1 Diffusion Models in Marketing: How to Incorporate the Effect of External Influence?

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Abstract

Diffusion models have been used traditionally in marketing for capturing the life-cycle dynamics of a new product, for forecasting the demand for a new product, and as a decision aid in making pre-launch, launch and post-launch strategic choices. Since their entrance into marketing, diffusion models have become increasingly complex. This complexity has been driven by the need to enhance the forecasting capability of these models and to improve their usefulness as a decision-making tool for managers. One of the challenges of diffusion modeling is to incorporate external influences in models, most notably the influence of marketing mix variables. This paper offers a framework for systematizing diffusion models in marketing, with a special emphasis on the role of marketing mix variables. Different models are compared and their advantages and disadvantages discussed. Suggestions for further research are also offered.

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1 Introduction

It has been documented that the natural growth of a number of phenomena can be described by an S-shaped pattern (Fisher and Pry, 1971; Meade and Islam, 1998). There are many varied examples including the number of future sales of durable products, future spreading of infectious diseases through a population, future adoption of an innovation, etc. (Rogers, 1990). The S-shaped pattern can be explained by using diffusion theory, which is a theory that concerns itself with communication channels, i.e. means through which the innovation is transmitted through the social system (Rogers, 1990). Models that rely on diffusion theory to predict the adoption of an innovation are called diffusion models.

This paper will focus on diffusion modeling of the growth pattern of new durable products, which represents an important topic in marketing. Diffusion models have entered the marketing discipline with the publication of the first mathematical model of new product diffusion by Bass (1969), who realized that it is possible to use diffusion theory to mimic the S-shaped growth pattern of new durables and technologies.

In marketing, diffusion models have been used traditionally for capturing the life-cycle dynamics of a new product, for forecasting the demand for a new product, and as a decision aid in making pre-launch, launch and post-launch strategic choices. Since their entrance into marketing, diffusion models have become increasingly complex. This complexity has been driven by the need to enhance the forecasting capability of these models and to improve their usefulness as a decision-making tool for managers.

Diffusion models describe how sales of a new product depend on time. Since in reality sales depend on a variety of external influences, such as the level of product advertising, changes in product price and variations in distribution intensity, it is of considerable importance to create such models that can include those external variables. Without them, the practical use of diffusion models would be limited indeed. Actually, one of the challenges of diffusion modeling is to incorporate external influences in models, most notably the influence of marketing mix variables such as price and advertising. As we will see in this paper, this is not an easy task. There are basically two approaches to this problem. Some authors incorporate marketing mix variables in a pre-specified way, so that model parameters remain constant and unaffected (Robinson and Lakhani, 1975; Bass, 1980; Kalish, 1985; Kamakura and Balasubramanian, 1988; Dockner and Jorgenson, 1988; Horsky, 1990; Bass, Jain and Krishnan, 1994), while others allow for parameters to change with time (Putsis, 1998; Von Bertalanffy, 1957; Easingwood, Mahajan and Muller, 1981, 1983; Horsky and Simon, 1983; Bewley and Fiebig, 1988).

Although theoretically more tractable, the assumption of parameter constancy seems to present a serious limitation. These models assume that we can predict how the external variables will change with time over a longer time period. However, in their unpredictability markets are more stochastic than deterministic; it is not possible to predict what changes markets may undergo several years from now, prompting managers to introduce alterations in their plans for marketing mix variables such as advertising, price, and distribution intensity. Unexpected changes in a variable would naturally be reflected in unexpected parameter changes. An advanced and realistic model would be expected to allow unplanned changes in the level of advertising, competitive actions and changes in consumer tastes to significantly impact model parameters. In order to be a good managerial decision-making tool, a model must recognize and incorporate changes that happen in the market. This illustrates the need for models that allow parameters to change with time.

Time variation in parameters is very important and represents the least understood aspect of diffusion models (Putsis, 1998). These models are difficult and research papers on the topic are not numerous. Again we have two approaches here: some authors allow that parameters change with time in a pre-specified way, while other authors resort to stochastic modeling. The authors who allow diffusion parameters to vary with time so that this variation is defined in a pre-specified way are Von Bertalanffy (1957), Easingwood, Mahajan and Muller (1981, 1983), Horsky and Simon (1983) and Bewley and Fiebig (1988). However, only Horsky and Simon (1983) link this parameter variation directly with marketing mix variables. Early evidence shows that allowing for parameter variation improves the model fit over traditional diffusion model (Easingwood, 1987, 1988, 1989). However, since markets can alter in unexpected ways, the assumption that we could pre-specify the way in which parameters change over time may present an oversimplification that might result in a less than desirable model fit. An alternative to this pre-specified parameter variation is a more difficult stochastic modeling (Putsis 1998), where parameters are allowed to stochastically vary with time. This model incorporates external influences through their effect on the remaining market potential. In such a way these variables have an indirect influence on diffusion parameters.

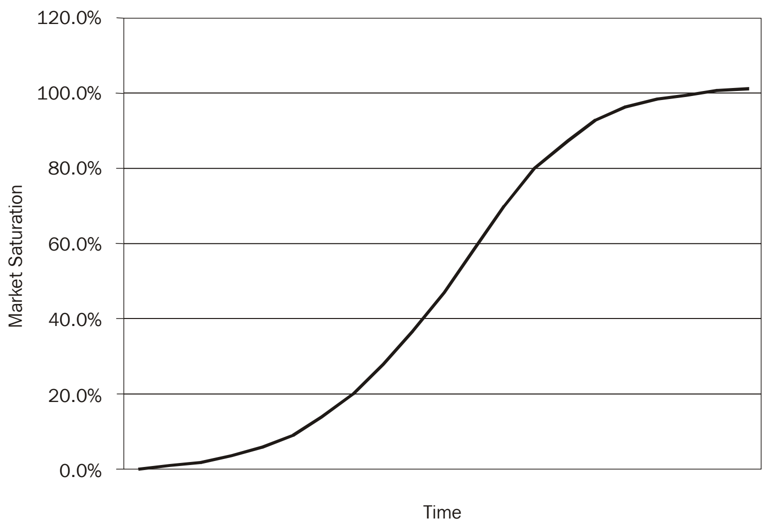
This paper presents a literature review of this exciting area in marketing. The purpose of the review is to look at the extant body of diffusion modeling literature from the aspect of incorporating external influence variables in the model. Since this is an important problem for both the marketing scientists who do diffusion modeling for sales forecasting purpose and for the managers who commission such work, a review paper of this area is very much needed. This paper is envisioned as a roadmap whose intention is to let the reader know what models are available and what their advantages and disadvantages with respect to modeling external influences are, so that the reader can choose the one model that most suits her or his need and data. In this paper we provide only the basic information on the details of models (an interested reader will easily find more in referenced papers), because the focus is on the external variables framework and on how all these models fit together. In addition, as is the tradition in review papers in this area of marketing (Mahajan, Muller and Bass, 1990; Bass, Jain and Krishnan, 2000), we will not estimate any of the models although we will discuss the pros and cons of various estimation methods.

This paper is structured in the following way. First, we discuss the basic Bass model to develop intuition before turning our attention to diffusion models with constant parameters. Then we proceed to the survey models with time varying parameters, starting with the models with a pre-specified parameter change and then going to more sophisticated stochastic models. After that model estimation issues are discussed. Finally, areas for further research are identified.

2 Basic Bass Model

A large body of literature on marketing research strongly demonstrates that product sales life cycles follow an S-curve pattern. An S-curve pattern implies that new product sales initially grow at a rapid rate, but then the rate of growth tapers off and finally declines with time. An example of an S-shaped curve is presented in Figure 1. In marketing, it is considered that the channels of communication include both the mass media and interpersonal communications. Members of a social system have different propensities for relying on mass media or interpersonal channels when seeking information about the innovation, and that presents an important influence in determining the speed and shape of an S-shaped pattern (Mahajan et al., 2000). The consumer product adoption process based on relative adoption time categorizes individuals as innovators, early adopters, early majority, late majority, and laggards.

Figure 1. **Example of an S-shaped curve**



The first diffusion model used in marketing was the Bass diffusion model. Bass (1969) suggested that the probability of a current purchase, by someone still in the market, is a linear function of the number of prior purchases. He interpreted the linear coefficients as the propensity to innovate and imitate. More precisely, the likelihood that someone would adopt a new product at time *t* (given that s/he has not adopted it before) is represented by the equation $\frac{f(t)}{1-F(t)} = p + qF(t)$, where parameters *p* and *q* represent the coefficient of innovation and imitation respectively, while *F(t)* is the cumulative distribution function (probability of adoption by time *t)* and *f(t)* is the probability density function of the random variable *t*, the adoption time of the new product. In the case when *F(t)* is differentiable (as it always is in all practical meaningful situations), this is equivalent to $f(t) = \frac{dF(t)}{dt}$ · Knowing this, the equation $\frac{f(t)}{1-F(t)} = p + qF(t)$ could be rewritten as $\frac{dF(t)}{dt} = p + (q-p)F(t) - qF(t)$ · Parameter p represents the external influence (that is usually media), while the parameter q depicts the influence of interpersonal channels (i.e. the influence of other people around the potential adopter).

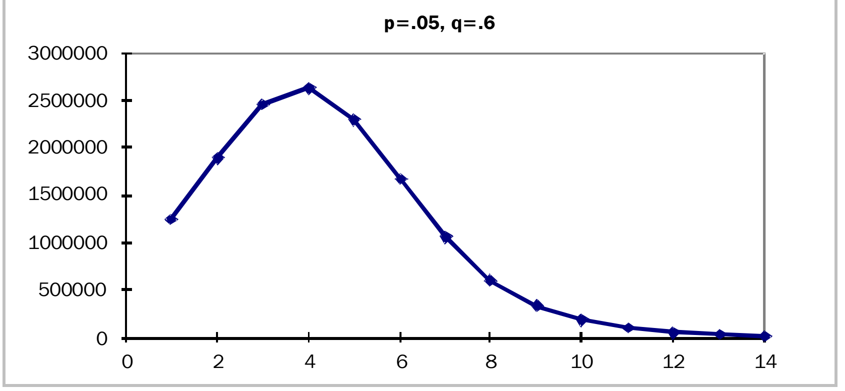
Let *S(t)* and *Y(t)* denote the sales and the cumulative sales of the new product, respectively, at time *t*, and let *m* be the total market potential (*m* represents all potential buyers of the product ever). Assuming that the product sales are *S(t)=mf(t)*, from the above equation we can derive that the sales can be expressed as $S(t) = pm + (q-p)mF(t) - qm(F(t))^2 = pm + (q-p)Y(t) - \frac{q}{m}Y(t)^2$.

The Bass model can assume two basic shapes. When $q \geq p$ the graph of adoptions has a bell shape as in example 1, while when $q \leq p$ the shape is downward sloping as in example 2.

Example 1

Here we present a Bass model with parameters p=0.05, q=0.6, and market potential m=1500000. This example illustrates that the coefficient of imitation (or word of mouth) generates the effect that is captured by the bell shaped curve.

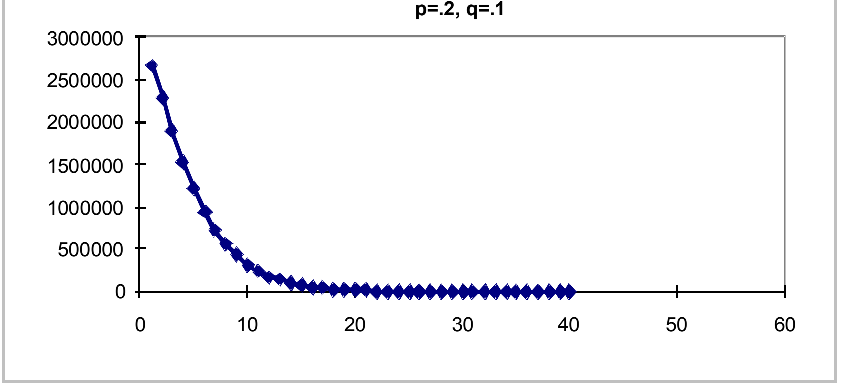
Example 1



Example 2

Here we present a Bass model with parameters p=0.2, q=0.1, and market potential m=1500000. This example illustrates that when the imitation component is smaller than media influence, sales decline steadily.

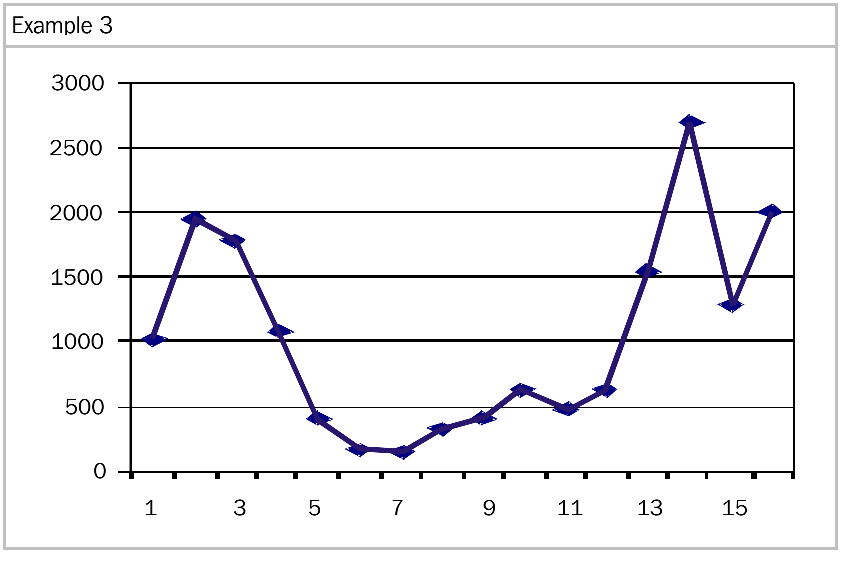
Example 2



In order for the basic Bass model to have a good fit with the actual sales data, the sales data should have one of two basic shapes. However, this is rarely so, as sales reflect the decisions made on many marketing variables. For example, if advertising slows down, the sales might follow as well. If the price of the product decreases, the number of new adopters is likely to go up. These problems are exacerbated if we revert to shorter time intervals (for example, from yearly data to quarterly or monthly data), for then we can observe more changes in the data. The realistic sales month by month might look more like Example 3 than either of the smooth and nice graphs in Examples 1 and 2.

Example 3

This graph presents real monthly sales data of an existing durable product. It is obvious that there are jumps and kinks in the data that do not conform to the classical Bass model. These jumps come from influences of external variables. For this particular product, important external variables are the level of advertising and the level of product promotion in a certain month.



This example illustrates that we really need the models that can incorporate external influences, and thus would be capable of producing more realistic forecasts. A number of researchers were trying to accomplish this goal; their models are presented and compared in the remainder of this paper.

3 Diffusion Models with Constant Parameters

A major limitation of the Bass model is that it does not incorporate marketing-mix variables, and that restricts the model's suitability for marketing planning. As a response to this shortcoming, several researchers generalized the Bass original framework by introducing marketing mix variables (please see Table 1 for mathematical details). The most common marketing mix variables considered in extant research were price and advertising.

One approach to dealing with marketing variables is to include them in an additional term. Robinson and Lakhani (1975) were the first to introduce decision variables into diffusion modeling. They modified the Bass model by including a product's price as an exponential term that multiplies the original Bass expression. Although very difficult to estimate, the model was employed by Dolan and Jeuland (1981) and Jeuland and Dolan (1982) to produce some normative implications. In a similar way price was treated in models by Dockner and Jorgenson (1988) and Teng and Thompson (1983), but these models also had empirical limitations (Bass, Jain and Krishnan, 2000). Although Bass (1980) also included price in an additional term, estimation problems were eased by assuming that firms set the price by equating marginal revenue and marginal cost, which follows the experience curve. This specific form for the cost function and the optimal price was used in empirical estimation.

Another approach was to model the impact of marketing variables through their effect on market potential by using a pre-specified functional form suggested by theory. Kamakura and Balasubramanian (1988) introduced an extension of the Bass model in which they considered both the price index, population change and the need for replacement sales. These variables affect the market potential and the remaining market potential, while diffusion force is unaffected. Horsky (1990) modified the Bass model by introducing the wage rate, the reservation price and the price of the new product. Similarly to Kamakura and Balasubramanian (1988), Horsky suggested that these variables drive the adoption by affecting the market potential, while they do not affect the term representing the diffusion force. Jain and Rao (1990) considered the influence of price on diffusion. They proposed two alternative models, one in which price impacts the market potential, and another in which price impacts the potential of remaining sales, while the diffusion part is unaltered.

Bass, Jain and Krishnan (1994) presented a model based on Bass (1969) with incorporated price and advertising. Unlike the research reviewed above, Bass, Jain and Krishnan assumed that these variables affect the hazard rate. The final form of the model is equivalent to the traditional Bass model, except for a multiplicative term containing marketing variables.

Kalish (1985) deviates from the original form of the Bass model and builds a two-step theoretical model grounded in the utility maximization principle. The model is divided into two stages based on how a new product is perceived. The first stage models a diffusion of awareness, which depends on cumulative sales of the product (denoted $Y(t)$), initial potential market (denoted m), advertising (denoted $A(t)$) and the information that potential adopters have about the new product (denoted I in Table 1). The second stage models adoption, which depends on cumulative sales of the product, initial potential market, and information that potential adopters have about the new product and price. Kalish tests his model on an unspecified durable good, but he only uses the adoption part of this model since awareness data is not available. In other words, in the empirical application of the model Kalish did not include advertising (i.e. he assumed that $I = 1$), and he estimated the model that includes only price.

Table 1. **Diffusion models with constant parameters**

	CONSTANT PARAMETERS
Bass (1969)	$\frac{dF(t)}{dt} = p + (q-p)F(t) - qF(t)^2$ $F(t)$ = cumulative distribution function (probability of adoption by time t) $f(t)$ = probability density function p = coefficient of innovation q = coefficient of imitation m = market potential
Robinson and Lakhani (1975)	$\frac{dF(t)}{dt} = \left[p + (q-p)F(t) - qF(t)^2\right] \cdot e^{-k\,\mathrm{Pr}(t)}$ $Pr(t)$ = price k = coefficient $F(t)$ = cumulative distribution function (probability of adoption by time t) $f(t)$ = probability density function p = coefficient of innovation q = coefficient of imitation m = market potential
Bass (1980)	$\frac{dF(t)}{dt} = \left[p + (q-p)F(t) - qF(t)^2\right] \cdot c \cdot \left[\mathrm{Pr}(t)\right]^2$ $Pr(t)$ = price η = price elasticity c = cost function $F(t)$ = cumulative distribution function (probability of adoption by time t) $f(t)$ = probability density function p = coefficient of innovation q = coefficient of imitation m = market potential
Kalish (1985) (This model is the one Kalish estimated in his empirical work and it does not include advertising)	$\frac{dY}{dt} = \left[g\left(\frac{\mathrm{Pr}(t)}{uY(t)/m}\right) \cdot I - Y(t)\right] \cdot k$ $\frac{dI}{dt} = \left[1 - I\right] \cdot \left[f(A(t)) + bI + b' \frac{Y(t)}{m}\right]$ $Y(t)$ = cumulative sales up to time t $Pr(t)$ = price at time t m = the initial market potential $A(t)$ = advertising at time t I = information or awareness level g and u are functional operators b, b', k = parameters (Kalish chose g to be exponential and u to be quadratic)
Horsky and Simon (1983)	$S(t) = \left[\alpha + \beta \ln(A(t)) + qY(t-1)\right]\left[m - Y(t)\right]$ $A(t)$ = advertising at time t β = effectiveness of advertising $Y(t)$ = cumulative sales up to time t $S(t)$ = sales at time t m = market potential

Kamakura and Balasubramanian (1988)	$S(t)=\left[p+qY(t)\right]\Pr(t)^{\beta_1}\left[\Theta M(t)\Pr(t)^{\beta_2}-Y(t)\right]$ *Y(t)* = cumulative sales up to time *t* *S(t)* = sales at time *t* *Pr(t)* = price at time *t* *M(t)* = population at time *t* Θ = ultimate penetration level β_1 = parameter (impact of price on adoption speed) β_2 = parameter (impact of price on market potential) *p* = coefficient of innovation *q* = coefficient of imitation
Horsky (1990)	$S(t)=\left[\dfrac{\theta M(t)}{1+\exp\left\{-\dfrac{K+w(t)-k\Pr(t)}{\delta(t)}\right\}}-Y(t)\right]\left[p+qY(t)\right]$ *w(t)* = average wage rate of the population *Pr(t)* = average market price of the product δ = dispersion of both distributions *K* = time saving attribute of the new product *k* = utility saving attribute of the new product *Y(t)* = cumulative sales up to time *t* *S(t)* = sales at time *t* *M(t)* = population at time *t* θ = fraction of potential buyers in the population *p* = coefficient of innovation *q* = coefficient of imitation
Jain and Rao (1990)	Alternative 1. $S(t)=\left[m\Pr(t)^{-\eta}-Y(t-1)\right]\dfrac{F(t)-F(t-1)}{1-F(t-1)}$ Alternative 2. $S(t)=\left[m-Y(t-1)\right]\Pr(t)^{-\eta}\dfrac{F(t)-F(t-1)}{1-F(t-1)}$ *Y(t)* = cumulative sales up to time *t* *S(t)* = sales at time *t* *Pr(t)* = price at time *t* η = price elasticity *m* = market potential *F(t)* = cumulative distribution function (probability of adoption by time *t*)
Bass, Jain and Krishnan (1994)	$\dfrac{dF(t)}{dt}=\left[p+(q-p)mF(t)-qm\left(F(t)\right)^2\right]x(t)$ where $x(t)=1+\beta_1\dfrac{\Pr'(t)}{\Pr(t)}+\beta_2\dfrac{A'(t)}{A(t)}$ *Pr(t)* = price at time *t* *Pr'(t)* = rate of change in price at time *t* *A(t)* = advertising at time *t* *A'(t)* = rate of change in advertising at time *t* β_1 = coefficient β_2 = coefficient *F(t)* = cumulative distribution function (probability of adoption by time *t*) *f(t)* = probability density function *p* = coefficient of innovation *q* = coefficient of imitation *m* = market potential

4 Diffusion Models with Parameters Changing with Time

Although all the reviewed papers represent a worthy contribution to the literature on diffusion processes by recognizing and modeling the impact of marketing variables, they all assume that model parameters do not change over time. There are many reasons why we should consider parameters as changing with time. Everyday experience teaches us that markets are never constant for long stretches of time. Different levels of competitive activity, changes in advertising level and changes in price elasticity, among other factors, all have a significant impact on diffusion and its parameters. Allowing parameters to vary with time would permit diffusion models to better match real data. In fact, evidence shows that models allowing for parameter variation provide an excellent fit to diffusion data (Putsis, 1998). According to Putsis (1998), comprehending parameter variation is very important for gaining insight into the nature of the diffusion process, as parameter variation is the least understood aspect of diffusion models. In further text relevant models will be discussed while their mathematical details are provided in Table 2.

Parameter variation can be modeled in such a way that researchers determine in advance how parameters will change over a product's life. Usually, this is accomplished by using a specific functional form postulated from theory. In such studies a transition from period to period varies according to this pre-specified form. This approach has received much attention and is employed in work by Von Bertalanffy (1957), Easingwood, Mahajan and Muller (1981, 1983), Horsky and Simon (1983) and Bewley and Fiebig (1988). Such models are usually referred to as flexible diffusion models. These models suggest that outside influences affect either innovation parameter p or imitation parameter q from the Bass model.

Horsky and Simon (1983) assumed that advertising has an impact on the innovative characteristics of adopters, so they represented innovation parameter p as a logarithmic function of advertising that changes with time. Other papers considered the impact of outside influences through imitation parameter q. Von Bertalanffy (1957) and Easingwood, Mahajan and Muller (1983) suggested that q changes systematically over time as a function of the penetration level. Bewley and Fiebig (1988) also assume that there is a systematic variation in parameter q that depends on time. In all three models this systematic variation in q is pre-specified. It is important to note that Von Bertalanffy (1957), Easingwood, Mahajan and Muller (1983) as well as Bewley and Fiebig (1988) do not explicitly link marketing variables to diffusion. Easingwood (1987, 1988, 1989) showed that in comparison with basic diffusion models, flexible diffusion models provide a better fit to diffusion data. Introducing the time variation to diffusion parameters improves the model fit, as expected.

Although these models present an improvement by allowing one of the parameters to be a pre-specified function of time, there are several shortcomings. For example, the number of variables is limited to one or two, because otherwise it would become too difficult to specify the functional form and estimate the model. In addition, although we can be led by theory in choosing the functional form, we can never be sure that the selected form is good for our particular data. Another feature of these models is that they assume either the innovation or the imitation component to be time dependent, but not both. In reality, we would expect that marketing mix variables might influence both innovation and imitation among the population.

Stochastic modeling represents an alternative to pre-specified time varying parameters. In this approach parameters are allowed to change stochastically from one period to another. The stochastic parameter variation has received the least attention to date (Putsis, 1998).

Putsis (1998) presented a stochastic diffusion model with time-varying parameters. The model includes marketing mix variables and replacement sales. Putsis divides total purchases into first time purchases and replacement purchases. He considers the proportion of the population who purchase for the first time, and the proportion of the population who purchase a replacement. These proportions are assumed to be influenced by variables such as income, price, prior durable stock and demographic and demand shifting variables. The proportions are then modeled as linear functions of these variables and the saturation level. The Putsis model contains the Bass model as a special case. The Putsis model exhibits a better fit than Bass (1969), Easingwood, Mahajan and Muller (1983), Horsky (1990) and Kamakura and Balasubramanian (1988), making a convincing case for the time-varying parameter modeling. However, the Putsis model does not establish an explicit link between marketing mix variables, and innovation and imitation parameters.

Table 2. **Diffusion models with time-varying parameters**

	Pre-specified parameter variation
Von Bertalanffy (1957)	$\frac{dF(t)}{dt} = \frac{q}{1-\theta} F^{\theta}(t)\left[1 - F(t)^{1-\theta}\right]$ θ = constant
Easingwood, Mahajan and Muller (1983)	$\frac{dF(t)}{dt} = \left[p + qF^{\delta}(t)\right]\left[1 - F(t)\right]$ δ = constant *F(t)* = cumulative distribution function (probability of adoption by time *t*) *f(t)* = probability density function *p* = coefficient of innovation *q* = coefficient of imitation *m* = market potential
Horsky and Simon (1983)	$S(t) = \left[\alpha + \beta \ln(A(t)) + qY(t-1)\right]\left[m - Y(t)\right]$ *A(t)* = advertising at time *t* β = effectiveness of advertising *Y(t)* = cumulative sales up to time *t* *S(t)* = sales at time *t* *m* = market potential
Bewley and Fiebig (1988)	$F(t) = \frac{1}{1 + e^{-\alpha + \beta t(\mu,k)}}$ α, β, *k*, μ = constants $t(\mu,k) = \left[\left[(1+kt)^{1/k}\right]^{\mu} - 1\right]/\mu$ when $\mu \neq 0, k \neq 0$ $t(\mu,k) = (1/k)\log(1+kt)$ when $\mu = 0, k \neq 0$ $t(\mu,k) = (e^{\mu t} - 1)/\mu$ when $\mu \neq 0, k = 0$ $t(\mu,k) = t$ when $\mu = 0, k = 0$ *F(t)* = cumulative distribution function (probability of adoption by time *t*) *f(t)* = probability density function *p* = coefficient of innovation *q* = coefficient of imitation *m* = market potential

Stochastic variation	
Putsis (1998)	$(\text{Purchases/Household})_t = a_t + b_t + c_t Y_t + d_t SL^{i\neq j}{}_{t-1} + (e-a)_t SL_{t-1} + e_t SL^2{}_{t-1} + e_t$ $(1-SL_{t-1}) =$ the percentage of non-owners out of total households for chosen product j at time t $(1-SL^{i\neq j}{}_{t-1}) =$ the percentage of non-owners out of total households for other products in the durable stock at time t $Y_t =$ income at time t $p_t =$ price of product j at time t

5 Estimation of Diffusion Models

Apart from giving normative prescriptions to managers, the aim of diffusion modeling is to predict future product sales. This is the part where we need to use estimation techniques to derive model parameters from the given data.

Most models based on the Bass model cause empirical difficulties with parameter estimation in the early stages of the product life cycle, when the available data streams are short. Mahajan, Muller and Bass (1990) noted that "parameter estimation for diffusion models is primarily of historical interest; by the time sufficient observations have been developed for reliable estimation, it is too late to use the estimates for forecasting purposes". Bass (1969) and most of the early work on diffusion models based on the Bass model employed the statistical technique known as ordinary least squares estimation (in further text OLS estimation), which proved to have problems such as the instability of parameter estimates when few data points were used. Additional problems of OLS estimation included the fact that standard errors of the parameter estimates of p, q and m are not readily available (Mahajan, Mason and Stuart, 1986), and there is also a time-interval bias since it is created by attempting to estimate the equation $S(t) = p + (q-p)mF(t) - qm(F(t))^2$ by using discrete data (Putsis, 1996). As a result of OLS shortcomings, MLE (maximum likelihood estimation) and NLLS (non-linear least squares estimation) were proposed as alternative estimation techniques. Both techniques have the advantage over OLS in that they do not suffer from the time-interval bias problem. In addition, they both provide standard errors for the parameters p,q and m. MLE was proposed by Schmittlein and

Mahajan (1982), who showed that under very general regularity conditions MLE is consistent, asymptotically normal and asymptotically efficient. NLLS, introduced by Srinivasan and Mason (1986), is the estimation technique that has recently become the standard in diffusion research. Comparing the two approaches, Putsis (2000) says that evidence suggests that the NLLS approach by Srinivasan and Mason (1986) "...will do well in most settings and may be preferred to MLE" (however, Putsis (2000) warns that in the context of nonlinear models with covariates it is not clear whether MLE or NLLS should be preferred, and that should be resolved by future research). Van den Bulte and Lilien (1997) showed that although preferred to OLS and MLE, NLLS is not immune to parameter estimation biases either, since it underestimates m and p and overestimates q. Dekimpe et al. (1998) noted that this problem is largely due to the model's estimating without external constraints on the parameter ranges. When such constraints were introduced, Dekimpe et al. (1998) showed that the adjusted diffusion model could be safely used for estimation even in the early stages of product adoption.

6 Suggestions for Further Research

From the discussion of currently available models, we can discern that there are two major avenues where improvement in diffusion modeling can take place. One is further work on time varying parameters, and the other is improvement in model estimation.

Regarding parameters, they should vary with time but in a flexible way, which precludes the modeling with pre-specified functional forms. Flexibility can be obtained by using stochastic modeling, whose structure may follow Putsis (1998). However, as the Putsis model is quite complicated, creating a simpler stochastic model would be desirable.

We have seen that one concern in modeling a growth pattern of new products is to explain and, therefore, predict the impact that marketing variables and actions have on the speed of diffusion. The issue of introducing such variables in the model is very relevant, especially since measuring the effect of marketing actions is of importance to managers. It would be especially valuable to create a model that would link marketing variables and parameters p and q, so that the impact of marketing actions on diffusion speed may be visible. Having that direct link may provide us with more information about the effect of marketing variables on innovation and imitation parameters.

Another avenue for improvement is the model estimation. That goal could be achieved in several ways. For example, the limitations on some parameters could be incorporated in the model, thus limiting computational complexity. Still another approach is to build on the approach taken by Dekimpe et al. (1998) in their treatment of marketing variables, as they have shown that their approach provides for better estimation. It would also be advisable to use new estimation methods in statistics and introduce them to diffusion modeling.

Regardless of parameter variation and estimation issues, there is another way in which we can improve models. Namely, the Bass model, as other diffusion models, does not account for seasonal variations in sales. One way to remove seasonality from data is to use yearly data, as it has often been done in the past. However, increasing global competition and the resulting shortening of product life cycles do not allow managers to wait for several years before attempting to forecast the life cycle. Crucial decisions have to be made very soon after the product's launch, so models that require several months of data vs. several years of data would be much more useful to managers. Such models should account for seasonal variations in sales predictions. Therefore, one suggestion for further research is to incorporate seasonality in diffusion models (one method of incorporating seasonality in any dynamic model is introduced in Radas and Shugan, 1998).

All these suggestions for further research have as a common goal the creation of diffusion models that would be more flexible, easier to use and easier to estimate, and could thus provide managers with necessary tools for better decision-making.

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