$ It was shown that the Hellmann-Feynman theorem is essentially satisfied when we add the first derivative bases to the basis set conventionally used. The method has been applied to studies of molecular structure, molecular vibration and chemical reaction. ${ }^{15-17}$ In the present model system of chemisorption, $\mathrm{Pd}-\mathrm{H}_{2}$, we are primarily interested in the force acting on the adsorbed molecule $\mathrm{H}_{2}$, and not in the force acting on the metal, since the metal atom represents here a surface atom of the metal catalyst. Therefore, we have added the first derivatives only to the $\mathrm{H}_{2}$ basis set which is the [2s] set of Dunning. ${ }^{18}$ The added derivative bases describe
well the polarization of the $\mathrm{H}_{2}$ electron cloud and lead to the satisfaction of the Hellmann-Feynman theorem for the force acting on the protons of the $\mathrm{H}_{2}$ molecule. Wang et al. ${ }^{8}$ found for PtH that the polarization functions centered on the H atom are more important than those centered on the Pt atom.
II. INTERACTION OF THE H $H_{2}$ MOLECULE WITH THE Pd ATOM IN THE $1 S$ AND 3 D STATES

The ground state of the Pd atom is the closed-shell ${ }^{1}$ S state with the configuration $4 d^{10}$. The first excited state is the ${ }^{3} D$ state with the configuration $4 d^{9} 5 s^{1}$. It lies $19 \mathrm{kcal} / \mathrm{mol}$ above the ground state. We have approached the $\mathrm{H}_{2}$ molecule onto the Pd atom keeping the $\mathrm{C}_{2 \mathrm{v}}$ symmetry (side-on), because, as will be shown below, this is a favorable approach. The $\mathrm{H}-\mathrm{H}$ lenoth was kept fixed to $0.74144 \AA$ and $1.0 \AA$. The former is the equilibrium distance of the free hydrogen molecule.

In Figure 1, we have shown the potential energy curves for the side-on approach of the $\mathrm{H}_{2}$ molecule to the Pd atom. The figure on the left hand side corresponds to the fixed $\mathrm{H}-\mathrm{H}$ distance of $0.74144 \AA$ and the one on the right hand side corresponds to the $\mathrm{H}-\mathrm{H}$ distance of $1.0 \AA$. The lower curve was obtained from the interaction of the singlet $4 d^{10}$ state of the Pd atom with the $\mathrm{H}_{2}$ molecule, and the upper curve was obtained from the interaction of the triplet $4 d^{9} 5 s^{1}$ state of the Pd atom with the $\mathrm{H}_{2}$ molecule. Figure 1 shows that the singlet $\mathrm{Pd}\left({ }^{1} S\right)-\mathrm{H}_{2}$ system is attractive but the triplet $\mathrm{Pd}\left({ }^{3} D\right)-\mathrm{H}_{2}$ system is repulsive. The energy difference of the ${ }^{1} S$ and ${ }^{3} D$ states of the Pd atom was taken from the experimental value ( $19 \mathrm{kcal} / \mathrm{mol}$ ). The calculated value for the atom was $-3 \mathrm{kcal} / \mathrm{mol}$, (i.e., the ${ }^{3} \mathrm{D}$ state was calculated to be lower than the ${ }^{1} S$ state) because of the lack of electron correlation. The correlation energy is larger for the closed-shell singlet state than for the open-shell triplet state,


Figure 1. Potential energy curves for the interactions of the $\mathrm{H}_{2}$ molecule with the ${ }^{1} S\left(4 d^{10}\right)$ and ${ }^{3} D\left(4 d^{9} 5 s^{1}\right)$ states of the Pd atom. The figures on the left and right hand sides are for the fixed $\mathrm{H}_{2}$ distances of $0.74144 \AA$ (equilibrium length) and $1.0 \AA$, respectively.

The existence of the repulsive curve, not far from the attractive curve, is very interesting. It may work in the detachment process of the hydrogen molecule from the Pd metal.

When the $\mathrm{H}-\mathrm{H}$ length is kept fixed at $0.74144 \AA$, the stabilization energy for the $\operatorname{Pd}\left({ }^{1} S\right)-\mathrm{H}_{2}$ system was calculated to be $3.6 \mathrm{kcal} / \mathrm{mol}$. It is small in comparison with the experimental value, $9.6 \mathrm{kcal} / \mathrm{mol}$, which is the heat of adsorption of the hydrogen molecule on the bulk palladium metal. ${ }^{1}$ The stable $\mathrm{Pd}-\mathrm{H}_{2}$ distance was calculated at $1.90 \AA$. When the $\mathrm{H}-\mathrm{H}$ length is elongated to $1.0 \AA$, the stabilization energy relative to the Pd atom and the elongated $\mathrm{H}_{2}$ molecule becomes $15.9 \mathrm{kcal} / \mathrm{mol}$ and the stable $\mathrm{Pd}-\mathrm{H}_{2}$ distance becomes $1.62 \AA$, though $20.2 \mathrm{kcal} / \mathrm{mol}$ is necessary for this elongation of the free hydrogen molecule.

During the course of the approach, the $\mathrm{H}_{2}$ molecule fixed at the equilibrium length, $0.74144 \AA$, receives the force, as shown below, which works to elongate the $\mathrm{H}-\mathrm{H}$ distance. Therefore, in an actual process, the $\mathrm{H}_{2}$ molecule approaches the Pd atom while gradually elongating its $\mathrm{H}-\mathrm{H}$ distance. For this optimal approach the stabilization energy would become larger than the present value. However, in order to obtain a value comparable to the experimental value, inclusion of the electron correlation effect would be necessary. The roles of the second, third,... Pd atoms of the metal surface are also of interest.

The triplet interaction of the $\mathrm{Pd}\left({ }^{3} \mathrm{D}\right)$ atom and the $\mathrm{H}_{2}$ molecule is repulsive, independent of the $\mathrm{H}-\mathrm{H}$ distance, though the slope becomes smaller for the interaction with the elongated $\mathrm{H}_{2}$. We will show later in the force theoretic analysis that the Pd atom in the $4 d^{9} 5 s^{1}$ configuration is more repulsive than that in the $4 d^{10}$ configuration.

## III. FORCE AND DENSITY ORIGIN OF THE INTERACTION

As seen from Eq. (1), the Hellmann-Feynman force depends on the three dimensional distribution of the electron density and nuclei. In the present $\mathrm{Pd}-\mathrm{H}_{2}$ system, we have two distinct subsystems which are the Pd atom and the $\mathrm{H}_{2}$ molecule. We therefore divide the electron density regionally into the one belonging to the Pd atom, the one belonging to the $\mathrm{H}_{2}$ molecule, and the one belonging to the overlap region of the Pd atom and the $\mathrm{H}_{2}$ molecule. This regional partitioning of the electron density naturally leads to the analysis of the force acting on the proton of the hydrogen molecule as

$$
\begin{align*}
& F=\sum_{r}^{\text {on } \mathrm{Pd}} \sum_{s} P_{r s}\langle r| f_{\mathrm{H}}|s\rangle-Z_{\mathrm{H}} Z_{\mathrm{Pd}} R_{\mathrm{H}-\mathrm{Pd}} / R_{\mathrm{H}-\mathrm{H}}^{3} \quad ; F(\mathrm{Pd}) \\
& \text { on } \mathrm{H}_{2} \\
& +\sum_{r s} \sum_{s} P_{r s}\langle r| f_{\mathrm{H}}|s\rangle-Z_{\mathrm{H}} Z_{\mathrm{H}^{\prime}} R_{\mathrm{H}-\mathrm{H}^{\prime}} / R_{\mathrm{H}-\mathrm{H}^{\prime}}^{3} \quad ; F\left(\mathrm{H}_{2}\right) \\
& +2 \sum_{r}^{\text {ouPd }} \sum_{s}^{\mathrm{onH}_{2}} P_{r s}\langle r| f_{\mathrm{H}}|s\rangle \quad ; F\left(\mathrm{H}_{2}-\mathrm{Pd}\right) \tag{2}
\end{align*}
$$

where $P_{r s}$ is the bond-order density matrix with respect to the bases $r$ and $s$. The first term, $F(\mathrm{Pd})$, represents the force acting on the proton due to the electrons and nucleus of the Pd atom, the second term, $F\left(\mathrm{H}_{2}\right)$ represents the force due to the electron density and the other proton of the $\mathrm{H}_{2}$ molecule
adsorbed on the Pd atom, and the last term, $F\left(\mathrm{H}_{2}-\mathrm{Pd}\right)$, represents the force due to the electron density accumulated in the overlap region of the AO's of the $\mathrm{H}_{2}$ molecule and the Pd atom.

## III A. Preference of the Side-on Attack

We first examine a preferable way of attack of the $\mathrm{H}_{2}$ molecule on the Pd atom. We have put the $\mathrm{H}_{2}$ molecule $2.0 \AA$ apart from the $\mathrm{Pd}\left({ }^{1} S\right)$ atom and declined it by $10^{\circ}$ from the $C_{2 v}$ side-on position. Table I shows the force

TABLE I
Force Acting on the $H_{a}$ and $H_{b}$ Atoms of the Hydrogen Molecule which is Side-on Slantwise on the Pd Atom. The H-H Distance is $0.74144 \AA$ and the Distance Between $P d$ and the Center of $H_{2}$ is $2.0 \AA$

|  | $\mathrm{H}_{\mathrm{a}}$ |  |  | $\mathrm{H}_{\mathrm{b}}$ |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
|  | $\mathrm{F}_{\perp}$ |  | $\mathrm{F}_{\text {// }}$ |  | $\mathrm{F}_{\perp}$ |
| $F(\mathrm{Pd})$ | 0.0210 | 0.0104 |  | $\mathrm{~F}_{/ /}$ |  |
| $F\left(\mathrm{H}_{2}\right)$ | -0.0046 | 0.0225 |  | 0.0301 | 0.0068 |
| $F\left(\mathrm{H}_{2}\right.$-Pd) | -0.0188 | -0.0230 |  | 0.0220 | -0.0130 |
| $F($ total $)$ | -0.0024 | 0.0099 | 0.0013 | 0.0111 |  |


analysis and Figure 2 shows the contour map of the density difference defined by

$$
\begin{equation*}
\Delta \varrho=\varrho\left(\mathrm{Pd}-\mathrm{H}_{2}\right)-\varrho\left(\mathrm{Pd},{ }^{1} S\right)-\varrho\left(\mathrm{H}_{2}\right) \tag{3}
\end{equation*}
$$

which shows the reorganization of the electron density due to the interaction between the $\mathrm{Pd}\left({ }^{1} \mathrm{~S}\right)$ atom and the $\mathrm{H}_{2}$ molecule.

From the force perpendicular to the $\mathrm{H}-\mathrm{H}$ axis, $\mathrm{F}_{\perp}$, we see that the hydrogen molecule receive the force which acts to recover the $C_{2 v}$ approach. This recovery seems to occur with slipping down the $\mathrm{H}_{2}$-rod onto the right-hand side, since the force paralell to the bond, $F_{\|}$, is larger for $H_{b}$ than for $H_{a}$. The $\mathrm{H}_{2}$ molecule tends to elongate the bond as seen from the values of $F_{\|}$.

The origin of the recovering force on the $\mathrm{H}_{\mathrm{a}}$ atom, $F_{\mathrm{H}_{\mathrm{a}} \perp}$ is the sum of the forces $F\left(\mathrm{H}_{2}-\mathrm{Pd}\right)$ and $F\left(\mathrm{H}_{2}\right)$, which overcomes the repulsion due to the Pd atom, $F(\mathrm{Pd})$. As seen from Figure 2, the force $F\left(\mathrm{H}_{2}-\mathrm{Pd}\right)$ reflects an increase in the electron density between the $\mathrm{H}_{\mathrm{a}}$ and Pd atoms and $F\left(\mathrm{H}_{2}\right)$ reflects a polarization of the electron density near the $\mathrm{H}_{a}$ atom. The origin of the recovering force acting on $\mathrm{H}_{\mathrm{b}}, F_{H_{\mathrm{b}} \perp}$, is the repulsion due to the Pd atom. Though


Figure 2. Density difference contour map for the singlet $\mathrm{Pd}\left({ }^{1} \mathrm{~S}\right)-\mathrm{H}_{2}$ system in which the $\mathrm{H}_{2}$ molecule is side-on slantwise on the Pd atom. The $\mathrm{H}-\mathrm{H}$ distance is $0.74144 \AA$ and the distance between Pd and the center of $\mathrm{H}_{2}$ is $2.0 \AA$. The definition of the density difference is given by Eq. (3). The real lines correspond to an increase in density and broken lines to a decrease, with the contour values of $0, \pm 1, \pm 2$, $\pm 3, \pm 4$, and $\pm 5$ corresponding to $0.0, \pm 0.002, \pm 0.005, \pm 0.01, \pm 0.02$, and $\pm 0.05$ a. u., respectively.
a polarization of the electron density near the $\mathrm{H}_{\mathrm{b}}$ atom and a bonding interaction between the $\mathrm{H}_{\mathrm{b}}$ and Pd atoms are seen from the density difference map, the extent is less than the repulsion due to the Pd atom.

## III B. Force and Density Origin of the Interaction for the Pd ( ${ }^{1}$ S $)-\mathrm{H}_{2}$ System

We now consider the side-on approach of the $\mathrm{H}_{2}$ molecule onto the $\mathrm{Pd}\left({ }^{1} S\right)$ atom. The force acting on the $\mathrm{H}_{2}$ molecule is divided into the two components, $F_{z}$ and $F_{y}$, shown in Figure 3. The negative of the force $F_{z}$ is an attractive force of chemisorption, and the force $F_{y}$ is the force which prolongs the $\mathrm{H}_{2}$ molecule. Figure 4 shows the contour map of the density difference defined by Eq. (3) for several points of the side-on approach. The $\mathrm{H}-\mathrm{H}$ distance was kept fixed at $0.74144 \AA$. In Figure 5, we have shown an analysis of the force of chemisorption, $F_{z}$, into the components defined by Eq. (2). The left and right hand sides correspond to the fixed H-H distances of $0.74144 \AA$ and $1.0 \AA$, respectively. Figure 6 shows a similar analysis of the stretching force, $F_{y}$, along the adsorption process.

From the total force curve shown in Figure 5, we can estimate an equilibrium $\mathrm{Pd}-\mathrm{H}_{2}$ distance. It is $1.92 \AA$ and $1.64 \AA$ for the systems with the fixed $\mathrm{H}-\mathrm{H}$ distances of $0.74144 \AA$ and $1.0 \AA$, respectively. These results are in close agreement with those obtained from the potential energy curves shown in Figure 1 ( $1.90 \AA$ and $1.62 \AA$, respectively). This is a matter of course since the present wavefunctions essentially satisfy the Hellmann-Feynman theorem.

From Figure 4, we can see a density origin of the 'chemisorption' of the $\mathrm{H}_{2}$ molecule on the $\mathrm{Pd}\left({ }^{1} S\right)$ atom. As the $\mathrm{H}_{2}$ molecule approaches, the Pd atom


Figure 3. Definition of the force acting on the $\mathrm{H}_{2}$ molecule which approaches side-on to the Pd atom.
extends its right and left lobes of the density from the outside of the $\mathrm{H}_{2}$ molecule. These lobes grow up to form a bond between Pd and each hydrogen. It becomes conspicuous at $1.7 \AA$. From Figure 5, we confirm that the origin of the attractive force is the $F\left(\mathrm{H}_{2}-\mathrm{Pd}\right)$ term. The electron density accumulated in the overlap region between $\mathrm{H}_{2}$ and Pd attracts $\mathrm{H}_{2}$ to chemisorb on the Pd atom. The $\mathrm{Pd}-\mathrm{H}$ bonds are formed as the $\mathrm{H}_{2}$ molecule approaches.

Further, as seen from Figure 4, the electron density decreases in the internuclear region of the $\mathrm{H}_{2}$ molecule, so that the bond between the hydrogen atoms is weakened as the $\mathrm{H}_{2}$ molecule approaches the Pd atom. As seen from Figure 6, the proton of the $\mathrm{H}_{2}$ molecule always receive the force which prolongs the $\mathrm{H}_{2}$ distance. The origin of this force is $F\left(\mathrm{H}_{2}\right)$. Namely, the decrease in the electron density in the $\mathrm{H}_{2}$ molecule causes a deshielding of the inter-proton repulsion. The repulsion from the Pd atom, $F(\mathrm{Pd})$, also works to elongate the $\mathrm{H}-\mathrm{H}$ distance.

We will see later that the increase in the electron density in the overlap region of Pd and $\mathrm{H}_{2}$ and the decrease in the density in the $\mathrm{H}_{2}$ region are mainly due to an electron-transfer interaction from the bonding MO of the $\mathrm{H}_{2}$ molecule to the vacant $5 s$ and $5 p_{z}$ AO's of the Pd atom.

When the $\mathrm{H}-\mathrm{H}$ length is elongated, in response to the force shown in Figure 6, the electron density reorganizes itself as shown in Figure 7. It is for the $\mathrm{H}-\mathrm{H}$ distance of $1.0 \AA$. The distance between Pd and $\mathrm{H}_{2}$ is $2.0 \AA$. Comparing Figure 7 with the corresponding contour map shown in Figure 4, we see that an elongation of the $\mathrm{H}_{2}$ distance very much facilitates the formation of the $\mathrm{Pd}-\mathrm{H}$ bond. The left and right lobes of the Pd atom extend up to the two protons to form the $\mathrm{Pd}-\mathrm{H}$ bond. In Figure 5, the effect of elongating the $\mathrm{H}-\mathrm{H}$ distance on the force of interaction $F_{z}$ is shown. The left and right figures correspond to the $\mathrm{H}-\mathrm{H}$ distances of $0.74144 \AA$ and $1.0 \AA$, respectively. By an increase in the $\mathrm{H}-\mathrm{H}$ distance, the $\mathrm{H}_{2}$ system receives more attractive force from the Pd atom. The origin is an increase in the $F\left(\mathrm{H}_{2}-\mathrm{Pd}\right)$ force. Namely, the electron density more accumulated in the bond region of Pd and H as

$1.4 \AA$
Figure 4. Density difference contour maps for the singlet $\mathrm{Pd}\left(d^{10},{ }^{1} S\right)-\mathrm{H}_{2}$ system. The $\mathrm{H}-\mathrm{H}$ distance is kept at $0.74144 \AA$ and the distance between Pd and $\mathrm{H}_{2}$ is given below each map. The definition of the density difference is given by Eq. (3). The real lines correspond to an increase in density and broken lines to a decrease, with the contour values of $0, \pm 1, \pm 2, \pm 3, \pm 4$, and $\pm 5$ corresponding to $0.0, \pm 0.002$,
$\pm 0.005, \pm 0.01, \pm 0.02$, and $\pm 0.05$ a. u., respectively.
shown in Figure 7 attracts more the protons to the Pd atom. The other components of the force, $F\left(\mathrm{H}_{2}\right)$ and $F(\mathrm{Pd})$, do not change much between the two systems.

The $\mathrm{H}_{2}$ molecule receives the force $F_{y}$ which prolongs the $\mathrm{H}-\mathrm{H}$ distance when it is kept at $0.74144 \AA$. However, when it is elongated to $1.0 \AA$, the


Figure 5. The analysis of the force $\mathrm{F}_{\mathrm{z}}$ acting on the $\mathrm{H}_{2}$ molecule of the singlet. $\operatorname{Pd}\left({ }^{1} S\right)-\mathrm{H}_{2}$ system. The definition of the forces is given by Eq. (2). The left and right hand sides correspond to the fixed $\mathrm{H}-\mathrm{H}$ distances of $0.74144 \AA$ and $1.0 \AA$, respectively.
protons receive the reverse force. At the $\mathrm{Pd}-\mathrm{H}_{2}$ separation of the $2.0 \AA$, the force $F_{\mathrm{y}}$ is 0.0037 a . u. for $\mathrm{H}_{2}$ with $R_{\mathrm{HH}}=0.74144 \AA$ and -0.0828 a . u. for $\mathrm{H}_{2}$ with $R_{\mathrm{HH}}=1.0 \AA$. The equilibrium length of $\mathrm{H}_{2}$ at this $\mathrm{Pd}-\mathrm{H}_{2}$ separation is estimated to be $0.772 \AA$.

The interaction between the $\mathrm{H}_{2}$ molecule and the Pd atom may further be understood in terms of the electron transfer and back-transfer interactions as in the Dewar-Chatt-Duncanson model ${ }^{20}$ of the interaction between platinum and the olefine double bond. In Figure 8, we have shown the population analysis of the $\mathrm{Pd}\left({ }^{1} S\right)-\mathrm{H}_{2}$ system along the $\mathrm{Pd}-\mathrm{H}_{2}$ distance. The upper side shows the changes in the gross atomic charges of the Pd and H atoms and the lower side shows the atomic orbital populations.

As the hydrogen molecule approaches the Pd atom, the electron is transferred from the $\mathrm{H}_{2}$ molecule tc the Pd atom. Qualitatively speaking, the orbital interactions between the $\mathrm{H}_{2}$ molecule and the Pd atom are that the bonding MO of $\mathrm{H}_{2}, \sigma_{\mathrm{H}_{2}}$ donates an electron to the metal AO's and that the antibonding MO of $\mathrm{H}_{2}, \sigma_{\mathrm{H}_{2}}^{*}$ receives the back-donated electron from the Pd atom. The $4 d_{z^{2}}{ }^{2}$, $5 s$, and $5 p_{z}$ AO's of the Pd atom can interact with the bonding $\sigma_{\mathrm{H}_{2}} \mathrm{MO}$, and only the $4 d_{y z} \mathrm{AO}$ interacts with the antibonding $\sigma_{\mathrm{H}_{2}}^{*} \mathrm{MIO}$. In Figure 9, we have given an MO interaction diagram. As the $\mathrm{H}_{2}$ molecule approaches, the $5 \mathrm{~s}, 5 p_{z}$, and $4 d_{z^{2}}$ AO's hybridize very rapidly and form $4 d_{z^{2}}{ }^{2}-\left(5 s+5 p_{z}\right)$ hybrid and ( $5 s+$ $\left.+5 p_{z}\right)+4 d_{z^{2}}$ hybrid with appropriate mixing coefficients (prehybridization step on the left hand side of Figure 9). The two electrons originally in the $d_{z^{2}}{ }^{2}$ AO occupy the former hybrid and the latter one is left unoccupied. The electron of the $\sigma_{\mathrm{H}_{2}} \mathrm{MO}$ is transferred to the unoccupied hybrid and forms a bond


Figure 6. The analysis of the force $F_{y}$ acting on the $\mathrm{H}_{2}$ molecule of the singlet Pd. $\left({ }^{1} S\right)-\mathrm{H}_{2}$ system with the fixed $\mathrm{H}-\mathrm{H}$ length of $1.0 \AA$. The definition of the forces is given by Eq. (2).


Figure 7. Density difference contour map for the singlet $\operatorname{Pd}\left(d^{10},{ }^{1} S\right)-\mathrm{H}_{2}$ system with the $\mathrm{H}-\mathrm{H}$ length of $1.0 \AA$ and the $\mathrm{Pd}-\mathrm{H}_{2}$ distance of $2.0 \AA$. The density difference is defined by Eq. (3). The real lines correspond to an increase in density and broken lines to a decrease, with the contour values of $0, \pm 1, \pm 2, \pm 3, \pm 4$, and $\pm 5$ corresponding to $0.0, \pm 0.002, \pm 0.005, \pm 0.01, \pm 0.02$, and $\pm 0.05$ a. u., respectively.


Figure 8. Gross atomic charge (upper side) and atomic orbital population (lower side) of the singlet $\mathrm{Pd}\left({ }^{1} \mathrm{~S}, 4 \mathrm{~d}^{10}\right)-\mathrm{H}_{2}$ system against the $\mathrm{Pd}-\mathrm{H}_{2}$ distance. The $\mathrm{H}-\mathrm{H}$ distance was kept at $0.74144 \AA$ throughout the approach.
between Pd and $\mathrm{H}_{2}$. The increase in the $5 s$ and $5 p_{z} \mathrm{AO}$ population and the decrease in the $4 d_{z^{2}}{ }^{2}$ AO population shown in Figure 8 are due to this MO interaction. The back-transfer interaction from the $\operatorname{Pd} 4 d_{v z} \mathrm{AO}$ to the $\sigma \mathrm{H}_{2} \quad \mathrm{MO}$ seems to be smaller than the transfer interaction. ${ }^{10}$ The slope of the $4 d_{y z} A O$ population is smaller than that of the $5 s$ and $5 p_{z}$ AO populations and the gross charge of the hydrogen atom is positive.

From the above discussion, we conclude that the bonding between the Pd atom and the $\mathrm{H}_{2}$ molecule is primarily due to the overlap between the $5 s-5 p$ hybrid of the Pd atom and the $1 s$ AO's of the $\mathrm{H}_{2}$ molecule. The participations of the $4 d$ AO's are secondary.

We note that the decrease in the electron density in the $4 d_{z}{ }^{2} \mathrm{AO}$ is also understood as being due to the induced inner excitation of the electrons from the $4 d_{z^{2}}$ AO to the $5 s$ and $5 p_{z}$ AO's caused by an interaction between Pd and $\mathrm{H}_{2}$. Such an interaction is usually called the 'polarization' term. ${ }^{21}$


Figure 9. An MO interaction diagram for the singlet $\operatorname{Pd}\left({ }^{1} S, 4 d^{10}\right)-\mathrm{H}_{2}$ system.
III C. Force and Density Origin of the Repulsive Interaction Between Pd $\left({ }^{3} D\right)$ and $\mathrm{H}_{2}$

Though the singlet ground state of the palladium is attractive for $\mathrm{H}_{2}$, the lowest triplet state is repulsive. Figure 10 shows the density difference map for the interaction between the $\mathrm{H}_{2}$ molecule and the Pd atom in the ${ }^{3} D\left(d^{9} s^{1}\right)$ state. It is defined by

$$
\begin{equation*}
\Delta \varrho=\varrho\left(\mathrm{Pd}-\mathrm{H}_{2}\right)-\varrho\left(\mathrm{Pd},{ }^{3} \mathrm{D}\right)-\varrho\left(\mathrm{H}_{2}\right) \tag{4}
\end{equation*}
$$

where $\varrho\left(\mathrm{Pd},{ }^{3} \mathrm{D}\right)$ is calculated for the averaged electronic configuration, $d_{x y}^{2} d_{z x}^{2} d_{y z}^{5 / 3} d_{z^{2}}^{5 / 3} d_{x^{2}-y^{2}}^{5 / 3} 4 s^{1}$, which has circular symmetry on the $y z$ plane. Figure 11 shows the force $F_{z}$ and its analysis.

Even when the $\mathrm{H}_{2}$ molecule approaches the $\operatorname{Pd}\left({ }^{3} \mathrm{D}\right)$ atom up to $2.3-2.0 \AA$, the electron density does not accumulate well in the overlap region. At $2.0 \AA$,


Figure 10. Density difference contour maps for the triplet $\mathrm{Pd}\left(4 d^{9} 5 s^{1},{ }^{3} D\right)-\mathrm{H}_{2}$ system. The $\mathrm{H}-\mathrm{H}$ distance is kept at $0.74144 \AA$ and the distance between Pd and $\mathrm{H}_{2}$ is given below each map. The density difference is defined by Eq. (4). The real lines correspond to an increase in density and broken lines to a decrease, with the contour values of $0, \pm 1, \pm 2, \pm 3, \pm 4$, and $\pm 5$ corresponding to $0.0, \pm 0.002, \pm 0.005, \pm 0.01$, $\pm 0.02$, and $\pm 0.05 \mathrm{a} . \mathrm{u}$. , respectively.
we find a line of $0.0 \mathrm{a} . \mathrm{u}$. intersecting the Pd atom and the $\mathrm{H}_{2}$ molecule. In comparison with the case of the attractive system, $\operatorname{Pd}\left({ }^{1} S\right)-\mathrm{H}_{2}$ shown in Figure 4, the electron density of the Pd atom shown in Figure 10 is less af-


Figure 11. The analysis of the force $F_{z}$ acting on the $H_{2}$ molecule of the triplet $\mathrm{Pd}\left({ }^{3} \mathrm{D}\right)-\mathrm{H}_{2}$ system. The definition of the forces is given by Eq. (2).
fected by an approach of the $\mathrm{H}_{2}$ molecule and tends to keep its spherical symmetry. Comparing the force analysis shown in Figure 11 with that of Figure 5, we see that the repulsion due to the Pd atom, $F(\mathrm{Pd})$, increases more rapidly in the present case than in the previous case. The attractive part of the force, $F\left(\mathrm{H}_{2}-\mathrm{Pd}\right)$ and $F\left(\mathrm{H}_{2}\right)$, can not overcome this repulsive force. The reason that the Pd atom in the $4 d^{9} 5 s^{1}$ configuration is more repulsive than that in the $4 d^{10}$ configuration is as follows. We first note that the $5 s$ orbital of Pd is more diffuse than the $4 d$ orbital. The atomic radii $\langle r\rangle$ of the Hartree-Fock AO's of the ${ }^{3} D\left(d^{9} s^{1}\right)$ state of Pd are ${ }^{6}$

$$
\langle r\rangle_{4 d}=0.765 \AA, \quad\langle r\rangle_{5 s}=1.945 \AA
$$

Since the $5 s$ orbital is more diffuse than the $4 d$ orbital, the electron in the $5 s$ orbital is less able to shield its central nucleus than the electron in the $4 d$ orbital, when the attacking $\mathrm{H}_{2}$ molecule is not far away. Therefore, the Pd atom in the $4 d^{9} 5 s^{1}$ configuration is more repulsive than the Pd atom in the $4 d^{10}$ configuration.

## IV. CONCLUSION

We studied the interaction of the hydrogen molecule with the palladium atom in the singlet ground state $\left(4 d^{10},{ }^{1} S\right)$ and in the lowest triplet state $\left(4 d^{9} 5 s^{1},{ }^{3} D\right)$, as a model of the chemisorption of the $\mathrm{H}_{2}$ molecule on the Pd metal. We used the effective core potential method and calculated the accurate Hellmann-Feynman force of the $\mathrm{H}_{2}$ system adding the first derivatives of the hydrogen basis set. We used an intuitive force concept for studying the origin of the interaction.

We found that the singlet $\operatorname{Pd}\left({ }^{1} S\right)-\mathrm{H}_{2}$ system is attractive but the triplet $\mathrm{Pd}\left({ }^{3} \mathrm{D}\right)-\mathrm{H}_{2}$ system is repulsive. The side-on approach is found to be the preferable path. For the $\operatorname{Pd}\left({ }^{1} S\right)-\mathrm{H}_{2}$ system, the Pd-H bonds are gradually formed as the $\mathrm{H}_{2}$ molecule approaches. The electron density is accumulated in the overlap region between $\mathrm{H}_{2}$ and Pd and it causes the $\mathrm{F}\left(\mathrm{H}_{2}-\mathrm{Pd}\right)$ force which pulls the $\mathrm{H}_{2}$ molecule onto the metal atom. As the hydrogen molecule approaches, the protons receive the force which prolongs the $\mathrm{H}-\mathrm{H}$ distance. This is due to the decrease in the electron density in the bonding region of the two hydrogens. In an MO interaction picture, this reorganization of the electron density is due to an electron transfer from the bonding MO of the hydrogen molecule to the empty hybrid orbital of the $5 s, 5 p_{z}$, and $4 d_{z^{2}}$ AO's of the Pd atom. The back-transfer of electrons from the $4 d_{y z} \mathrm{AO}$ to the antibonding orbital of the $\mathrm{H}_{2}$ molecule is small.

For the triplet $\mathrm{Pd}\left({ }^{3} \mathrm{D}\right)-\mathrm{H}_{2}$ system, the electron density of the Pd atom is less affected by the approach of the $\mathrm{H}_{2}$ molecule and tends to keep its spherical symmetry. Further, the Pd atom in the $4 d^{9} 5 s^{1}$ configuration is more repulsive than that in the $4 d^{10}$ configuration. This is because the electron density in the $4 d^{9} 5 s^{1}$ configuration is less able to shield its Pd nucleus than the electron density in the $4 d^{10}$ configuration, since the $5 s$ orbital is more diffuse than the $4 d$ orbital. Therefore, the $\mathrm{H}_{2}$ molecule is repelled by the Pd atom.

## REFERENCES

1. The Palladium-Hydrogen System, F. A. Lewis, New York, Academic Press, 1967.
2. D. M. Roundhill, Adv. Organomet. Chem. 13 (1975) 273; P. M. Henry, Adv. Organomet. Chem. 13 (1975) 273.
3. The Organic Chemistry of Palladium, Vols. 1, 2, P. M. Maitlis, New York, Academic Press, 1971.
4. Advanced Inorganic Chemistry, F. A. Cotton and G. Wilkinson, New York, John Wiley \& Sons, 1980.
5. H. Basch and S. Topiol, J. Chem. Phys. 71 (1979) 802.
6. P. S. Bagus and C. Björkman, Phys. Rev. A 23 (1981) 461.
7. G. Pacchioni, J. Koutecký, and P. Fantucci, Chem. Phys. Lett. 92 (1982) 486.
8. S. W. Wang and K. S. Pitzer, J. Chem. Phys. 79 (1983) 3851.
9. Molecular Spectra and Molecular Structure. IV Constants of Diatomic Molecules, K. P. Huber and G. Herzberg, New York, Van Nostrand Reinhold Co., 1979.
10. A. A. Bagatur'yants, N. A. Anikin, G. M. Zhidomirov, and V. B. Kazanskii, Russ. J. Phys. Chem. (Engl. Transl.) 55 (1981) 1157.
11. L. R. Kahn, P. Baybutt, and D. G. Truhlar, J. Chem. Phys. 65 (1976) 3826.
12. P. J. H a y, J. Amer. Chem. Soc. 103 (1981) 1390.
13. H. Nakatsuji, J. Amer. Chem. Soc. 95 (1973) 345, 354, 2084; 96 (1974) 24, 30; H. Nakatsuji and T. Koga, J. Amer. Chem. Soc. 96 (1974) 6000; chapter 3 of reference 14 .
14. The Force Concept in Chemistry, B. M. Deb (Ed.), New York, Van Nostrand Reinhold Co., 1981.
15. H. Nakatsuji, K. Kanda, and T. Yonezawa, Chem. Phys. Lett. 75 (1980) 340 ; H. Nakatsuji, T. Hayakawa, and M. Hada, Chem. Phys. Lett. 80 (1981) 94; H. Nakatsuji, K. Kanda, M. Hada, and T. Yonezawa, J. Chem. Phys. 77 (1982) 3109; 79 (1983) 2493.
16. H. Nakatsuji, M. Hada, and T. Yonezawa, Chem. Phys. Lett. 95 (1983) 573.
17. H. Nakatsuji, K. Kanda, and T. Yonezawa, J. Chem. Phys. 77 (1982) 1961; H. Nakatsuji, M. Hada, K. Kanda, and T. Yonezawa, Int. J. Quantum Chem. 23 (1983) 387; H. Nakatsuji and K. Kanda, in: J. P. Dahl and J. Avery (Eds.), Local Density Approximations in Quantum Chemistry and Solid State Physics, New York 1984, pp. 771-784.
18. T. H. Dunning, Jr., J. Chem. Phys. 53 (1970) 2823.
19. Atomic Energy Levels, Vol. III, C. E. Moore, Washington D. C., National Bureau of Standard, 1971.
20. M. J. S. Dewer, Bull. Soc. Chim. Fr. (1951) 79; J. Chatt and L. A. Duncanson, J. Chem. Soc. (1953) 2939.
21. K. Fukui and H. Fujimoto, Bull. Chem. Soc. Japan, 41 (1968) 1989; 42 (1969) 3399.

## SAZ̆ETAK

## Interakcija molekule $\mathbf{H}_{2}$ sa atomom paladija primjenom koncepta Hellmann-Feynmanove sile

## Hiroshi Nakatsuji i Masahiko Hada

Kao model za kemisorpciju razmotrena je interakcija molekule $\mathrm{H}_{2}$ i atoma Pd u singuletnom ${ }^{1} S\left(4 d^{10}\right)$ i tripletnom ${ }^{3} D\left(4 d^{9} 5 s^{1}\right)$ stanju. Sila kojom atom Pd djeluje na $\mathrm{H}_{2}$ izračunana je vrlo točno. Ustanovljeno je da je sila privlačna za singulet, a odbojna za triplet. Pri veznoj interakciji dolazi do gomilanja elektronske gustoće $u$ regiji prekrivanja atoma $\operatorname{Pd}$ i molekule $H_{2}$, što se događa na račun smanjenja gustoće duž veze $\mathrm{H}-\mathrm{H}$. Pri tome se tvori nova veza $\mathrm{Pd}-\mathrm{H}$, a slabi veza između atoma vodika. Dolazi i do migracije naboja iz vezne MO $\mathrm{H}_{2}$ u prazne $5 s 5 p_{z}$ orbitale atoma Pd.

