Electromagnetic Modeling of Wire-Ground Structures by using a MTL Based Approach

A multi-conductor transmission lines-based approach is developed to model electromagnetic coupling of wires above metallic structures. With this approach, global impedances and local distributions are accessible. According to the frequency range and the geometry of the studied structures, it is possible to only take into account well-chosen parts. In that case, the shading concept provides an efficient way to determine which part is sufficient, thereby allowing reduction in model size. Global quantities results are validated against measurements, and local ones by theoretical knowledge. This work is usefulness in both the nomadic applications (automotive and aeronautic wiring . . . ) and the usual electrical applications (home and industrial wiring).

Key words: Electromagnetic coupling, ground planes, Shade of conductors, transmission line modeling (MTL), wiring

1 INTRODUCTION

In modern electro-system facilities containing different kind of high-speed and high technology electrical/electronic equipment, the electromagnetic interactions studies between participants require consideration of the following three main parts: the metallic environment (ground planes), the cable harnesses, and the equipment (loads). Many electromagnetic interference problems (EMI) associated with electrical wiring connecting devices may prevent systems from satisfying stringent electromagnetic compatibility criteria (EMC).

For global modeling in EMC/EMI, it is usual to consider the loads extension as small compared to signals wavelengths, so that they can be modeled using lumped elements. For high frequencies, when the sizes of the studied structures are long enough, the use of 3D full-wave analysis in EMC design leads to an enormous computational cost because the required mesh size for wires is too small.

Moreover, the metallic environment used to minimize the return common impedance, is usually assumed to be a perfect ground plane with infinite sizes. Unfortunately, this is not always true, since for many nomadic (vehicle, aeronautic, mobile . . . ) applications, the ground planes – chassis are not enough perfect and/or also have finite dimensions. It is then necessary to take into account its physical and geometrical characteristics in the modeling process. The EM emissions are principally generated by the various components within a system and they propagate and spread between the equipment mostly via the wiring and the common ground plane. So it is essential to identify current loops, shared paths, and dominant coupling mechanisms between different circuits during the design phase in order to ensure a good working of the equipment.

[1-4] propose numerical based-PEEC-FEM approaches (Partial Electric Equivalent Circuit – Finite Element Method) to model wire-ground structures and to take into account the metallic environment return effects. While [5-
use analytical and/or hybrid approaches based on the MTL method (Multi-Conductor Transmission Line) with the image concept, to extract an equivalent common mode circuit.

Practically, no single modeling tool is able to capture all the effects from different parts. The main reason is the difficulty to handle very different geometric scales, related to the three main parts of the system: the ground planes, the cable harnesses and the components. These parts can be modeled and analyzed using 3D-full-wave, 2D-transmission line (MTL) or circuit approaches. An accurate modeling tool requires various simulation techniques combining models of various types to capture the physical effects from the different parts of the system.

This paper presents an MTL modeling approach for EMC-EMI frequency analysis of wire-ground structures with finite geometrical and physical proprieties. The ground plane is considered as a full-participant conductor. Its contribution on coupling effects will be expressed by the level and the spreading of its current distribution. As a part of this work, an easily reduced model can be extracted from MTL modeling by using the concept of shading [10], i.e. intense or dominant coupling parts are kept while the remaining ones are eliminated. Then a reduced simulation can be achieved with very similar input-output behavior, compared to the original model. The approach may be useful as a decision-design help tool, it doesn’t replace full-wave 3D tools, but it can provide reduced input data for those tools, this may reduce treatment costs.

2 THE MTL MODELING APPROACH

In the case of coupled wire-ground plane, which is the most common configuration in nomadic applications, the quasi-TEM (Transverse Electromagnetic) mode becomes preponderant as compared to antenna mode [9]. Therefore, a MTL modeling generally provides good estimations for currents and voltages of multi-wire bundle above ground plane. Assuming some physical and geometrical conditions, non-uniform parts can be decomposed into small uniform segments. Thus, a 3D wiring system can be represented by a cascade of 2.5D problems, in the sense that it is formed by the juxtaposition of 2D MTLs problems. This allows the use of analytical closed-form exponential matrix solution, which is available for MTLs only.

2.1 The ground plane approximation

The ground plane is considered as a conductor with large rectangular cross section. It carries non uniform distribution of common and/or differential modes currents. TEM mode assumes that currents flow in the same direction, parallel to cable bundle.

By dividing the ground plane and all tick conductors into bars of smaller rectangular cross section, the current inside is assumed to be uniformly distributed but with unknown level from one bar to the other (Fig. 1). Only the total current is known. This essentially approximates current distribution over the entire cross section as a piecewise-constant approximation. The division must comply with the frequency skin effect constraints in both cross section dimensions. Then we will have the situation of multi-conductor transmission line with rectangular and circular cross sections.

The system is divided into $n$ rectangular and circular sub-conductors forming a coupled multi-conductor transmission line. With such configuration, and assuming infinitely long conductors, the per unit length matrices can be computed analytically. They are constants along the spatial dimension.

2.2 Per unit length matrices

For a flexible choice of a reference conductor, a remote (fictitious) reference ($o$), with no current flowing, located at infinity is assumed; the return to a local reference is performed, thereafter, by means of matrix operations in the final mathematical model.

Fig. 2 shows a 2D generic configuration to calculate the per unit length parameters between rectangular and/or circular conductors. Since the current is assumed to be uniformly distributed over each sub-conductor cross section;
the per unit length parameters are frequency independent. Then, the per unit length resistance of a conductor \( i \) is expressed by

\[
R_i = \frac{\rho_i}{s_i}
\]

With \( \rho_i \) the resistivity of the material of conductor \( i \) and \( s_i \) its cross section

The per unit length mutual inductance between two filamentary open loops (po) and (qo) is given by (2).

\[
l_{pq} = \frac{\mu}{s_{pq}} \log \left( \frac{1}{r_{pq}} \right)
\]

We note that \( l_{pq} \) is not the total per unit inductance but the contribution of sub-conductor’s \( p \) and \( q \) on the total per unit mutual inductance, between two closed loops without return filaments.

The mean mutual inductance between conductors \( i \) and \( j \) is obtained by integration over their cross sections (3).

\[
L_{ij} = \frac{\mu}{s_{ij}} \cdot \frac{1}{2\pi} \int_{s_j} \int_{s_i} \log \left( \frac{1}{r_{pq}} \right) ds_p ds_q
\]

From the generic configuration, three cases can be considered: both sections are circular, both sections are rectangular, or one is circular and the other is rectangular. In the first case we obtain a simple expression, identical to (2) with \( r_{pq} \) equals to the distance between the centers. In the second case, no simplification is possible and expression (3) remains. In the third case, the integration is reduced to only over the section \( s_j \) (the rectangular one) with the point \( p \) fixed at the center of the circular section (4).

\[
L_{ij} = \frac{\mu}{s_{ij}} \cdot \frac{1}{2\pi} \int_{s_j} \int_{s_i} \log \left( \frac{1}{r_{pq}} \right) ds_p ds_q
\]

The mean self-inductance is obtained as if \( i \rightarrow j \) in the expression (3). Analytic integration over orthogonally configurations is proposed in [11-13].

If the surrounding medium is homogeneous with permittivity \( \epsilon \), permeability \( \mu \), and conductivity \( \sigma \) then, we admit that matrices \( L, C \) and \( G \) satisfy approximately the well know identities (5) and (6).

\[
LC = \mu \epsilon I
\]

\[
LG = \mu \sigma I
\]

From which the per unit length capacitance matrix is obtained

\[
C = \mu \epsilon L^{-1}
\]

Because the surrounding medium is assumed lossless, then the conductance \( G \) is zero. Finally, the “partials” per unit length impedance and admittance matrices are respectively defined as in (8)

\[
\begin{align*}
Z &= R + j\omega L \\
Y &= G + j\omega C
\end{align*}
\]

Such quantities are essential to build the MTL model.

### 2.3 Short summary on the MTL method

In frequency-domain analysis (steady state, sinusoidal excitation), for a per unit length of \( n \) coupled-conductor line, the coupled first order MTL equations are (9a).

\[
\begin{align*}
dV(z)/dz &= -ZI(z) \\
dI(z)/dz &= -YV(z)
\end{align*}
\]

Where \( V(z) \) and \( I(z) \) are \( n \times 1 \) column vectors containing voltages (with respect to a fictitious reference) and currents, respectively. The \( n \times n \) complex matrices of per-unit-length impedance \( Z \) and admittance \( Y \) are symmetric.

The uncoupled second order MTL equations obtained from (9a) by differentiation with respect to \( z \) dimension are given by (10).

\[
\begin{align*}
d^2V(z)/dz^2 &= ZYV(z) \\
d^2I(z)/dz^2 &= YZI(z)
\end{align*}
\]

Using similarity transformation (11) that verifies the condition (12) matrices \( ZY \) and \( YZ \) are diagonal [14].

\[
\begin{align*}
V(z) &= T_v V_m(z) \\
I(z) &= T_i I_m(z)
\end{align*}
\]

\[
T_v^T T_i = T_i^T T_v = I_n
\]

This yields to decoupled modes of the system (10) and gives (13) and (14).

\[
\begin{align*}
\frac{d^2}{dz^2}V_m(z) &= (T_v^{-1} Z Y T_i) V_m(z) = \gamma^2 V_m(z) \\
\frac{d^2}{dz^2}I_m(z) &= (T_i^{-1} Y Z T_v) I_m(z) = \gamma^2 I_m(z)
\end{align*}
\]

With:

\[
\gamma^2 = T_v^{-1}Z YT_v = T_i^{-1} Y Z T_i
\]

Thus, the solutions governing the mode voltages and currents in (13) are decoupled and are given by (15).

\[
\begin{align*}
V_m(z) &= e^{-\gamma z} V_m^+ + e^{+\gamma z} V_m^- \\
I_m(z) &= e^{-\gamma z} I_m^+ - e^{+\gamma z} I_m^-
\end{align*}
\]

Where \( \gamma \) is a diagonal \( n \times n \) matrix of propagation mode coefficients, \( V_m^\pm \) and \( I_m^\pm \) are \( n \times 1 \) undetermined
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vectors related to forward/backward travelling waves of modes.

Transforming back to the phasor voltages and currents via (11) gives the general solution to the MTL equations containing a total of \(4n\) undetermined constants in the \(n \times 1\) vectors \(V^+, V^-\), \(I^+, I^-\) as in (16).

\[
V (z) = T_v (e^{-\gamma z} V^+_m + e^{+\gamma z} V^-_m) \quad (16a)
\]

\[
I (z) = T_i (e^{-\gamma z} I^+_m + e^{+\gamma z} I^-_m) \quad (16b)
\]

Defining the characteristic impedance matrix (17), thereby reducing the number of undetermined constant to only \(2n\). Substituting (16b) into (9b) yields (18).

\[
Z_c = Y^{-1} T_i \gamma T_i^{-1}
\]

Finally, as for multiport, to evaluate the final solution which involves \(2n\) undetermined constants, we need \(2n\) additional constraint equations provided by the terminal conditions.

2.4 Reference handling and terminal condition

Herein the reasoning is only detailed for the sending end; the same approach can be undertaken for the receiving end.

First, the solution (18) is written in a compact form at the sending end of the system (19).

\[
\begin{bmatrix}
V_0 \\
I_0
\end{bmatrix} =
\begin{bmatrix}
A_0 & B_0 \\
C_0 & D_0
\end{bmatrix}
\begin{bmatrix}
I^+_m \\
I^-_m
\end{bmatrix} \quad (19)
\]

Two ways to manipulate reference are presented herein. The first way (Fig. 3a) consists in increasing the system size by creating new variables. A high impedance, which represents the vacuum between the remote reference and the terminal network, is added and creates a voltage point \(N\) where the sum of current is zero. Mathematically, this can be expressed by (20) and (21). These expressions are inserted in (19) to give (22).

\[
E_s = V_0 + Z_s I_0 - 1V_N \quad (20)
\]

\[
0 = Z_\infty \sum I_0 + V_N \quad (21)
\]

\[
\begin{bmatrix}
E_s \\
I_0
\end{bmatrix} =
\begin{bmatrix}
(A_0 + Z_s C_0) & (B_0 + Z_s D_0) \\
C_0 & D_0
\end{bmatrix}
\begin{bmatrix}
I^+_m \\
I^-_m
\end{bmatrix} - \begin{bmatrix}
1 \\
0
\end{bmatrix} V_N \quad (22)
\]

Where \(1_n\) is a \((1 \times n)\) row vector (23).

\[
1_n = \begin{bmatrix}
1 & 1 & \ldots & 1_n
\end{bmatrix} \quad (23)
\]

An impedance value \(Z_\infty\) of some hundred M\(\Omega\)'s is sufficient to have a very similar vacuum situation.

In the second way (Fig. 3b) no increase in system size is necessary, only translation of the reference is sufficient. If a conductor \(k\) is chosen as a local reference, then we subtract from all rows, in \(V_0\) and in the product \(Z_c I_0\), the row corresponding to conductor \((k)\), and then, we replace the latter by a new row expressing the null sum of all currents.

Then, for each conductor \(i\) excepting \(k\) we can write (24).

\[
E_s(i) - E_s(k) = V_0(i) - V_0(k) + Z_s(i) I_0(i) - Z_s(k) I_0(k) \quad (24)
\]

Its compact form is (25).

\[
E'_s = V_0 + (Z'_s - I_0) \quad (25)
\]

For the \(k^{th}\) conductor (reference) equation (24) is zero and will be replaced by the null sum of current (26).

\[
\sum I_0 = 0 \quad (26)
\]

Finally we obtain (27) for the starting end.

\[
\begin{bmatrix}
E'_s \\
I_0
\end{bmatrix} =
\begin{bmatrix}
(A'_0 + (Z_s C_0))' & (B'_0 + (Z_s D_0))' \\
C_0 & D_0
\end{bmatrix}
\begin{bmatrix}
I^+_m \\
I^-_m
\end{bmatrix} \quad (27)
\]

Where a matrix \(X'\) is a matrix with \((n - 1)\) row defined in (24) and (25).

Forming systems (22) or (27) for sending and receiving ends, and combining adequate terminal equations, the solution gives the unknown modal currents, \(I^+_m\) and \(I^-_m\), to be used in (18) to get the voltage and current throughout the lines.
3 VALIDATION OF THE APPROACH

In this section, the capability and the limits of such approach are tested by comparing its results to those of known situations, either by theory or through measurements.

3.1 Impedances prediction

The test setup is shown in Fig. 4a. A large copper ground plane (1m × 1m × 0.001m) with two cooper conductors of 0.35mm radius and 1m length above. They are supported by Styrofoam blocks at heights respectively 1.0 cm and 5.0 cm. Conductor 1 is straight while conductor 2 can be bent by 90° at its middle. The equivalent relative permittivity of the surrounding medium is taken close to that of the vacuum, 1.25. A network analyzer is used to give the impedances over a wide frequency band (100 kHz to 500 MHz) via the S-parameters measurements. In simulation process, the system is decomposed into three connected MTL problems as shown in Fig. 4b. The ground plane is divided into 200 bars of 5mm width and 1mm thickness.

At each operating frequency and for each MTL part, systems (22) or (27) are established with specified terminal conditions and solved for \( I^{\pm} \). Actual currents and voltages are then evaluated using (18), the input mutual impedance \( Z_{ij} \) between two ports \( i \) and \( j \) is then the ratio \( V_i/I_j \) at origin coordinates. We note here that, for the adopted piecewise-constant approximation, the transformation matrix \( T_i \) in (11) is frequency independent. Particular attention is made when connecting models for terminal parts; input impedances for some parts become terminal impedances for other parts. For example in Fig. 4b, input impedances of parts MTL-2 and MTL-3 become terminal impedances for MTL-1.

Through input impedances prediction, simulation results shown in Fig. 5 and Fig. 6, show good agreement with measurement, both in appearance and in magnitude. Of course, this is only valid for configurations with bends close to 0 or 90 degrees, in other words, only for configurations that can be decomposed into quasi TEM sections.

3.2 Current distribution

Contrary to image modeling concept, this approach also gives the current distribution in the ground plane, which is very important for local EM field calculations.

For the setup in Fig. 4b, each ground is divided into 200 bars; terminal resistances are 0.5 Ohm, and 10 volts voltage source. Fig. 7 shows the current distribution over the entire ground plane at two-spaced frequency, 50 Hz and 5 MHz.

Qualitatively this agrees well with theory, which asserts that at low frequency, return currents spread in the entire return conductor domain, Fig. 7a, but at high frequency, return currents will concentrate more beneath associated
3.3 Shading effect – Equivalent reduced ground plane

In the case of a reference conductor being a large ground plane, each current returning in the ground plane will be concentrated, in an area as a shade, beneath the “going” conductor as illustrated in Fig. 8 and Fig. 9. The transverse spreading of this shading area depends on various parameters, of which the position (horizontal and vertical) of driver conductor, the operating frequency, the ground plane thickness, etc. Fig. 8 shows horizontal position effect where Fig. 9 shows frequency excitation effect on the currents spreading.

The distribution of these currents is used to determine the rectangular cross section of an equivalent ground plane conductor – shading area - that has, nearly, the same distribution, and thus, will give roughly the same effects (behav-
The term “nearly” means an adopted approximation with respect to the maximum distribution in a given situation. For example, if we adopt an approximation of 90% of the maximum value, all ground area that corresponds to values less than 10% of the maximum distribution will be removed. Fig. 10 shows how delimiting the equivalent “shading area” and the similarity between entire and shading current distributions is quite similar.

Depending on approximation degree, the current which was under removed parts should then be redistributed in such a fashion that the current density is larger at the edges of the shaded area (Fig. 10). Poor approximations greatly affect and distort local behaviors; high approximations preserve behaviors but reduce less the area study.

Figure 11 shows the width variation of the equivalent ground plane, “shading area”, when moving horizontally the driver conductor from one edge to the other. It shows that the shading width increases in the vicinity of edges, then, remains relatively constant between them; this constant value can be adopted for all horizontal positions. It shows also that when increasing conductor height, the shading width rapidly increases.

For middle horizontal position, Fig. 12 shows that the shading width “exponentially” decreases when frequency increases. While Fig. 13 shows that the shaded area width “parabolically” increases with the conductor height.

It should be noted that for a given situation of conductors above ground plane, the shading area is also valid for all situations in which frequencies are higher and heights are less. Since the model is linear, the shading for multiple conductors is the superposition of each conductor shade.

3.4 Influence upon simulation performances

For the case of two straight conductors, Fig. 14 shows a 20 cm reduced ground plane equivalent to previous original ground plane in Fig. 4a. This reduced one reproduces, at least, 90% of the original behavior for all frequencies upper than 10 kHz and conductors heights lower than 5 cm. Herein, the simple example of two straight conductors is tested. All proper input impedances, measured or simulated, are quite similar and are not presented here, slight differences are observed only for the mutual ones. Fig. 15 shows a comparison between the measured input mutual impedance for both the entire ground and the reduced one and the simulation results in the case of open terminations. A slight increase of the mutual impedance, relative to entire case, is due to decrease, by truncation, in coupling paths.
For this example, Table I shows, for a unique frequency, a comparison of some simulation performances between the two cases: the entire ground plane and the reduced one, using a processor Intel® Core™ 2 Quad CPU Q9650 3GHz. Since the number of unknowns is twice the number of elements, it is evident that the consumed time, calculations and solving, of the reduced model in this example is only 3.43% of the counterpart of the entire model. This is a high gains related to a simple example. The simplicity does not diminish the relevance of the concept for more complex situations.

### Table 1. Comparison of simulation costs

<table>
<thead>
<tr>
<th></th>
<th>Full model</th>
<th>Reduced model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground plane Size</td>
<td>1000x1000x1</td>
<td>200x1000x1</td>
</tr>
<tr>
<td>(Width-Length-Thickness)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Element Size [mm]</td>
<td>1x1000x1</td>
<td>1x1000x1</td>
</tr>
<tr>
<td>Number of elements</td>
<td>1002</td>
<td>202</td>
</tr>
<tr>
<td>(bars)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elapsed Time [sec]</td>
<td>21.85</td>
<td>0.75</td>
</tr>
<tr>
<td>Time Ratio</td>
<td>0.0343</td>
<td></td>
</tr>
</tbody>
</table>

Using these results, it is possible to conclude that the determination of the equivalent reduced rectangular conductor to a given ground plane depends on several parameters: the desired degree of approximation, the original ground plane size, the position and size of driver conductors, the operating frequency, etc. To compile all these parameters in a same relationship is a tricky task that is under future investigation. Nevertheless such a concept can be used before any 3D full-wave study, this can provides rational description of the studied field.

### 4 CONCLUSION

In this paper, an MTL based-approach for modeling electromagnetic coupling in wire-ground structures was presented. The fundamental assumption associated with this approach is that the structure must be composed of quasi-TEM parts. This assumption appears to be a weakness for the approach, but most closely wire-ground structures can be decomposed into typically related TEM parts. The metallic environment, ground plane, chassis... etc.; was considered as a certain distribution of sub-conductors, which allowed not only access to global quantities as input impedances, but also to local quantities as current distributions which are essential for any electromagnetic field mapping. The electromagnetic analysis of local quantities has shown that it is not required to take into account the entire metallic environment; only a well-chosen restricted part will be sufficient. The shading concept was presented and has allowed a convenient way to choose this restricted part, which has significantly improved treatment costs with very similar behavior. Using this approach, a good agreement between measurements and simulation is observed.

The simplicity of the examples discussed in this paper does not diminish the generalization of the approach.
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REFERENCES


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