GEOFIZIKA VOL. 12 1995

Original scientific paper
UDC 550.341

Broadband *b*: scaling law of P-wave broadband radiated energy

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Received 15 September 1994, in final form 2 February 1996

We analysed the NEIC broadband radiated energy catalogue and found that the scaling law of earthquake energy deduced from Gutenberg-Richter's law is not valid in a quantitative sense. The analysis of broadband radiated energy, however, also shows a scaling law, which may be represented by a broadband *b* value.

Širokopojasni *b*: razdioba broja potresa prema energiji P-valova

Analiziran je NEIC-ov katalog energije potresa određene analizom širokopojasnih seizmograma. Nađeno je da razdioba broja potresa u odnosu na njihovu energiju kvantitativno ne odgovara Gutenberg-Richterovoj relaciji, nego se nagib odgovarajućeg pravca mora opisati posebnom, »širokopojasnom« vrijednošću parametra *b*.

1. Introduction

The study and interpretation of the scaling law of earthquake energy is one of the most important parts in the application of nonlinear dynamics. Traditionally the discussion is conducted as follows: starting from the Gutenberg-Richter's law

$$\log N = a - bm \tag{1}$$

in which N is the number of earthquakes with magnitude m, and using the empirical relation between earthquake energy E and magnitude m

$$\log E = c + dm \tag{2}$$

with c and d being constants, it is easy to get the occurrence frequency distribution function

$$N \propto E^{-f} \tag{3}$$

where f is constant. This function, having the typical form of scaling law, has been related to the fractal characteristics of fault system (*e.g.* King, 1983), and recently has been interpreted as a manifestation of the self-organized critical behaviour of the dynamic lithosphere (Bak and Tang, 1989).

The scaling law of earthquake energy described above is crucial in the theoretical modelling of seismicity and the applications in earthquake prediction (*e.g.*, Ito and Matsuzaki, 1990; Sornette et al., 1990; Lomnitz-Adler, 1993). Unfortunately, despite that these models are mainly initiated by the development of modern physics, the observational evidence for these »modern« models still remains »classical«. In »classical« seismology, the estimation of radiated energy of earthquakes was conducted through the measurements of seismic signals within a limited frequency band. Such a process is just the measurement of magnitudes. The energy is obtained by the extrapolation to the whole frequency band using certain theoretical or empirical rupture models. These estimations, contained in the »classical« earthquake catalogues being analysed in many studies, have significant uncertainties due to the complexity of the spectral composition of earthquakes.

Recently, the development of broadband seismology has made it possible to estimate the energy of earthquakes over a broad frequency band, and actually since 1987 such estimation has become a job of some seismological agencies, such as the NEIC, on routine basis. The information over a wide frequency band provide an opportunity to test the scaling law of earthquake energy and have a new look at the physical significance of Gutenberg-Richter's relation. However, the importance of the broadband radiated energy as a new source parameter of earthquake is yet to be recognized both in seismology and in physics.

To test the scaling law of earthquakes with broadband energy, we analysed the broadband energy estimations by NEIC, published in the Preliminary Determination of Epicenters (PDE) Monthly Listing and contained in the NEIC source parameter database. From the analysis we obtained a pessimistic result that the scaling law deduced from Gutenberg-Richter's relation seems not valid in a quantitative sense, and an optimistic result that in the view of broadband radiated energy, there still exists a scaling law which indicates the existence of a broadband b value.

2. Broadband radiated energy

Since November 1986, NEIC has been doing and publishing the estimations of broadband radiated energy on routine basis. The broadband radiated energy is estimated from the energy spectral density of the vertical broadband P waves, using the method described by Boatwright and Choy (1986), where the energy flux in the P wave is integrated over a broad frequency band. Data used are either direct P waves for deep focus earthquakes or the P wave group consisting of P, pP, and sP for shallow earthquakes. The raw long-period (LP) data and short period (SP) data and/or intermediate-period (IP or BB) data are processed using the method of Harvey and Choy (1982) to produce the broadband seismograms. The broadband waveforms, produced by such a process, have flat response over a wide frequency band from low frequencies, say, 0.01 Hz, near to the »zero-frequency limit«, to at least 2.0 Hz, higher than the corner frequency of large and intermediate strength earthquakes. The effect of attenuation is considered with the frequency-dependent t^* of Choy and Cormier (1986). The focal mechanisms used are either the P-wave first motion solution, the USGS moment tensor, or the Harvard CMT solution, with little difference between different solutions.

Figure 1 demonstrates the relation between the broadband radiated energy E and the surface wave magnitude M_S for shallow earthquakes (earthquakes with surface wave magnitude estimations) worldwide from November 1986 to December 1994. The fitting of the data gives

$$\log E = 6.62 + 1.15\, M_S$$

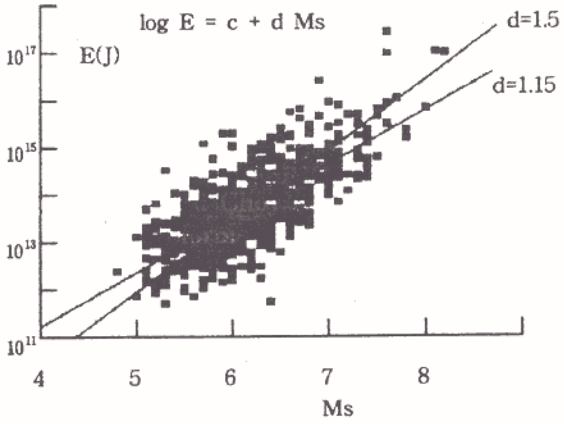

Figure 1. Relation between broadband radiated energy E and surface wave magnitude M_S for shallow earthquakes worldwide from November 1986 to December 1994. In the figure the theoretical relation $\log E \sim 1.5\, M_S$ (Scholz, 1990) is also plotted as a reference.

as shown by the straight line in the figure, with a low correlation coefficient $r_{xy} = 0.73$ and a large residual $s = 0.61$. The meanings of the parameters are given in the Appendix. In this result and hereafter the unit of the broadband radiated energy E is Nm or J. Figures 2(a)–2(d) present the relation between the broadband radiated energy E and body wave magnitude m_b and surface wave magnitude M_S for shallow earthquakes (earthquakes with surface wave magnitude estimations) in Japan and the continental United States from November 1986 to December 1994. The data gives

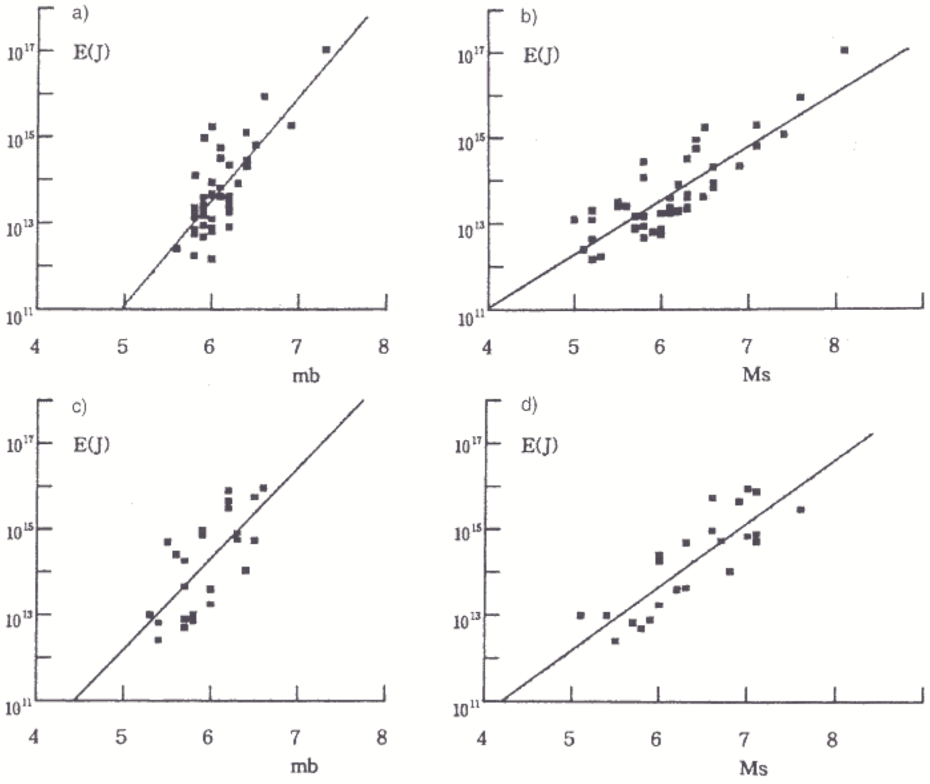

Figure 2. Relation between broadband radiated energy E and a) body wave magnitude m_b, b) surface wave magnitude M_S for shallow earthquakes in Japan (November 1986 – December 1994). Parts c) and d) correspond to parts a) and b) for earthquakes in the continental United States.

$$\log E = -1.05 + 2.43\, m_b, \quad r_{xy} = 0.77, \quad s = 0.62$$

$$\log E = 6.02 + 1.25\, M_S, \quad r_{xy} = 0.84, \quad s = 0.52$$

for Japan region, and

$$\log E = 1.54 + 2.13\, m_b \qquad r_{xy} = 0.71, \quad s = 0.82$$

$$\log E = 4.74 + 1.49\, m_b, \qquad r_{xy} = 0.85, \quad s = 0.61$$

for the continental United States, as shown by the straight lines in the figures. As we expected, there does exist a trend that higher energy corresponds to larger magnitude.

However, the data is much more scattered than could be explained by the errors of the energy and magnitude estimation, implying that in a quantitative sense, the relation described by (2) is not valid. In fact, this phenomenon is not surprising because the spectral composition of large and intermediate strength earthquakes, especially of shallow ones that occur in the crust and upper mantle, is very complex. In contrast, the magnitudes are band-limited and sometimes even »monotonic«. Another important reason for this scattering is that the E–M_S relation has a strong dependence on the environmental stress field (George Choy, 1994; Peishan Chen, 1994, personal communications). Before the development of the broadband seismology, the method mentioned in (2) was the only way to estimate the energy of earthquakes. At present, however, the situation is different.

3. Scaling law associated with broadband radiated energy

Since the relation (2) seems not valid, it is necessary to look at the energy directly. One of the technical problems is the completeness of the catalogue. Normally radiated energy is estimated for earthquakes with $m_b > 5.6$ or so. For some events, however, the estimations are not available because of their unacceptable uncertainties, even if the magnitudes are quite large. To avoid the incompleteness of earthquake catalogue, we first analysed the worldwide incompleteness of the catalogue and found that it depends on the global distribution of seismic stations. On the other hand, we can also choose some special regions as »windows« with ideal completeness for the test. In this approach the two »windows« around Japan islands (30–46 °N, 137–149 °E) and the continental United States (26–50 °N, 111–130 °W) were chosen for the test.

In equation (1) originally N is the number of earthquakes with magnitude m. When dealing with a limited data set, to obtain more stability in calculation of the b value, equivalently one can use N as the accumulation number, which indicates the number of earthquakes with magnitudes larger than or equal to m. In this case the b value is the same, but the value of constant a is different.

Figure 3(a) shows the relation between broadband energy E and the accumulation number of events N with energy larger than and/or equal to E for the shallow earthquakes (those with surface wave magnitude estimations) in Japan region. In figure 3(b) the result for shallow earthquakes in the continental United States is shown. The data spans the time period from November 1986 to December 1994. It may be seen that, for the broadband radiated energy there still exists a scaling law. The data gives

$$\log N = 8.15 - 0.50 \log E, \quad r_{xy} = 0.99, \quad s = 0.03$$

for Japan region, and

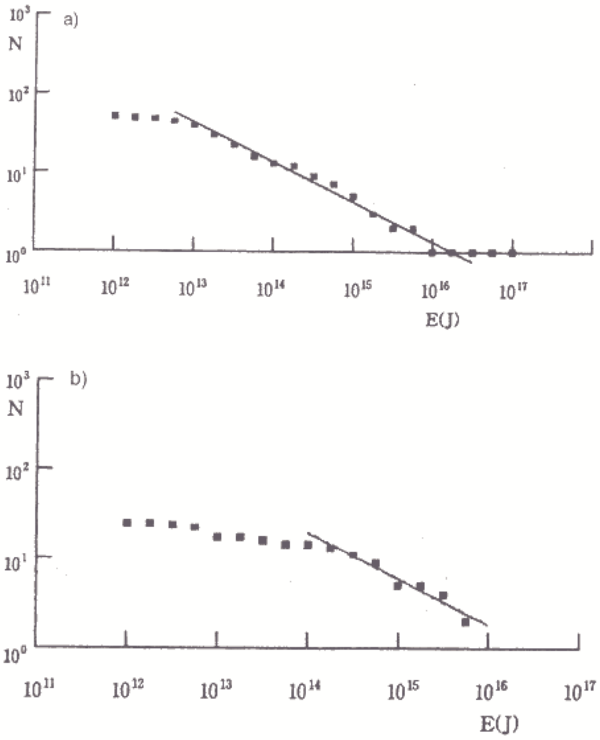

Figure 3. (a) Accumulation number for events with broadband radiated energy larger than or equal to E for shallow earthquakes in Japan from November 1986 to December 1994. b) The same for continental United States.

$$\log N = 8.15 - 0.51 \log E, \quad r_{xy} = 0.97, \quad s = 0.04$$

for the continental United States, as represented by the straight lines in the figures, showing the broadband b values of 0.50 and 0.51, respectively. As a comparison, Figures 4(a) and 4(b) show the results for surface wave magnitudes. The data give

$$\log N = 5.99 - 0.76\, M_S, \quad r_{xy} = 0.99, \quad s = 0.02$$

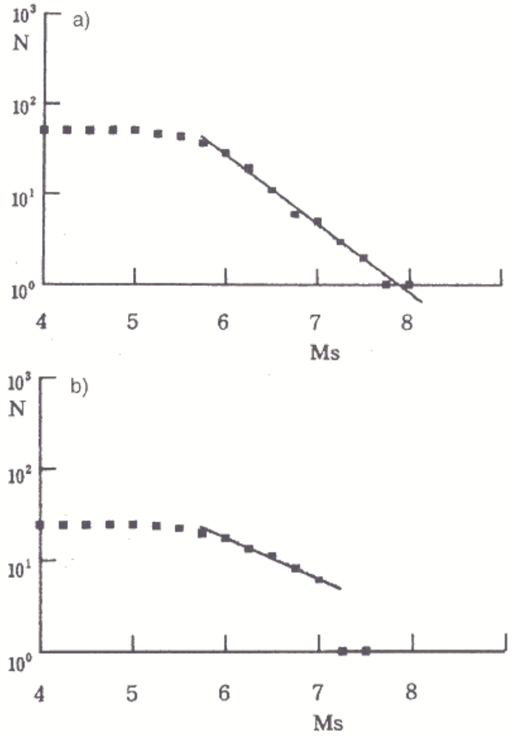

Figure 4. (a) Accumulation number of events with surface wave magnitude larger than or equal to M_S for shallow earthquakes in Japan from November 1986 to December 1994. b) The same for continental United States.

for Japan. For the continental United States, there is a clear break in the curve. The fitting of one part of the data (as shown in the figure) gives that

$$\log N = 3.91 - 0.45\, M_S, \quad r_{xy} = 1.00, \quad s = 0.01.$$

It may be seen that the scaling relation revealed by broadband radiated energy is quite different from that revealed by surface wave magnitude.

It should be pointed out that, considering the errors of the energy estimation, it is meaningless to study the detailed variations of the scaling behaviour revealed in the analysis, even though there do exist some clues that the scaling law for broadband energy exists locally and varies with scale. Also, because of the incompleteness associated with lower magnitudes which resulted from the arbitrariness of event selection, we chose only the events above a certain energy threshold, say, 10^{13} Nm in Figure 3(a) and 10^{14} Nm in Figure 3(b), for the analysis.

A property of the accumulation number is interesting. Although the E–M_S relation is quite scattering, as shown in Figures 1 and 2, the accumulation numbers have the scaling laws for E and M_S, respectively. Figures 5(a) and

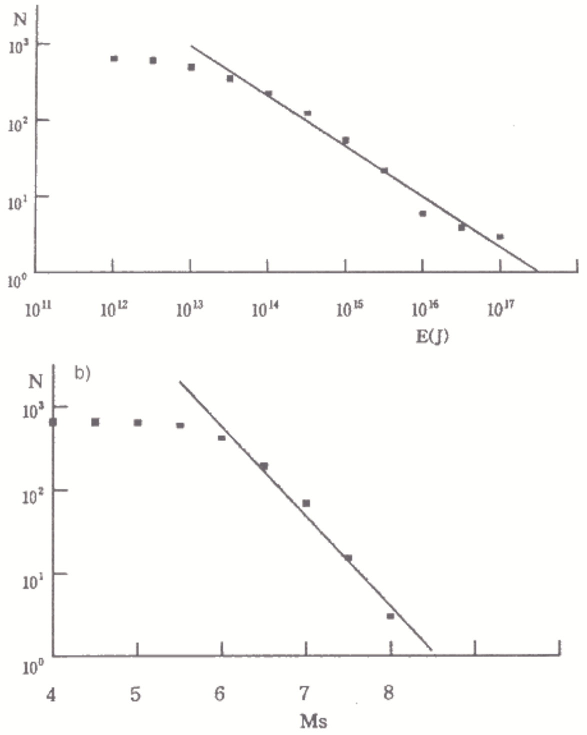

Figure 5. (a) Accumulation number of events with a) broadband radiated energy larger than or equal to E, b) surface wave magnitude larger than or equal to M_S for shallow earthquakes worldwide from November 1986 to December 1994.

5(b) show the result for worldwide shallow earthquakes from November 1986 to December 1994. The data give

$$\log N = 11.53 - 0.66 \log E, \quad r_{xy} = 0.99, \quad s = 0.01$$

$$\log N = 9.25 - 1.08\, M_S, \quad r_{xy} = 0.99, \quad s = 0.01$$

as shown by the straight lines in the figures. The linear relations are ideal for the two parameters, respectively. What is interesting is that theoretically, for worldwide earthquakes, one has (Scholz, 1990).

$$\log \mathrm{E} \sim 1.5\, M_S \tag{4}$$

$$\log M_o \sim 1.5\, M_S \tag{5}$$

$$N \sim M_o^{-2/3} \tag{6}$$

where M_o is the seismic moment. In this case, theoretically there should be

$$\log N \sim -0.67 \log E \tag{7}$$

$$\log N \sim M_S\,. \tag{8}$$

What is impressive is that although relation (4) is not satisfied in a quantitative sense, in spite of the incompleteness of the worldwide broadband energy catalogue, relations (7) and (8) are satisfied very well.

4. Discussion

Recently, increasing attention is being paid to the hypothesis of the self-organized criticality of earthquakes (Bak and Tang, 1989). Such an approach is discussed both in seismology and in physics because it is supposed to be helpful to lead to a »new deal« for the study on the forecasting of large and intermediate strength earthquakes (see *e. g.* Keilis-Borok, 1990). Besides the impact of the recent developments of nonlinear dynamics on Earth sciences, one reason why such models attracted much attention in seismology is that they have provided a strong support to the validity of simple dynamic models in the study of seismicity and earthquake prediction. It is believed that since near to the critical state there exist some characteristics which are system independent, it would be possible that even very simple models, say, the mass-spring model or cellular automata models, can help reveal some important features of seismicity. Hence by studying the behavior of the synthetic seismicity produced by such dynamical models, it is possible to obtain some useful experience for earthquake predictions in the real world, which founded the physical basis of the »pattern dynamics of earthquakes« (Yong Chen, 1993, personal communication). Moreover, based on the concept of self-organized criticality, two important changes in the basic understanding of the regularities of earthquake occurrence and the prediction of large earthquakes have to be made. One change is that, theoretically, the forecasting of large and intermediate strength earthquakes for a long time scale cannot be deterministic; and the other change is that the possible precursors preceding large earthquakes may exist in a large area much bigger than the source region. Actually this is the physics in some algorithms used for the diagnosis of the time of increased probability (TIP) of strong earthquakes (*e. g.* Keilis-Borok, 1990). It is clear that here the scaling law of earthquake energy is one of the most important links between physics and seismology. And in fact the derivation from (1) to (3) has become a »standard« demonstration of the scaling law in both the phenomenology and the mechanics of earthquakes. Seen from this point of view, it may be argued that the importance of our approach, reevaluating the basic hypothesis of the nonlinear dynamics of earthquakes in the perspective of broadband seismology, is by no means only a simple earthquake statistics.

From the results of our analysis it may be concluded that, seen from a broad frequency band, the relation described by (2) is not valid in a quantitative sense. Fortunately, however, the analysis of broadband energy shows that there still exists a scaling law, and this scaling relation seems quite different from that revealed by »classical magnitudes«, at least for the special regions under discussion.

It is a pity that the NEIC broadband energy catalogue is yet to be improved to have better completeness and longer time span. For further studies more and better data are needed, but right now at least we are confident that modifications to some statements have to be made in the context of nonlinear dynamics.

Acknowledgment – This work is supported by the National Natural Science Foundation of China. We thank Prof. George L. Choy for his help in broadband seismology. Acknowledgments are also due to our friends G. Dunphy and Pamela Benfield for their helps in using the NEIC database. One of the authors (W.Z.L.) thanks the Seismological Institute, Hanyang University, Korea for its support and help during his visit.

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Appendix: Terminology used in the text

The terminology used in the text follows the standard definitions in statistics. For two variables $\{x_i, i = 1,2,...N\}$ and $\{y_i, i = 1,2,...N\}$, when studying the relation

$$y_i = f(x_i)$$

we use the linear function

$$y = a + bx$$

to fit the relation $y_i = f(x_i)$ in a sense of least squares, *i.e.*, to let

$$R = \sum_{i=1}^{N} \left[y_i - (a + bx_i) \right]^2$$

to get to its minimum. In this case

$$b = \frac{l_{xy}}{l_{xx}}, \qquad a = \bar{y} - b\bar{x}$$

in which

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i, \qquad \bar{y} = \frac{1}{N} \sum_{i=1}^{N} y_i,$$

$$l_{xx} = \sum_{i=1}^{N} (x_i - \bar{x})^2 \qquad l_{xy} = \sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})$$

The correlation coefficient r_{xy} is defined by

$$r_{xy} = \frac{l_{xy}}{\sqrt{l_{xx}\, l_{yy}}}$$

where

$$l_{yy} = \sum_{i=1}^{N} (y_i - \bar{y})^2$$

The residual s is defined by

$$s = \sqrt{\frac{l_{yy} - b\, l_{xy}}{N - 2}} = \sqrt{\frac{(1 - r)^2\, l_{yy}}{N - 2}}$$

indicating that for every x point, 95.4% of y points fall into the region confined by

$$y' = a + bx - 2s, \qquad y'' = a + bx + 2s$$

and 99.7% of y points fall into the region confined by

$$y' = a + bx - 3s, \qquad y'' = a + bx + 3s.$$

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