VOLATILITY PATTERNS OF THE LARGEST POLISH COMPANIES: SOME EVIDENCE FROM HIGH-FREQUENCY DATA

Krzysztof Drachal

Krzysztof Drachal
Faculty of Economic Sciences,
University of Warsaw, Poland,
kdrachal@wne.uw.edu.pl

Article info
Paper category: Original Scientific Paper
Received: 18.1.2018.
Accepted: 22.3.2018.
JEL classification: C32, C58, G10, G17

Keywords
Diamond-Verrecchia hypothesis; HAR, high-frequency data; UHF-GARCH; Warsaw Stock Exchange

ABSTRACT

Purpose. *The article is focused on the empirical properties of the high-frequency data of 20 selected stocks from the Warsaw Stock Exchange (in particular the ones listed on WIG 20). The intraday data from at least more than 1 year were analysed. In particular, correlation between returns and durations were checked.*

Methodology. *Also, the heterogeneous autoregressive model for realized volatility (HAR) was analysed and an attempt to construct the UHF-GARCH model was taken. The HAR model is a linear model and the UHF-GARCH is based on a certain adjustment of physical durations. Then, the standard ARMA-GARCH approach can be considered. Moreover, the hypothesis of Diamond and Verrecchia predicting a negative correlation between price changes and the time passed between transactions was checked. The analysis was done in R statistical software.*

Findings. *The presented research can serve as an introduction to some further (and more thorough and narrow) researches. Except a direct presentation of outcomes from the study of the selected stocks from the Warsaw Stock Exchange, the paper contains quite an extensive literature review on high-frequency data.*

1. INTRODUCTION

As the capabilities of data collecting and storing are increasing very rapidly, the usage of high frequency data in finance becomes more and more popular. Such a usage is also connected with the increase of the popularity of, for example, algorithmic trading and other practical applications. In such techniques even ultra-high frequency data are analysed. Of course, in a situation, when fast computational machines are unavailable, then the enormous amount of data cannot be quickly analysed and high frequency data are not so convenient.

Nowadays, transactions on stock exchanges are executed in majority by computers. Whereas not long ago most of them were done, at least in principle, by the human being. Currently, computers are capable not only of submitting, but also of managing, the stock exchange orders. It is estimated that between 50% and 77% of the trading volume in the US equities markets is nowadays done by the algorithmic trading (Cartea and Jaimungal, 2013).

Of course, such changes resulted in a vast literature on fundamentals of high frequency data analysis (for example: Ait-Sahalia and Jacod, 2014; Dacorogna et al., 2001; Mariano and Yiu-Kuen, 2008).

However, high frequency data are not just a mere increase in a computational challenge for statisticians, econometricians and programmers. They are not merely a source of more records with no new insight into the structure of the market. Tick data unveil some specific patterns and effects, which diminish even if daily data are used. In other words, high frequency data are very useful in studying the microstructure of the market (O'Hara, 1997; Hautsch, 2012; Hasbrouck, 2007).

The microstructure foundations of the Warsaw Stock Exchange were thoroughly discussed by Orłowski (2009) and Bień (2005). Yet, their researches were done for a very specific stock or for the period just after EU accession. Therefore, it seems interesting to analyse also a bit more recent period, relatively large period and for selected stocks. Usually, most researches for Polish stock exchange, cited later in this paper, were based solely on the market index. Despite the fact that the stock market index is a fair representative of the overall market, it still gives just the overall information and in certain situations may give biased results. For example, the stock market index was found to have a stronger effect on stocks than stock have on the index (Kennet et al., 2013). It seems very natural then that researches interested in the microstructure of the market should analyse also particular selected stocks.

Secondly, the developed markets were quite extensively studied in case of high-frequency data. On the other hand, less attention was given to the developing stock exchanges. However, the Warsaw Stock Exchange is the biggest in the region. As a result, it is tried herein to fill a certain literature gap and provide more detailed study on its market microstructure.

In particular, the aim of this research is to analyse certain relationships from the Warsaw Stock Exchange in the context of the market microstructure and high-frequency approach. Moreover, the aim is to formulate the results basing on the data from single stocks, instead of studying just the behaviour of the stock market index. In a more narrow sense the aim is to check the correlation between stock returns and durations between the transactions. Secondly, the volatility patterns are studied with the heterogeneous autoregressive model for realized volatility (HAR) and the UHF-GARCH model. Additionally, the standard ARMA-GARCH approach is also considered. Thirdly, the hypothesis of Diamond and Verrecchia (supposing a negative correlation between price changes and the time passed between transactions) is checked.

2. LITERATURE REVIEW

High-frequency data contain a lot of information useful for market players. For example, Andersen and Teräsvirta (2009) stressed that this information can be used for better volatility forecasting. In other words, the more frequent data can result in more accurate forecasts. Secondly, high-frequency data contain information which can be important in the context of the distribution of returns under the conditions of non-arbitrage. Moreover, this kind of data can be interestingly joint with the concept of multivariate measures of the quadratic variation. Finally, they noticed the advantages of high-frequency data in estimating realized volatility, as well as, generally in the specification and estimation of the econometric model.

As it was just aforementioned, high frequency data unveil a lot of interesting patterns and effects not seen under the analysis of less frequent records. On the contrary to daily data, tick data are characterized by unequal durations between the time of sequent transactions (i.e., a non-synchronous trading occurs). Including this information in the statistical analysis of returns shows that there is a negative serial autocorrelation between the returns. Moreover, tick data allow to include the information about the bid-ask spread, which imposes a negative first lag autocorrelation of price changes. This effect is usually named a bid-ask bounce (Bauwens, 2001; Campbell et al., 1996; Tsay, 2005). As a result, for intraday transaction data a price reversal can be observed.

Indeed, based on numerous empirical findings it is commonly agreed that the transaction data are characterised by unequally spaced time intervals between the transactions and also by the existence of a daily periodic or diurnal pattern (Ghysels, 2000). In particular, most of the transactions are executed at the beginning and at the end of the session. The number of transactions is usually the lowest in the middle of the session. Such a pattern is clearly seen as an U-shaped curve of the transactions intensity (Wood et al., 1985; Admati and Pfleiderer, 1988).

Also, the fact that multiple transactions can be executed within a single second and prices have only discrete values plays a significant role in the statistical analysis (Cartea and Meyer-Brandis, 2010; Dufour and Engle, 2000; Easley and O'Hara, 1992; Engle and Russell, 1998; Manganelli, 2005).

For example, Gramming and Wellner (2002) noticed an interesting fact that the duration between transactions (or quote updates) can significantly affect the volatility. On the other hand, they questioned if the volatility does feedback the durations. Finally, they found that volatility shocks can lead to longer transaction durations.

The above observation is linked with the stimulative hypothesis of Diamond and Verrecchia (1987) who predicted a negative correlation between price changes and the time passed between transactions. Their conclusions were drawn from modelling and analysing the role of information on the market. According to their considerations, durations should be negatively correlated with price changes, because of short-selling constraints which prevent from the trading on private bad news, whereas there are no similar constraints to prevent from trading on private good news.

Their paper has been very stimulating for the further discussion and the development of numerous researches on the role of information for traders. Indeed, the presented relations are usually discussed in the context of information-based models or inventory-based models (Bowe et al., 2007; Tsay and Ting, 2008). However, it is worth to mention that basing also on the analysis of the role of information, Easley and O'Hara (1992) derived the conclusions opposite to Diamond and Verrecchia (1987).

But Tay et al. (2011) found that after long transaction durations it is more likely that prices will decline, which is consistent rather with the hypothesis of Diamond and Verrecchia (1987) than with that of Easley and O'Hara (1992). Yet, it should be mentioned, that the research of Tay et al. (2011) was based on the extension of ACD (autoregressive conditional duration) model initially proposed by Engle and Russel (1998). This model is currently one of the fundamental tool in high frequency data analysis. However, Dufour and Engle (2000) argued that long durations should have lower impact on prices than short durations.

Yet, staying with the hypothesis of the negative correlation, one can also find some reminiscences of the Elliot waves theory. According to this hypothesis, it is believed that the increases of the price are usually lasting for a longer time and are more stable than the price declines, which happen in relatively shorter periods (Frost and Prechter, 1998; Poser and Plummer, 2003). The Elliot wave theory is deeply rooted in behavioural aspects, which are also very important on stock exchanges. Nevertheless, this paper is focused only on quantitative analysis, so it will not be more discussed.

As far as now, there is no consensus amongst researchers whether a sudden price decrease should be the sing for an investor to sell the stocks as soon as possible and prevent from further price decline or should an investor wait till the price correction (i.e., to benefit from the price reversal effect). For example, a sudden price decline can result in the decrease in the liquidity and, as a result, in further obstacles against selling stocks.

In this context, some researchers acknowledge the disposition effect. It is a name for the situation, in which investors keep their stocks even though the price has significantly fallen. Investors can be more willing to trade after price increases. Then, it can be assumed that a decrease in the liquidity is connected with a price decrease (Griffin et al., 2007). Similarly, a threat to the liquidity can result in a price decrease. But the impact between the price and the liquidity can be asymmetric. The trading volume is usually considered as the liquidity measure, as it is the simplest and the most fundamental one (Gabrielsen et al., 2011).

On the contrary to the above described effects, Ammann and Kessler (2009) analysed certain types of stock price crashes on the US market. They based on the intraday data. They found that during the crash, the liquidity is in general relatively higher, similarly as the trading volume and the number of transactions in a fixed time interval.

Jones (2002) analysed the Dow Jones index for the whole 20th century and showed that high liquidity can result in low returns. Another problem, indicated by numerous researchers is that low liquid stocks can generate relatively a bit higher returns in order to compensate relatively higher transaction costs. Yet, this liquidity premium was found rather small (Baker and Filbeck, 2015; Lhabitant and Gregoriou, 2008).

For the Warsaw Stock Exchange it was found by Doman and Doman (2010) that the liquidity is the most important factor influencing the process of discovering information by uninformed traders.

Baker and Stein (2004) introduced a certain model of various investor types. They found that lower bid-ask spread (being an indicator of the increasing liquidity) can result in lower returns. Similarly, Engle (2000) basing on ACD and GARCH type models, found that variances are high in the periods of longer durations between transactions. The usual explanation for such a relationship is derived on the basis of the asymmetry of information available for traders.

The liquidity is not observable. Therefore, there exist various liquidity measures. Unfortunately, Baker (1996) found that different measures can lead to mutually exclusive conclusions, which vary depending on which liquidity measure is used.

Yet, there has not been much research in the above direction on the Warsaw Stock Exchange. Although, Gluzicka (2013) and Doman (2008) analysed stock market indexes on the Warsaw Stock Exchange in this context. They found that there is a significant and strong correlation between the price and the trading volume during the boom. However, this correlation was significant, strong and positive only during the boom periods. On the other hand, on the bear market and shortly afterwards the sign of the correlation depended on which market index was analysed. The linear test of the Granger causality proved that the price change results in the change in the trading volume. But it could not have been concluded that a change in the trading volume results in a change in the price. Moreover, the causality between the trading volume and returns was found significant only for few market indexes.

It is also worth to mention that for a developed market, Foster and Viswanathan (1990; 1993) observed that the trading volume should exhibit some degree of autocorrelation.

Another interesting problem is the forecasting of the volatility. Similarly, like the liquidity, it is not observable. One of the modern ways of modelling the volatility is based on the GARCH model framework. This model was introduced by Bollerslev (1986) as a generalization of the model of Engle (1982). The Engle model allows the error term to have a characteristic size of variance. The GARCH model additionally allows to include the lagged conditional variance terms as autoregressive. Then, the variance depends both on past shocks (through the lagged residuals) and the past values of itself.

The simplest type of the above family of models is the GARCH(1,1) model. In particular, x_t follows the GARCH(1,1) process, if:

$$xt = c + f1\, y1 + f2\, y2 + \ldots + fk\, yk + ut, \quad (1)$$

$$ut = zt \sqrt{ht}, \quad (2)$$

$$ht = w + a\,(ut-1)2 + b\, ht-1, \quad (3)$$

where c, $f_1, \ldots, f_k$ are parameters, $y_1, \ldots, y_k$ are explanatory variables and u is the error term. It is assumed that z_t follows the standard normal distribution with zero mean and variance one. It should be noted that there are various modifications of the simple GARCH(1,1) model. Therefore, one should rather speak of the GARCH type family of models.

For example, Xekalaki and Degiannakis (2010), Craioveanu (2008) and Doman and Doman (2009) provided thorough reviews of the applicability of various GARCH type models in finance. One of the great advantages of the GARCH model is that such a model can be quite easily fitted depending on the market specification and the research context. On the other hand, this fact can also be a drawback, as one can describe the concrete case in various ways (Ai et al., 2011; Berra and Higgins, 1993; Engle, 2001). An extensive list of various GARCH type models was presented by Bollerslev (2008) in a quite concise manner. Empirical results of the application of GARCH type models for the Warsaw Stock Exchange were described, for example, by Fiszeder (2009), Bień (2005), Doman and Doman (2005) and Ferenstein and Gšsowski (2004). It should also be mentioned that some researchers criticized the GARCH models (Starica, 2006).

Bauwens (2001) discussed the usage of GARCH(1,1) type models for intraday data and found that the influence of trade related variables is important for 5min, 10min, 15min and 30min data. However, Dacorogna et al. (2001) presented an interesting review and arguments that the estimates of the GARCH model based on a physical time can lead to spurious estimates if high frequency data are used. Shortly, this is due to the need of incorporating the statistically significant, previously mentioned, seasonal intraday pattern (Andersen and Bollerslev, 1997); but then the aggregation properties of the model break down.

Christoffersen (2012) and Francq and Zakoian (2010) provided an interesting discussion, comparison of the GARCH based volatility with the realized volatility concept and references to the vast literature. Indeed, there are various approaches towards the estimation of the volatility of an asset (Andersen et al., 2010; 2012; Barndor-Nielsen et al., 2004). For example, if $r_{t,i}$ is the i-th intraday return on day t and n is the number of intraday returns per day, then the Eq. (4) defines the realized volatility.

$$RV_t = \Sigma_{i=1}^{n} r_{t,i}^{2} \quad (4)$$

The realized volatility and the GARCH framework (together with the stochastic volatility and the implied volatility) are one of the most important approaches towards the dynamic modelling of the volatility. It is worth to mention that the realized volatility is non-parametric, whereas the volatility derived from the GARCH framework is parametric. Moreover, the GARCH model provides a one step ahead forecast given the past, whereas the realized volatility does not use the information from the other days. The important advantage of the realized volatility is that in the case of high frequency data returns the classical time series methods can be applied (Andersen et al., 2003). The concept of volatility in various aspects was thoroughly and interestingly discussed in the collection edited by Knight and Satchell (2007). Short reviews in the context of the Warsaw Stock Exchange were presented by Ślepaczuk and Zakrzewski (2009a; 2009b).

Some solutions of the previously mentioned problems with non-synchronous trading and volatility modelling, yet staying with the GARCH framework approach, were proposed, inter alia, by Rossi and Fantazzini (2014) and Chen et al. (2009; 2011). For example, Dionee et al. (2009) proposed a successful modification for the case of the Toronto Stock Exchange, Antola and Vuorenmaa (2013) for Nasdaq and Martens (2002) for S&P 500 index. Andersen and Bollerslev (1998a) decomposed the conditional variance of intraday returns into multiplicative product of diurnal components. Chanda et al. (2005) and Engle and Sokalska (2012) focused on modelling and forecasting intraday returns. The volatility was decomposed into different components, i.e., the conditional variance was expressed as a product of daily, diurnal and stochastic intraday volatility. Such a methodology was applied to 10 min returns of over 2700 US equities. Muller et al. (1997) proposed an approach connected with the hypothesis of a heterogeneous market. They used the heterogeneous interval ARCH model, in which the conditional variance depends on past squared returns taken at different frequencies. Such an approach was a try to fix the problem of the asymmetry in the standard GARCH approach.

In case of tick data Engle (2000) proposed to consider time-adjusted returns. In other words, r_t is defined to be the return in time t divided by the square root of the duration interval time. Next, it is assumed that r_t follows the ARMA process, i.e.,

$$r_t = c + f\, r_{t-1} + g\, u_{t-1} + u_t\,, \tag{5}$$

where c, f and g are parameters and u is the error term. It is assumed that

$$u_t = z_t \sqrt{h_t}\,, \tag{6}$$

where z_t follows the standard normal distribution with zero mean and variance one and

$$h_t = w + a\,(u_{t-1})^2 + b\,h_{t-1}\,. \tag{7}$$

As a result, the ARMA(1,1)-GARCH(1,1) model can be estimated. It is an example of the simplest UHF-GARCH model (ultra-high frequency GARCH). Its advantage is that it can be easily estimated as the standard ARMA-GARCH type model. The obtained outcomes allow the estimation of the variance per unit time.

A quite different approach was proposed by Ghysels and Jasiak (1998). They modelled unequal durations by the variation of the Engle and Russell (1998) ACD model and the volatility by the GARCH(1,1) model. The discussion of this approach in the context of the UHF-GARCH type model was presented by Meddahi et al. (2006). An extensive review of ACD models, both from theoretical and applied point of view, was presented by Pacurara (2008).

However, Coen and Racicot (2004) and Racicot et al. (2008) argued that the integrated volatility method of Andersen and Bollerslev (1998a) and Bollerslev and Wright (2001) measured by the squared value of intraday returns outperforms UHF-GARCH type models.

On the other hand, an interesting and successful application of UHF-GARCH model to the Portuguese money market was done by Sol Murta (2007). Zongxin and Xiao (2011) applied similar model to 14 stocks from the Shanghai Stock Exchange. Also, Wang et al. (2010) discussed the GARCH model for the Shanghai Stock Exchange. Alfonso-Cifuentes and Serna-Cortes (2012) successfully studied the Colombian Exchange Market index with such an approach. Another interesting model was given by Darolles et al. (2000). The Bayesian inference for certain UHF-GARCH models based on the tick data for the Warsaw Stock Exchange was studied by Huptas (2009; 2013).

Later, Engle and Sun (2005) expanded the UHF-GARCH model by incorporating autocorrelations caused by the microstructure effect. Their approach was applied to the problem of a one-day volatility forecasting of the tick data.

In case of the realized volatility Andersen et al. (2007) and Corsi (2009) proposed the HAR model (heterogeneous autoregressive model for realized volatility). This model is easy to estimate and can reproduce the long memory and fat tails. The model is given by the Eq. (8).

$$RV_{t+1} = c_0 + c_1\, RV_t + c_2\, RV_{t-5,t} + c_3\, RV_{t-22,t} + u_{t+1}\,, \tag{8}$$

where RV is defined as in Eq. (4).

3. METHODOLOGY

The periodicity in diurnal patterns for high frequency data can be analysed by various methods (Martens et al., 2002; Franse, 1992; Ghysels and Osborn, 2001). For example, with a help of the Fourier flexible form approximation, by sampling moments for each intraday bin or by the dynamic cubic spline (Creal et al., 2011 and 2013; Harvey, 2013; Harvey and Koopman, 1993; Ito, 2013). The last method allows for the dynamic in the periodicity, an inclusion of the day of the week effect with no seasonal dummy variables, and the fact that the overnight news can change the next day trading pattern.

But, for example, the problem of the occurrence of the day of the week effect on the Warsaw Stock Exchange is not clearly solved yet (Gajdosowva et al., 2011; Heryan and Stavarek, 2012; Jamróz and Koronkiewicz, 2014). Therefore, and also due to the simplicity, it was not taken under considerations in the present research.

It can also be assumed that the pattern of the periodicity is fixed in time. Such an assumption was made in this research, following numerous previous studies, for example, Andersen and Bollerslev (1998b), Brownlees et al. (2011), Campbell and Diebold (2005), Engle and Rangel (2008), Engle and Russell (1998) and Engle and Sokalska (2012). Therefore, the deseasonalized durations are understood as the physical time durations divided by the seasonal factor. Similarly as in the paper of Allen et al. (2006), the seasonal factor was estimated by the cubic smoothing splines with 4 effective degrees of freedom for the spline (Nychka et al., 2015) applied to the mean daily aggregated data.

Following Engle and Sokalska (2012) and Sokalska (2010) the overnight returns were excluded from the analysis. In particular, logarithmic returns were used. If the sample was aggregated into fixed time intervals, the median price was taken as a representative for a particular bin, except for the daily data, for which the last transaction price was taken. Usually, the last observation is taken in many studies also for short intervals. However, the median seems to be more representative from the point of view of an investor who can deal with this price. Nevertheless, as a bin is taken more and more smaller, the difference between different aggregation methods should diminish (Brownlees and Gallo, 2006). However, it should be mentioned that Henker and Wang (2006) argued that even small changes in the specification of time interval can significantly affect the outcomes through the severe biases in the estimated parameters. But their research was based on a very developed market, i.e., the New York Stock Exchange, and contradicted some previous findings.

Interestingly, Diebold (1988) suggested that the conditional heteroskedasticity should be smaller, when the width of an aggregate bin becomes smaller. Additionally, De Luca (2006) stated that very high frequency processes are close to the integrated GARCH(1,1) model, i.e., the IGARCH(1,1) model.

Many empirical analyses focused on stock exchange indexes, whereas separate time series for particular stocks are less often analysed. It is true that the behaviour of a stock in encoded in the stock exchange index, merely by definition. On the other hand, it seems interesting to analyse the concrete stocks as the fundamental activity on a stock exchange is to trade the stocks. It should also be mentioned that the composition of a market index is changed from time to time. Moreover, the investor can trade on particular stocks, but a short-selling is not allowed, which significantly affects the situation. This constraint is relaxed for index contracts, but the transaction durations of an index contract are not meaningful for the behaviour of durations of a particular stock. All in all, it seems very reasonable to analyse some concrete portfolio of stocks.

In this research 20 selected stocks from the Warsaw Stock Exchange were analysed: ALIOR, ASSECOPOL, BOGDANKA, BZWBK, EUROCASH, JSW, KERNEL, KGHM, LOTOS, LPP, MBANK, ORANGEPL, PEKAO, PGE, PGNIG, PKNORLEN, PKOBP, PZU, SYNTHOS and TAURONPE. These stocks constituted WIG 20 index in the end of 2014. WIG 20 is composed of 20 largest stocks from the Warsaw Stock Exchange. No more than 5 stocks out of a particular market sector can be included in it, nor can an investment fund be included in it (GPW, 2015). The tick data were obtained from BOŚ (2015). Table 1. presents the number of observations for particular time series. The date of the last observation is March 5, 2015 for all analysed stocks.

The computational part was done in R (Gentleman and Ihaka, 1996; R Core Team, 2014) with a help of "highfrequency" (Boundt et al., 2014) and "rugarch" (Ghalanos, 2014) packages.

If not stated otherwise, the 5% significance level was assumed.

Table 1.: Data details

Stock name	Day of first observation	
ALIOR	14/12/12	172954
ASSECOPOL	28/10/05	610690
BOGDANKA	22/07/09	320374
BZWBK	25/06/01	866861
EUROCASH	04/02/05	382480
JSW	06/07/11	599300
KERNEL	23/11/07	368693
KGHM	17/11/00	3847988
LOTOS	09/06/05	1064840
LPP	16/05/01	128928
MBANK	26/11/13	140587
ORANGEPL	14/01/14	197071
PEKAO	17/11/00	2026850
PGE	15/12/09	1202889
PGNIG	20/10/05	1416429

Stock name	Day of first observation	
PKNORLEN	17/11/00	2361739
PKOBP	10/11/04	3086038
PZU	12/05/10	1343152
SYNTHOS	02/11/07	491488
TAURONPE	30/06/10	661875

Source: Own elaboration.

4. RESULTS

First, the mentioned Diamond and Verrecchia (1987) hypothesis was tested for the unadjusted durations. The results are presented in Table 2. It presents the correlation between price changes and durations. The correlation was checked by the Pearson coefficient. It can be seen that the correlation occurred to be statistically significant at 5% level only in just a bit more than a half of the stocks. But if it happened to be significant, then (in 73% of such cases) it was negative, as predicted by Diamond and Verrecchia (1987). Nevertheless, it has to be mentioned that in some cases a significant positive correlation was found, but such a behaviour is rather unusual according to the analysed data.

Table 2.: Price change and duration correlation

Stock name	Correlation	P-value
ALIOR	0.0041	0.0863
ASSECOPOL	0.0024	0.0648
BOGDANKA	0.0010	0.5564
BZWBK	0.0005	0.6362
EUROCASH	-0.0100	0.0000
JSW	0.0063	0.0000
KERNEL	-0.0097	0.0000
KGHM	-0.0018	0.0003
LOTOS	0.0007	0.4618
LPP	0.0029	0.2953
MBANK	-0.0124	0.0000
ORANGEPL	-0.0091	0.0001
PEKAO	-0.0009	0.2059
PGE	-0.0023	0.0106
PGNIG	-0.0162	0.0000
PKNORLEN	-0.0003	0.6810
PKOBP	0.0018	0.0013
PZU	-0.0050	0.0000
SYNTHOS	-0.0005	0.7482
TAURONPE	0.0058	0.0000

Source: Own elaboration.

Table 3. presents the results of testing the first lag autocorrelations for price changes and logarithmic returns. Similarly as claimed by the already cited literature sources, in every analysed case the correlation was found negative and statistically significant.

Table 3.: First lag autocorrelations of price changes and logarithmic returns

Stock name	Price change autocorrelation	P-value	Log returns autocorrelation	P-value
ALIOR	-0.2289	0.0000	-0.2307	0.0000
ASSECOPOL	-0.0611	0.0000	-0.1391	0.0000
BOGDANKA	-0.2480	0.0000	-0.2475	0.0000
BZWBK	-0.3411	0.0000	-0.3614	0.0000
EUROCASH	-0.2493	0.0000	-0.2565	0.0000
JSW	-0.2698	0.0000	-0.2536	0.0000
KERNEL	-0.2487	0.0000	-0.2473	0.0000
KGHM	-0.3090	0.0000	-0.3463	0.0000
LOTOS	-0.2569	0.0000	-0.2517	0.0000
LPP	-0.2466	0.0000	-0.2002	0.0000
MBANK	-0.4184	0.0000	-0.4176	0.0000
ORANGEPL	-0.2843	0.0000	-0.2853	0.0000
PEKAO	-0.2888	0.0000	-0.2961	0.0000
PGE	-0.2855	0.0000	-0.2829	0.0000
PGNIG	-0.3861	0.0000	-0.3958	0.0000
PKNORLEN	-0.2910	0.0000	-0.3251	0.0000
PKOBP	-0.3150	0.0000	-0.3276	0.0000
PZU	-0.3338	0.0000	-0.3358	0.0000
SYNTHOS	-0.0106	0.0000	-0.0669	0.0000
TAURONPE	-0.3548	0.0000	-0.3527	0.0000

Source: Own elaboration.

Table 4.: Correlations of trading volume with other indicators

Stock name	Durations	P-value	De-seasonalised durations	P-value	Log returns	P-value	First lag trading volume autocorrelation	P-value
ALIOR	0.0376	0.0000	0.0443	0.0000	-0.0237	0.0000	0.0696	0.0000
ASSECOPOL	0.0240	0.0000	0.0430	0.0000	-0.0008	0.5580	0.0278	0.0000
BOGDANKA	0.0200	0.0000	0.0240	0.0000	-0.0043	0.0139	0.0317	0.0000
BZWBK	0.0217	0.0000	0.0173	0.0000	0.0016	0.1435	0.0623	0.0000
EUROCASH	0.0552	0.0000	0.0577	0.0000	-0.0099	0.0000	0.0460	0.0000

Stock name	Durations	P-value	Deseasonalised durations	P-value	Log returns	P-value	First lag trading volume autocorrelation	P-value
JSW	0.0406	0.0000	0.0553	0.0000	0.0103	0.0000	0.0108	0.0000
KERNEL	0.0262	0.0000	0.0336	0.0000	-0.0197	0.0000	0.0139	0.0000
KGHM	0.1414	0.0000	0.1891	0.0000	-0.0031	0.0000	0.0500	0.0000
LOTOS	0.0647	0.0000	0.0846	0.0000	-0.0005	0.6161	0.0688	0.0000
LPP	0.0345	0.0000	0.0347	0.0000	0.0115	0.0000	0.2574	0.0000
MBANK	0.0950	0.0000	0.1790	0.0000	-0.0369	0.0000	0.0476	0.0000
ORANGEPL	0.0789	0.0000	0.1196	0.0000	-0.0013	0.5598	0.0175	0.0000
PEKAO	0.1042	0.0000	0.1567	0.0000	-0.0015	0.0338	0.0611	0.0000
PGE	0.1112	0.0000	0.1559	0.0000	0.0244	0.0000	0.1090	0.0000
PGNIG	0.1207	0.0000	0.1336	0.0000	0.0060	0.0000	0.1171	0.0000
PKNORLEN	0.1173	0.0000	0.1338	0.0000	0.0066	0.0000	0.0548	0.0000
PKOBP	0.1752	0.0000	0.2202	0.0000	-0.0002	0.7421	0.0546	0.0000
PZU	0.1325	0.0000	0.1315	0.0000	0.0215	0.0000	0.0314	0.0000
SYNTHOS	0.0088	0.0000	0.0150	0.0000	-0.0008	0.5914	0.0026	0.0635
TAURONPE	0.0741	0.0000	0.1030	0.0000	0.0092	0.0000	0.0677	0.0000

Source: Own elaboration.

In every case the correlation of the trading volume and the duration is positive and statistically significant. This is consistent with previously cited references. The same relationship holds, if deseasonalised durations are considered. On the other hand, there is no clear relationship between the trading volume and logarithmic returns. The correlation is statistically significant in 70% of analysed stocks. In 7 cases a statistically significant negative correlation was found. In 7 cases a statistically significant positive correlation was found. Therefore, it seems that there is no definite relationship between returns and the trading volume. On the other hand, in every case a positive first lag autocorrelation of the trading volume was found. This is consistent with the already cited references. Only in one case this relationship was not found statistically significant at 5% significance level. But if 10% level is assumed, then the relationship is statistically significant for every analysed stock.

Consistently with the cited references it was also checked if the significance of ARCH effects for logarithmic returns possibly vanishes within some intervals. It was found that significant ARCH effects are present for 5 min., 10 min., 15 min., 30 min. and 1 hour intervals for every analysed stock. The result are presented in Table 5.

Table 5.: P-values of LM ARCH effects test

Stock name	5 min	10 min	15 min	30 min	60 min
ALIOR	0.0000	0.0000	0.0000	0.0000	0.0000
ASSECOPOL	0.0000	0.0000	0.0000	0.0000	0.0000
BOGDANKA	0.0000	0.0000	0.0000	0.0000	0.0000
BZWBK	0.0000	0.0000	0.0000	0.0000	0.0000
EUROCASH	0.0000	0.0000	0.0000	0.0000	0.0000
JSW	0.0000	0.0000	0.0000	0.0000	0.0000
KERNEL	0.0000	0.0000	0.0000	0.0000	0.0000
KGHM	0.0000	0.0000	0.0000	0.0000	0.0000
LOTOS	0.0000	0.0000	0.0000	0.0000	0.0000
LPP	0.0000	0.0000	0.0000	0.0000	0.0000
MBANK	0.0000	0.0000	0.0000	0.0000	0.0000
ORANGEPL	0.0000	0.0000	0.0000	0.0000	0.0000
PEKAO	0.0000	0.0000	0.0000	0.0000	0.0000
PGE	0.0000	0.0000	0.0000	0.0000	0.0000
PGNIG	0.0000	0.0000	0.0000	0.0000	0.0000
PKNORLEN	0.0000	0.0000	0.0000	0.0000	0.0000
PKOBP	0.0000	0.0000	0.0000	0.0000	0.0000
PZU	0.0000	0.0000	0.0000	0.0000	0.0000
SYNTHOS	0.0000	0.0000	0.0000	0.0000	0.0000
TAURONPE	0.0000	0.0000	0.0000	0.0000	0.0000

Source: Own elaboration.

Following Engle (2000) who proposed to consider time-adjusted returns, logarithmic returns were divided by the square root of the duration interval time. Such returns were checked for the existence of ARCH effects. Only in three cases no significant ARCH effects were found. However, for the majority of the analysed stocks such effects were statistically significant. The results are presented in Table 6. As a result, it was justified to perform UHF-GARCH analysis for these stocks, as described by Eqs (5) - (7).

Table 6.: Results of LM ARCH effects test for time-adjusted returns

Stock name	Statistic	P-value
ALIOR	3.3066	0.9930
ASSECOPOL	0.0024	1.0000
BOGDANKA	44.8897	0.0000
BZWBK	594.6586	0.0000
EUROCASH	692.7784	0.0000
JSW	286.5236	0.0000
KERNEL	334.7558	0.0000
KGHM	74.3607	0.0000

Stock name	Statistic	P-value
LOTOS	54.5645	0.0000
LPP	28.1457	0.0053
MBANK	9413.2983	0.0000
ORANGEPL	87.5038	0.0000
PEKAO	135.2232	0.0000
PGE	478.2346	0.0000
PGNIG	1356.6808	0.0000
PKNORLEN	106.0851	0.0000
PKOBP	360.1966	0.0000
PZU	62.4195	0.0000
SYNTHOS	0.0000	1.0000
TAURONPE	110.2270	0.0000

Source: Own elaboration.

UHF-GARCH models were estimated according to Eqs (5) - (7). Of course, for the previously mentioned three cases with no significant ARCH effects the UHF-GARCH model was not estimated. The results are presented in Tables 7. and 8. In Table 7. there are estimated coefficients of Eqs (5) - (7) and in Table 8. the corresponding p-values indicating t-test that these coefficients are indeed not equal to zero. The parameter c in Eq. (5) for every stock was set to 0 (i.e., models without mean were estimated). In addition, in Table 7. there are p-values from LM test checking whether there remain any significant ARCH effects in standardized residuals. In all cases there are no remaining ARCH effects, meaning that GARCH(1,1) specification was enough.

It is interesting to notice that the estimated coefficients for GARCH equations are similar amongst different stocks. The coefficients of UHF-GARCH models are quite similar to the ones of Będowska-Sójka and Kliber (2010). On the other hand, values of w in Eq. (7) are much smaller than those found by Sokalska (2012). In this context the paper of Strawiński and Ślepaczuk (2008) is also interesting. They analysed high frequency data from WIG20 index futures in the context of efficient market hypothesis.

Table 7.: Estimated coefficients of UHF-GARCH and LM test for residuals

Stock name	f	g	w	a	b	LM test
BOGDANKA	0.0219	-0.1164	0.0000	0.0552	0.9417	1.0000
BZWBK	-0.1246	-0.1205	0.0000	0.0436	0.9461	0.9959
EUROCASH	-0.0708	-0.0595	0.0000	0.0188	0.9772	1.0000
JSW	-0.0301	-0.1457	0.0000	0.0562	0.9104	1.0000
KERNEL	-0.0714	-0.0687	0.0000	0.0542	0.9286	1.0000
KGHM	-0.1223	-0.1522	0.0000	0.0576	0.9110	1.0000
LOTOS	-0.0259	-0.1782	0.0000	0.0531	0.9217	1.0000
LPP	-0.0770	-0.0688	0.0000	0.0123	0.9844	0.9993

Stock name	f	g	w	a	b	LM test
MBANK	-0.3497	0.0207	0.0000	0.0502	0.9295	1.0000
ORANGEPL	-0.1675	-0.0406	0.0000	0.0598	0.9134	1.0000
PEKAO	-0.0795	-0.0798	0.0000	0.0565	0.9130	0.9998
PGE	-0.0495	-0.1362	0.0000	0.0659	0.9099	1.0000
PGNIG	-0.0686	-0.2251	0.0000	0.0393	0.9455	0.9892
PKNORLEN	-0.0549	-0.1224	0.0000	0.0550	0.9102	0.9989
PKOBP	-0.0206	-0.2270	0.0000	0.0566	0.9096	1.0000
PZU	-0.1253	-0.2020	0.0000	0.0620	0.9114	1.0000
TAURONPE	-0.0595	-0.2368	0.0000	0.0572	0.9166	1.0000

Source: Own elaboration.

Table 8.: P-values of estimated coefficients of UHF-GARCH

Stock name	f	g	w	a	b
BOGDANKA	0.6494	0.0164	0.5367	0.0000	0.0000
BZWBK	0.0000	0.0000	0.2586	0.0000	0.0000
EUROCASH	0.0000	0.0003	0.6150	0.0000	0.0000
JSW	0.0001	0.0000	0.6382	0.0000	0.0000
KERNEL	0.0084	0.0084	0.4381	0.0000	0.0000
KGHM	0.0000	0.0000	0.2995	0.0000	0.0000
LOTOS	0.0000	0.0000	0.4940	0.0000	0.0000
LPP	0.0070	0.0148	0.8787	0.0000	0.0000
MBANK	0.0000	0.0000	0.6067	0.0000	0.0000
ORANGEPL	0.0000	0.0026	0.5584	0.0000	0.0000
PEKAO	0.0000	0.0000	0.5612	0.0000	0.0000
PGE	0.1831	0.0002	0.9243	0.0000	0.0000
PGNIG	0.0000	0.0000	0.1860	0.0000	0.0000
PKNORLEN	0.0000	0.0000	0.3649	0.0000	0.0000
PKOBP	0.0000	0.0000	0.5886	0.0000	0.0000
PZU	0.0000	0.0000	0.7272	0.0000	0.0000
TAURONPE	0.0000	0.0000	0.4855	0.0000	0.0000

Source: Own elaboration.

From Table 8. it can be seen that estimated MA coefficient in ARMA equation was statistically significant for every stock. On the other hand, AR term was not statistically significant in two cases. Statistical non significance of constant term in variance equation for every stock is not problematic. As seen from Table 7. this coefficient should be assumed equal to zero. The other terms in variance equation are statistically significant for every analysed stock.

Table 9. presents the coefficients of HAR model, i.e., the one given by Eqs (8) and (4). Also, adjuster R-squared coefficients are given. For two stocks adjusted R-squared coefficient is very low. It indicated that HAR model in these cases poorly

fits the data. In a few cases this coefficient is quite high (JSW, KGHM, PEKAO, PGE, PKNORLEN, PKOBP, SYNTHOS). For these stocks HAR model can quite reasonably describe the real data. For every stock the constant term is quite small. In most cases the coefficient representing weekly realized volatility is significantly higher than the one representing the daily volatility. Yet, in many cases the coefficient representing monthly volatility is smaller even that the one for daily variance. This ca be a sign that after one month the market "forgets" about price fluctuations.

Table 9.: Estimated coefficients of HAR model and adjusted R-squared coefficients

Stock name	c0	c1	c2	c3	Adj. R-squared
ALIOR	0.0010	0.0141	0.0559	0.0241	0.0000
ASSECOPOL	0.0024	0.0059	0.0066	-0.0063	0.0010
BOGDANKA	0.0003	0.2480	0.4104	0.0503	0.2572
BZWBK	0.0003	0.2016	0.2946	0.3577	0.3140
EUROCASH	0.0006	0.1074	0.3022	0.2440	0.1137
JSW	0.0002	0.4229	0.3375	0.0442	0.4473
KERNEL	0.0007	0.0921	0.2490	0.3408	0.1023
KGHM	0.0002	0.2337	0.4331	0.2228	0.4664
LOTOS	0.0003	0.1482	0.3385	0.2791	0.2068
LPP	0.0003	0.1087	0.2683	0.3344	0.1339
MBANK	0.0006	0.3231	0.0644	0.2182	0.1438
ORANGEPL	0.0005	0.3305	0.0673	0.1104	0.1580
PEKAO	0.0001	0.2119	0.4573	0.2115	0.4369
PGE	0.0002	0.3739	0.4069	0.0390	0.4530
PGNIG	0.0005	0.1888	0.3697	0.2162	0.2463
PKNORLEN	0.0001	0.3640	0.2597	0.2626	0.4697
PKOBP	0.0001	0.2462	0.4758	0.1694	0.5192
PZU	0.0001	0.1586	0.5533	0.1003	0.3483
SYNTHOS	0.0003	0.3140	0.5702	0.0005	0.6085
TAURONPE	0.0004	0.3780	0.3156	0.0069	0.3243

Source: Own elaboration.

Table 10. presents p-values for the coefficients from Table 9. The great majority of the estimated coefficients for HAR models are statistically significant.

Table 10.: P-values for estimated coefficients of HAR model

Stock name	c0	c1	c2	c3
ALIOR	0.0000	0.7746	0.6365	0.9120
ASSECOPOL	0.0309	0.7991	0.9089	0.9540
BOGDANKA	0.0000	0.0000	0.0000	0.4021
BZWBK	0.0000	0.0000	0.0000	0.0000
EUROCASH	0.0000	0.0000	0.0000	0.0000

Stock name	c0	c1	c2	c3
JSW	0.0004	0.0000	0.0000	0.4446
KERNEL	0.0000	0.0008	0.0000	0.0000
KGHM	0.0000	0.0000	0.0000	0.0000
LOTOS	0.0000	0.0000	0.0000	0.0000
LPP	0.0000	0.0000	0.0000	0.0000
MBANK	0.0125	0.0000	0.5943	0.1786
ORANGEPL	0.0005	0.0000	0.5530	0.4605
PEKAO	0.0000	0.0000	0.0000	0.0000
PGE	0.0000	0.0000	0.0000	0.4062
PGNIG	0.0000	0.0000	0.0000	0.0000
PKNORLEN	0.0000	0.0000	0.0000	0.0000
PKOBP	0.0000	0.0000	0.0000	0.0000
PZU	0.0001	0.0000	0.0000	0.0861
SYNTHOS	0.0000	0.0000	0.0000	0.5612
TAURONPE	0.0000	0.0000	0.0000	0.9097

Source: Own elaboration.

5. CONCLUSIONS

The empirical properties of the high-frequency data of 20 selected stocks from the Warsaw Stock Exchange (in particular the ones listed on WIG 20) were examined. In every case a large amount of data was analysed. All analysed data were intraday one.

It was found that in many cases there is a significant negative correlation between price change and duration. The stylized facts of negative first lag autocorrelation of price change and logarithmic returns were also confirmed.

In case of trading volume it was found that it is significantly positively correlated with durations. There is also a positive significant first lag autocorrelation. On the other hand, the relationship between the reading volume and logarithmic returns is inconclusive. For some stocks it is significantly positive, but some significant and negative. It seems therefore interesting, for the future researches, to include in the considerations larger sample. In other words, to consider the analysis basing on more stocks. As the Warsaw Stock Exchange is expanding and with a time more data are collected, this should be possible.

ARCH effects are present for every analysed stock. However, the UHF-GARCH model could be constructed only for 85% of the considered stocks. The outcomes were quite similar for every stock. GARCH(1,1) specification for the variance equation was enough in every case.

The HAR model was also estimated. In many cases quite high R-squared coefficient was obtained. Both AR(1)MA(1)-GARCH(1,1) and HAR models had statistically significant coefficients in most cases.

Although the already applied methods are based in the specific econometric modelling, i.e., suitable to catch the high-frequency properties; the core methodology is still well grounded in the conventional econometric theory. Nevertheless, still some kind of a drawback can erase, if the forecasting would be the only aim. The conclusions derived in this paper base on the analysis of the whole available data sample. In reality, the market player can only use the past information; but can update his or her state of knowledge as the new information arrives on the market. In this context the Bayesian econometrics and conventional, but recursive and rolling window approaches, might be worth to consider. However, they were yet not applied much in a joint way with the concept of high-frequency data. Therefore, for the future researches such a combination could be interesting (Du et al., 2016).

REFERENCES

Admati, A.R., Pfleiderer. P., *A theory of intraday patterns: volume and price variability,* in: *Review of Financial Studies*, Vol. 1, No. 1, (1988): 3-40

Ai, D., Xu, J., Zhang, Z., Zhao, L., *The application review of GARCH model*, in: *Proceedings of International Conference on Multimedia Technology, IEEE, Hangzhou*, (2011): 2658-2662

Ait-Sahalia, Y., Jacod, J., (2014), *High-Frequency Financial Econometrics,* Princeton University Press.

Alfonso-Cifuentes, J.C., Serna-Cortes, M., *Intraday-patterns in the Colombian Exchange Market index and VaR: Evaluation of different approaches,* in: *Revista Colombiana de Estadistica*, Vol. 35, No. 1, (2012): 109-129

Allen, D., Lazarov, Z., McAleer, M., *Modeling intra-day seasonality and forecasting densities in financial duration data,* in: *Journal of Financial Forecasting,* Vol. 1, No. 1, (2007): 1-25

Ammann, M., Kessler, S.M., *Intraday characteristics of stock price crashes,* in: *Applied Financial Economics*, Vol. 19, No. 15, (2009): 1239-1255

Andersen, T.G., Bollerslev, T., I*ntraday periodicity and volatility persistence in financial markets*, in: *Journal of Empirical Finance*, Vol. 4, (1997): 115-158

Andersen, T.G., Bollerslev, T., *Answering the sceptics: yes, standard volatility models do provide accurate forecasts*, in: *International Economic Review*, Vol. 39, No.4, (1998a): 885-905

Andersen, T.G., Bollerslev, T., *Deutsche mark-dollar volatility: intraday activity patterns, macroeconomic announcements, and longer run dependencies*, in: *Journal of Finance*, Vol. 53, No.1, (1998b): 219-265

Andersen, T.G., Bollerslev, T., Diebold, F., *Roughing it up: including jump components in the measurement, modelling and forecasting of return volatility,* in: *The Review of Economics and Statistics*, Vol. 89, (2007): 701-720

Andersen, T.G., Bollerslev, T., Diebold, F.X., *Parametric and nonparametric volatility measurements*, in: Ait-Sahalia, Y., Hansen, L.P. (eds.), *Handbook of Financial Econometrics: Tools and Techniques*, Elsevier, (2010): 67-137

Andersen, T.G., Bollerslev, T., Diebold, F., Labys, P., *Modeling and forecasting realized volatility,* in: *Econometrica,* Vol. 71, (2003): 579-625

Andersen, T.G., Dobrev, D., Schaumburg, E., *Jump-robust volatility estimation using nearest neighbor truncation,* in: *Journal of Econometrics,* Vol. 169, (2012): 75-93

Andersen T.G., Teräsvirta T., *Realized volatility*. In: Mikosch, T., Kreiß, J. P., Davis, R., Andersen, T. (eds.), *Handbook of Financial Time Series*. Springer, (2009): 555-575

Antola, M., Vuorenmaa, T.A., (2013), *Predicting intraday price distributions at high frequencies,* in: *SSRN Working Paper*, http://dx.doi.org/10.2139/ssrn.2293860.

Baker, H.K., (1996), *Trading Location and Liquidity: An Analysis of U.S. Dealer and Agency Markets for Common Stocks,* New York University.

Bakera, M., Steinb, J.C., *Market liquidity as a sentiment indicator*, in: *Journal of Financial Markets*, Vol. 7, (2004): 271-299

Bauwens, L. (ed.), (2001), *Econometric Modelling of Stock Market Intraday Activity,* Kluwer Academic Publishers.

Baker, H.K, Filbeck, G. (eds.), (2015), *Investment Risk Management,* Oxford University Press.

Barndor-Nielsen, O.E., Hansen, P.R., Lunde, A., Shephard, N., (2004), *Regular and modified kernel-based estimators of integrated variance: The case with independent noise*, in: *Working Paper of Nuffield College*.

Będowska-Sójka, B., Kliber, A., *Realized volatility versus GARCH and stochastic volatility models. The evidence from the WIG20 index and the EUR/PLN foreign exchange market*, in: *Przeglad Statystyczny*, Vol. 57, (2010): 105-127

Bera, A., Higgins, M., *ARCH models: properties, estimation and testing*, in: *Journal of Economic Surveys*, Vol. 7, No. 4, (1993): 305-362

Bień, K., (2005), *Wybrane modele ekonometryczne finansowych szeregów czasowych o ultrawysokiej częstotliwości*, Warsaw School of Economics. (in Polish).

Bollerslev, T., *Generalized Autoregressive Conditional Heteroskedasticity*, in: *Journal of Econometrics*, Vol. 31, No. 3, (1986): 307-327

Bollerslev, T., (2008), *Glossary to ARCH (GARCH)*, in: *CREATES Research Paper*, Vol. 49.

Bollerslev, T., Wright, J.H., *High-frequency data, frequency domain inference, and volatility forecasting*, in: *Review of Economics and Statistics*, Vol. 83, No. 4, (2001): 596-602

BOŚ, (2015), http://bossa.pl.

Boudt, K., Cornelissen, J., Payseur, S., Nguyen, G., Schermer, M., (2014), *highfrequency: tools for highfrequency data analysis (R package version 0.4)*, http://CRAN.R-project.org/package=highfrequency.

Bowe, M., Hyde, S., McFarlane, L., (2007), *Duration, trading volume and the price impact of trades in an emerging futures market*, in: *Manchester Business School Working Paper*, Vol. 519.

Brownlees, C.T., Cipollini, F., Gallo, G.M., *Intra-daily volume modelling and prediction for algorithmic trading*, in: *Journal of Financial Economics*, Vol. 9, No. 3, (2011): 489-518

Brownlees, C.T., Gallo, G.M., *Financial econometric analysis at ultra-high frequency: Data handling concerns*, in: *Computational Statistics & Data Analysis*, Vol. 51, (2011): 2232-2245

Campbell, J.Y., Lo, A.W., MacKinlay, A.C., (1996), *The Econometrics of Financial Markets*, Princeton University Press.

Campbell, S.D., Diebold, F.X., *Weather forecasting for weather derivatives*, in: *Journal of American Statistical Association*, Vol. 100, No. 469, (2005): 6-16

Cartea, A., Jaimungal, S., *Modelling asset prices for algorithmic and high frequency trading*, in: *Applied Mathematical Finance*, Vol. 20, No. 6, (2013): 512-547.

Cartea, A., Meyer-Brandis, T., *How duration between trades of underlying securities affects option prices*, in: *Review of Finance*, Vol. 14, No. 4, (2010): 749-785

Chanda, A., Engle, R.F., Sokalska, M.E., (2005), *High-frequency multiplicative component GARCH*, in: *Working Paper of Center for Financial Econometrics, New York University*.

Chen, X., Ghysels, E., Wang, F., (2009), *The class of HYBRID GARCH models*, in: *Discussion Paper UNC*.

Chen, X., Ghysels, E., Wang, F., *HYBRID GARCH models and intra-daily return periodicity*, in: *Journal of Time Series Econometrics*, Vol. 3, No. 1, (2011): 1941-1928

Christoffersen, P.F., (2012), *Elements of Financial Risk Management*, Academic Press.

Coen, A., Racicot, F.-E., (2004), *Integrated volatility and UHF-GARCH models: A comparision using high frequency financial data*, in: *EFMA 2004 Basel Meetings Paper*.

Corsi, F., *A simple approximate long memory model of realized volatility*, in: *Journal of Financial Econometrics*, Vol. 7, (2009): 174-196.

Craioveanu, M.O., (2008), *Essays on Models for Financial Volatility,* The Department of Economics of the Louisiana State University.

Creal, D.D., Koopman, S.J., Lucas, A., *A dynamic multivariate heavy-tailed model for time-varying volatilities and correlations,* in: *Journal of Business and Economic Statistics*, Vol. 29, No. 4, (2011): 552-563

Creal, D.D., Koopman, S.J., Lucas, A., *Generalized autoregressive score models with applications*, in: *Journal of Applied Econometrics*, Vol. 28, No. 5, (2013): 777-795.

Dacorogna, M.M., Gencay, R., Muller, U., Olsen, R.B., Pictet, O.V., (2001), *An Introduction to High-Frequency Finance,* Academic Press.

Darolles, S., Gourieroux, C., Le Fol, G., *Intraday transaction price dynamics,* in: *Annales D'Economie et De Statistique,* Vol. 60, (2000): 207-238

De Luca, G., *Forecasting volatility using high-frequency data*, in: *Statistica Applicata*, Vol. 18, No. 2, (2006): 407-422

Diamond, D.W., Verrecchia, R.E., *Constraints on short-selling and asset price adjustment to private information,* in: *Journal of Financial Economics,* Vol. 18, (1987): 277-311

Diebold, F.X., (1988), *Empirical Modelling of Exchange Rate Dynamics*, Springer.

Dionne, G., Duchesne, P., Pacurar, M., *Intraday Value at Risk (IvaR) using tick-by-tick data with application to the Toronto Stock Exchange,* in: *Journal of Empirical Finance,* Vol. 16, No. 5, (2009): 777-792

Doman, M., *Zależności pomiędzy zmiennościš, wolumenem i czasem trwania ceny na Giełdzie Papierów Wartościowych w Warszawie*, in: *Studia i Prace Wydziału Nauk Ekonomicznych i Zarzšdzania*, Vol. 9, (2008): 185-199 (in Polish)

Doman, M., Doman, R., *Prognozowanie dziennej zmienności indeksu WIG określonej za pomocš danych o wyższej częstotliwości,* in: *Acta Universitatis Lodziensis. Folia Oeconomica,* Vol. 166, (2003): 37-50 (in Polish)

Doman, M., Doman, R., (2009), *Modelowanie zmienności i ryzyka: Metody ekonometrii finansowej*, Wolters Kluwer Polska. (in Polish)

Doman, M., Doman, R., *Dependencies between price duration, volatility, volume and return on the Warsaw Stock Exchange* , in: *Journal of Modern Accounting and Auditing*, Vol. 6, No. 10, (2010): 27-38

Du, B., Zhu, H., Zhao, J. *Optimal execution in high-frequency trading with Bayesian learning*, in: *Physica A: Statistical Mechanics and its Applications*, Vol. 461, (2016): 767-777

Dufour, A., Engle, R.F., *Time and the price impact of a trade*, in: *The Journal of Finance*, Vol. 55, No. 6, (2000): 2467-2498

Easley, D., O'Hara, M., *Time and the process of security price adjustment*, in: *The Journal of Finance*, Vol. 47, No. 2, (1992): 577-605

Engle, R.F., *Autoregressive conditional heteroskedasticity with estimates of the variance of the United Kingdom inflation*, in: *Econometrica*, Vol. 50, (1982): 987-1006

Engle, R.F., *The econometrics of ultra-high frequency data*, in: *Econometrica*, Vol. 68, No. 1, (2000): 1-22

Engle, R., *The use of ARCH/GARCH models in applied econometrics*, in: *Journal of Economic Perspectives*, Vol. 15, No. 4, (2001): 157-168

Engle, R.F., Rangel, J.G., *The spline-GARCH model for low-frequency volatility and its global macroeconomic causes*, in: *Review of Financial Studies*, Vol. 21, No. 3, (2008): 1187-1222

Engle, R. F., Russell, J. R., *Autoregressive conditional duration: A new model for irregularly spaced transaction data*, in: *Econometrica*, Vol. 66, No. 5, (1998): 1127-1162

Engle, R.F., Sokalska, M.E., *Forecasting intraday volatility in the US equity market. Multiplicative component GARCH*, in: *Journal of Financial Econometrics*, Vol. 10, No. 1, (2012): 54-83

Engle, R.F., Sun, Z., (2005), *Forecasting volatility using tick by tick data*, in: *SSRN Working Paper*, http://dx.doi.org/10.2139/ssrn.676462.

Ferenstein, E., Gšsowski, M., *Modelling stock returns with AR-GARCH processes*, in: *Statistics and Operations Research Transactions*, Vol. 28 , No. 1, (2004): 55-68

Fiszeder, P., (2009), *Modele klasy GARCH w empirycznych badaniach finansowych*, Wydawnictwo Naukowe Uniwersytetu Mikołaja Kopernika.

Foster, F.D., Viswanathan, S., *A theory of the interday variations in volume, variance, and trading costs in securities markets*, in: *Review of Financial Studies*, Vol. 3, No. 4, (1990): 593-624

Foster, F.D., Viswanathan, S., *Variations in trading volume, return volatility, and trading costs: evidence on recent price formation models*, in: *Journal of Finance*, Vol. 48, No. 1,(1993): 187-211

Franse, P.H., Testing for seasonality, in: *Economic Letters*, Vol. 38, (1992): 259-262

Frost, A.J., Prechter, R.R., (1998), *Elliot Wave Principle: Key to Market Behavior*, New Classics Library.

Francq, C., Zakoian, J.-M., (2010), *GARCH Models: Structure, Statistical Inference and Financial Applications*, Wiley.

Gabrielsen, A., Marzo, M., Zagaglia, P., (2011), *Measuring market liquidity: an introductory survey*, in: *MPRA Working Papers*, Vol. 35829.

Gajdosova, K., Heryan, T., Tufan, E., *Day of the week effect in the European emerging stock markets: recent evidence from the financial crisis period*, in: *Scientific Papers of the University of Pardubice*, Vol. 19, (2011): 38-51.

Gentleman, R., Ihaka, R., *R: a language for data analysis and graphics*, in: *Journal of Computational and Graphical Statistics*, Vol. 5, No. 3, (1996): 299-314

Ghalanos, A., (2014), *Package rugarch*, http://CRAN.R-project.org/package=rugarch.

Ghysels, E., *Some econometric recipes for high-frequency data cooking*, in: *Journal of Business & Economic Statistics*, Vol. 18, No. 2, (2000): 154-163.

Ghysels, E., Jasiak, J., *GARCH for irregularly spaced financial data: the ACD-GARCH model*, in: *Studies in Nonlinear Dynamics and Econometrics*, Vol. 2, (1998): 133-149

Gysels, E., Osborn, D.R., (2001), *The Econometric Analysis of Seasonal Time Series*, Cambridge University Press.

Gluzicka, A., *Analiza zależności zachodzšcych między wielkościš obrotów a indeksami Giełdy Papierów Wartościowych w Warszawie*, in: *Studia Ekonomiczne, Uniwersytet Ekonomiczny w Katowicach*, Vol. 163, (2013): 13-28. (in Polish)

GPW, (2015), http://www.gpw.pl.

Grammig, J., Wellner, M., *Modeling the interdependence of volatility and inter-transaction duration processes*, in: *Journal of Econometrics*, Vol. 106, No. 2, (2002): 369-400

Griffin, J.M, Nardari, F., Rene, Stulz, M., *Do investors trade more when stocks have performed well? Evidence from 46 countries*, in: *The Review of Financial Studies*, Vol. 20, No. 3, (2007): 905-951

Harvey, A.C., (2013), *Dynamic Models for Volatility and Heavy Tails: with Applications to Financial and Economic Time Series*, Cambridge University Press.

Harvey, A.C., Koopman, S.J., *Forecasting hourly electricity demand using time-varying splines*, in: *Journal of the American Statistical Association*, Vol. 88, No. 424, (1993): 1228-1236

Hasbrouck, J., (2007), *Empirical Market Microstructure*, Oxford University Press.

Hautsch, N., (2012), *Econometrics of Financial High-Frequency Data,* Springer.

Henker, T., Wang, J.-X., *On the importance of timing specifications in market microstructure research,* in: *Journal of Financial Markets*, Vol. 9, (2006): 162-179.

Heryan, T., Stavarek, D., (2012), *Day of the week effect in Central European stock markets*, in: *MPRA Working Papers*, Vol. 38431.

Huptas, R., *Intraday seasonality in analysis of UHF financial data: Models and their empirical verification,* in: *Dynamic Econometric Models*, Vol. 9, (2009): 129-138.

Huptas, R., (2013), *The UHF-GARCH model in analysis of intraday volatility and durations - Bayesian approach and example form the Polish stock market*, in: *Macromodels International Conference*, Warsaw, Poland.

Ito, R., (2013), *Modeling dynamic diurnal patterns in high frequency financial data*, in: *Cambridge Working Papers in Economics,* Vol. 1315.

Jamróz, P., Koronkiewicz, G., *The occurrence of the day-of-the-week effects on Polish and major world stock markets*, in: *Studies in Logic, Grammar and Rhetoric*, Vol. 37, No. 50, (2014): 71-88

Jones, C. M., (2002), *A century of stock market liquidity and trading costs*, in: *Working Papers of Columbia University.*

Kenett, D. Y., Ben-Jacob, E., Stanley, H. E., and gur-Gershgoren, G. (2013), *How high frequency trading affects a market index,* in: *Scientific Reports*, Vol. 3, http://doi.org/10.1038/srep02110.

Knight, J., Satchell, S. (eds.), (2007), *Forecasting Volatility in the Financial Markets*, Elsevier.

Lhabitant, F.-S., Gregoriou, G.N., (2008), *Stock Market Liquidity: Implications for Market Microstructure and Asset Pricing,* Wiley.

Manganelli, S., *Duration, volume and volatility impact of trades*, in: *Journal of Financial Markets*, Vol. 8, No. 4, (2005): 377-399

Mariano, R.S., Yiu-Kuen, T. (eds.), (2008), *Econometric Forecasting and High-frequency Data Analysis,* World Scientific.

Martnes, M., *Measuring and forecasting S&P 500 index-futures volatility using high-frequency data,* in: *Journal of Futures Markets,* Vol. 22, No. 6, (2002): 497-518

Martens, M., Chang, Y.-C., Taylor, S.J., *A comparison of seasonal adjustment methods when forecasting intraday volatility,* in: *The Journal of Financial Research,* Vol. 25, No. 2, (2002): 283-299

Meddahi, N., Renault, E., Werker, B., *GARCH and irregularly spaced data*, in: *Economic Letters*, Vol. 90, (2006): 200-204

Muller, U.A., Dacorogna, M.M., Dave, R.D., Olsen, R.B., Picteto, O.V., Weizsacker, J.E.V., *Volatilities at different time resolution - Analyzing the dynamics of market components*, in: *Journal of Empirical Finance*, Vol. 4, (1997): 213-239

Nychka, D., Furrer, R., Sain, S., (2015), Package "*fields*", http://www.image.ucar.edu/Software/Fields.

O'Hara, M., (1997), *Market Microstructure Theory,* Wiley.

Orłowski, P., (2009), *Verification of selected market microstructure hypotheses for a Warsaw Stock Exchange traded stock,* in: Department of Applied Econometrics Working Papers, No. 4-09

Pacurar, M., *Autoregressive conditional duration (ACD) models in finance: A survey of the theoretical and empirical literature*, in: *Journal of Economic Surveys*, Vol. 22, No. 4, (2008): 711-751

Poser, S.W., Plummer, P.J., (2003), *Applying Elliot Wave Theory Profitably,* Wiley.

R Core Team, (2014), *R: A Language and Environment for Statistical Computing,* R Foundation for Statistical Computing, http://www.R-project.org.

Racicot, F.-E., Theoret, R., Coen, A., *Forecasting irregularly spaced UHF financial data: Realized volatility vs UHF-GARCH models,* in: *International Advances in Economic Research,* Vol. 14, No. 1, (2008): 112-124

Rossi, E., Fantazzini, D., *Long memory and periodicity in intraday volatility*, in: *Journal of Financial Econometrics,* doi:10.1093/jjfinec/nbu006. (in press)

Sokalska M., *Intraday volatility modeling: The example of the Warsaw Stock Exchange*, in: *Quantitative Methods in Economics,* Vol. 11, (2010): 139-144

Sokalska M., *Comparison of intraday volatility forecasting models for Polish equities*, in: *Quantitative Methods in Economics,* Vol. 13, (2012): 107-114

Sol Murta, F., *The money market daily session: An UHF-GARCH model applied to the Portuguese case before and after the introduction of the minimum reserve system of the single monetary policy,* in: *Brussels Economic Review,* Vol. 50, No. 3, (2007): 1-28

Starica, C., (2006), *Is GARCH(1,1) as good a model as the accolades of the Nobel Prize would imply?*, in: *Chalmers University of Technology Working Paper*.

Ślepaczuk, R., Zakrzewski, G., (2009a), *Emerging versus developed volatility indices. The comparison of ViW20 and VIX indices*, in: *Working Papers of Faculty of Economic Sciences, University of Warsaw*, Vol. 21, No. 11.

Ślepaczuk, R., Zakrzewski, G., (2009b), *High-frequency and model-free volatility estimators*, in: *Working Papers of Faculty of Economic Sciences, University of Warsaw*, Vol. 23, No. 13.

Strawiński, P., Ślepaczuk, R., *Analysis of high frequency data on the Warsaw Stock Exchange in the context of efficient market hypothesis,* in: *Journal of Applied Economic Sciences,* Vol. 3, (2008): 306-319

Tay, A.S., Ting, C., Tse, Y.K., Warachka, M., *The impact of transaction duration, volume and direction on price dynamics and volatility,* in: *Quantitative Finance,* Vol. 11, No. 3, (2011): 447-457

Tay, A.S., Ting, C., *Intraday stock prices, volume, and duration: a nonparametric conditional density analysis,* in: Bauwens, L., Pohlmeier, W., Veredas, D. (eds.), *High Frequency Financial Econometrics. Recent Developments,* Physica-Verlag, (2008): 253-268

Tsay, R.S., (2005), *Analysis of Financial Time Series,* Wiley.

Wang, H., Yu, Y., Li, M., *On intraday Shanghai Stock Exchange index,* in: *Journal of Data Science,* Vol. 8, (2010): 413-427

Wood, R.A., McInish, T.H., Ord, J.K., *An investigation of transaction data for NYSE stocks*, in: *Journal of Finance,* Vol. 40, (1985): 723-739

Xekalaki, E., Degiannakis, S. (2010), *ARCH Models for Financial Applications,* Wiley.

Zongxin, Z., Xiao, Z., *Trading duration, mutual funds behavior and stock market shock: Based on ACD model to mine mutual funds investment behavior,* in: *China Finance Review International,* Vol. 1, No. 3, (2011): 220-240