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An elementary model of density distribution, thermohaline circulation and quasigeostrophic flow in land-locked seas

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Density distribution and currents generated by the surface and coastal buoyancy flux in a land-locked basin are considered. A simple conceptual model is developed for the case when the surface buoyancy loss (gain) is locally balanced by the coastal buoyancy gain (loss). The model predicts cross-shore density gradient, coast-to-surface directed hydraulic flow, and thermohaline circulation characterized by upwelling (downwelling) along the coasts and downwelling (upwelling) prevailing over the greater part of the basin. Due to deflecting influence of the Coriolis force, long-shore currents appear as well, both barotropic (related to the hydraulic effect) and baroclinic (connected with the thermohaline circulation). The model shows some similarity with the winter residual dynamics of the Adriatic Sea. In particular, it predicts surface cyclonic circulation under the surface buoyancy loss that balances coastal buoyancy gain, in fair agreement with the observations. It is stressed, however, that the primary purpose of the model is didactic, as it bridges the gap between qualitative interpretations of the buoyancy-driven processes in inland seas and complex models which allow for more sophisticated parameterizations of friction, nonlinear coupling of differently generated phenomena, joint effects of baroclinicity and relief, and/or spatial variability of buoyancy forcing.

Keywords: Seawater density, thermohaline circulation, quasigeostrophic flow, land-locked seas.

Jednostavan model razdiobe gustoće, termohaline cirkulacije i kvazigeostrofičkog strujanja u okrajnjim morima

U članku se razmatra razdioba gustoće i struje što ih u okrajnjim bazenima uzrokuju površinski i obalni protoci vlage i topline. Razvijen je jednostavan konceptualni model za slučaj kad su površinski protoci lokalno uravnoteženi obalnim protocima. Model predviđa gradijent gustoće okomit na obale, hidraulički tok usmjeren od obala prema površini mora, kao i termohalinu cirkulaciju koja uključuje uzlazno (silazno) strujanje uz obale te silazno (uzlazno) strujanje u većem dijelu bazena. Uslijed otklanjajućeg djelovanja

Coriolisove sile javljaju se i dužobalne struje, kako barotropne (vezane uz hidraulički efekt) tako i barokline (povezane s termohalinom cirkulacijom). Teorijski rezultati pokazuju sličnost sa zimskom rezidulanom dinamikom Jadranskog mora. Tako npr. model predviđa površinsku ciklonalnu cirkulaciju, uzrokovanu površinskim gubitkom vlage i topline i odgovarajućim obalnim primitkom, što je u suglasju s opažanjima. Međutim, u članku je naglašeno da je prvenstvena namjena modela didaktička, budući da on premošćuje jaz između kvalitativnih objašnjenja termohalinih procesa u okrajnjim morima te složenih modela koji uvažavaju bolje parametrizacije trenja, nelinearno međudjelovanje različitih pojava, zajedničke efekte baroklinosti i topografije, i/ili prostorne promjene vanjskih utjecaja.

1. Introduction

Recently, I began to explore simple conceptual models which would enable senior undergraduate or beginning graduate students to get acquainted with response of the sea to the buoyancy forcing. In a previous paper seasonal variability related to the buoyancy flux across the sea surface has been considered (Orlić, 1993). The present paper addresses density distribution and circulation generated in a land-locked basin by both the surface and coastal buoyancy forcing.

More specifically, the paper focuses on cyclonic surface circulation which is usually observed in inland seas of the northern hemisphere. According to some early authors, it could be interpreted in terms of the river discharge and deflecting influence of the Coriolis force (Shtokman, 1967). Later on, a number of alternative mechanisms have been proposed in the literature (see Schwab et al., 1995, and references cited therein). Yet, the original interpretation is still considered valid for some basins: *e. g.* in the Adriatic Sea horizontal density gradients, generated by buoyancy loss from the sea surface on one hand and coastal fresh-water inflow on the other, are observed in winter and are deemed responsible for the counterclockwise surface circulation (Buljan and Zore-Armanda, 1976). Hendershott and Rizzoli (1976) developed a numerical model by allowing for both the wind and buoyancy forcing of the Adriatic, and showed that the latter is more important for the observed winter conditions than the former.

The prevalence of thermohaline residual dynamics in the Adriatic Sea during winter implies that an elementary model of buoyancy-driven processes in a land-locked basin might not only have a certain didactic value but may reproduce some real-world phenomena as well. A model that bridges the gap between the early interpretations which were not supported by formal means and a rather sophisticated numerical simulations performed by Hendershott and Rizzoli (1976) is therefore constructed. It rests on a number of assumptions, which render an analytical solution possible and thus enable the model to be easily compared with observations, but also pave the way to the existing and forthcoming models of greater complexity.

2. Model

A transverse section positioned in central part of an elongated basin of constant width (2b) and depth (H) is considered. Axes of the coordinate system are placed long-shore (x), cross-shore (y) and along the vertical (z), so that the domain of interest is defined by $-b \le y \le b$ and $-H \le z \le 0$ (Figure 1). Response of the sea to the spatially uniform and temporally invariant fluxes of heat and water across the sea surface and coast is analyzed. Assuming that there is no long-shore variability, the steady-state equations governing the motion, pressure and density fields may be written (*e. g.* Pond and Pickard, 1983; Apel, 1987):

$$v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - fv = F_x \tag{1a}$$

$$v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} + F_y \tag{1b}$$

$$v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + F_z \tag{1c}$$

$$v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z} + \rho\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \tag{1d}$$

$$\rho = \rho_o\,(1 - \alpha\, T + \beta\, S + \gamma\, p) \tag{1e}$$

$$v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = K_y \frac{\partial^2 T}{\partial y^2} + K_z \frac{\partial^2 T}{\partial z^2} \tag{1f}$$

$$v \frac{\partial S}{\partial y} + w \frac{\partial S}{\partial z} = M_y \frac{\partial^2 S}{\partial y^2} + M_z \frac{\partial^2 S}{\partial z^2} \tag{1g}$$

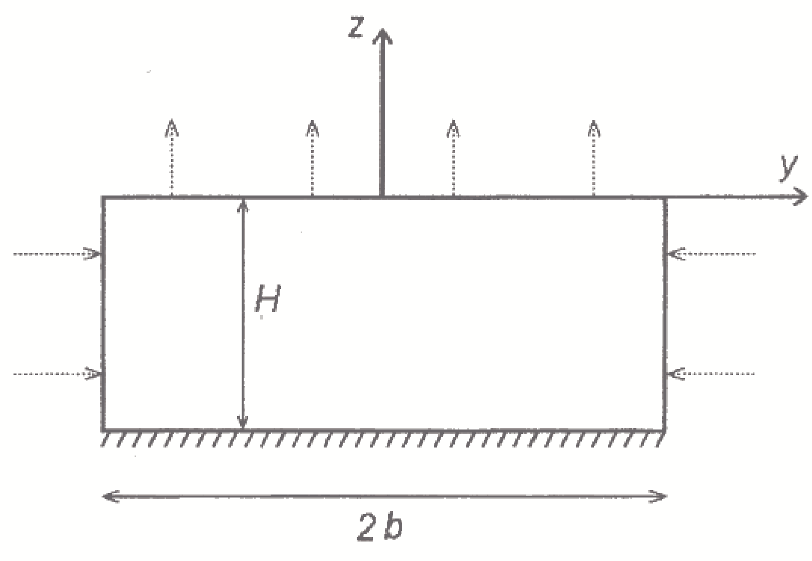

Figure 1. Central transverse section of the elongated basin modeled. Dotted arrows indicate surface buoyancy loss and coastal buoyancy gain.

where u, v and w are velocity components aligned with the x, y and z axis, respectively, p is pressure, ρ density, ρ_o reference density, T temperature, S salinity, f the Coriolis parameter (regarded as invariable), g acceleration due to gravity, F_i ($i = x, y, z$) are components of the frictional force, α, β and γ are coefficients of expansion/contraction, whereas K_i and M_i ($i = y, z$) are constant coefficients of eddy heat and salt exchange, respectively. The equation of state has been linearized.

All the dependent variables may be split into two parts, one representing the mean state that would occur in the absence of external forcing, the other standing for perturbations due to buoyancy driving:

$$\begin{aligned} u &= \bar{u} + u' = u' \\ v &= \bar{v} + v' = v' \\ w &= \bar{w} + w' = w' \\ p &= \bar{p} + p' \\ \rho &= \bar{\rho} + \rho' \\ T &= \bar{T} + T' \\ S &= \bar{S} + S'. \end{aligned} \tag{2}$$

From (1e) it then follows:

$$\bar{\rho} = \rho_o (1 - \alpha\, \bar{T} + \beta\, \bar{S} + \gamma\, \bar{p}) \tag{3a}$$

$$\rho' = \rho_o (-\alpha\, T' + \beta\, S' + \gamma\, p'). \tag{3b}$$

As $\rho_o \approx 10^3$ kg m^{-3}, $\alpha = 2 \times 10^{-4}$ °C^{-1}, $\beta = 8 \times 10^{-4}$ psu^{-1}, $\gamma = 4 \times 10^{-10}$ Pa^{-1}, for typical values of temperature (10–20 °C), salinity ($\approx$ 35 psu) and pressure ($\approx 10^6$ Pa) in the seas, (3) implies $\rho' \ll \bar{\rho} \approx \rho_o$.

The mean state is thus modeled by:

$$0 = -\frac{1}{\rho_o}\frac{\partial \bar{p}}{\partial y} \tag{4a}$$

$$0 = -\frac{1}{\rho_o}\frac{\partial \bar{p}}{\partial z} - g \tag{4b}$$

$$0 = K_y \frac{\partial^2 \bar{T}}{\partial y^2} + K_z \frac{\partial^2 \bar{T}}{\partial z^2} \tag{4c}$$

$$0 = M_y \frac{\partial^2 \bar{S}}{\partial y^2} + M_z \frac{\partial^2 \bar{S}}{\partial z^2} \tag{4d}$$

and hence the hydrostatic equation ($\bar{p} = p_a - g\rho_0 z$, where p_a is the atmospheric pressure) and constant values for both $\bar{T}$ and $\bar{S}$ are obtained.

The perturbations are captured by:

$$v\,\frac{\partial u}{\partial y} + w\,\frac{\partial u}{\partial z} - fv = F_x \tag{5a}$$

$$v\,\frac{\partial v}{\partial y} + w\,\frac{\partial v}{\partial z} + fu = -\frac{1}{\rho_o}\frac{\partial p'}{\partial y} + F_y \tag{5b}$$

$$v\,\frac{\partial w}{\partial y} + w\,\frac{\partial w}{\partial z} = -\frac{1}{\rho_o}\frac{\partial p'}{\partial z} + g\,\alpha\,T' - g\,\beta\,S' + F_z \tag{5c}$$

$$-\alpha\,v\,\frac{\partial T'}{\partial y} + \beta\,v\,\frac{\partial S'}{\partial y} - \alpha\,w\,\frac{\partial T'}{\partial z} + \beta\,w\,\frac{\partial S'}{\partial z} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{5d}$$

$$v\,\frac{\partial T'}{\partial y} + w\,\frac{\partial T'}{\partial z} = K_y\,\frac{\partial^2 T'}{\partial y^2} + K_z\,\frac{\partial^2 T'}{\partial z^2} \tag{5e}$$

$$v\,\frac{\partial S'}{\partial y} + w\,\frac{\partial S'}{\partial z} = M_y\,\frac{\partial^2 S'}{\partial y^2} + M_z\,\frac{\partial^2 S'}{\partial z^2} \tag{5f}$$

where it has been recognized that g = 9.81 m s^{-2} and $H \approx 100$ m and consequently $\gamma g \rho_o H \ll 1$. A scaling analysis indicates that the last two terms on the left-hand side of (5d) predominate. Moreover, putting:

$$\delta = -\rho_o\,(\alpha\,T' - \beta\,S') \tag{6}$$

and assuming $K_i = M_i = N_i$ ($i = y, z$), it follows that (5e) and (5f) may be compressed into an equation for density anomaly δ (first derived by Lineykin, 1955). Thus:

$$v\,\frac{\partial u}{\partial y} + w\,\frac{\partial u}{\partial z} - fv = F_x \tag{7a}$$

$$v\,\frac{\partial v}{\partial y} + w\,\frac{\partial v}{\partial z} + fu = -\frac{1}{\rho_o}\frac{\partial p'}{\partial y} + F_y \tag{7b}$$

$$v\,\frac{\partial w}{\partial y} + w\,\frac{\partial w}{\partial z} = -\frac{1}{\rho_o}\frac{\partial p'}{\partial z} - \frac{g}{\rho_o}\,\delta + F_z \tag{7c}$$

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{7d}$$

$$v\,\frac{\partial \delta}{\partial y} + w\,\frac{\partial \delta}{\partial z} = N_y\,\frac{\partial^2 \delta}{\partial y^2} + N_z\,\frac{\partial^2 \delta}{\partial z^2}\,. \tag{7e}$$

Linearization of the above equations results in:

$$-fv = F_x \tag{8a}$$

$$fu = -\frac{1}{\rho_o}\frac{\partial p'}{\partial y} + F_y \tag{8b}$$

$$0 = -\frac{1}{\rho_o}\frac{\partial p'}{\partial z} - \frac{g}{\rho_o}\delta + F_z \tag{8c}$$

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{8d}$$

$$0 = N_y \frac{\partial^2 \delta}{\partial y^2} + N_z \frac{\partial^2 \delta}{\partial z^2}\,. \tag{8e}$$

Advection of momentum and (especially) density may well be important for dynamics of a coastal basin. However, as emphasized by Salmon (1986) in a somewhat different context, the value of a nonlinear theory cannot be properly assessed without a complete understanding of the linear results. Linearization thus conforms to the proclaimed didactic purpose of the model, leading to an explicit solution which – as will transpire from the paper – resembles findings of a nonlinear study. Equation of continuity (8d) enables streamfunction ψ to be introduced:

$$v = -\frac{\partial \psi}{\partial z}, \quad w = \frac{\partial \psi}{\partial y}. \tag{9a,b}$$

Cross-differentiation of the equations of motion (8b) and (8c) gives:

$$\frac{\partial F_y}{\partial z} - \frac{\partial F_z}{\partial y} - f\frac{\partial u}{\partial z} = -\frac{g}{\rho_o}\frac{\partial \delta}{\partial y}. \tag{10}$$

Frictional forces are parameterized as first suggested by Guldberg and Mohn (1876, 1880):

$$F_x = -Ku, \quad F_y = -Kv, \quad F_z = -Kw, \tag{11a, b, c}$$

where K is a constant decay coefficient. Such a parameterization is certainly artificial. As pointed out by Salmon (1986), however, other approximations have their shortcomings too, and consequently one should discount any property of a solution that depends very sensitively on the particular choice of parameterization. The formulae used here greatly simplify the arithmetic, while – as will be shown – providing results which basically agree with those obtained via another approach. The equations to be solved finally read:

$$0 = N_y \frac{\partial^2 \delta}{\partial y^2} + N_z \frac{\partial^2 \delta}{\partial z^2} \tag{12a}$$

$$K^2 \frac{\partial^2 \psi}{\partial y^2} + (K^2 + f^2) \frac{\partial^2 \psi}{\partial z^2} = -\frac{gK}{\rho_o} \frac{\partial \delta}{\partial y} \tag{12b}$$

$$u = -\frac{f}{K} \frac{\partial \psi}{\partial z} . \tag{12c}$$

These equations enable density anomaly δ, streamfunction ψ and long-shore speed u to be determined in turn. From (8b) and (8c) the perturbed pressure field may then be obtained. Boundary conditions for density anomaly are given by:

$$\frac{\partial \delta}{\partial y}(y = -b) = -\rho_o\, \alpha \left(\frac{\partial T'}{\partial y}\right)_{-b} + \rho_o\, \beta \left(\frac{\partial S'}{\partial y}\right)_{-b} = \frac{\rho_o R}{N_y} (\alpha\, \Delta T + \beta\, S_o) \tag{13a}$$

$$\frac{\partial \delta}{\partial y}(y = b) = -\rho_o\, \alpha \left(\frac{\partial T'}{\partial y}\right)_{b} + \rho_o\, \beta \left(\frac{\partial S'}{\partial y}\right)_{b} = -\frac{\rho_o R}{N_y} (\alpha\, \Delta T + \beta\, S_o) \tag{13b}$$

$$\frac{\partial \delta}{\partial z}(z = -H) = -\rho_o\, \alpha \left(\frac{\partial T'}{\partial z}\right)_{-H} + \rho_o\, \beta \left(\frac{\partial S'}{\partial z}\right)_{-H} = 0 \tag{13c}$$

$$\frac{\partial \delta}{\partial z}(z = 0) = -\rho_o\, \alpha \left(\frac{\partial T'}{\partial z}\right)_{0} + \rho_o\, \beta \left(\frac{\partial S'}{\partial z}\right)_{0} = \frac{\rho_o}{N_z} \left[-\alpha \frac{Q_s}{\rho_o c_p} + \beta S_o (E - P) \right] \tag{13d}$$

where ΔT is temperature difference between the river water and the sea, S_o a mean salinity, R the rate of river discharge, $E - P$ the evaporation/precipitation difference, Q_s the surface heat flux into the sea, whereas c_p is the specific heat at constant pressure. Boundary conditions for the streamfunction imply:

$$\psi\, (y = -b) = -R\, (z + H) \tag{14a}$$

$$\psi\, (y = b) = R\, (z + H) \tag{14b}$$

$$\psi\, (z = -H) = 0 \tag{14c}$$

$$\psi\, (z = 0) = (E - P)\, y \tag{14d}$$

where it has been taken into account that R is constant along the vertical and $E-P$ across the basin, and that $(E-P)\,b = RH$ is valid in accordance with the supposed long-basin uniformity. Kinematic boundary condition at the sea surface has been linearized.

3. Density field

Introducing a new variable:

$$\tilde{y} = y\sqrt{\frac{N_z}{N_y}}$$

equation (12a) may be transformed into the Laplace equation:

$$\frac{\partial^2\delta}{\partial\tilde{y}^2} + \frac{\partial^2\delta}{\partial z^2} = 0$$

which has to be solved with boundary conditions stemming from (13):

$$\frac{\partial\delta}{\partial\tilde{y}}\left(\tilde{y} = -b\sqrt{\frac{N_z}{N_y}}\right) = \frac{\rho_0 R}{\sqrt{N_y N_z}}(\alpha\,\Delta T + \beta\,S_0)$$

$$\frac{\partial\delta}{\partial\tilde{y}}\left(\tilde{y} = b\sqrt{\frac{N_z}{N_y}}\right) = -\frac{\rho_0 R}{\sqrt{N_y N_z}}(\alpha\,\Delta T + \beta\,S_0)$$

$$\frac{\partial\delta}{\partial z}(z = -H) = 0$$

$$\frac{\partial\delta}{\partial z}(z = 0) = \frac{\rho_0}{N_z}\left[-\alpha\frac{Q_{S'}}{\rho_0\,c_p} + \beta\,S_0\,(E-P)\right].$$

The Neumann problem thus posed is solvable if:

$$\left[-\alpha\frac{Q_s}{\rho_0\,c_p} + \beta\,S_0\,(E-P)\right]b = R\,(\alpha\,\Delta T + \beta S_0)\,H$$

which – due to the previously assumed local water balance, $(E-P)\,b = RH$ – implies:

$$-\frac{Q_s}{\rho_0\,c_p}\,b = R\,\Delta T\,H.$$

Trying:

$$\delta = A\tilde{y}^2 + b(z+H)^2 + C$$

where A, B and C denote arbitrary constants, one gets from the Laplace equation $A = -B$. Coastal boundary conditions then give:

$$B = \frac{\rho_0 R}{2\, b\, N_z} (\alpha\, \Delta T + \beta\, S_o).$$

The trial solution obviously satisfies the surface and bottom boundary conditions. Recovering the original cross-basin coordinate, one arrives at:

$$\delta = \frac{\rho_0 R}{2\, b} (\alpha\, \Delta T + \beta\, S_o) \left[\frac{(z+H)^2}{N_z} - \frac{y^2}{N_y} \right] + C.$$

Since:

$$\frac{1}{2\, b\, H} \int_{-H}^{0} \int_{-b}^{b} \delta \; \mathrm{d}y \; \mathrm{d}z = 0$$

the solution finally reads:

$$\delta(y,z) = \frac{\rho_0 R}{2\, b} (\alpha\, \Delta T + \beta\, S_o) \left[\frac{(z+H)^2}{N_z} - \frac{H^2}{3N_z} - \frac{y^2}{N_y} + \frac{b^2}{3N_y} \right]. \qquad (15)$$

It reduces to a simple form if there is no heat flux across boundary surfaces ($Q_s = \Delta T = 0$). Then:

$$S'(y,z) = \frac{(E-P)\, S_0}{N_z} \left[\frac{(z+H)^2}{2H} - \frac{H}{6} \right] - \frac{R\, S_0}{N_y} \left(\frac{y^2}{2b} - \frac{b}{6} \right). \qquad (16)$$

The solution represents salinity field controlled by the water flux across the sea surface and coasts, under a purely diffusive regime.

In order to illustrate solution (16), typical values for various parameters are chosen: H = 100 m, $b = 10^5$ m, $E - P = 10^{-8}$ m s^{-1}, $R = 10^{-5}$ m s^{-1}, S_o = 35 psu, N_y = 10 m^2 s^{-1}, and $N_z = 10^{-2}$ m^2 s^{-1}. Eddy diffusion coefficients are the same as found by Fischer (1980) on the Atlantic continental shelf, under dynamic conditions which resemble those prevailing in the Adriatic during winter. Other numerical values are order-of-magnitude estimates for the Adriatic Sea. The computed field of salinity anomaly is shown in Figure 2. It is similar to the winter salinity distribution in the Adriatic (Buljan and Zore-Armanda, 1976), in that an offshore directed salinity gradient of correct order of magnitude is found, and that vertical uniformity prevails. Of course, there are also some discrepancies, mostly related to the fact that river dis-

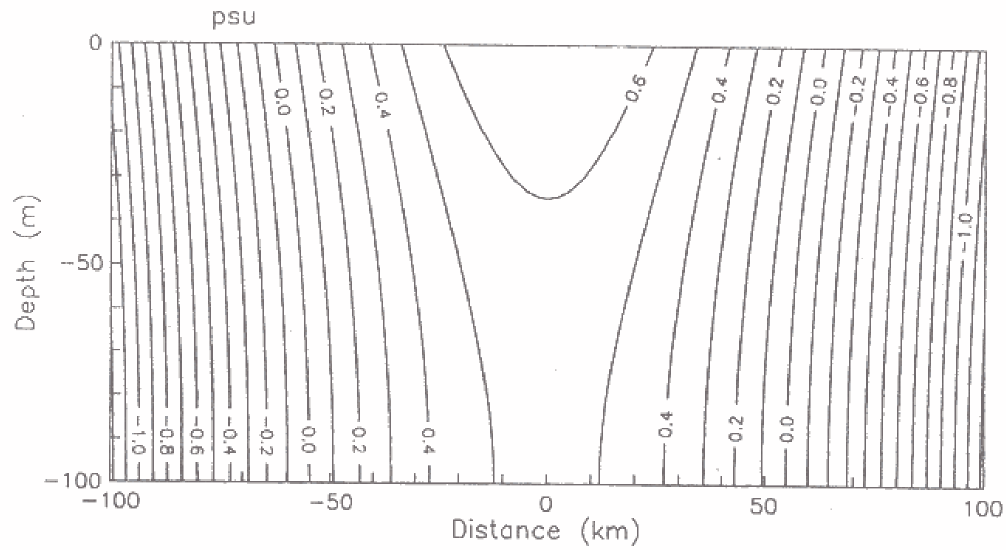

Figure 2. Modeled salinity anomaly, due to the coastal fresh-water inflows and upward water flux from the sea surface. Parameters assumed in constructing the figure are listed in the text.

charges are not evenly distributed over the coastal walls, and that they are not locally balanced by the upward water flux from the sea surface. Also unrealistic is mid-basin surface salinity maximum, which would most probably disappear if convection were allowed for. Neglect of advection by cross-shore flow seems acceptable, possibly because the process is indirectly taken into account through vigorous horizontal mixing (on the so-called shear diffusion phenomenon one may consult the pioneering laboratory and theoretical study by Taylor, 1954, and reviews of subsequent geophysical applications by Fischer, 1973, and Csanady, 1982).

4. Circulation

Substituting a transformed vertical coordinate:

$$\tilde{z} = z \sqrt{\frac{K^2}{K^2 + f^2}}$$

and recalling (15), equation (12b) becomes:

$$\frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial \tilde{z}^2} = A\, y$$

where:

$$A = \frac{g\, R}{K\, b\, N_y} (\alpha\, \Delta T + \beta\, S_o)$$

and which has to be solved subject to boundary conditions derived from (14):

$$\psi(y=-b) = -R\left(\tilde{z}\sqrt{\frac{K^2+f^2}{K^2}} + H\right)$$

$$\psi(y=b) = R\left(\tilde{z}\sqrt{\frac{K^2+f^2}{K^2}} + H\right)$$

$$\psi\left(\tilde{z} = -H\sqrt{\frac{K^2}{K^2+f^2}}\right) = 0$$

$$\psi(\tilde{z}=0) = (E-P)\,y.$$

Solution of the Poisson equation may be split into two parts:

$$\psi = \psi_1 + \psi_2$$

where:

$$\psi_1(y,\tilde{z}) = \frac{R}{b}\,y\left(\tilde{z}\sqrt{\frac{K^2+f^2}{K^2}} + H\right)$$

is solution of the corresponding Laplace equation with the boundary conditions given above, whereas ψ_2 is obtained by solving the original equation with streamfunction equalled to zero along boundary. Let it be shown that the latter is an odd function. Introducing:

$$\tilde{\psi}_2(y,\tilde{z}) = -\psi_2(-y,\tilde{z})$$

i. e. function which obviously satisfies homogeneous boundary conditions, one gets:

$$\frac{\partial^2\tilde{\psi}_2}{\partial y^2} + \frac{\partial^2\tilde{\psi}_2}{\partial \tilde{z}^2} = -\frac{\partial^2\psi_2}{\partial y^2}(-y,\tilde{z}) - \frac{\partial^2\psi_2}{\partial \tilde{z}^2}(-y,\tilde{z}) = -A(-y) = Ay.$$

Due to uniqueness of solution of the Poisson equation, it follows that:

$$\psi_2 = \tilde{\psi}_2, \quad \psi_2(0,\tilde{z}) = 0.$$

Consequently, solution may be sought on the transverse subsection defined by $[0, b] \times [-h, 0]$, the transformed bottom depth being:

$$h = H\sqrt{\frac{K^2}{K^2 + f^2}}.$$

Putting:

$$\psi_2 = \psi_P + \psi_G$$

where:

$$\psi_P(y, \tilde{z}) = \frac{A}{2}\, y\, \tilde{z}^2$$

is a particular solution, ψ_G has to be harmonic function subject to:

$$\psi_G(y = 0) = 0$$

$$\psi_G(y = b) = -\frac{A\,b}{2}\,\tilde{z}^2$$

$$\psi_G(\tilde{z} = -h) = -\frac{A\,h^2}{2}\,y$$

$$\psi_G(\tilde{z} = 0) = 0.$$

Splitting further:

$$\psi_G = \psi_{G1} + \psi_{G2}$$

where:

$$\psi_{G1}(y,\tilde{z}) = \frac{A}{2}\,h\,y\,\tilde{z}$$

it follows that ψ_{G2} is harmonic function constrained by:

$$\psi_{G2}(y = 0) = 0$$

$$\psi_{G2}(y = b) = -\frac{A\,b}{2}\,\tilde{z}^2 - \frac{A}{2}\,h\,b\,\tilde{z}$$

$$\psi_{G2}(\tilde{z} = -h) = 0$$

$$\psi_{G2}(\tilde{z} = 0) = 0.$$

Separation of variables yields:

$$\psi_{G2} = (a \operatorname{sh} \lambda y + b \operatorname{ch} \lambda y)(c \sin \lambda \tilde{z} + d \cos \lambda \tilde{z})$$

where a, b, c, d and λ are arbitrary constants. Application of three homogeneous boundary conditions gives:

$$\psi_{G2}(y, \tilde{z}) = \sum_{n=1}^{\infty} a_n \operatorname{sh}\left(\frac{n\pi}{h} y\right) \sin\left(\frac{n\pi}{h} \tilde{z}\right)$$

and the remaining boundary condition requires that:

$$\sum_{n=1}^{\infty} a_n \operatorname{sh}\left(\frac{n\pi}{h} b\right) \sin\left(\frac{n\pi}{h} \tilde{z}\right) = -\frac{A\,b}{2} \tilde{z}^2 - \frac{A}{2} h\, b\, \tilde{z}$$

wherefrom:

$$a_n = \frac{2\,A\,b\,h^2}{n^3 \pi^3 \operatorname{sh}\left(\frac{n\pi}{h} b\right)} [(-1)^n - 1] .$$

By collecting all the elements of solution and returning to the original vertical coordinate, one finally arrives at:

$$\psi(y, z) = \frac{R}{b} y\,(z + H) + \frac{A}{2} \frac{K^2}{K^2 + f^2} y z^2 + \frac{A}{2} H \frac{K^2}{K^2 + f^2} y z + \sum_{n=1}^{\infty} a_n \operatorname{sh}\left(\frac{n\pi}{H} \sqrt{\frac{K^2 + f^2}{K^2}}\, y\right) \sin\left(\frac{n\pi}{H} z\right) \tag{17a}$$

where, let it be repeated:

$$A = \frac{g\,R}{K\,b\,N_y} (\alpha\, \Delta T + \beta\, S_0) \tag{17b}$$

$$a_n = \frac{2A\,b\,H^2 K^2}{n^3 \pi^3 (K^2 + f^2) \operatorname{sh}\left(\frac{n\pi}{H} \sqrt{\frac{K^2 + f^2}{K^2}}\, b\right)} [(-1)^n - 1] . \tag{17c}$$

Separation of variables yields:

$$\psi_{G2} = (a \operatorname{sh} \lambda y + b \operatorname{ch} \lambda y)\,(c \sin \lambda \tilde{z} + d \cos \lambda \tilde{z})$$

where a, b, c, d and λ are arbitrary constants. Application of three homogeneous boundary conditions gives:

$$\psi_{G2}(y, \tilde{z}) = \sum_{n=1}^{\infty} a_n \operatorname{sh}\left(\frac{n\,\pi}{h}\,y\right) \sin\left(\frac{n\,\pi}{h}\,\tilde{z}\right)$$

and the remaining boundary condition requires that:

$$\sum_{n=1}^{\infty} a_n \operatorname{sh}\left(\frac{n\,\pi}{h}\,b\right) \sin\left(\frac{n\,\pi}{h}\,\tilde{z}\right) = -\frac{A\,b}{2}\,\tilde{z}^2 - \frac{A}{2}\,h\,b\,\tilde{z}$$

wherefrom:

$$a_n = \frac{2\,A\,b\,h^2}{n^3\,\pi^3 \operatorname{sh}\left(\frac{n\,\pi}{h}\,b\right)}\,[(-1)^n - 1]\,.$$

By collecting all the elements of solution and returning to the original vertical coordinate, one finally arrives at:

$$\psi(y, z) = \frac{R}{b}\,y\,(z + H) + \frac{A}{2}\,\frac{K^2}{K^2 + f^2}\,yz^2 + \frac{A}{2}\,H\,\frac{K^2}{K^2 + f^2}\,yz + \sum_{n=1}^{\infty} a_n \operatorname{sh}\left(\frac{n\,\pi}{H}\sqrt{\frac{K^2 + f^2}{K^2}}\,y\right) \sin\left(\frac{n\,\pi}{H}\,z\right) \tag{17a}$$

where, let it be repeated:

$$A = \frac{g\,R}{K\,b\,N_y}\,(\alpha\,\Delta T + \beta\,S_0) \tag{17b}$$

$$a_n = \frac{2A\,b\,H^2\,K^2}{n^3\,\pi^3\,(K^2 + f^2) \operatorname{sh}\left(\frac{n\,\pi}{H}\sqrt{\frac{K^2 + f^2}{K^2}}\,b\right)}\,[(-1)^n - 1]\,. \tag{17c}$$

This completes solution for the streamfunction. The solution simplifies in the case $Q_s = \Delta T = 0$, since then:

$$A = \frac{g\, R\, \beta\, S_o}{K\, b\, N_y}\,. \tag{18}$$

This special case is illustrated in Figure 3, with $f = 10^{-4}\ \mathrm{s}^{-1}$, $K = 10^{-5}\ \mathrm{s}^{-1}$, and the other parameters as already specified. The figure shows the first term on the right-hand side of (17a) – *i. e.* ψ_1 (Figure 3a), the remaining three terms – *i. e.* ψ_2 (Figure 3b), and their sum (Figure 3c). Obviously, ψ_1 represents hydraulic flow forced by coastal inflows and surface water loss, whereas ψ_2 in this case stands for purely haline circulation generated by coastal buoyancy sources and offshore buoyancy sink. As the hydraulic flow in the present example is small compared to the haline circulation, the resulting field is dominated by upwelling concentrated close to the coasts and compensated by downwelling which is distributed over the greater part of the basin.

Equations (12c) and (17a) yield:

$$u(y, z) = u_1 + u_2 + u_3 + u_4 \tag{19a}$$

where:

$$u_1 = -\frac{f\, R}{K\, b}\, y \tag{19b}$$

$$u_2 = -A\, \frac{f\, K}{K^2 + f^2}\, yz \tag{19c}$$

$$u_3 = -\frac{A}{2}\, \frac{H\, f\, K}{K^2 + f^2}\, y \tag{19d}$$

$$u_4 = -\sum_{n=1}^{\infty} a_n \frac{n\, \pi\, f}{H\, K}\, \mathrm{sh}\!\left(\frac{n\, \pi}{H} \sqrt{\frac{K^2 + f^2}{K^2}}\; y\right) \cos\left(\frac{n\, \pi}{H}\, z\right) \tag{19e}$$

with A and a_n as being given above. Figure 4 shows – for the case when the sea is forced solely by water flux across boundaries – the four fields contributing to the long-shore speed. The first of them, barotropic flow u_1, is related to the hydraulic effect and is obviously insignificant for the present choice of parameters. The second contribution, u_2, represents relative current, u_3 stands for slope current, whereas the last field – u_4 – is controlled by frictional effects and has nonzero values only close to the coasts. Superposition of the four fields is illustrated in Figure 5. It displays cyclonic circulation close to the sea surface and anticyclonic circulation near the bottom, related to the

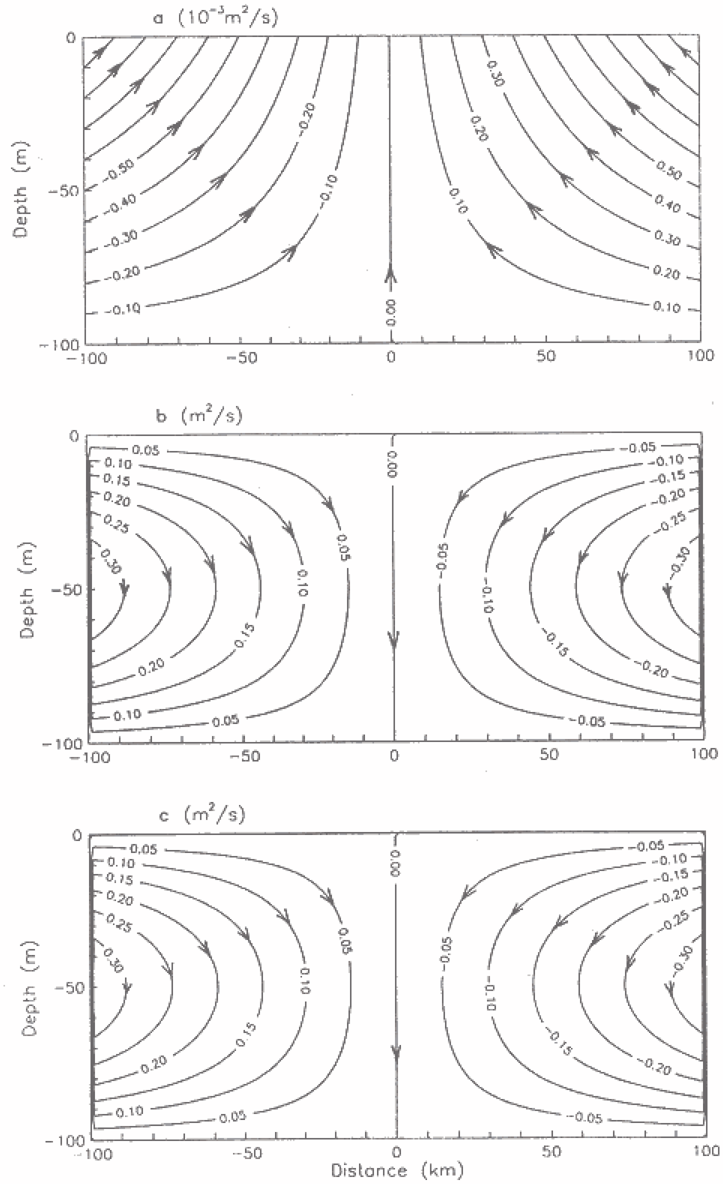

Figure 3. Cross-basin flow related to the salinity field of Figure 2. Shown is coast-to-surface directed hydraulic flow (a), haline circulation (b) and their superposition (c).

quasigeostrophic rectification of the surface offshore flow and the bottom onshore currents, respectively.

The above solution for streamfunction and long-shore speed is sensitively dependent on the relative importance of the Coriolis and frictional effects. When $f \ll K$ the long-shore speed is small and the current field is dominated by cross-basin circulation. On the other hand, when $f \gg K$ only hydraulic contribution to the cross-basin flow survives whereas quasigeostrophic cur-

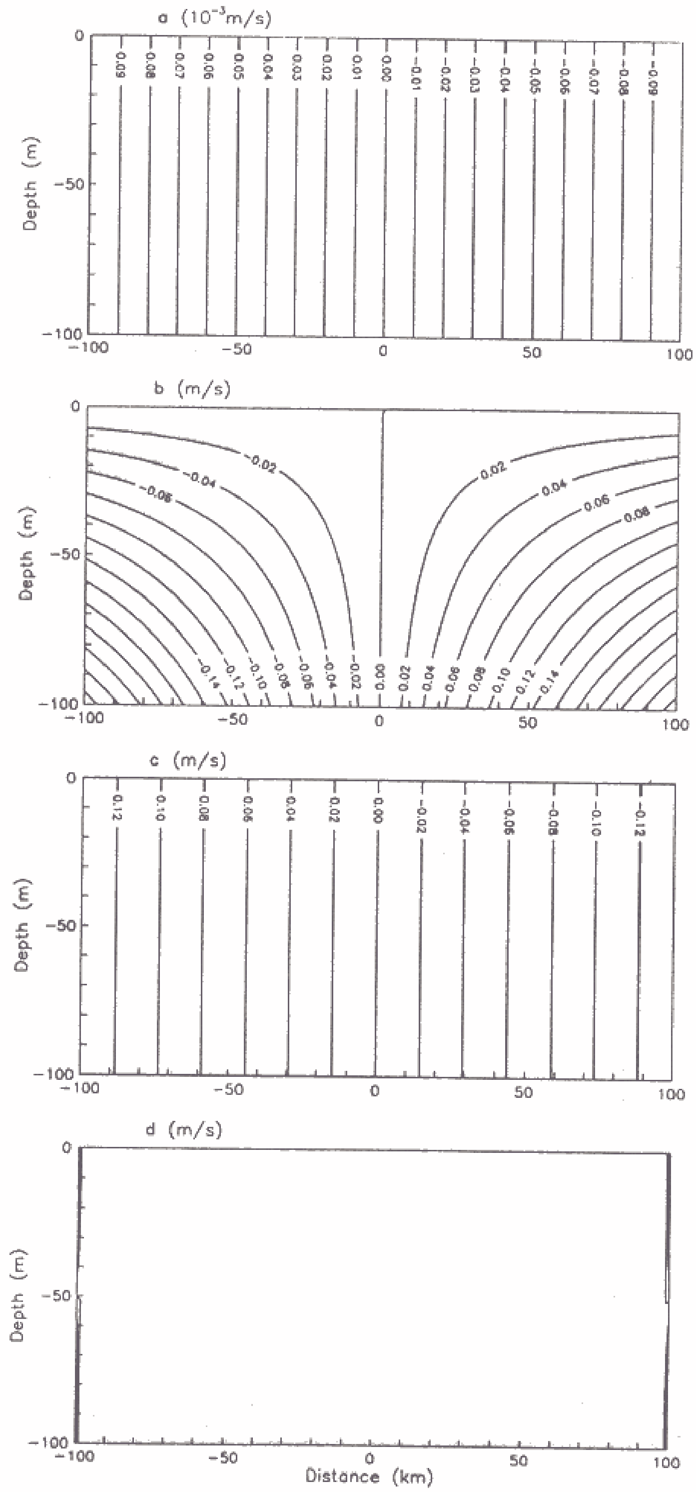

Figure 4. Long-shore currents due to the quasigeostrophic rectification of the cross-basin flow depicted in Figure 3. The four sections illustrate barotropic flow related to the hydraulic effect (a), as well as relative (b), slope (c) and frictional (d) currents connected with the haline circulation. In the last section nonzero values appear only close to the coasts.

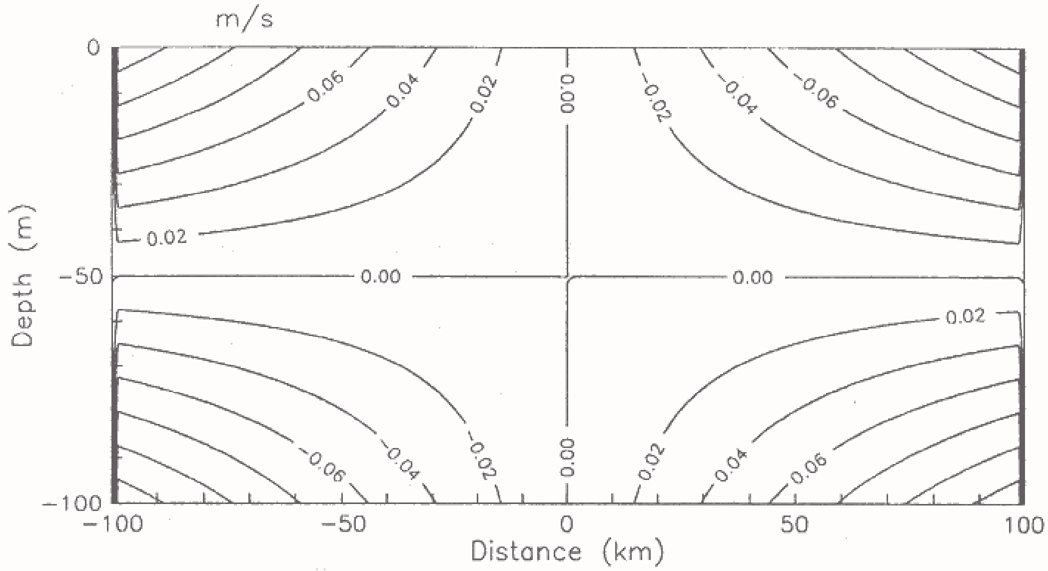

Figure 5. Superposition of the four contributions to the long-shore flow, shown in Figure 4.

rents are well developed in the long-basin direction. It is interesting to note that in the latter case long-basin barotropic flow may become significant and may modify the slope current. However, as there are no reliable estimates of K, either for the Adriatic Sea or any other basin, it is difficult to assess the reality of these limiting cases. Yet, they emphasize didactic value of the model: it shows how, due to the buoyancy driving across the sea surface and coasts, the cross-basin circulation is developed and the long-basin flow appears as a result of deflecting influence of the Coriolis force. Moreover, the model enables the barotropic and baroclinic currents to be related to a particular source – the buoyancy forcing. The weak point of the present model is that some of its results are probably influenced by the employed parameterization of friction. In particular, the obtained barotropic and slope currents, while qualitatively interesting, may in a quantitative sense heavily depend on modeled friction, and thus the determined level of no long-shore motion should be treated with caution.

The model developed in this paper is close to at least two previously published works. Huang (1971) theoretically analyzed winter dynamics in a lake in which temperature is everywhere below 4 °C. By imposing maximum surface temperature in central part of the basin and minima close to the coasts, he arrived at cross-basin circulation and long-basin flow resembling the fields of Figure 3 and 5, respectively. As Huang considered nonlinear processes and parameterized friction after J. Boussinesq with no slip at the bottom, his results support validity of the present, simpler model. The other related study was published by Heaps (1972). Inspired by a series of papers authored by Nomitsu (1933a, 1933b) and Nomitsu and Takegami (1933), Heaps analytically modeled currents – due to the cross-shore density gradient – off a straight coast across which rivers discharge into the sea. Offshore surface currents and onshore near-bottom flow obtained by Heaps support the present findings. On the other hand, his long-shore currents diminish from the surface to the bottom, without changing their direction. Although Heaps assumed hydrostatic balance along the vertical, more important for this result is probably parameterization of friction as per J. Boussinesq and assumed linear slip at the bottom. This again signals that the barotropic and slope components of the long-shore flow obtained here should be taken at their qualitative value only.

Does the present model bear any similarity with the observed winter dynamics of the Adriatic Sea? Surface cyclonic circulation of about 10 cm s^{-1} and offshore/onshore cross-basin flow in vicinity of river mouths are well documented features of the Adriatic (Buljan and Zore-Armanda, 1976), which obviously agree with some of the results obtained here (Figures 3, 5). Yet, the bottom anticyclonic circulation, displayed in Figure 5, has never been observed in the Adriatic Sea. Whether this is due to the slope current being reinforced by long-basin barotropic flow, frictional processes differing from the modeled phenomena or long-shore variability neglected here, is an open question.

5. Conclusion

An elementary model, which reproduces density distribution and circulation generated by the surface and coastal buoyancy flux in a land-locked basin, has been developed. When the coastal fresh-water inflows are locally balanced by the upward water flux from the sea surface, the model predicts offshore salinity increase, coast-to-surface directed hydraulic flow, and haline circulation characterized by upwelling along the coasts and downwelling prevailing over the greater part of the basin. Due to the Coriolis force, long-shore currents appear as well, those related to the hydraulic effect being barotropic, the haline currents comprising relative, slope and frictional contributions which combine to produce cyclonic circulation at the sea surface. Although not specifically discussed, cross-basin thermal variability and circulation and long-basin thermal flow are covered by the model as well, supposing that heat fluxes across the sea surface and the coasts balance each other.

The assumptions introduced while formulating the present model suggest the way more complex models may be built. As already pointed out, other parameterizations of friction are of interest, and some effort already went into this direction (Huang, 1971; Heaps, 1972). Still another problem pose different forcing mechanisms and possible nonlinear couplings of resulting phenomena; this line of investigation has been pioneered by Stommel and Leetmaa

(1972) for open continental shelves. Also important may be joint effects of baroclinicity and bottom relief, as shown by Hendershott and Rizzoli (1976) in their modeling study of the Adriatic Sea. Last but not least, the fact that river discharges are not evenly distributed along the coastal walls, but tend to be concentrated at few points and close to the sea surface, and that they are not locally balanced by the surface buoyancy flux, is responsible for a plethora of phenomena which necessitate special theoretical consideration (for a recent review one may consult Garvine, 1995).

Thus, there is a number of ways to improve the proposed model, some of which have already been tackled in the literature. The question, however, is whether the model can be simplified without losing its ability to simulate basics of the response of a land-locked sea to buoyancy forcing. If the answer is negative, the purpose of the present paper has been achieved.

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