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Note

Green's Function for the Classical System

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Bogoliubov and Sadovnikov[1] were the first to introduce the Green's function formalism into the statistical mechanics of classical systems. The aim of this work is to present a slightly different approach of its introduction and to describe the decoupling of Green's function hierarchy by the moment conserving method.

With the formal approach[1] ($\hbar \rightarrow 0$) from quantum mechanical description to the classical one the following spectral representation can be obtained:

$$<[A(t),\ B(t')]> = -\frac{i}{KT}\int_{-\infty}^{+\infty} I(\omega)\, e^{-i\omega(t-t')}\, d\omega \tag{1}$$

$$\ll A; B \gg_E^{r,\,a} = \frac{1}{2\pi KT}\int_{-\infty}^{+\infty} I(\omega)\,\omega\, \frac{d\omega}{E-\omega \pm i\varepsilon} \tag{2}$$

The symbols and the notation are the conventional ones[1,2]. The equation of motion for the dynamic quantity

$$A(t) = A\{q_1(t) \ldots q_N(t);\ p_1(t) \ldots p_N(t)\}$$

is

$$\frac{dA}{dt} = [A, H] \tag{3}$$

where [ ] denotes a Poisson bracket. In analogy with the quantum mechanical approach the following equation results:

$$\frac{dG(t, t')}{dt} = \delta(t-t') <[A(t), B(t')]> + \ll [A, H]; B \gg \tag{4}$$

or with

$$G(t, t') = \int_{-\infty}^{+\infty} G(E)\, e^{-iE(t-t')}\, dE$$

$$-iE \ll A; B \gg_E = \frac{1}{2\pi} <[A, B]> + \ll [A, H]; B \gg_E \tag{5}$$

The evaluation of the right side of eq. (5) gives

$$\ll A; B \gg_E = \frac{1}{2\pi}\left\{\frac{i <[A, B]>}{E} + \frac{i^2}{E^2} <[[A, H], B]> + \right.$$

$$\left. + \frac{i^3}{E^3} <[[[A, H], H], B]> + \ldots \right\} \tag{6}$$

With the notation

$$M_0 = \langle [A, B] \rangle \qquad M_1 = \langle [[A, H], B] \rangle \qquad etc.$$

the eq. (5) takes the form

$$\langle\langle A; B \rangle\rangle_E = \frac{1}{2\pi} \left\{ \frac{i}{E} M_0 + \frac{i^2}{E^2} M_1 + \frac{i^3}{E^3} M_2 + \ldots \right\} \tag{7}$$

The moments M_n at time $t = t'$ can be generated from eq. (1)

$$M_0 = \langle [A, B] \rangle = -\frac{i}{KT} \int_{-\infty}^{+\infty} I(\omega)\, \omega\, d\omega$$

by differentiation. Thus

$$M_n = \frac{(-i)^{n+1}}{KT} \int_{-\infty}^{+\infty} I(\omega)\, \omega^{n+1}\, d\omega$$

Introducing

$$m_n = \int_{-\infty}^{+\infty} I(\omega)\, \omega^{n+1}\, d\omega$$

the eq. (7) takes the form

$$\langle\langle A; B \rangle\rangle_E = \frac{1}{2\pi KT} \left[\frac{m_0}{E} + \frac{m_1}{E^2} + \ldots + \frac{m_n}{E^{n+1}} + \ldots \right] \tag{8}$$

The procedure for decoupling eq. (8) known[3] as the moment conserving method *via* Pade approximants can be used. To show the utility of this approach we used the Pade approximant [2,1] known as the geometric approximation. The moments m_1, m_3 of the transform of the correlation function are known[4,5]

$$m_1 = \frac{k^2}{m \beta}$$

and

$$m_3 = 3 \frac{k^4}{m^2 \beta^4} + \frac{k^2 n}{m^2 \beta} \int \frac{\partial^2 V}{\partial z^2} g(r)(1 - \cos kz)\, dr = \frac{k^2}{\beta m} \left(3 \frac{k^2}{m \beta} + W_k^2 \right)$$

The index k determines the *quasi*-descrete levels and was previously omitted from eq. (4).

As an example we want to describe the geometric approximation as applied to plasma oscillation. W_k^2 in this case is the plasma frequency[5] $W_k^2 = \frac{4 \pi^2 l^2 n}{m}$. The geometric approximation gives the dispersion relation:

$$E(k) = \sqrt{W_k^2 + \frac{3 k^2}{m \beta}}$$

which is a well known result first obtained by Vlasov[6]. The above described procedure can be of some use in the study of liquid state to discuss the problem as collective motion in it.

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IZVLEČEK

Greenove funkcije klasičnega sistema delcev

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V delu so zapisane gibalne enačbe Greenovih funkcij za klasični sistem delcev. Z uporabo momentov in Padejevo aproksimacijo je izračunana energijska disperzijska relacija za geometrijski približek razklopitve sistema Greenovih funkcij.

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