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Note on reducibility of parabolic induction for Hermitian quaternionic groups over p-adic fields*

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Abstract. In this paper, we study reducibility of a certain class of parabolically induced representations of p-adic Hermitian quaternionic groups. We use Jacquet modules techniques and the theory of R-groups to extend reducibility results of Tadić for split classical groups to the case of an arbitrary Hermitian quaternionic group.

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1. Introduction

The purpose of this note is to extend the results of Tadić [24] regarding reducibility of parabolically induced representations of split symplectic and special orthogonal groups over a p-adic field to the case of arbitrary p-adic Hermitian quaternionic groups. Hermitian quaternionic groups over a p-adic field F are the isometry groups of Hermitian and antihermitian forms on finite-dimensional right vector spaces over the quaternion division algebra D central over F. These are classical groups, and their structure resembles the structure of split classical groups, but they are not quasi-split. See [12] for a classification of such groups.

Parabolic induction is one of the most important constructions of representations of reductive groups. Hence, reducibility of parabolically induced representations is always among the first steps in the study of the representation theory for a given group. In the case of split and quasi-split classical groups over a p-adic field, parabolic induction was studied extensively by many authors, among which Bernstein and Zelevinsky [2, 27], Harish-Chandra [9], Casselman [4], Knapp and Stein [10], Shahidi [15, 16, 17], Silberger [18], and Tadić [22, 23, 24, 25]. Many techniques and approaches to the reducibility question have been developed in that context. However, since Hermitian quaternionic groups are not quasi-split, we must choose our methods carefully, as some of them, such as Shahidi's theory of automorphic L-functions [16], are not available in our setting.

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In this paper, we study reducibility of representations of p-adic Hermitian quaternionic groups that are parabolically induced from cuspidal and (essentially) square-integrable representations of the Levi factors of standard parabolic subgroups. With slight modifications, the results are very much parallel to the reducibility results of Tadić [24]. The proofs rely on the theory of R-groups [6, 10], extended to cover the case of Hermitian quaternionic groups and Jacquet module techniques, in particular the structural formula [23], which may be applied to any classical p-adic group.

The paper is organized as follows. In Section 2, we introduce the p-adic Hermitian quaternionic groups and their structure. Section 3 recalls some basic facts from representation theory of reductive p-adic groups. In Section 4, the theory of Jacquet modules is introduced and the main technical tools are verified in the case of Hermitian quaternionic groups. Section 5 contains the results regarding reducibility of certain parabolically induced representations of Hermitian quaternionic groups.

2. Hermitian quaternionic groups

In this section, we define the Hermitian quaternionic groups considered in the paper and recall their structure. The main references are [12, 14, 7, 11].

2.1. Definition

Let F be a p-adic field, i.e., a non-archimedian local field of characteristic zero. Let D be the unique (up to isomorphism) quaternion algebra, central over F, with an involution τ of the first kind.

For a positive integer m, we write $GL(m, D)$ for the group of invertible elements of the algebra $M_m(D)$ of $m \times m$ matrices with entries in D. It is an inner form of the general linear group $GL(2m, F)$ over F.

Let V be either Hermitian or antihermitian right vector space of dimension N over D. We denote by $(\, , \,)$ the form on V. Let X be a maximal isotropic subspace of V. Let Y be another maximal isotropic subspace, such that X and Y form a total polarization. Let n denote the dimension of X and Y. Then, there is a (possibly trivial) anisotropic subspace of dimension l, orthogonal to X and Y, such that there is a direct sum decomposition

$$V = X \oplus W \oplus Y.$$

Note that we have $N = 2n + l$.

We fix ordered bases $\{e_1, \ldots, e_n\}$ and $\{e_{n+1}, \ldots, e_{2n}\}$ of spaces X and Y, respectively, and an ordered basis $\{f_1, \ldots, f_l\}$ of the anisotropic space W, such that

$$(e_i, e_{2n+1-j}) = \delta_{ij},$$

for $i, j = 1, \ldots, n$, where δ_{ij} is the Kronecker δ symbol. Then the form on V is given by

$$(v, v') = \epsilon\tau\big((v', v)\big), \quad (vx, v'x') = \tau(x)(v, v')x',$$

for $v, v' \in V$ and $x, x' \in D$, where $\epsilon = 1$, if the form is Hermitian and $\epsilon = -1$ if the form is antihermitian.

We recall the classification of Hermitian and antihermitian spaces over (D,τ) from [12, Chapter 1]. The Hermitian spaces are completely classified by their dimension, while the antihermitian spaces are classified by their dimension and the determinant of the form. The classification is summarized in Tables 1 and 2.

Dimension N	Short description
$2n$	The anisotropic space W is trivial.
$2n+1$	The anisotropic space W is the unique anisotropic Hermitian space of dimension $l=1$.

Table 1: *Classification of Hermitian spaces according to* [12]

Dimension N	Determinant d	Short description
$2n$	$d \in (\mathbb{F}^{\times})^2$	The anisotropic space W is trivial.
$2n+2$	$d \notin (\mathbb{F}^{\times})^2$	The anisotropic space W is one of 3 non-isomorphic anisotropic antihermitian spaces of dimension $l=2$.
$2n+1$	$d \notin -(\mathbb{F}^{\times})^2$	The anisotropic space W is one of 3 non-isomorphic anisotropic antihermitian spaces of dimension $l=1$.
$2n+3$	$d \in -(\mathbb{F}^{\times})^2$	The anisotropic space W is the unique anisotropic antihermitian space of dimension $l=3$.

Table 2: *Classification of antihermitian spaces according to* [12]

Let

$$J_n = \begin{pmatrix} & & 1 \\ & \cdot^{\cdot^{\cdot}} & \\ 1 & & \end{pmatrix},$$

and let J_W be the matrix of the form restricted to W in the fixed basis for W. Then

$$J = \begin{pmatrix} 0 & 0 & J_n \\ 0 & J_W & 0 \\ \epsilon J_n & 0 & 0 \end{pmatrix},$$

is the matrix of the form on V in the fixed basis of V, where J_W does not appear if the anisotropic space W is trivial.

Let $G_n = G_{n,l}(D,\epsilon)$ be the group of isometries of the form (,) on V. We refer to these groups as the Hermitian quaternionic groups and their representation theory is the subject of this paper. The group G_n is an algebraic group defined over F of F-rank n. The group of its F-rational points is

$$G_n(F) = \{g \in GL(N,D) \mid g^* J g = J\},$$

where for the matrix $g = (g_{ij}) \in GL(N,D)$, the matrix g^* is defined as $(\tau(g_{ji}))$. The group G_n is an inner form of the symplectic group Sp_{2N} if the space is Hermitian, i.e., $\epsilon = 1$, and an inner form of the special orthogonal group SO_{2N} if the space is antihermitian, i.e., $\epsilon = -1$.

Note that if $l = 0$, then $G_0(F)$ is just a trivial group. If $l \neq 0$, then the group $G_0(F)$ is an anisotropic group, and thus compact. Its irreducible representations are finite-dimensional.

2.2. Parabolic subgroups

We fix the maximal F-split torus A_0 of G_n with the group of F-rational points

$$A_0(F) = \left\{ \mathrm{diag}(\lambda_1, \ldots, \lambda_n, I_l, \lambda_n^{-1}, \ldots, \lambda_1^{-1}) \mid \lambda_i \in F^\times \right\},$$

where the identity matrix I_l is omitted if $l = 0$. Let $\Phi(G_n, A_0)$ be the relative root system of G_n with respect to A_0. It is a root system of type B_n for Hermitian quaternionic groups of Hermitian space, i.e., for $\epsilon = 1$, and of type C_n for Hermitian quaternionic groups of antihermitian space, i.e., for $\epsilon = -1$.

We fix once and for all a minimal parabolic subgroup $P_{\min}$ defined over F. In our matrix realization we may and will take $P_{\min}$ to be the group of upper-triangular matrices in G_n. Let Δ be the set of simple roots determined by the choice of $P_{\min}$. If ε_i denotes the projection to the ith component of A_0, i.e.,

$$\varepsilon_i(\mathrm{diag}(\lambda_1, \ldots, \lambda_n, I_l, \lambda_n^{-1}, \ldots, \lambda_1^{-1})) = \lambda_i,$$

then

$$\Delta = \{\alpha_1, \ldots, \alpha_n\},$$

where $\alpha_i = \varepsilon_i - \varepsilon_{i+1}$, for $i = 1, \ldots, n-1$, and $\alpha_n = \varepsilon_n$ if $\epsilon = 1$, $\alpha_n = 2\varepsilon_n$ if $\epsilon = -1$.

The standard parabolic subgroups of G_n are in bijective correspondence with subsets θ of Δ. Subsets θ of Δ can be parameterized by sequences $(\alpha) = (n_1, \ldots, n_k)$ of positive integers such that $n_1 + \cdots + n_k \leq n$. More precisely, a sequence $(\alpha) = (n_1, \ldots, n_k)$ corresponds to the subset

$$\theta = \Delta \setminus \{\alpha_{n_1}, \alpha_{n_1+n_2}, \ldots, \alpha_{n_1+\cdots+n_k}\}.$$

For $\theta \subset \Delta$ parameterized by a sequence $(\alpha) = (n_1, \ldots, n_k)$, we write $P_\theta = P_{(\alpha)} = M_{(\alpha)} N_{(\alpha)}$ for the Levi decomposition of the corresponding standard parabolic subgroup. Then the Levi factor $M_{(\alpha)}(F)$ consists of block-diagonal matrices of the form

$$\mathrm{diag}(g_1, \ldots, g_k, h, J_{n_k}(g_k^{-1})^* J_{n_k}, \ldots, J_{n_1}(g_1^{-1})^* J_{n_1}),$$

where $g_i \in GL(n_i, D)$ and $h \in G_r(F)$ with $r = n - \sum_{j=1}^k n_j$ a non-negative integer. It is isomorphic to the product $GL(n_1, D) \times \cdots \times GL(n_k, D) \times G_r(F)$. Note that if $G_n = G_{n,l}(D, \epsilon)$, then $G_r = G_{r,l}(D, \epsilon)$, i.e., the anisotropic space remains the same.

3. Preliminaries from representation theory

In this section, we recall some basic facts from the representation theory of general linear groups over p-adic division algebras, and rephrase in our setting the Langlands quotient theorem. All considered representations are smooth.

3.1. Parabolic induction

Let $(\alpha) = (n_1, \ldots, n_k)$ be a sequence of positive integers such that $\sum_{j=1}^{k} n_j \leq n$, and $r = n - \sum_{j=1}^{k} n_k$, as above. If $\pi_1, \ldots, \pi_k$ are representations of the groups $GL(n_1, D), \ldots, GL(n_k, D)$, respectively, and σ is a representation of $G_r(F)$, we denote by

$$\pi_1 \times \cdots \times \pi_k \rtimes \sigma = \mathrm{Ind}_{P_{(\alpha)}(F)}^{G_n(F)} (\pi_1 \otimes \cdots \otimes \pi_k \otimes \sigma)$$

the representation of $G_n(F)$ obtained by (normalized) parabolic induction from the representation of the Levi factor $M_{(\alpha)}(F)$ of $P_{(\alpha)}(F)$, which is isomorphic to $GL(n_1, D) \times \cdots \times GL(n_k, D) \times G_r(F)$.

3.2. Cuspidal reducibilities for $GL(k, D)$

For $g \in GL(k, D)$, we let $\nu(g) = |RN(g)|_F$, where RN is the reduced norm on the algebra $M_k(D)$ as in [14, Sect. 8.5]. For each irreducible cuspidal representation ρ of $GL(k, D)$, let ρ' be the essentially square-integrable representation of $GL(2k, F)$ attached to ρ by the local Jacquet–Langlands correspondence [5]. If ρ is an irreducible cuspidal representation of $GL(k, D)$, we define, as in [26], an integer

$$s(\rho) = \begin{cases} 1, & \text{if } \rho' \text{ is cuspidal,} \\ 2, & \text{otherwise,} \end{cases}$$

and we put $\nu_\rho = \nu^{s(\rho)}$.

The following theorem from [20] completely describes the cuspidal reducibility of parabolic induction for $GL(k, D)$.

Theorem 1. *Let ρ_1 and ρ_2 be irreducible cuspidal representations of $GL(k_1, D)$ and $GL(k_2, D)$, respectively. Then the induced representation $\rho_1 \times \rho_2$ of $GL(k_1 + k_2, D)$ reduces if and only if $k_1 = k_2$, $s(\rho_1) = s(\rho_2)$ and $\rho_1 \cong \nu_{\rho_2}^{\pm 1} \rho_2$.*

3.3. Essentially square-integrable representations

Sets of the form

$$\left\{\rho, \nu_\rho \rho, \ldots, \nu_\rho^m \rho\right\},$$

where ρ is an irreducible cuspidal representation of $GL(k, D)$ and m a non-negative integer, are called segments of irreducible cuspidal representations of the general linear group over a division algebra D. The segment above is denoted by

$$\Delta = [\rho, \nu_\rho^m \rho].$$

According to [20], for any segment Δ, the induced representation $\nu_\rho^m \rho \times \nu_\rho^{m-1} \rho \times \cdots \times \nu_\rho \rho \times \rho$ has a unique irreducible subrepresentation, denoted by

$$\delta(\Delta) = \delta([\rho, \nu_\rho^m \rho]),$$

and a unique irreducible quotient, denoted by

$$\zeta(\Delta) = \zeta([\rho, \nu_\rho^m \rho]).$$

Irreducible representation $\delta(\Delta)$ is essentially square-integrable and the representation $\nu_\rho^m \rho \otimes \cdots \otimes \nu_\rho \rho \otimes \rho$ appears in its Jacquet module (see Sect. 4).

For a symmetric segment $[\nu_\rho^{-(m-1)/2}\rho, \nu_\rho^{(m-1)/2}\rho]$, we use a shorter notation

$$\delta(\rho, m) = \delta\left([\nu_\rho^{-(m-1)/2}\rho, \nu_\rho^{(m-1)/2}\rho]\right).$$

By [24, Sect. 9], for every $\alpha \in \mathbb{R}$ we have $\nu_\rho^\alpha \delta(\rho, m) \cong \delta(\nu_\rho^\alpha \rho, m)$.

3.4. Langlands quotient theorem

Following [8], we describe now the Langlands classification [3] in the setting of the Hermitian quaternionic group $G_n(F)$. For every essentially square-integrable representation δ of the group $GL(n, D)$, there exists a unique real number $e(\delta)$ and a unique square-integrable representation δ^u of $GL(n, D)$ such that

$$\delta = \nu^{e(\delta)}\delta^u.$$

We say that an ordered multiset $(\delta_1, \ldots, \delta_k)$ of irreducible esentially square-integrable representations of general linear groups over D is in standard order if $e(\delta_1) \geq e(\delta_2) \geq \cdots \geq e(\delta_k)$.

Theorem 2 (Langlands quotient theorem, see [3])**.** *Let $(\delta_1, \ldots, \delta_k)$ be an ordered multiset of essentially square-integrable representations of general linear groups over D in standard order and such that $e(\delta_k) > 0$. Let τ be an irreducible tempered representation of the group $G_r(F)$. Then the parabolically induced representation*

$$\delta_1 \times \cdots \times \delta_k \rtimes \tau$$

has a unique irreducible quotient, the so-called Langlands quotient, which is denoted by $L(\delta_1, \ldots, \delta_k; \tau)$. It appears with multiplicity one in the parabolically induced representation. Conversely, every irreducible representation of the group $G_n(F)$ is obtained as a Langlands quotient in this way.

4. Jacquet modules, the structural formula and R-groups

This section forms the technical heart of the paper, as we introduce here the theory of Jacquet modules, and check whether the structural formula and the theory of R-groups can be applied in our setting.

Let $(\alpha) = (n_1, \ldots, n_k)$ be a sequence of positive integers such that $r = n - \sum_{j=1}^k n_j \geq 0$. For an irreducible admissible representation σ of the Hermitian quaternionic group $G_n(F)$, we denote by $s_{(\alpha)}(\sigma)$ the normalized Jacquet module of σ with respect to the standard parabolic subgroup $P_{(\alpha)}$.

4.1. Square-integrability criterion

We now recall from [4] (see also [8]) the Casselman's square-integrability criterion for irreducible admissible representations of $G_n(F)$.

Let σ be an irreducible admissible representation of $G_n(F)$. Let $P_{(\alpha)}$ be one of the associate parabolic subgroups that are minimal in the set of standard parabolic subgroups P of G_n with the property that the Jacquet module of σ with respect to P is non-zero. Then every irreducible subquotient of $s_{(\alpha)}(\sigma)$ is a cuspidal representation of $M_{(\alpha)}(F)$.

As in Section 3, for a cuspidal representation ρ_i of $GL(n_i, D)$, let $e(\rho_i)$ be a real number such that $\rho_i \cong \nu^{e(\rho_i)}\rho_i^u$, where ρ_i^u is unitary. For a cuspidal representation $\rho \cong \rho_1 \otimes \cdots \otimes \rho_k \otimes \tau$ of the Levi factor $M_{(\alpha)}(F) \cong GL(n_1, D) \times \cdots \times GL(n_k, D) \times G_r(F)$, where $(\alpha) = (n_1, \ldots, n_k)$, we define a character of A_0 as the n-tuple

$$e(\rho) = (e(\rho_1), \ldots, e(\rho_1), e(\rho_2), \ldots, e(\rho_2), \ldots, e(\rho_k), \ldots, e(\rho_k), 0, \ldots, 0) \in X(A_0) \otimes \mathbb{R},$$

where $e(\rho_i)$ appears n_i times, and 0 appears $r = n - \sum_{j=1}^{k} n_j$ times, and $X(A_0)$ denotes the $\mathbb{Z}$-module of F-rational characters of A_0.

Theorem 3 (The square integrability criterion, see [4])**.** *An irreducible admissible representation σ of the Hermitian quaternionic group $G_n(F)$ is square-integrable if and only if for every irreducible subquotient $\rho = \rho_1 \otimes \cdots \otimes \rho_k \otimes \tau$ of the Jacquet module $s_{(\alpha)}(\sigma)$, where as above (α) is such that $P_{(\alpha)}$ is minimal with non-zero Jacquet module of σ, the following inequalities hold*

$$(e(\rho), \beta_{n_1+n_2+\cdots+n_i}) > 0,$$

for all $i = 1, \ldots, k$, where $(\alpha) = (n_1, \ldots, n_k)$ and β_j is the jth fundamental weight for G_n with respect to A_0.

Recall that fundamental weights form the basis dual to the basis consisting of simple coroots for the root system of G_n with respect to A_0. The ordering of fundamental weights is consistent with the ordering of simple roots as in Section 2. Hence, the square-integrability condition of Theorem 3 can be written explicitly as

$$\begin{aligned} n_1 e(\rho_1) &> 0 \\ n_1 e(\rho_1) + n_2 e(\rho_2) &> 0 \\ &\vdots \\ n_1 e(\rho_1) + \cdots + n_k e(\rho_k) &> 0, \end{aligned}$$

where n_i and $e(\rho_i)$ are as introduced before the theorem was stated.

4.2. Structural formula

On the direct sum $R = \oplus_{k \geq 0} R_k$ of the Grothendieck groups R_k of the category of smooth finite length representations of $GL(k, D)$, there is a well-known Hopf algebra structure [2]. The multiplication $m : R \otimes R \to R$ is given by parabolic induction and denoted by $\times$. The comultiplication $m^* : R \to R \otimes R$ is defined using Jacquet modules as

$$m^*(\pi) = \sum_{i=0}^{k} \mathrm{s.\,s.}(r_{(i,k-i)}(\pi)),$$

where π is a representation of $GL(k, D)$, and $r_{(i,k-i)}(\pi)$ denotes the normalized Jacquet module with respect to the standard parabolic subgroup of $GL(n, D)$ with the Levi factor consisting of two blocks of sizes i and $k - i$.

Let $R(G_n)$ be the Grothendieck group of the category of smooth finite length representations of the Hermitian quaternionic group $G_n(F)$. Let $R(G) = \oplus_{n\geq 0} R(G_n)$. Parabolic induction gives the R-module structure on $R(G)$, where the left multiplication by elements of R is denoted by $\rtimes$. The comodule structure is defined in the same way as in the case of general linear groups. The $\mathbb{Z}$-linear map $\mu^* : R(G) \to R \otimes R(G)$ is defined on the basis consisting of irreducible admissible representations by

$$\mu^*(\sigma) = \sum_{k=0}^{n} \mathrm{s.\,s.}(s_{(k)}(\sigma)),$$

where σ is a representation of $G_n(F)$, and $s_{(k)}(\sigma)$ is the Jacquet module with respect to the parabolic subgroup corresponding to $(\alpha) = (k)$, and s. s. stands for semi-simplification.

If σ is a finite length representation of $G_n(F)$ and $(\alpha) = (n_1, \ldots, n_k)$, we consider semi-simplification $\mathrm{s.\,s.}(s_{(\alpha)}(\sigma))$ as an element of $R_{n_1} \otimes \cdots \otimes R_{n_k} \otimes R(G_r)$, where $r = n - \sum_{j=1}^{k} n_j$. Denote by $s : R \otimes R \to R \otimes R$ a linear map given by $s(\pi_1 \otimes \pi_2) = \pi_2 \otimes \pi_1$. We define a ring homomorphism $\psi^* : R \to R \otimes R$ by

$$\psi^* = (m \otimes 1) \circ (\tilde{} \otimes m^*) \circ s \circ m^*,$$

where $\tilde{}$ denotes taking the contragredient.

Theorem 4 (Structural formula)**.** *For a smooth representation π of $GL(k, D)$ of finite length and a smooth representation σ of $G_r(F)$ of finite length, we have*

$$\mu^*(\pi \rtimes \sigma) = \psi^*(\pi) \rtimes \mu^*(\sigma).$$

Proof. The proof of this structural formula for μ^* of a parabolically induced representation is based on the Geometric Lemma [2, 27]. The filtration of Jacquet modules is determined by representatives of minimal length of certain double cosets in the associated Weyl groups. More precisely, we need a convenient description of representatives of minimal length for double cosets in $W_\theta \setminus W / W_\Omega$, where W is the Weyl group of the relative root system $\Phi(G_n, A_0)$, and W_θ, for $\theta \subset \Delta$, is the Weyl group of the relative root system of the Levi factor M_θ of the parabolic subgroup P_θ with respect to A_0.

Since the relative root systems of Hermitian quaternionic groups are either of type B_n or C_n, the description of these double cosets is essentially the same as in the case of split classical groups $SO(2n+1, F)$ and $Sp(2n, F)$. Hence, we may follow closely the description of double cosets of [23], and the proof reduces to the proof of Theorems 5.2 and 5.4 in *loc. cit.* □

4.3. R-groups

The theory of R-groups is a general approach to the study of reducibility and composition series of representations parabolically induced from square-integrable representations of the Levi factors. The R-group provides a combinatorial description of the structure of the induced representations in question.

Theorem 5 (R-groups)**.** *Let $(\alpha) = (n_1, \ldots, n_k)$ be a sequence of positive integers such that $r = n - \sum_{j=1}^{k} n_j \geq 0$. For $j = 1, \ldots, k$, let δ_j be a square-integrable representation of $GL(n_j, D)$ and τ a square-integrable representation of $G_r(F)$. Then the induced representation*

$$\delta_1 \times \cdots \times \delta_k \rtimes \tau$$

of $G_n(F)$ decomposes into a direct sum of 2^d non-isomorphic irreducible tempered representations, where d is the number of non-isomorphic representations δ_j such that $\delta_j \rtimes \tau$ reduces.

Proof. The intertwining algebra of the induced representation is determined by its R-group. In our case, it is isomorphic to the group algebra of the R-group. Hence, the R-group completely determines the structure of the induced representation.

The computation of the R-group reduces to the computation in the Weyl group of G_n. It is a subgroup of the stabilizer in the Weyl group of the inducing representation of the Levi subgroup. The subgroup is determined by vanishing of the Plancherel measure at zero, which is equivalent to reducibility of the representations $\delta_j \rtimes \tau$, as stated in the theorem. Hence, the computation of the Weyl group essentially reduces to the split case, which is handled in [6]. See also [7] for the result in the case of Hermitian quaternionic groups with trivial anisotropic space.

For the convenience of the reader, we recall from [7] the action of the Weyl group on the Levi factors and their representations. Since the Weyl group is generated by simple reflections, it is sufficient to provide the action in the case of maximal parabolic subgroups. Let M be the Levi factor of a maximal parabolic subgroup, which is isomorphic to $GL(k, D) \times G_r(F)$. It consists of block-diagonal matrices of the form

$$\mathrm{diag}\left(g, h, J_k (g^{-1})^* J_k\right),$$

where $g \in GL(k, D)$ and $h \in G_r(F)$, as in Section 2. Then the unique non-trivial element w of the Weyl group of G_n modulo the Weyl group of M acts as

$$w \,\mathrm{diag}\left(g, h, J_k (g^{-1})^* J_k\right) w^{-1} = \mathrm{diag}\left((g^{-1})^*, h, J_k g J_k\right).$$

Hence, if $\sigma \cong \delta \otimes \tau$ is an irreducible representation of $M(F)$, where δ and τ are irreducible representations of $GL(k, D)$ and $G_r(F)$, respectively, then its conjugate by w is isomorphic to

$$\sigma^w \cong \delta^* \otimes \tau,$$

where δ^* denotes the representation of $GL(k, D)$ given by $\delta^*(g) = \delta((g^{-1})^*)$. However, according to [13, Lemma 1.1], the representation δ^* is isomorphic to the contragredient representation $\widetilde{\delta}$ of δ. We remark that this is where the self-duality condition in Section 5.1 comes from.

It turns out that the R-group is a direct product of R-groups for the case of so-called basic parabolic subgroups, which are parabolic subgroups with the Levi factor of the form $GL(m, D)^l \times G_r(F)$. For these, the R-group is isomorphic to $(\mathbb{Z}/2\mathbb{Z})^{d_m}$, where d_m is the number of non-isomorphic representations δ_j of $GL(m, D)$ such that $\delta_j \rtimes \tau$ reduces. This gives the claim. □

5. Reducibility of induced representations - fascicule de résultats

In this section, reducibility of certain induced representations of Hermitian quaternionic groups is described. The proofs are almost entirely omitted, as they follow in the same way as in the split case, once that the applicability of technical tools is checked in Section 4. Thus the *fascicule de résultats* in the section title.

Throughout this section, let ρ be an irreducible unitarizable cuspidal representation of $GL(k, D)$, and σ an irreducible cuspidal representation of the Hermitian quaternionic group $G_r(F)$.

5.1. Condition $(C\alpha)$

According to [18], if ρ is self-dual (i.e., isomorphic to its contragredient $\widetilde{\rho}$), then there is a unique $\alpha \geq 0$ such that the induced representation $\nu_\rho^\beta \rho \rtimes \sigma$ reduces if and only if $\beta \in \{\pm\alpha\}$, where ν_ρ is as defined in Section 3. On the other hand, if ρ is not self-dual, then the induced representation $\nu_\rho^\beta \rho \rtimes \sigma$ is irreducible for all $\beta \in \mathbb{R}$.

Definition 1. *We say that the pair (ρ, σ) satisfies condition $(C\alpha)$, for $\alpha \geq 0$, if the induced representation $\nu_\rho^\alpha \rho \rtimes \sigma$ reduces.*

Observe that, if ρ is self-dual, then the pair (ρ, σ) satisfies condition $(C\alpha)$ for a unique $\alpha \geq 0$, and if ρ is not self-dual, then the pair (ρ, σ) does not satisfy $(C\alpha)$ for any $\alpha \geq 0$. If a pair (ρ, σ) satisfies $(C\alpha)$ with $\alpha = 0$, $\alpha = 1/2$ or $\alpha = 1$, we say that it has generic cuspidal reducibility.

5.2. Generalized Steinberg representation

Suppose that the pair (ρ, σ) satisfies condition $(C\alpha)$ with $\alpha > 0$. Then for every integer $m \geq 0$ the induced representation

$$\nu_\rho^{\alpha+m} \rho \times \nu_\rho^{\alpha+m-1} \rho \times \cdots \times \nu_\rho^\alpha \rho \rtimes \sigma$$

has a unique irreducible subrepresentation and it is square-integrable. It is called a generalized Steinberg representation. The main properties of the generalized Steinberg representation are given in the following theorem.

Theorem 6. *Let ρ be an irreducible unitarizable cuspidal representation of the group $GL(k, D)$, and let σ be a cuspidal representation of the Hermitian quaternionic group $G_r(F)$. Suppose that the pair (ρ, σ) satisfies $(C\alpha)$ for some $\alpha > 0$, that is, $\nu_\rho^\alpha \rho \rtimes \sigma$ reduces for some $\alpha > 0$. Then the following holds.*

(a) The representation ρ is self-dual.

(b) For all $m \geq 0$, the representation

$$\nu_\rho^{\alpha+m} \rho \times \nu_\rho^{\alpha+m-1} \rho \times \cdots \times \nu_\rho^\alpha \rho \rtimes \sigma$$

has a unique irreducible subrepresentation denoted by $\delta([\nu_\rho^\alpha\rho, \nu_\rho^{\alpha+m}\rho], \sigma)$. *It is square-integrable and we have*

$$\mu^*(\delta([\nu_\rho^\alpha\rho, \nu_\rho^{\alpha+m}\rho], \sigma)) = \sum_{i=-1}^{m} \delta([\nu_\rho^{\alpha+i+1}\rho, \nu_\rho^{\alpha+m}\rho]) \otimes \delta([\nu_\rho^\alpha\rho, \nu_\rho^{\alpha+i}\rho], \sigma),$$

where we take formally that $\delta(\emptyset, \sigma) = \sigma$, *and for the contragredient*

$$\delta([\nu_\rho^\alpha\rho, \nu_\rho^{\alpha+m}\rho], \sigma)\tilde{} = \delta([\nu_\rho^\alpha\rho, \nu_\rho^{\alpha+m}\rho], \widetilde{\sigma}).$$

(*c*) *For all* $m \geq 0$, *the representation*

$$\nu_\rho^{\alpha+m}\rho \times \nu_\rho^{\alpha+m-1}\rho \times \cdots \times \nu_\rho^\alpha\rho \rtimes \sigma$$

has a unique irreducible quotient denoted by $\zeta([\nu_\rho^\alpha\rho, \nu_\rho^{\alpha+m}\rho], \sigma)$. *We have*

$$\mu^*(\zeta([\nu_\rho^\alpha\rho, \nu_\rho^{\alpha+m}\rho], \sigma)) = \sum_{i=-1}^{m} \zeta([\nu_\rho^{-\alpha-m}\rho, \nu_\rho^{-\alpha-i-1}\rho]) \otimes \zeta([\nu_\rho^\alpha\rho, \nu_\rho^{\alpha+i}\rho], \sigma),$$

where we take formally that $\zeta(\emptyset, \sigma) = \sigma$. *In the Langlands classification*

$$\zeta([\nu_\rho^\alpha\rho, \nu_\rho^{\alpha+m}\rho], \sigma) = L(\nu_\rho^\alpha\rho, \nu_\rho^{\alpha+1}\rho, \ldots, \nu_\rho^{\alpha+m}\rho, \sigma),$$

and it can be characterized as the unique irreducible subquotient π *of the induced representation* $\nu_\rho^{m+\alpha}\rho \times \nu_\rho^{m-1+\alpha}\rho \times \cdots \times \nu_\rho^\alpha\rho \rtimes \sigma$ *that satisfies*

$$s_{(k)^{m+1}}(\pi) = \nu_\rho^{-\alpha-m}\rho \otimes \nu_\rho^{-\alpha-(m-1)}\rho \otimes \cdots \otimes \nu_\rho^{-\alpha-1}\rho \otimes \nu_\rho^{-\alpha}\rho \otimes \sigma,$$

where $(k)^{m+1}$ *stands for the sequence* $(k, \ldots, k)$ *with* k *appearing* $m+1$ *times.*

Proof. For (a) see [19]. The proof of (b) is analogous to the quasi-split case in [25, Prop. 3.1]. Applying Aubert involution [1] to (b) gives (c). □

5.3. Unitary induction of $GL-$type

We consider here reducibility of the induced representations $\delta \rtimes \sigma$, where δ is an irreducible square-integrable representation of $GL(k, D)$.

Theorem 7. *Let* ρ *be an irreducible unitarizable cuspidal representation of the group* $GL(k, D)$, *and let* σ *be a cuspidal representation of the Hermitian quaternionic group* $G_r(F)$. *Then the following holds.*

(*a*) *Suppose that the pair* (ρ, σ) *does not satisfy* $(C\alpha)$ *for any* $\alpha \in 1/2 + \mathbb{Z}$. *Then the representation* $\delta([\nu_\rho^{-m-1/2}\rho, \nu_\rho^{m+1/2}\rho]) \rtimes \sigma$ *is irreducible for all integers* $m \geq 0$.

(*b*) *Suppose that the pair* (ρ, σ) *does not satisfy* $(C\alpha)$ *for any* $\alpha \in \mathbb{Z}$. *Then the representation* $\delta([\nu_\rho^{-m}\rho, \nu_\rho^m\rho]) \rtimes \sigma$ *is irreducible for all integers* $m \geq 0$.

(c) *Suppose that the pair* (ρ, σ) *satisfies* $(C1/2)$. *Then for all integers* $m \geq 0$, *the representation* $\delta([\nu_\rho^{-m-1/2}\rho, \nu_\rho^{m+1/2}\rho]) \rtimes \sigma$ *reduces to the sum of two non-isomorphic irreducible representations.*

(d) *Suppose that the pair* (ρ, σ) *satisfies either* $(C0)$ *or* $(C1)$. *Then for all integers* $m \geq 1$, *the representation* $\delta([\nu_\rho^{-m}\rho, \nu_\rho^{m}\rho]) \rtimes \sigma$ *reduces to the sum of two non-isomorphic irreducible representations.*

Proof. The proof of all claims is based on reducibility criteria for parabolically induced representations established in [24, Section 3]. These criteria are based on the existence of the so-called coherent decompositions of Jacquet modules introduced in *loc. cit.* Since this technique is developed for arbitrary p-adic reductive groups, it can be directly applied to the case of Hermitian quaternionic groups. The main tools that we use are [24, Lemma 3.8] and the reducibility criterion of [24, Lemma 3.1 and Remark 3.2].

Then the proofs of (a) and (b) follow as the proofs of [24, Prop. 4.1] and [24, Prop. 4.2]. In (c), for reducibility we use [24, Lemma 3.1 and Remark 3.2], and the rest follows as [21, Theorem 4.2]. Claim (d) follows as [21, Theorems 5.4 and 6.4]. □

Using Aubert involution [1], the same theorem is obtained for representations of the form $\zeta(\Delta)$ for symmetric segments Δ.

5.4. Irreducibility of some induced representations

We now consider induced representations $\nu_\rho^\beta\rho \rtimes \delta(\nu_\rho^\beta\rho, \sigma)$ and $\nu_\rho^\beta\rho \rtimes \zeta(\nu_\rho^\beta\rho, \sigma)$, for $\beta \in (1/2)\mathbb{Z}$ with $\beta \geq 1$, and induced representations $\delta([\rho, \nu_\rho\rho]) \rtimes \sigma$ and $\zeta([\rho, \nu_\rho\rho]) \rtimes \sigma$.

Proposition 1. *Let* ρ *be an irreducible unitarizable cuspidal representation of the group* $GL(k, D)$, *and let* σ *be a cuspidal representation of the Hermitian quaternionic group* $G_r(F)$. *Then the following holds:*

(a) *Suppose that the pair* (ρ, σ) *satisfies* $(C\beta)$ *for* $\beta \in (1/2)\mathbb{Z}$ *with* $\beta \geq 1$. *Then induced representations* $\nu_\rho^\beta\rho \rtimes \delta(\nu_\rho^\beta\rho, \sigma)$ *and* $\nu_\rho^\beta\rho \rtimes \zeta(\nu_\rho^\beta\rho, \sigma)$ *are irreducible.*

(b) *Suppose that the pair* (ρ, σ) *satisfies neither* $(C0)$ *nor* $(C1)$. *Then the induced representations* $\delta([\rho, \nu_\rho\rho]) \rtimes \sigma$ *and* $\zeta([\rho, \nu_\rho\rho]) \rtimes \sigma$ *are irreducible.*

Proof. As in Theorem 7, the proof of (a) is based on the coherent decompositions of Jacquet modules. It follows as the proof of [24, Prop. 5.1].

The proof of (b) closely follows the proof of [24, Lemmas 6.2 and 6.3]. It uses the theory of R-groups, which holds for Hermitian quaternionic groups as checked in Theorem 5, to establish irreducibility of the induced representation $\rho \times \delta([\nu_\rho^{-1}\rho, \nu_\rho\rho]) \rtimes \sigma$. Then the structural formula, checked in Theorem 4 for Hermitian quaternionic groups, is applied to show that the representation $\delta([\rho, \nu_\rho\rho]) \times \delta([\rho, \nu_\rho\rho]) \otimes \sigma$ is of multiplicity four in the representations $\mu^*(\rho \times \rho \times \nu_\rho\rho \times \nu_\rho\rho \rtimes \sigma)$ and $\mu^*(\rho \times \delta([\nu_\rho^{-1}\rho, \nu_\rho\rho]) \rtimes \sigma)$. □

5.5. Cuspidal reducibilities at 1

In this subsection, we consider the pair (ρ, σ) satisfying condition $(C1)$. The following theorem holds for any $\alpha \in \mathbb{R}$, but we state it only for $\alpha \geq 0$, because the case of negative α follows from the equality $\pi \rtimes \sigma = \widetilde{\pi} \rtimes \sigma$ in the Grothendieck group.

Theorem 8. *Let ρ and ρ_0 be an irreducible unitarizable cuspidal representation of $GL(k, D)$ and $GL(k_0, D)$, respectively, and let σ be an irreducible cuspidal representation of $G_r(F)$. Suppose that the pair (ρ, σ) satisfies $(C1)$. Let m be a positive integer and $\alpha \geq 0$ a real number. Then the following holds:*

(*a*) *If $\rho_0 \not\cong \rho$, then $\nu_{\rho_0}^{\alpha}\rho_0 \rtimes \delta([\nu_\rho\rho, \nu_\rho^m\rho], \sigma)$ reduces if and only if (ρ_0, σ) satisfies condition $(C\alpha)$. In that case,*

- *if (ρ_0, σ) satisfies condition $(C0)$, then $\rho_0 \rtimes \delta([\nu_\rho\rho, \nu_\rho^m\rho], \sigma)$ is the sum of two non-isomorphic irreducible tempered representations;*
- *if (ρ_0, σ) satisfies condition $(C\alpha)$ for some $\alpha > 0$, then the representation $\nu_{\rho_0}^{\alpha}\rho_0 \rtimes \delta([\nu_\rho\rho, \nu_\rho^m\rho], \sigma)$ contains a unique square-integrable subquotient denoted by $\delta(\nu_{\rho_0}^{\alpha}\rho_0, [\nu_\rho\rho, \nu_\rho^m\rho], \sigma)$, and we have*

$$\nu_{\rho_0}^{\alpha}\rho_0 \rtimes \delta([\nu_\rho\rho, \nu_\rho^m\rho], \sigma) = \delta(\nu_{\rho_0}^{\alpha}\rho_0, [\nu_\rho\rho, \nu_\rho^m\rho], \sigma) + L(\nu_{\rho_0}^{\alpha}\rho_0, \delta([\nu_\rho\rho, \nu_\rho^m\rho], \sigma))$$

  in the Grothendieck group.

(*b*) *If $\rho_0 \cong \rho$, then $\nu_\rho^{\alpha}\rho \rtimes \delta([\nu_\rho\rho, \nu_\rho^m\rho], \sigma)$ reduces if and only if $\alpha \in \{0, m+1\}$. The representation $\rho \rtimes \delta([\nu_\rho\rho, \nu_\rho^m\rho], \sigma)$ is the sum of two non-isomorphic irreducible tempered representations. In the Grothendieck group we have*

$$\nu_\rho^{m+1}\rho \rtimes \delta([\nu_\rho\rho, \nu_\rho^m\rho], \sigma) = \delta([\nu_\rho\rho, \nu_\rho^{m+1}\rho], \sigma) + L(\nu_\rho^{m+1}\rho, \delta([\nu_\rho\rho, \nu_\rho^m\rho], \sigma)).$$

(*c*) *If $\alpha > 0$ and if $\nu_{\rho_0}^{\alpha}\rho_0 \rtimes \delta([\nu_\rho\rho, \nu_\rho^m\rho], \sigma)$ is irreducible, then*

$$\nu_{\rho_0}^{\alpha}\rho_0 \rtimes \delta([\nu_\rho\rho, \nu_\rho^m\rho], \sigma) \cong L(\nu_{\rho_0}^{\alpha}\rho_0, \delta([\nu_\rho\rho, \nu_\rho^m\rho], \sigma)).$$

Proof. The proof of [24, Theorem 7.1] may be applied in our case, as it only uses the structural formula, which is verified for Hermitian quaternionic groups in Theorem 4. □

The analogous result, under the same assumptions, for the dual representation $\zeta([\nu_\rho\rho, \nu_\rho^m\rho], \sigma)$ follows by using Aubert involution [1]. For the description of the composition series in that case we may follow the proof of [24, Theorem 7.2].

5.6. Cuspidal reducibilities at 1/2

Now we consider the pair (ρ, σ) satisfying condition $(C1/2)$. Note that, as in the previous subsection, the result is stated only for $\alpha \geq 0$, although it also holds for negative α.

Theorem 9. *Let ρ and ρ_0 be irreducible unitarizable cuspidal representations of $GL(k, D)$ and $GL(k_0, D)$, respectively, and let σ be an irreducible cuspidal representation of $G_r(F)$. Suppose that the pair (ρ, σ) satisfies $(C1/2)$. Let m be a positive integer and $\alpha \geq 0$ a real number. Then the following holds:*

(*a*) *If $\rho_0 \not\cong \rho$, then $\nu_{\rho_0}^{\alpha}\rho_0 \rtimes \delta([\nu_\rho^{1/2}\rho, \nu_\rho^{1/2+m}\rho], \sigma)$ reduces if and only if (ρ_0, σ) satisfies condition $(C\alpha)$. In that case,*

- *if (ρ_0, σ) satisfies condition $(C0)$, then $\rho_0 \rtimes \delta([\nu_\rho^{1/2}\rho, \nu_\rho^{1/2+m}\rho], \sigma)$ is the sum of two non-isomorphic irreducible tempered representations;*
- *if (ρ_0, σ) satisfies $(C\alpha)$ for some $\alpha > 0$, then the representation $\nu_{\rho_0}^{\alpha}\rho_0 \rtimes \delta([\nu_\rho^{1/2}\rho, \nu_\rho^{1/2+m}\rho], \sigma)$ contains a unique square-integrable subquotient denoted by $\delta(\nu^\alpha\rho_0, [\nu_\rho^{1/2}\rho, \nu_\rho^{1/2+m}\rho], \sigma)$, and we have*

$$\begin{aligned}&\nu^\alpha\rho_0 \rtimes \delta([\nu_\rho^{1/2}\rho, \nu_\rho^{1/2+m}\rho], \sigma)\\ &\qquad = \delta(\nu^\alpha\rho_0, [\nu_\rho^{1/2}\rho, \nu_\rho^{1/2+m}\rho], \sigma) + L(\nu^\alpha\rho_0, \delta[\nu_\rho^{1/2}\rho, \nu_\rho^{1/2+m}\rho], \sigma))\end{aligned}$$

in the Grothendieck group.

(*b*) *If $\rho_0 \cong \rho$, then $\nu_\rho^\alpha\rho \rtimes \delta([\nu_\rho^{1/2}\rho, \nu_\rho^{1/2+m}\rho], \sigma)$ reduces if and only if $\alpha \in \{1/2, m+3/2\}$.*

- *For $\alpha = m + 3/2$, we have*

$$\begin{aligned}&\nu_\rho^{m+3/2}\rho \rtimes \delta([\nu_\rho^{1/2}\rho, \nu_\rho^{1/2+m}\rho], \sigma)\\ &\qquad = \delta([\nu_\rho^{1/2}\rho, \nu_\rho^{3/2+m}\rho], \sigma) + L(\nu_\rho^{m+3/2}\rho, \delta([\nu_\rho^{1/2}\rho, \nu_\rho^{m+1/2}\rho]), \sigma))\end{aligned}$$

in the Grothendieck group.

- *For $\alpha = 1/2$ and $m > 0$, there exists a unique irreducible square-integrable subquotient in the induced representation $\nu_\rho^{-1/2} \rtimes \delta([\nu_\rho^{1/2}\rho, \nu_\rho^{1/2+m}\rho], \sigma)$ denoted by $\delta([\nu_\rho^{-1/2}\rho, \nu_\rho^{1/2+m}\rho]_+, \sigma)$, and we have*

$$\begin{aligned}&\nu_\rho^{1/2}\rho \rtimes \delta([\nu_\rho^{1/2}\rho, \nu_\rho^{1/2+m}\rho], \sigma)\\ &\qquad = L(\nu_\rho^{1/2}\rho, \delta([\nu_\rho^{1/2}\rho, \nu_\rho^{1/2+m}\rho], \sigma) + \delta(([\nu_\rho^{-1/2}\rho, \nu_\rho^{1/2+m}\rho]_+, \sigma)\end{aligned}$$

in the Grothednieck group.

(*c*) *If $\alpha > 0$ and if $\nu_{\rho_0}^{\alpha}\rho_0 \rtimes \delta([\nu_\rho^{1/2}\rho, \nu_\rho^{1/2+m}\rho], \sigma)$ is irreducible, then*

$$\nu_{\rho_0}^{\alpha}\rho_0 \rtimes \delta([\nu_\rho^{1/2}\rho, \nu_\rho^{1/2+m}\rho], \sigma) = L(\nu_{\rho_0}^{\alpha}\rho_0, \delta[\nu_\rho^{1/2}\rho, \nu_\rho^{1/2+m}\rho], \sigma)).$$

Proof. This theorem is stated without proof as [24, Theorem 8.2]. However, its proof is very similar to the proof of [24, Theorem 8.1], which is an analogous result for the representations of the form $\zeta([\nu_\rho^{1/2}\rho, \nu_\rho^{1/2+m}\rho], \sigma)$. Since the proof uses only the structural formula, it may be applied in our case due to Theorem 4. □

5.7. Reducibility points of the representation $\nu_\rho^\alpha\delta(\rho,m)\rtimes\sigma$

Recall from Section 3 the notation $\delta(\rho,m)=\delta([\nu_\rho^{-(m-1)/2}\rho,\nu_\rho^{(m-1)/2}\rho])$, which is a square-integrable representation of $GL(mk,D)$, where m is a positive integer. We assume here that $m\geq 2$, because for $m=1$ we have $\delta(\rho,1)=\rho$, and reducibility points are just cuspidal reducibilities of $\nu_\rho^\alpha\rho\rtimes\sigma$.

Theorem 10. *Let ρ be an irreducible unitarizable cuspidal representation of the group $GL(k,D)$, and σ an irreducible cuspidal representation of the Hermitian quaternionic group $G_r(F)$. Let $m\geq 2$ be an integer, and $\alpha\in\mathbb{R}$. Then the induced representation $\nu_\rho^\alpha\delta(\rho,m)\rtimes\sigma$ has the following reducibility points:*

(*a*) *If ρ is not self-dual, then $\nu_\rho^\alpha\delta(\rho,m)\rtimes\sigma$ is irreducible for all $\alpha\in\mathbb{R}$.*

(*b*) *If the pair (ρ,σ) satisfies $(C1/2)$, then the representation $\nu_\rho^\alpha\delta(\rho,m)\rtimes\sigma$ is reducible if and only if*

$$\alpha\in\left\{\frac{-m}{2},\frac{-m}{2}+1,\ldots,\frac{m}{2}\right\}.$$

(*c*) *If the pair (ρ,σ) satisfies $(C0)$, then the representation $\nu_\rho^\alpha\delta(\rho,m)\rtimes\sigma$ is reducible if and only if*

$$\alpha\in\left\{\frac{-m+1}{2},\frac{-m+1}{2}+1,\ldots,\frac{m-1}{2}\right\}.$$

(*d*) *If the pair (ρ,σ) satisfies $(C1)$, then the representation $\nu_\rho^\alpha\delta(\rho,m)\rtimes\sigma$ is reducible if and only if*

$$\alpha\in\left\{\frac{-m-1}{2},\frac{-m-1}{2}+1,\ldots,\frac{m+1}{2}\right\}.$$

Proof. The proof of [24, Theorem 9.1] for the split groups is applicable to our case. □

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