Study of Transient Interaction in a System with Transformer Supplied from Network through a Cable: Assessment of Interaction Frequencies and Resonance Evolvement

SUMMARY

Transformer together with its windings is a complex oscillatory system. The interaction between the transformer and an electric network during transients can cause the development of resonance phenomenon in the windings leading to overvoltages and the risk of transformer fault.

This report presents the results of studies of resonance phenomena in transformer windings, caused by interaction with an electric network containing the feeder cable. The approach to a simple assessment of dominant oscillation frequency of a voltage in the system “feeder cable – transformer” and estimation of the resonant frequencies of transformer winding is considered. The report also describes the technique for measurement of winding resonance voltages. The resonance phenomenon evolution in transformer windings is considered and the impact of decaying oscillating applied voltage on maximum ratio of resonance overvoltages is estimated.

Key words: Transformer windings, resonance overvoltages, winding natural oscillation frequencies, frequency response analysis, FRA, voltage measurement in transformer windings.

1. INTRODUCTION

Power transformers are the key and crucial elements in the chain of transmission and distribution of electrical energy. In transient conditions caused by switching or short-circuit faults in power networks the transformers actively interact with other elements of electrical network [1, 2]. This electrical interaction is complex and it may result in transformer insulation faults under adverse circumstances.

One of the instances of electrical interaction with the supplying network is an internal resonance in transformer windings in the case when the oscillation frequency of the voltage at its input terminals is close to one of the windings natural oscillation frequencies. If the secondary winding of the transformer is not loaded, such resonant phenomena can be accompanied by significant overvoltages in internal insulation of transformer in which the voltage on the certain parts of the winding insulation is comparable or exceeds the voltage at transformer input terminals. The harmfulness of the resonance phenomenon in transformer windings is aggravated by the fact that the external arresters installed at the input terminals of the transformer are not able to limit such overvoltages that occur on the certain elements inside the transformer.
For the above reasons resonant overvoltages potentially pose a threat to the internal insulation of transformer. In general, resonant overvoltages impact on both the longitudinal insulation of windings (primarily in the entrance area) and the main insulation (at some distance from the entrance area of the winding). These resonant overvoltages can manifest themselves by damaging corresponding winding insulation elements.

It should be noted that the internal insulation of oil-immersed transformers is a composition consisting of many layers. For example, the insulation of wires may consist of three or more layers (per one side) of craft paper; the main insulation of windings, in general, may comprise more than one insulation barrier. Repetitive exposure of resonant overvoltages can lead to subsequent degradation of the solid insulation and to significant decrease of the transformer internal insulation electric strength. The cumulative effect may cause the damage of the transformer internal insulation under normal operating voltage after some time since this transformer had been put into operation.

To assess the possibility of the resonant overvoltages development in the transformer windings and the degree of their severity for the internal insulation it is necessary, first of all, to determine the dominant frequencies of possible voltage oscillations in the network, the resonant frequencies of the transformer windings and to make an assessment of the voltages on the elements of the windings insulation.

2. VOLTAGE OSCILLATION IN THE SYSTEM “FEEDER CABLE – TRANSFORMER”

The necessary conditions of the internal resonance evolution in the transformer are the absence of load on the transformer secondary side and the presence of voltage oscillations on the supply side with a frequency close to one of the natural frequencies of the transformer windings.

Natural frequencies of high voltage and intermediate voltage windings of medium and high power transformers are typically tens of kHz. One of the main sources of voltage oscillations at these frequencies are oscillations in feeder cables or overhead lines having length of the order of hundreds of meters, caused by multiple reflections of waves in these lines due to any switching. In this regard, the networks, in which a transformer is energized together with a feeder line from its remote end, are a subject to potential danger.

The following typical situations in power facilities [3] causing high-frequency voltage oscillations in the system “feeder cable – transformer” may be identified:

1) short-circuit earth fault on one of the phases at the beginning of the cable (including faults at the substation busbars);
2) energization of the system “feeder cable – transformer” from the busbar to which several cable lines with a low total surge impedance are connected;
3) energization of the system “feeder cable – transformer” from the busbar to which one or more cables with similar parameters and lengths are connected.

In the considered systems “feeder cable – transformer” an approximate evaluation of the dominant oscillation frequency may be performed by solving the following transcendental equation [4]:

\[ f = \frac{v}{4L} \left[ 1 - \frac{2}{\pi} \arctan\left( \frac{2\pi f C_T Z_c}{v} \right) \right], \]  

where \( v \) – wave propagation speed in the feeder cable; \( L \) – length of the feeder cable; \( C_T \) – transformer surge capacitance; \( Z_c \) – characteristic impedance of the feeder cable.

It follows from the expression (1) that the dominant frequency in the system is determined primarily by the parameters of the feeder cable and by the transformer surge capacitance. Meanwhile, the greater the oscillation frequency and the characteristic impedance of the cable, the greater the effect of transformer surge capacitance. In the case of long cables with low characteristic impedance the last term in the expression (1) can be neglected for the approximate estimation of the oscillation frequency.

3. ESTIMATION OF TRANSFORMER WINDINGS NATURAL OSCILLATION FREQUENCIES

3.1. Theoretical evaluation of the windings natural frequencies

The evaluation of the transformer windings natural frequencies is often one of the main stages of impulse transients calculation and longitudinal insulation design. To date, there are a large number of publications which address the issues of theoretical determination of natural frequencies, for example [5]. However, due to a complex mutual magnetic and electrical interaction between the individual elements of a single winding and/or adjacent windings the expressions for natural frequencies cited in the literature typically are based on a number of significant simplifying assumptions, therefore they are suitable only for qualitative evaluation.
For a uniform winding with earthed neutral by neglecting the mutual magnetic influence of individual elements of this winding and the influence of adjacent windings the following expression for the $k$-th natural frequency ($k$-th harmonic) can be obtained [5]:

$$f_k = \frac{k}{2\sqrt{LC\left(1 + \frac{K}{C}(k\pi)^2\right)}}$$

(2)

where $L$ – inductance of winding; $C$ and $K$ – capacitance to earth and series capacitance of winding.

Considering the fact that the natural frequency $f_k$ is related to the winding wave propagation speed $V_k$ by the formula

$$f_k = \frac{kV_k}{2l},$$

(3)

where $l$ – the “electrical length” of the winding. From the expression (2) the formula for $V_k$ may be obtained:

$$V_k = \frac{l}{\sqrt{LC\left(1 + \frac{K}{C}(k\pi)^2\right)}}$$

(4)

It follows from the expression (2) that for windings with low series capacitance the first few natural frequencies are determined mainly by the capacitance to earth and are close to the oscillation frequencies of the line with the same electrical length grounded at the far end.

It follows from the expression (4) that in the case of a winding with low series capacitance the wave propagation speed is equal to

$$V_k = \frac{c}{\sqrt{LC\left(1 + \frac{K}{C}(k\pi)^2\right)}}$$

where $c$ – speed of light, $\varepsilon_s$ – equivalent dielectric permittivity of the winding surrounding medium which determines the capacitance of the winding to the ground. Meanwhile, the harmonic’s wave propagation speed decreases with the increase of the harmonic’s number.

It should also be noted that estimation of the first natural frequencies can be made by the expression (3) provided that the estimations of the wave propagation speeds and the electrical length of the winding are known, which often happens in practice. For example, in oil-immersed transformers, in general, the first natural frequency corresponds to the propagation speed of about 140–180 m/µs, and in dry-type transformers with open windings – 240–300 m/µs.

3.2. Evaluation of the winding natural frequencies by means of numerical modeling

For a more accurate evaluation (as compared with theoretical formulae) of the winding natural frequencies a high-frequency model of the transformer can be used. It is based on an equivalent circuit with distributed or lumped parameters which is implemented in software programs for calculation of impulse transients in transformer windings.

![Figure 1 - A high frequency transformer model in the EMTLab software program: calculation of frequency response](image)
In order to determine natural frequencies by means of numerical modeling authors use an approach described in [4]. It is based on a combined application of two software programs. The first one (TT) is used for generation of a high-frequency lumped parameters model of a transformer which is afterwards exported to the second program (EMTLab). EMTLab is designed for simulation of transients in electrical networks (EMTLab, figure 1) and features the frequency analyzer module which calculates transfer functions in frequency domain (figure 2).

Figure 2 - The EMTLab’s frequency analyzer module and calculated winding admittance

3.3. **Experimental determination of windings natural frequencies**

When a power transformer is fed by AC voltage with frequency equal to one of its resonant frequencies relatively high resonant oscillating voltages develop in its windings. Corresponding currents which flow mainly within these windings are high too. These oscillating currents are accompanied by loss of energy and, therefore, the highest active power is consumed from power source in the resonance mode. While changing the frequency of the applied voltage in a wide range it becomes close to one of the resonant frequencies and the energy losses in the transformer grow up. Thus, the local maximum of consumed active power may be used as an indication of achievement of one of the transformer resonant frequencies.

The problem of consumed power maximum determination in a certain frequency range can be reduced to the problem of determination of maximum real part of input admittance of a fed transformer winding [1]. To solve this problem in practice it is quite convenient to use Frequency Response Analysis (FRA) measuring systems.

The principle of FRA systems is based on determination of frequency response of transformer windings by means of signal injection through one of transformer terminals and measurement of an input voltage $U_1$ and an output voltage $U_2$ induced at a certain transformer terminal [6]. The result is a transfer function represented by corresponding absolute value $A = |U_2 / U_1|$ (usually expressed in dB) and phase angle $\phi = \angle(U_2, U_1)$. The transfer function is defined in a broad frequency range (from tens of Hz to thousands of kHz).

In FRA systems measuring cables are about 5–15 m long; to eliminate their possible influence on measurement results at high frequencies these cables are terminated by matching resistors with resistance equal to cable surge impedance (usually 50 Ohms). The measured output voltage is proportional to an output current $I_2$ flowing to earth through a shunt (matching resistor of 50 Ohms), which resistance is small enough compared to the impedance of the winding at high frequencies. Thus, the winding admittance is defined as

$$Y_{12} = \frac{U_2}{Z_c (U_1 - U_2)} = \frac{1}{Z_c \left( \frac{1}{A \angle \phi} - 1 \right)}.$$  \hspace{1cm} (5)

It is necessary to note that when measuring the frequency response of individual windings the matching impedance of 50 Ohms is shunted by capacitance to earth $C_s$ of high voltage bushing and leads that connect the winding with bushing and other windings as well as by capacitance of measuring cables and wires. The value of this shunt capacitance may have order of hundreds or thousands of pF. At frequencies of several MHz the impedance of this capacitance $C_s$ becomes comparable with the
resistance of a matching resistor and, therefore, output current of measured winding partially flows through capacitance $C_s$ instead of 50 Ohm resistor. Taking into account the shunt capacitance $C_s$, formula (5) takes the form:

$$\Gamma_{12} = \frac{1}{Z_0 + j\omega C_s}.$$  \hspace{1cm} (6)

Thus, neglecting the earth capacitance leads to an error in active and reactive component of input admittance of the winding achieved by FRA measurements and expression (5). However, the first natural frequencies of high-voltage transformer windings typically are few tens or few hundreds of kHz while the effect of shunt capacitance on the active component of input admittance is generally not so significant in this frequency range. So the first natural frequencies can be evaluated from FRA measurement but it needs to be mentioned that this is just an estimation rather than accurate measurement.

As an illustration of abovementioned statements the results of continuous disc-type winding transfer functions and frequency response measurements are presented. The winding model (figure 3) has an average diameter of about 1000 mm, height of 1100 mm and consists of 52 discs of 10 turns in each. For the purpose of measurements the winding has intermediate terminals at outer transitions between every pair of discs. The measurement results are shown in figure 4, which shows the transfer function voltages at the terminals of junctions between discs 14 and 15, 26 and 27 and 38 and 39 (hereinafter, terminal “14-15”, “26-27”, “38-39”), roughly corresponding to 1/4, 2/4 and 3/4 of the electrical length of the winding.

It should be noted that this model is continuous disc-type and has relatively low longitudinal capacitance. For these reasons, the obtained frequency response contains a pronounced resonance peaks corresponding to the winding natural frequencies. As can be seen from figure 4, the resonance peaks of the frequency responses of the winding are located at the same frequencies as the resonant peaks of the transfer functions of the terminal voltages.

In general, the resonant frequencies represented in the frequency response of the individual winding correspond not only to winding natural frequencies. The resonance frequency of individual windings can be distinguished from the frequencies corresponding to the interaction between windings as follows.

The transient voltages in any winding can be expanded as a combination of standing waves at self-frequencies. For the winding to which the voltage is applied the standing wave at the first winding
natural frequency has the nodes at the ends of this winding. Current distribution of this standing wave at the first natural frequency is shifted from voltage distribution by 1/4 of spatial period [7], thus, the current has a node in the middle of the winding and different polarities in the upper and lower halves of the fed winding. This leads to the fact that the EMF induced in the discs of the secondary winding are mutually compensated and the magnetic flux generated by the primary winding flows through the magnetic core without reaction from the secondary winding. Thus, at the frequency corresponding to the first natural frequency of the primary winding, the short circuiting of secondary winding has almost no influence on the input impedance of this primary winding. As a result, the frequency responses of the primary winding of the transformer obtained for open and short-circuit secondary winding are very close to each other starting from frequencies close to the first natural frequency of the primary winding.

However at frequencies much lower than the first natural frequency the open or short-circuit state of the secondary winding affects the magnetic flux and the flux return path and, therefore, significantly affects the value of input admittance of the primary winding and resonant frequency of interaction between windings.

Thus, the first natural frequency of the primary winding can be determined using FRA-measurements of this winding with respect to the input terminals with open and short-circuit secondary winding and subsequent identification of the first resonant frequency which achieves a local maximum of the real part of input admittance of the primary winding and the coincidence of frequency responses for open and short-circuit states of secondary winding.

Example below presents the frequency response and the real part of admittance $G_{12}$ of high-voltage winding (HV) of two-winding transformer (figures 5а and 5в, respectively), obtained when voltage is applied to the line terminal and measured at the neutral N of HV winding with open and short-circuited secondary low voltage winding (LV).

![Figure 5 - Frequency responses (a) and real part of admittance (b) for the primary winding](image)

As it can be seen from figure 5, the frequency responses of HV winding with open and short-circuited LV winding are significantly different up to the frequency of 8-10 kHz, both in magnitude and frequencies of resonance peaks; for the frequencies of about 13 kHz and higher frequency responses match each other. Thus, in this example, the resonant peaks at frequencies of about 5-7 kHz are related to frequencies of the interaction between windings and the resonant peak at a frequency of about 13 kHz corresponds to the first natural frequency of HV winding.

4. MEASUREMENTS OF WINDING VOLTAGES AT RESONANT FREQUENCIES

4.1. The technique

One of the crucial points in the resonant overvoltages problem is that the transformer in operation is exposed to the impact of decaying voltages. Therefore maximum possible values of overvoltage ratio may not have enough time to be achieved. However, these possible maximum ratios are definitely of interest as they represent the worst case.

In practice, the estimation of the maximum possible ratio of the resonant overvoltages in transformer windings can be performed by feeding the primary winding of the transformer by continuous AC voltage with a peak value of few tens of volts and by measuring the voltage at the intermediate points of the winding. Obviously, it is possible only in factory conditions. These measurements actually have a lot in common with repetitive surge oscillography (RSO) by low impulse voltages, and therefore they could be combined with it.
However, compared with RSO measurements the voltage measurement under resonant conditions has its own features, most important of them are listed below.

1) RSO measurements of oil-immersed transformers are usually performed without oil, resulting in roughly twice smaller values of equivalent dielectric permittivity of the environment (which determines the earth capacitance of the windings). Measured voltage peaks and resonant frequencies will be slightly higher than the values obtained in oil-filled conditions. Thus, the measurement results should be considered as an upper estimation.

2) As a rule, connection of measuring equipment to intermediate points of the winding is carried out by means of connection cables or wires, which leads to addition of their earth capacitances to these points. If the capacitance of connected objects is comparable with the capacitance to earth of individual sections of winding in these intermediate points, the distortion of values of resonant frequencies and voltage peaks is to be expected. As a consequence to ensure accurate measurements it is necessary to avoid the use of measurement cables with high capacitances and long connecting wires and, in general, to use the connection between measuring system and winding with the lowest possible capacitance to the earth.

3) With respect to the abovementioned statements we may state that the measurement of voltages at fixed frequencies corresponding to expected winding natural frequencies gives inaccurate estimations. In the presence of measuring wires the maximum voltage values at specific points of winding are achieved at slightly different frequencies. Due to the strong frequency dependence of the resonant voltages even a small shift in frequency leads to a significant change of measured voltages. In order to make more accurate determination of highest voltage values at certain intermediate points of the winding it is advisable to make measurements in fairly wide frequency range.

Considering the above, the measurement of resonant overvoltages in windings may be performed most correctly using FRA measurement systems, which output signal frequency varies in frequency range from a few Hz up to few MHz. Regular measuring cables and termination resistor of 50 Ohms are to be excluded from the winding response measurement circuit and replaced by digital oscilloscope probe with built-in divider (10:1 for example) with a bandwidth of tens or hundreds of MHz, input impedance of 10 MΩ and low input capacitance not bigger than 10–20 pF.

As an example, the results of measurements of transfer functions and voltages in the resonant conditions for the described above winding model are presented below. The completeness of measurement results is largely determined by the amount of winding intermediate terminals available for measurement. In the considered model the winding has intermediate terminals after each two discs. Point-by-point measurement of transfer functions for each terminal allows to build a spatial-frequency spectrum for maximum voltage values (figure 6) and the voltage distribution along the winding at natural frequencies of the winding (figure 7).

![Figure 6 - Spatial-frequency spectrum of maximum voltages in the winding](image)

Studies conducted on the winding model show that in case of connection to intermediate point of the winding made by the measurement circuit with the input capacitance not greater than 10-20 pF the expected voltage measurement error is not greater than 10%. The deviation of measured resonant frequencies from winding natural frequencies can be used as an indicator of measurement circuit influence.

4.2. Analysis of the resonant voltage increase in the windings under the influence of an alternating voltage

Resonant voltages in transformer windings may be thought of as a combination of standing waves. The sum of these waves leads to voltage rise in certain parts of the windings. To achieve the maximum overvoltage ratio in the windings it is necessary to apply periodic input voltage for a sufficient amount of time, greater than the winding wave travel time.

With this in view, the attenuation of voltage oscillations in the network plays an important role in the development of dangerous resonance phenomena in transformer windings. The time required to reach the steady state with maximum overvoltage ratio may be of the order of tens of periods of applied oscillating voltage. Thus, if the input voltage decays with time the maximum overvoltage ratio may just not have time to develop. Moreover, the greater is the attenuation of the applied voltage, the less is the ratio of resonant overvoltages.

Using the measurement technique described above and the generator of sinusoidal voltage with a frequency equal to one of the winding natural frequencies, it is possible to perform oscillography of resonant rise of voltages in certain parts of the winding.

For example, the oscillograms of voltages at model terminals are presented in figure 8. For the frequencies of applied voltages equal to the first (86.4 kHz) and to the third (210.5 kHz) winding natural frequencies voltages were measured in the middle of the winding. For the second natural frequency (153.4 kHz) the voltage was measured at ¼ of the winding length.

As may be seen from figure 8, the terminal voltages have the form of exponentially growing periodic voltage. Due to wave propagation the terminal voltage has time delay with reference to applied voltage. The voltage at antinode of standing wave (which is actually the maximum voltage) has time delay equal to $\frac{T_k}{4} + (m-1)\frac{T_k}{2}$, where $m$ – voltage antinode number.

Assuming that the applied voltage is $u_a(t) = U_s \sin(2\pi f_k t)$, the maximum terminal voltage at corresponding natural frequency may be approximately represented by the following expression

$$u_k(t) = U_s A_k \sin(2\pi f_k t + \phi_k) \cdot \left(1 - e^{-\frac{t}{\tau_k}}\right),$$

where $k$ – number of winding natural frequency; $t$ – time; $U_s$ – peak value of applied voltage; $A_k$ – maximum ratio of terminal voltage at $k$-th winding natural frequency; $f_k$ – $k$-th winding natural frequency; $\phi_k$ –
– represents time-delay in reference to applied voltage; \( \varphi_k \approx \pi/2 + (m - 1)\pi \) for terminal corresponding to voltage antinode; \( \tau_k \) – time constant of terminal voltage rise at the \( k \)-th winding natural frequency.

For the received waveforms the values of time constant \( \tau_k \) and its ratio to the period of winding natural frequencies \( T_k \) are given in Table 1.

Table 1 – Time constant of terminal voltage rise

<table>
<thead>
<tr>
<th>Number of harmonics ( k )</th>
<th>Frequency ( f_k ), kHz</th>
<th>Period ( T_k ), µs</th>
<th>Time constant ( \tau_k ), µs</th>
<th>( \tau_k / T_k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>86.4</td>
<td>11.6</td>
<td>56.7</td>
<td>~5</td>
</tr>
<tr>
<td>2</td>
<td>153.4</td>
<td>6.5</td>
<td>72.8</td>
<td>~11</td>
</tr>
<tr>
<td>3</td>
<td>210.5</td>
<td>4.7</td>
<td>71.9</td>
<td>~15</td>
</tr>
</tbody>
</table>

In order to estimate maximum value of terminal voltage for the case of decaying sinusoidal applied voltage \( u_d(t) = U_s \sin(2\pi f_k t) \cdot \exp(-t / \tau_d) \) using the Laplace transform and neglecting time-delay for simplicity, the following expression can be obtained for the terminal voltage

\[
u_{td}(t) = U_s A_k \left[ \sin(2\pi f_k t)e^{-\beta t} + \frac{A\cos(\omega_k t)(e^{-\alpha t} - e^{-\beta t}) + \sin(\omega_k t)(Be^{-\alpha t} - Ce^{-\beta t})}{(\beta - \alpha)(4\omega_k^2 + (\beta - \alpha)^2)} \right] \tag{8}
\]

where \( \tau_d \) – time constant of applied voltage damping; \( \omega_k = 2\pi f_k \); \( \alpha = 1/\tau_k \); \( \beta = 1/\tau_d \);

\( A = 2\omega_k\alpha\beta \); \( B = \beta \); \( C = \beta^2 - \alpha\beta + 4\omega_k^2 \).

Typically, the transient voltage at high frequencies is characterized by a fast decay, the value of \( \tau_d \) may be equal to several periods of transient voltage dominant frequency. At a known value of \( \tau_d \) expression (8) may be used to estimate the maximum terminal voltage that may develop with regard to the attenuation of applied voltage. For example, with \( A_k = 8 \) p.u., \( a = 1/(5T_k) \), \( \beta = 1/(4T_k) \) the terminal voltage under decaying supply voltage will reach the maximum value of 2.6 p.u.

5. CONCLUSION

To assess the possibility of resonant overvoltages in transformer windings and the degree of their severity for the internal insulation it is necessary, first of all, to determine the dominant frequency of possible voltage oscillations in the network, the resonant frequencies of the transformer and the voltages on the elements of winding insulation in resonance conditions.

Potential subjects to danger are networks in which transformers are commutated together with supplying lines to remote busbars with identical cables of similar length or with a large number of cables with low total surge impedance. An estimation of the dominant oscillation frequency of the system “feeder cable – transformer” can be made analytically on the basis of the wave propagation speed, characteristic impedance and length of the cable line, and the transformer surge capacitance.

Evaluation of the winding resonant frequencies with different degrees of accuracy may be performed analytically or with the help of numerical modeling. More precise determination of the natural frequencies can be accomplished experimentally by measuring the frequency response of the winding using FRA measurement systems and subsequent determination of frequencies corresponding to maxima of the real part of the transformer primary winding input admittance.

The recognizing of the resonant frequencies associated with winding natural frequencies can be performed by measuring the frequency response of this winding at the input terminals with open and short-circuited secondary winding with subsequent identification of frequencies, at which local maxima of the real part of input admittance coincide in both cases of open and short-circuited secondary winding.

For measurement of maximum ratio of resonant overvoltages it is advisable to use FRA systems that give the picture in a wide range of frequencies. For purposes of overvoltages ratio measurement the regular measuring cables and the termination impedance of 50 Ohms should be excluded.

Any connection to intermediate points of windings leads to inclusion of additional capacitances and to an error in the obtained maximum overvoltages ratio both in frequency and in peak value. To ensure high precision, it is necessary to use connections, with the lowest possible input capacitance of the measuring circuit, for example, high-frequency measuring probes of the digital oscilloscope with an input capacitance of less than 10 pF.

The time required to reach the steady state with maximum overvoltages ratio is of the order of tens of periods of the applied voltage. If primary winding is supplied by decaying oscillating voltage the overvoltages with maximum ratio may just not have enough time to develop. To evaluate overvoltages at
individual points of windings in this case the Laplace transform may be applied to the combination of the decaying input voltage and the continuous AC solution. It constitutes a simple analytical approach.

6. REFERENCES