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## ESTIMATION OF STRAY LOSSES IN POWER TRANSFORMERS USING 3D FEM AND STATISTICS

### SUMMARY

Total losses of a power transformer are subdivided in several distinctive parts. The  $I^2R$  losses are easy to calculate and can be precise to a level of measurement repeatability and tolerances of the guaranteed material properties. The additional losses inside the windings can be calculated almost with the same precision using analytic methods. The third part of the load losses that consists of stray losses is the smallest and the most difficult to estimate. However, stray losses estimation is very important in the design phase of a power transformer. Not only because the guaranteed design parameters have to be satisfied, but also the utmost care has to be taken that local losses density across the power transformer does not exceed levels that are permissible in the long term loading conditions of a transformer. The additional losses estimation process presented in this paper models the additional losses level in a transformer as a unit.

**Key words:** eddy losses, finite element method, power transformer, stray field, stray losses

### 1. INTRODUCTION

Estimation of stray losses in the power transformers is a well known topic that is being tackled from different angles continuously through time. Some considerations that make the calculation of the stray losses in power transformers so complicated are given in more recent literature (e.g. [1], [4]), and others are well known for some time, so analytic approaches are developed that make calculation of, for example, excess losses easier in special cases [5], [6]. In addition, studies about behavior of ferromagnetic material used in transformer construction parts are made with calculation of additional losses in mind [4]. Using the available computing power with modern 3D finite element software, coupled with the knowledge of material properties and behavior, a statistical model can be developed that can estimate the additional losses in a power transformer that is both - more accurate and more precise than the 2D estimation process. But, the test results of additional losses measurement on different transformers of the same type show that significant amount of statistical variation is already included in the measurements, i.e. the variation of losses inside this set can not be calculated using the computer model because their origin is of other nature.

## 2. ADDITIONAL OR STRAY LOSSES

The calculation of stray losses on complex geometries is usually done using numerical techniques and different software for field calculation. However, there are results in analytical approaches in eddy losses calculation that can be extended to the more general cases [1], [2] [5], [6], [7]. Different relations estimating the additional losses in the material have the same structure, but due to geometrical variations or different boundary conditions, some parts of the formulae used to calculate the losses change. Usually, the magnetic field is considered through magnetic field intensity variable which is mathematically connected to the losses. The field intensity and magnetic flux density across the transformer geometry varies with respect to time and spatial coordinates. For the statistical modeling to be applied to the transformer geometry, this behavior of the field has to be defined with respect to losses. The amount of losses is modeled to be directly proportional to the some power of the time maximum of the magnetic flux density at one point in space. Losses on the surface area are represented by the integral of the squared magnetic flux density vector across the surface as given in

$$B_{eq} = \sqrt{\frac{\iint_S (\vec{B}\vec{n})^2 dS}{\iint_S dS}}, \quad (1)$$

$$P'_{add} = \xi_j \cdot B_{eq,i,j}^{\eta_j} \cdot S_{i,j} \cdot \delta_{i,j} \cdot c_{i,j}, \quad (2)$$

where:  $B_{eq}$  denotes equivalent magnetic flux density,  $S$  is the surface of the element,  $\mathbf{n}$  is the surface normal vector,  $dS$  the surface differential,  $P'_{add}$  additional losses on an element,  $\xi$  and  $\eta$  are the unknown coefficients that need to be determined through statistics,  $i$  and  $j$  are indices that run through different construction elements and different measurements,  $\delta$  is the skin depth of the material,  $c$  is the number of skin-depths considered. The equivalent magnetic induction defined by (1) is used across the different surfaces (2) in the power transformer to calculate the overall stray losses. There are also stray losses that are result of to the stray magnetic field of the winding leads, which can readily be modeled by the following equation:

$$P'_{add,leads} = k \cdot \Phi_{FR,i}^2, \quad (3)$$

where:  $P'_{add,leads}$  represents the losses due to the stray field of leads,  $k$  is the coefficient of proportionality, while the  $\Phi_{FR}$  is the magnetic flux due to the regulating winding leads,  $i$  is a running index.

## 3. TRANSFORMER GEOMETRY AND FEM MODEL

The overall power transformer geometry can be very different with respect to the type of outer cooling system etc. Fig. 1 shows the geometry of a transformer with separate cooling bank. Although the appearance of the transformer varies, the situation with stray losses on different units is very similar. The active part of the transformer that is shown in Fig. 1b consists of the standard construction parts.

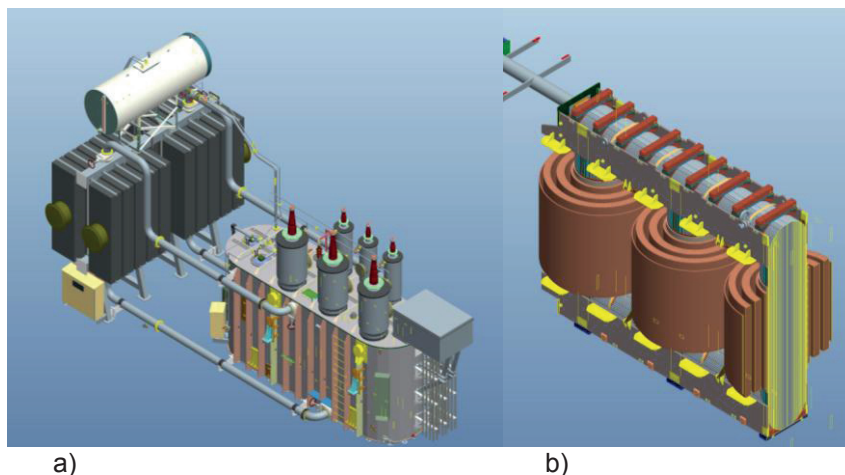


Fig. 1 Transformer geometry

For the purpose of stray-losses and stray field calculation only the inner geometry of the transformer is important. The losses appear in the conductive materials, so these parts of the geometry need to be modeled in more detail. Moreover, the analyzed part of the losses occurs outside of the winding, which ensures that the transformer windings can be modeled by cylinders without modeling each individual conductor or a strand. The sources of the stray field are current carrying conductors in the windings, while the field distribution outside of the windings is determined by the geometry of the core, clamping system and tank. These elements are predominantly made of constructional steel. The first step to modeling is to know the magnetic permeability of the material and its electrical conductivity. The parameters of the material are statistically distributed and, consequently, the losses are different, even on different units of the same type. The other sources of statistical variation are due to the small, but inevitable differences in the transformer production (within tolerances), and, finally, measurement. The constructional steel material used in transformer production, along with the tie-plate material have been studied recently to obtain the values that represent the real material properties the best in the stray losses calculation models [4]. The recommended values are used in the analyses presented here. The simplified power transformer geometry used in 3D FEM calculations is given in Fig. 2. Elements that predominantly influence the stray magnetic field distribution are the core, clamping plates, the tank and tie plates. The sources of the magnetic field are modeled with the overall nominal ampere-turns in different tap positions according to transformer data.

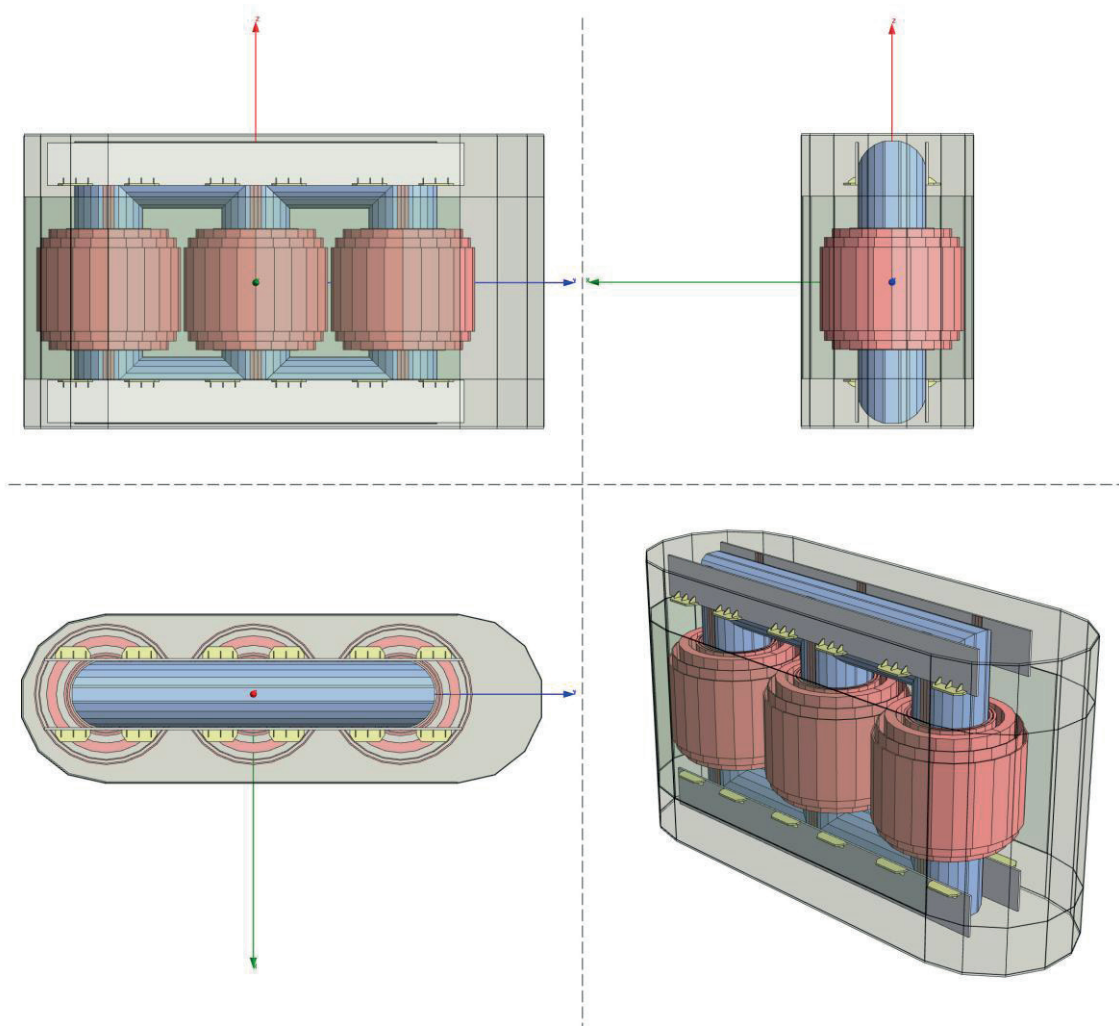


Fig. 2 The simplified 3D FEM model geometry of a typical power transformer in the analysis.

The fundamental element to the analysis is the solution to the magnetic stray field distribution using commercial Ansoft Maxwell v13 software for finite element method (FEM) calculations on a three-dimensional computer model. The approach is a natural extension to the analysis given in [1] with new parameters, and relies on similar statistical principles that govern estimation of stray losses that is currently in use in power transformer electrical design. Here, the connection is established between the

local magnetic field solution on the element, the geometrical properties and local losses on the particular element. Finally, the losses are added together and statistically correlated to the test values. The process of the calculation takes time, and for the time being is not suitable for use in optimization schemes.

#### 4. STRAY FIELD

After the basic elements required for the field calculations are defined (geometry, properties of materials, excitation, boundary conditions), the results obtained on one model of a power transformer are represented graphically in Fig. 3. The distribution of the stray field is the principle component behind the stray losses, so the distribution of the stray magnetic flux density in a way reflects the losses.

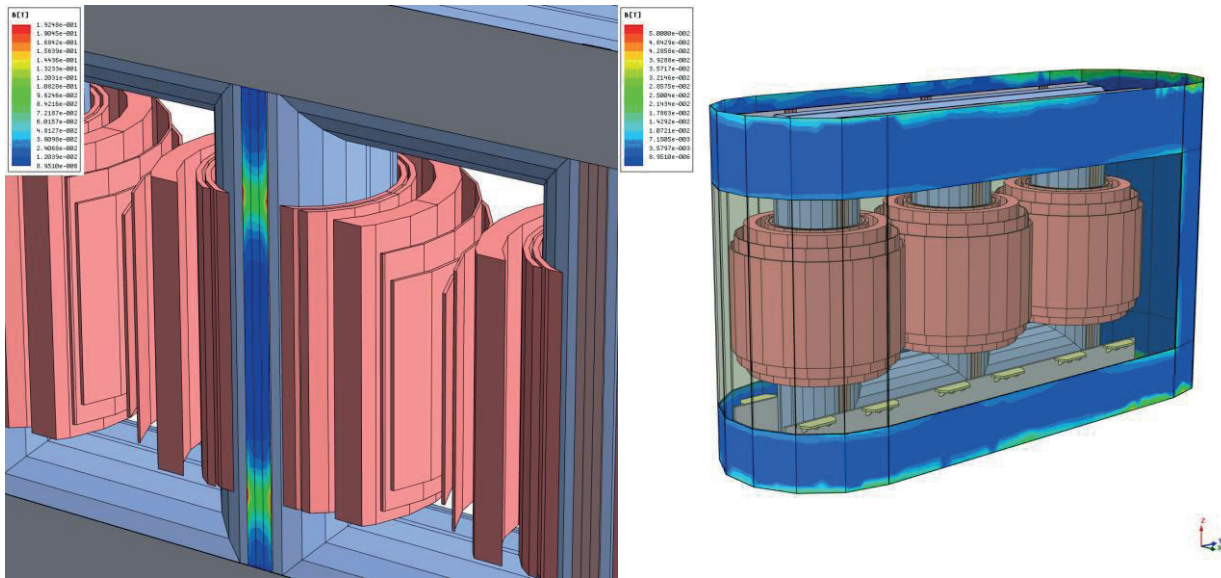


Fig. 3 Details of the power transformer geometry with the magnetic flux density field solution on a) transformer tie plates; b) transformer tank wall (parts not covered by magnetic shunts).

#### 5. STATISTICAL MODEL

The next step is the acquisition of the results and plugging them in the mathematical model of the following form

$$P'_{add,i} = \sum_{j=1}^N \xi_j \cdot B_{eq,i,j}^{\eta_j} \cdot S_{i,j} \cdot \delta_{i,j} \cdot c_{i,j} + k \cdot \Phi_{FR,i}^2, \quad (4)$$

where:  $i$  is index representing a single tap position of a transformer,  $P'_{add}$  are total additional losses outside of windings,  $N$  is the number of elements in the analysis,  $\xi$  is the shape factor of the element,  $B_{eq}$  is the equivalent induction,  $\eta$  is the exponent,  $S$  is the surface area of the element,  $\delta$  is the skin depth of the field,  $c$  is the number of skin depths in the volume,  $k$  is the coefficient modeling the stray losses due to the fine regulating winding current loop and  $\Phi_{FR}$  is the calculated magnetic flux produced by the leads of the fine regulating winding. These losses across elements are finally fed to mathematical minimization process of the objective function (5) that will result in unknown coefficients that model the losses statistically in the best possible way (i.e. the sum of squares of differences between test and calculation results is minimal).

$$F(\xi, \eta, k) = \sum_{i=1}^N (P'_{add,meas,i} - P'_{add,calc,i})^2, \quad (5)$$

where:  $F$  is the objective or cost function,  $N$  is the number of the measurements,  $P'_{add,meas}$  are measured additional losses,  $P'_{add,calc}$  are calculated losses (4), while  $i$  is the running index representing a single measurement. In this manner, the test results are linked through statistical model to the calculation results. Through this connection and the fact that the FEM 3D model is used (instead

of some form of 2D approximating geometry that cannot model structural parts of the transformer accurately) some merits over traditional methods of stray losses estimation are expected. These expectations include lowering statistical dispersion of the calculated vs. test results regarding stray losses outside of windings and ability to predict the stray losses outside of windings with greater statistical confidence.

## 6. RESULTS

The results of the statistical modeling process are given graphically in Fig. 4.

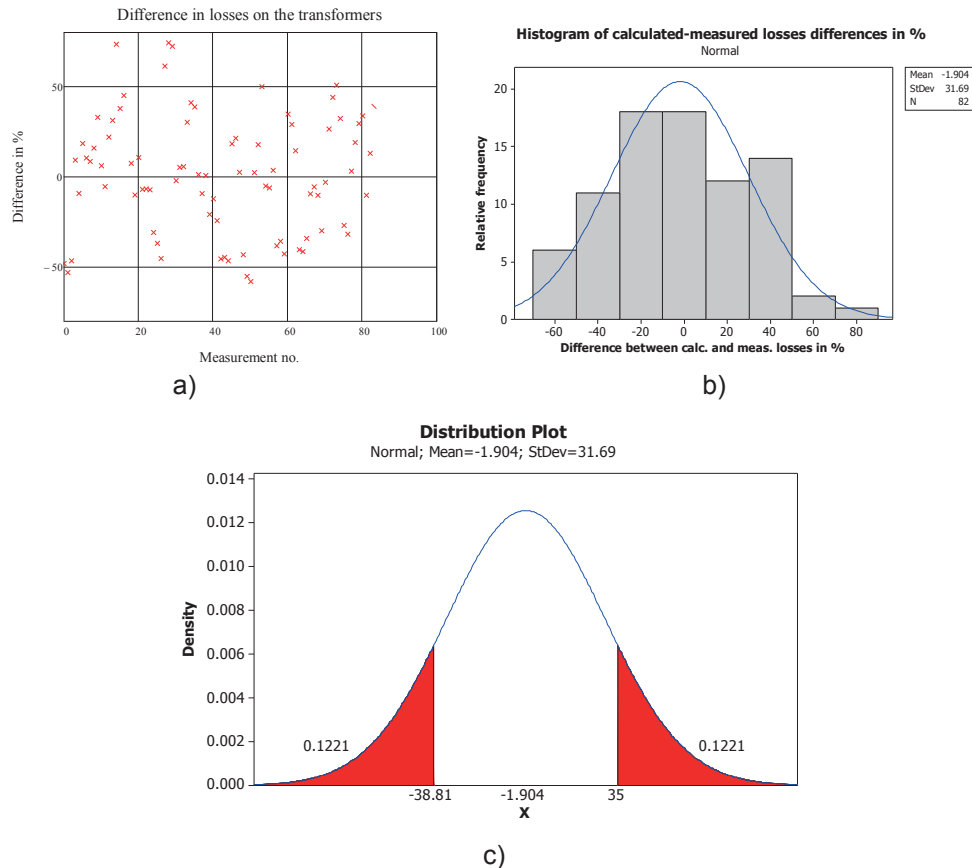


Fig. 4 Relative differences of the calculated and measured losses a); the statistical distribution of the differences b); illustration of the percentage of the results outside the  $\pm 35\%$  boundary c).

More than 75% of the results fall into the range between  $\pm 35\%$  from the measured values. This difference in losses may seem significant, but the fact has to be kept in mind that the measured additional losses do not represent only the statistical variation of the stray losses, but also reflect the statistical distribution of other components of losses that measured additional losses are calculated from. This is why this kind of results dispersion is not considered to be too high. When the results are compared through their relative frequencies, the resulting histogram exhibits the behavior of normal statistical distribution. This is a final confirmation that the minimization process finished correctly and that no strong trend is present in the residual values.

## 7. CONCLUSION

Because of their previously described behavior, the statistical approach to the estimation of additional losses is used. It was already stated that the measurements of additional losses exhibit statistical dispersion as a byproduct of several influences like the manufacturing tolerances, the measurement uncertainty, the range of material properties etc. Due to the stochastic nature of the processes involved, a statistical method of additional losses estimation based on 3D FEM is a natural extension to the widely used 2D methods in the losses calculation. It has its advantages and drawbacks.

The main drawbacks of the presented method are that it requires substantially more time in execution and that the transformer construction geometry should be known at the time of the calculation. The latter makes the application of the method not feasible for the quick and simple estimation during evaluation of the different design solutions. However, the advantage of 3D calculation over the 2D estimation of additional losses is in its inherent capability to model the geometry of the constructional parts of the transformer more accurately. Finally, it is a reasonable assumption that this approach can have more success in the additional losses estimation on new or different power transformer geometry.

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