on Historical simulation. The testing is performed out of the sample, with four different observation periods.

Key words: Basel II, Market risk, Value at Risk, Historical simulation

JEL classification: G15, G18, G20

1. INTRODUCTION

Financial institutions are subject to many sources of risk, among which credit risk and market risk are the most important. Risk can be broadly defined as the degree of uncertainty about future net returns (Alexander, 2000). Financial risk can be defined as a probability of occurrence of unwanted financial results and consequences. Market risk is a result of changing market prices of securities in capital markets (Bessis, 2002). Market risk arises from movement in the underlying risk factors of a particular security, such as: equity prices, interest rates, exchange rates and commodity prices. A single factor or a combination of these risk factors affects the value of a portfolio. Market risk exposure of a portfolio is determined by both the volatility of the underlying risk factors as well as the sensitivity of the portfolio to movements of these risk factors.

In this paper the authors investigate the applicability of measuring VaR for the purpose of forming bank's capital requirements by using historical simulation on stock market indexes from EU new member and candidate states. All of the calculations and backtesting are performed in MatLab 7.0. The paper is structured as follows: In section 2, an overview of regulatory guidelines regarding the forming of capital requirements is given. In section 3, the reasons behind the use of VaR methodology for forming capital requirements is presented. In section 4, the characteristics of stock markets in EU new member and candidate states are investigated. Section 5 presents the methodology of calculating
VaR via historical simulation, together with its advantages and disadvantages. In section 6, historical simulation models, with different observation periods are tested out-of-the-sample on stock indexes from EU new member and candidate states. The conclusions are summarized in Section 7.

2. MARKET RISK IN LIGHT OF BASEL COMMITTEE GUIDELINES

The increase in the relative importance of market risk in banks' portfolios has obliged regulators to devote more attention to these issues. The European Commission in its' Capital Adequacy Directive (CAD), agreed in 1993 and introduced at the beginning of 1996, and then later CAD II and the planned CAD III established EU minimum capital requirements for the trading books of banks and securities firms (Commission of the European Communities, 2004). The Basle Committee for Banking Supervision proposals regarding the treatment of market risk are summarized in a paper issued in January 1996 entitled "Overview of the Amendment of the Capital Accord to Incorporate Market Risks" (Basel Committee on Banking Supervision, 1996a). This and other papers (Basel Committee on Banking Supervision, 1996b) issued by the Basel Committee proposed a system comprising two alternative ways of calculating trading book capital requirements. Banks and investment firms can now decide whether they wished to be regulated under the standardized or the alternative - internal approach proposed by the Basle Committee. Under the Basle alternative approach, rather than laying down the percentage capital requirements for different exposures, regulators established standards for banks' internal risk models. These models form the basis for the calculation of capital requirements. This approach has the key additional advantage of aligning the capital calculation with the risk measurement approach of the particular bank. Using internal models to generate capital requirements is a radical change in approach but supervisors have for some time been moving steadily in this direction. In the CAD and the Basle standardized method, it is recognized that only by employing the banks' internal models can some positions be correctly processed for inclusion in the capital calculation. To ensure that the capital requirements calculated under the internal approach are adequate Basle Committee has laid down standards for the construction of these models. The distribution of losses must be calculated over a ten-day holding period using at least twelve months of data and must yield capital requirements sufficient to cover losses on 99% of times. Adopting general standards is necessary both to increase consistency between banks and to ensure that capital requirements really are adequate to the task. Basle Committee did not prescribe the exact type of model to be used in these calculations. The impact of these changes in banking regulation on the banks in less developed countries has not been well studied. In the European Union not even all the member of the EU-15 countries have systematically conducted research on the consequences and impact of these changes on their banking sector. New EU member states and EU member candidate states are even further behind in these issues. The group of EU new member and candidate states is comprised of the following countries: Bulgaria, Romania, Croatia and Turkey. Bulgaria and Romania became full EU members in January 2007. Croatia and Turkey started the accession negotiations in November of 2005. Croatia is expected to become a full EU member in 2009, which is not the case of Turkey, since it has a long journey still ahead of it. Although, very different and unique in its' own way, when looking through a financial prism these countries are similar in certain aspects. EU new member and candidate states are all significantly lagging behind the most developed EU countries in many fields but especially in matters of: financial legislation, market discipline, insider trading, disclosure of information (financial and other), embezzlement, fraud and knowledge of financial instruments and markets as well as the associated risks.

3. LITERATURE REVIEW

According to published research, VaR models based on moving average volatility models seem to perform the worst. Otherwise, there is no straightforward result, and it is impossible to establish a ranking among the models. The results are very sensitive to the type of loss functions used, the chosen probability level of VaR, the period being turbulent or normal etc. Hendricks (1996) in his famous study tested twelve VaR models (variance-
covariance VaR based on equally weighted moving average approach with 50, 125, 250, 500, and 1,250 days observation periods, variance-covariance VaR with varying exponentially weighted moving averages and historical simulation VaR with 125, 250, 500, and 1,250 days observation periods). Hendricks (1996) finds that in almost all cases the approaches cover the risk that they are intended to cover. In addition, the twelve approaches tend to produce risk estimates that do not differ greatly in average size, although historical simulation approaches yield somewhat larger 99th percentile risk measures than the variance-covariance approaches. Despite the similarity in the average size of the risk estimates, his investigation reveals differences, some times substantial, among the various VaR approaches. In terms of variability over time, the VaR approaches using longer observation periods tend to produce less variable results than those using short observation periods or those using weighting schemes. Jackson, Maude, Perraudin (1998) conclude that simulation-based VaR models yield more accurate measures of tail probabilities than parametric VaR models. They find that parametric VaR analysis tracks the time-series behaviour of volatility better and yield slightly superior volatility forecasts compared to non-parametric, simulation-based techniques (though the differences are generally not statistically significant). In their study the parametric VaR models that yield the best forecasts have relatively short window lengths and large weighting factors. But such models are very poor at fitting the tails of return distributions and capital requirements based on them tend to be too low. Lehar, Scheicher, Schittenkopf (2002) find that more complex volatility models (GARCH and Stochastic volatility) are unable to improve on constant volatility models for VaR forecast, although they do for option pricing. According to Brooks, Persand (2003), the relative performance of different models depends on the loss function used. However, GARCH models provide reasonably accurate VaR.

Although there is an abundance of research papers dealing with VaR and market risk measurement and management all of the existing VaR models are developed and tested in mature, developed and liquid markets. Quantitative testing of VaR models in other, less developed or developing financial market is scarce (e.g. Sinha, Chamu, 2000, Magnusson, Andonov, 2002, Cayon, Sarmiento, 2004, Žiković, 2005, 2006a, 2006b). Sinha, Chamu (2000) compare the performance of three different methods of calculating VaR in the context of Mexican and Latin American securities. They examine weaknesses of these methods by using five different tests: test based on the time until first failure, test based on failure rate, test based on expected value, test based on autocorrelation, and test based on (rolling) mean absolute percentage error. In their study BRW historical simulation performs better than the historical simulation method and they conclude that BRW VaR gives estimates as precise as the stochastic simulation method, but with lower analytical and computational resources. Furthermore, they find that historical simulation and RiskMetrics methodology can lead to serious errors in estimating VaR in the world of volatile markets. Magnusson and Andonov (2002) study some aspects of the influence of capital adequacy requirements (CAR) on financial stability in Iceland. They conclude that Icelandic market is characterised by relatively high volatility and relatively small diversification of the economy, suggesting that Icelandic banking sector should increase its capital coverage above the mandatory minimum during the upswing of the economy. They also find that tested approaches fail to provide universal methodology or hardly any guidance about the optimal size of the CAR. Cayon, Sarmiento (2004) by using coefficient of variation as a relative risk measure failed to provide conclusive evidence that the historical simulation VaR is a reliable for measuring risk at high confidence level in the Colombian stock market. Although, they could not reject the null hypothesis in all the cases, their finding can be explained by the fact that they did not use enough historical monthly observations to make it statistically significant, which can distort the results obtained at certain confidence levels. Žiković (2005) developed a semiparametric VaR model that uses EWMA volatility forecasting and tested it on Croatian VIN and CROBEX index and Slovenian SBI 20 index. The model performed far superiorly to historical simulation and BRW historical simulation but also failed to properly capture the dynamics of SBI 20 index at extreme confidence levels. Based on the performed tests on CROBEX and VIN index Žiković (2006a,b) concluded that historical simulation VaR models should not be used for high confidence level estimates (above 95%), especially VaR models based on shorter rolling windows. The obtained results show that although BRW VaR approach also has its flaws,
especially when testing for temporal dependence in the tail events, it brings significant improvement to historical simulation.

4. MEASURING MARKET RISK WITH VALUE AT RISK (VaR) METHODOLOGY

One of the most significant advances in the past two decades in the field of measuring and managing market risk is the development of Value at Risk (VaR) methodology and models for measuring market risk. VaR methodology was specifically developed for measuring and managing risk of portfolios across entire financial institution. VaR represents a method of assessing risk that uses standard statistical techniques, which are commonly used in other technical fields. VaR measures the worst expected loss over a given horizon under normal market conditions at a given confidence level (Alexander, 2000). Due to the approval by the Basel Committee for Banking supervision (Basel Committee on Banking Supervision, 1996a) of using internally developed VaR models for the purpose of measuring market risk, a large number of different approaches for calculating VaR figures has developed. The three main approaches to calculating VaR are (Jorion, 2001):

- Parametric approach
- Nonparametric approach and
- Monte Carlo simulation

Each of these approaches has its’ own advantages and disadvantages, and none of them performs superior to others in all the circumstances and markets (Hendricks, 1996). The main advantage of VaR as a risk measure is that it is theoretically simple, and can be used to summarize the risks of individual trading positions, or the risk of a large internationally diversified portfolio.

The errors in VaR estimation depend on the assumptions made when calculating VaR. Probably the most important assumption to be made is the choice of the theoretical distribution that describes the distribution of empirical data. The assumptions about the distribution of returns, as well as other assumptions made when calculating VaR, can be judged by whether the VaR measure provides the correct conditional and unconditional risk coverage (Manganelli, Engle, 2002). A VaR measure achieves the correct unconditional coverage if the portfolio losses exceed the $p$ percent VaR $1-p$ percent of the time. Because the losses are expected to exceed $p$ percent VaR $1-p$ percent of the time, a VaR measure that satisfies the unconditional coverage is correct on average. Correct conditional coverage means that as the risk of a portfolio changes daily, so should the VaR estimate change, and provide the correct VaR figure daily, and not on average. Although it is probably unrealistic for VaR to provide the exact coverage for every time period, a good VaR measure should at least go so far as to increase, when the risk of a portfolio appears to be increasing.

Testing of the VaR models in less developed or developing financial market is at best scarce. Research papers dealing with VaR calculation or volatility forecasting in the financial markets of EU new member or candidate states are extremely rare. Since candidate states are all exposed to similar processes of strong inflow of foreign direct and portfolio investments, and offer possibilities of huge profits for investors, these countries represent a very interesting opportunity for foreign investors, primarily banks. Banks and investment funds (domestic and foreign) that invest and operate in these countries employ the same risk measurement models for measuring market risk and forming of provisions that are used in the developed and liquid markets. Employing the VaR models in forming of bank’s provisions that are not suited to financial markets that they are used on, can have serious consequences for any investor, resulting in significant losses in trading portfolio that could pass undetected by the employed risk measurement models, leaving the investors unprepared for such events. Banks could also be penalized by the regulators, via higher scaling factor when forming their market risk provisions, due to the use of a faulty risk measurement model1.

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1 See Basel Committee on Banking Supervision: Supervisory framework for the use of “backtesting” in conjunction with the internal models approach to market risk capital requirements. Bank for International settlements, Jan 1996. The same applies to bank regulation in EU new member and candidate states; see for example Odluka o adekvatnosti kapitala banaka, NN 17/2003.
4. CHARACTERISTICS OF STOCK MARKETS IN EU NEW MEMBER AND CANDIDATE STATES

For transitional economies such as those of EU new member and candidate states a significant problem for a serious and statistically significant analysis is a short history of market economy and active trading in the financial markets. Because of the short time series of returns on individual stock and their highly variable liquidity it is practical to analyse the stock indexes of this countries. After all, stock index can be viewed as a portfolio of selected securities from an individual country. In this paper the characteristics of stock indexes of Croatian Zagreb stock exchange (CROBEX), Bulgaria (SOFIX), Romania (BBETINRM) and Turkey (XU100) will be tested. To determine if the analysed indexes have some common properties the values of the indexes and their natural logarithms are analysed and compared. The values of the analysed indexes and logs of their values are presented in figures 1 to 8.

Figure 1 - CROBEX index values in the period 24.10.2000 - 02.01.2007

Figure 2 - CROBEX index returns in the period 24.10.2000 - 02.01.2007

Figure 3 - SOFIX index values in the period 24.10.2000 - 02.01.2007

Figure 4 - SOFIX index returns in the period 24.10.2000 - 02.01.2007

Figure 5 - BBETINRM index values in the period 24.10.2000 - 03.01.2007

Figure 6 - BBETINRM index returns in the period 24.10.2000 - 03.01.2007

Figure 7 - XU100 index values in the period 24.10.2000 - 04.01.2007

Figure 8 - XU100 index returns in the period 24.10.2000 - 04.01.2007

It is obvious from the values of compared indexes that there is a strong positive trend present
in these indexes so the assumption of the stationarity of the time series is clearly violated. As was expected, the financial markets of EU new member and candidate states are experiencing a boom due to the catching up of these economies to the European standards and strong inflow of foreign direct and portfolio investments. From looking at the graphical representation of the realized index returns, in the analysed period, it can be concluded that volatility clustering and occurrence of extreme positive and negative returns characterize these observations.

To statistically determine if the daily returns of tested indexes are normally distributed, normality of distribution is tested in several ways. All of the tests and analysis are performed on the full set of observations to ensure the significance of the obtained results. The simplest test of normality is to analyse the third and fourth moment around the mean of the distribution. Third moment around the mean, asymmetry, in the case of normal distribution should be zero. Negative asymmetry means that the distribution is skewed to the left, which implies that there is a greater chance of experiencing negative returns, and vice versa. Fourth moment around the mean, kurtosis, in the case of normal distribution should be three. Most of the statistical software packages modify the equation for kurtosis to equal zero for normal distribution, to ease the interpretation. Kurtosis higher than zero means that the distribution has fatter tails than normal n, meaning that more extreme events occur more frequently in the analysed data than would be expected under normal distribution. More sophisticated tests for normality of distribution are Lilliefors test, Shapiro-Wilks test and Jarque-Bera test. Descriptive statistics for the CROBEX, SOFIX, BBETINRM and XU100 index are presented in table 1.

Table 1 – Descriptive statistics for CROBEX, SOFIX, BBETINRM and XU100 index in the period 24.10.2000 - 04.01.2007

<table>
<thead>
<tr>
<th></th>
<th>CROBEX index</th>
<th>SOFIX index</th>
<th>BBETINRM index</th>
<th>XU100 index</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.0009068</td>
<td>0.0016289</td>
<td>0.0017372</td>
<td>0.0006495</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>0.0006429</td>
<td>0.0009276</td>
<td>0.0011079</td>
<td>0.0013929</td>
</tr>
<tr>
<td><strong>Minimum</strong></td>
<td>-0.060324</td>
<td>-0.20999</td>
<td>-0.11902</td>
<td>-0.19979</td>
</tr>
<tr>
<td><strong>Maximum</strong></td>
<td>0.14979</td>
<td>0.21073</td>
<td>0.0920749</td>
<td>0.17774</td>
</tr>
<tr>
<td><strong>St. Dev</strong></td>
<td>0.013148</td>
<td>0.0195902</td>
<td>0.0148993</td>
<td>0.026573</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>0.009642</td>
<td>-0.32989</td>
<td>-0.050982</td>
<td>-0.052461</td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
<td>25.703</td>
<td>34.645</td>
<td>9.0329</td>
<td>10.225</td>
</tr>
</tbody>
</table>

In accordance with the graphical representation, stock indexes of the EU member candidate states have a significant positive mean that indicates a positive trend in the movement of the indexes. The highest positive mean is present in the Romanian BBETINRM index, and the smallest in the Turkish XU100 index. Mean and median of stock indexes are not equal, which is assumed under normality. Third moment around the mean (skewness) for all the indexes is different from the zero, with CROBEX index deviating the most and BBETINRM index being the closest to the value assumed under normality. Another significant difference between CROBEX index and other indexes of the EU member candidate states is the strong positive skewness of the returns as opposed to the negative skewness of all other indexes. This fact is very important for all the investors meaning that the probability of positive returns is far greater on CROBEX index than it is on other tested indexes. In fact all the analysed indexes with the exception of CROBEX are negatively skewed meaning that there is a greater probability of experiencing negative returns. The fourth moment around the mean (excess kurtosis) for all the indexes is different from the zero, assumed under normality. Indexes with the highest kurtosis are SOFIX and CROBEX. The reason for this can be seen from the values of their maximum and minimum. Both indexes experienced extreme positive daily returns in the observed period, SOFIX index +21.07% and CROBEX index +14.98%, as well as extreme negative daily returns, SOFIX index -20.90% and CROBEX index -9.03%. The high value of kurtosis for these indexes indicates that banks and other investors investing in these markets can expect unusually high both positive and negative returns on their portfolios. Normality tests for the CROBEX, SOFIX, BBETINRM and XU 100 index are presented in table 2.

Table 2 – Tests of normality of distribution for CROBEX, SOFIX, BBETINRM and XU100 index returns in the period 24.10.2000 - 04.01.2007

<table>
<thead>
<tr>
<th>Normality tests</th>
<th>CROBEX index</th>
<th>SOFIX index</th>
<th>BBETINRM index</th>
<th>XU100 index</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Jarque-Bera</strong></td>
<td>31914</td>
<td>63102</td>
<td>2272.4</td>
<td>3348.7</td>
</tr>
<tr>
<td><strong>(p value)</strong></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Lilliefors</strong></td>
<td>0.10143</td>
<td>0.17065</td>
<td>0.07389</td>
<td>0.057236</td>
</tr>
<tr>
<td><strong>(p value)</strong></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Shapiro-Wilk</strong></td>
<td>0.95653</td>
<td>0.95945</td>
<td>0.95889</td>
<td>0.95615</td>
</tr>
<tr>
<td><strong>(p value)</strong></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
All of the employed normality tests are unanimous in the claim that the hypothesis of the normality of returns for the EU member candidate states stock indexes should be rejected. Probability values for the CROBEX, SOFIX, BBETINRM and XU100 indexes under all the normality test are zero, strongly indicating that there is no possibility that the returns on these indexes are normally distributed.

Returns on financial assets themselves are usually not dependent (correlated), otherwise traders could forecast daily returns. Returns squared are dependent, this meaning that volatility can be forecasted, but not the direction of the change of a variable. Two random variables are uncorrelated if $\rho_{XY} = 0$. In addition, if both $X$ and $Y$ are normal random variables, then $\rho_{XY} = 0$ if and only if $X$ and $Y$ are independent. When the linear dependence within a univariate time series (between $r_t$ and $r_{t-l}$) is measured, the concept of correlation is generalized to autocorrelation. Autocorrelation measures the linear dependence between observations of a same variable across time. The autocorrelation coefficient between $r_t$ and $r_{t-l}$ is called lag – $l$ autocorrelation of $r_p$ which under the weak stationarity assumption is a function of $l$ only and is invariant to time (Šošić, Serdar, 1994):

$$\rho_l = \frac{Cov(r_t^2, r_{t-l}^2)}{\sqrt{\sigma^2(r_t^2)\sigma^2(r_{t-l}^2)}} = \frac{Cov(r_t^2, r_{t-l}^2)}{\sigma^2(r_t)}$$

where the weak stationarity property implies that $\sigma^2(r_p) = \sigma^2(r_{t-l})$, meaning that the variance of returns is time invariant. From the above equation it can be seen that $\rho_0 = 1$, $\rho_l = \rho_{-l}$, and $-1 \leq \rho_l \leq 1$. Weakly stationary series is not serially correlated if and only if $\rho_l = 0$ for all $l > 0$. The sample autocorrelation and partial autocorrelation functions of squared returns of EU member candidate states stock indexes are presented in figures 9 to 12.

Figure 9 - Sample autocorrelation and sample partial correlation function of squared CROBEX index returns in the period 22.11.2004 - 02.01.2007

The figures 9 to 12 show that at 5% significance level the presence of autocorrelation and partial autocorrelation in the squared returns is significant. This points to the conclusion that the returns are not IID i.e. that the volatilities tend to cluster together (periods of low volatility are followed by further periods of low volatility and vice versa).

A wide established approach to detecting volatility clusters, which is autoregression in the data, is the Ljung-Box Q-statistic calculated on the squared returns. Ljung-Box Q-statistic is the $I$th autocorrela-
tion of the $T$-squared returns, and calculates whether the size of the movement at time $t$ has any useful information to predict the size of the movement at time $t+1$. Ljung-Box Q-statistic is used as a lack-of-fit test for a departure from randomness. Under the null hypothesis that the model fit is adequate, the test statistic is asymptotically chi-square distributed with $m$ degrees of freedom. Engle’s hypothesis test for the presence of autoregressive conditional heteroskedasticity (ARCH) effects tests the null hypothesis that a time series of sample residuals consists of independently and identically distributed (IID) Gaussian disturbances, i.e., that no ARCH effects exist. Given sample residuals from a curve fit (e.g., a regression model), Arch test tests for the presence of $M^p$ order ARCH effects by regressing the squared residuals on a constant and the lagged values of the previous $M$ squared residuals. Under the null hypothesis, the asymptotic test statistic, $T(R^2)$, where $T$ is the number of squared residuals included in the regression and $R^2$ is the sample multiple correlation coefficient, asymptotically chi-square distributed with $M$ degrees of freedom. The values of the Ljung-Box Q-statistic and Engle’s Arch test are presented in table 3.

The Ljung-Box Q-statistic and Engle’s Arch test reaffirm that there is significant autocorrelation and ARCH effects present in the analysed indexes, meaning that the returns on these indexes are not independently and identically distributed. The results are that much more indicative when considering that the hypothesis of IID was rejected for all the tested time lags (5, 10, 15 and 20 days) and all of the indexes, with the exception of

<table>
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<tr>
<th>Table 3 – Test of independency for CROBEX, SOFIX, BBETINRM and XU100 index returns</th>
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<tr>
<td><strong>Mean adjusted squared returns (BBETINRM index)</strong></td>
</tr>
<tr>
<td>Ljung-Box-Pierce Q-test (BBETINRM index)</td>
</tr>
<tr>
<td><strong>Period (days)</strong></td>
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<tr>
<td>5</td>
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<tr>
<td>10</td>
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<td>20</td>
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<tr>
<td><strong>ARCH test (BBETINRM index)</strong></td>
</tr>
<tr>
<td><strong>Period (days)</strong></td>
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<td>5</td>
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<tr>
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<td>20</td>
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<tr>
<td><strong>Mean adjusted squared returns (SOFIX index)</strong></td>
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<tr>
<td>Ljung-Box-Pierce Q-test (SOFIX index)</td>
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<td><strong>Period (days)</strong></td>
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<tr>
<td><strong>ARCH test (SOFIX index)</strong></td>
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<td><strong>Period (days)</strong></td>
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CROBEX index Engle’s Arch test at 20 day lag. This discovery is very indicative for risk managers in banks and other financial institutions, meaning that because the elementary assumption of historical simulation is not satisfied, the VaR figures obtained via historical simulation are not to be completely trusted and at best provide only unconditional coverage. Because of the significant presence of ARCH effect, a better way of forecasting VaR in these markets would probably be to model the time series of returns with an ARCH or GARCH process and then calculate the VaR by using either a parametric or nonparametric approach updated by ARCH/GARCH volatility forecasts.

5. MEASURING VAR USING HISTORICAL SIMULATION

The most commonly used VaR models in the world, including the J.P. Morgan’s RiskMetrics system are parametric, and assume in advance a particular theoretical shape of the cumulative distribution of a variable (stock price, interest rate, etc.). The most frequently used distribution in finance is the normal (Gaussian) distribution. A random variable \(X\) is normally distributed with mean \(\mu\) and variance \(\sigma^2\) if the probability that the value \(x\), which is a function of \(f(x)\), obeys the following probability density function (Šošić, Serdar, 1994):

\[
f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{(x-\mu)^2}{2\sigma^2} \right)
\]

where \(X\) is defined over \(-\infty < x < \infty\).

Every random variable \(X\) that is normally distributed can be transformed into a standardized normal random variable \((Z)\) if variable \(X\) is linearly transformed into \(X = \mu + z\sigma\) (Šošić, Serdar, 1994).

\[
Z = \frac{X - \mu}{\sigma} \quad X \sim N(\mu, \sigma^2) \quad Z \sim N(0,1)
\]

The mean of a standardized distribution is 0, and standard deviation is equal to 1. With the help of standardized variable \(Z\), the standardized normal distribution can be written as (Šošić, Serdar, 1994):

\[
f(Z) = \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{Z^2}{2} \right)
\]

which does not depend on the unknown parameters \(\mu\) and \(\sigma\). The implication is that it is very simple to calculate the probabilities of any state of the variable \(X\) by using the linear transformation to \(Z\). The probability that the value of variable \(Z\) is in interval \([z_x,z_y]\) is (Šošić, Serdar, 1994):

\[
P(z_x < Z \leq z_y) = \int_{z_x}^{z_y} f(z)dz = F(z_y) - F(z_x)
\]

Because of the fact that normal distribution uses only the mean and standard deviation of the variable to describe its’ distribution it is very simple to work with. The parametric mean-variance model of VaR on an aggregate portfolio level is simply calculated as (Jorion, 2001):

\[
VaR(h_p,cl) = Z_{cl} \sqrt{\sigma^2} - hp \mu,
\]

where \(hp\) is the holding period and \(cl\) is confidence level.

The use of normal distribution in finance is very questionable, especially in developing and shallow markets such those of EU member candidate states. As stated before the normally distributed mean-variance VaR takes into account only the first two moments of the distributions, and completely neglects the third and fourth moment around the mean (skewness and kurtosis). It is a well-documented fact that distributions of stock returns are asymmetric (negatively skewed) and leptokurtotic (have fatter tails than described by the normal distribution) (Mandelbrot, 1963, Bollerslev, 1986). Because of these drawbacks, VaR calculation based on assumption of normality of distribution, including the Monte Carlo simulation, when faced with empirical distribution that clearly is not normal, perform poorly.

Historical simulation is a member of nonparametric family of methods of calculating VaR. The main characteristic of nonparametric approach is the calculation of VaR without making apriori assumptions about the shape of the distribution of the realized returns. Nonparametric approach, unlike the parametric approach that apriori assigns a theoretical distribution to a random variable, empirically determines the distribution of the observed variable, and the VaR figure is easily computed via order statistics from the desired quantile of the cumulative distribution function. Historical simulation is based on two elementary assumptions:

1) future will be similar to the past, and that from the data obtained from the recent past,
the risk in the near future can be calculated (Hendricks, 1996).

2) realized returns are independently and identically distributed (IID) through time (Manganelli, Engle, 2002).

Unfortunately, even these assumptions do not hold in practice, as it is tested and proven in this paper. When comparing only classical historical simulation and normally distributed mean-variance VaR, it is the authors’ opinion that historical simulation approach to calculating VaR would be better suited for calculating market risk on capital market in EU new member and candidate states for many reasons (Žiković, 2005):

1) volatilities of stocks are time varying (heteroskedastic),
2) coefficients of correlation between stocks are not stationary, they often change very dramatically and suddenly in very short time intervals,
3) distribution of returns of stocks is asymmetric and has fat tails,
4) existence of sufficient number of extreme events

The main advantages of historical simulation compared to the other methods of estimating VaR are (Dowd, 2002, Žiković, 2005):

- the method is theoretically simple,
- it is easy to implement in practice,
- data used can be easily obtained from the stock exchange or from specialized companies, such as Bloomberg, Reuters and DataStream,
- obtained VaR figures are simple to present to the top management,
- since it is not parametric in its’ nature, asymmetry and kurtosis can be easily included in the calculation of VaR,
- there is no need for the calculation of the variance-covariance matrices, which greatly lowers the computational and time burden.

Besides all the stated advantages, historical simulation also exhibits some serious problems when compared to other methods of calculating VaR. The principle disadvantage of historical simulation method is that it computes the empirical cumulative distribution function of the portfolio returns by assigning an equal probability weight of 1/N to each day’s return. This is equivalent to assuming that the risk factors, and hence the historically simulated returns are independently and identically distributed (IID) through time. This assumption is unrealistic because it is known that the volatility of asset returns tends to change through time, and that periods of high and low volatility tend to cluster together (Bollerslev, 1986). One of the most serious critiques on account of historical simulation is the fact that it completely depends on the past events and data that is used as a basis for the calculation of VaR. Another serious problem that is not noticeable in the developed markets but is clearly present in the transitional countries is the lack of a larger number of observations that is required for the historical simulation. Other potential drawbacks of historical simulation are (Dowd, 2002, Žiković, 2005):

- if the time period used for the calculation of VaR is characterized by low volatility and includes no extreme events, historical simulation can underestimate the true level of risk,
- if the time period used for the calculation of VaR is characterized by high volatility and includes numerous extreme events, historical simulation can overestimate the true level of risk,
- historical simulation is known to react poorly to one-time changes that happen in the observation period, such as currency devaluation,
- the method can react very slowly to sudden changes in the market, especially if the observation period used for the calculation of VaR is long,
- the method is known to suffer from “ghost effect”, meaning that high losses that occurred in relatively distant past continue to effect the level of VaR until they disappear from the observation period,
- VaR is limited to the highest losses that happened in the observation period disregarding the current market volatility.

Banks often rely on VaR’s from historical simulations (HS VaR). The value of VaR is calculated as the $100p^{th}$ percentile or the $(T+1)p^{th}$ order statistic of the set of pseudo portfolio returns. In principle it is easy to construct a time series of historical portfolio returns using current portfolio holdings and historical asset returns. The returns on the indexes in this paper are calculated as (Dowd, 2002):
\[ r_t = \ln(1 + R_t) = \ln \frac{P_t}{P_{t-1}} \]

In practice, however, historical asset prices for the assets held today may not be available. Examples where difficulties arise include derivatives, individual bonds with various maturities, private equity, new public companies, merged companies and so on. For these cases “pseudo” historical prices must be constructed using either pricing models, factor models or some ad hoc considerations. The current assets without historical prices can for example be matched to “similar” assets by capitalization, industry, leverage, and duration. Historical pseudo asset prices and returns can then be constructed using the historical prices on these substitute assets (Dowd, 2002):

\[ r_{i,t} = \sum_{j=1}^{N} w_{i,t} r_{i,j} \equiv W^{i,t} R_t, \quad t = 1, 2, \ldots, T \]

Following the Basle Committee recommendations for the use of VaR in internal market risk measurement models 99th percentile should be used (Basel Committee on Banking Supervision, 1996b). Historical simulation VaR can be expressed as (Dowd, 2002):

\[ HS - VaR^{p}_{r_{\cdot \cdot \cdot \cdot}} = r_n((T + 1)p) \]

where \( r_n((T + 1)p) \) is taken from the set of ordered pseudo returns \( \{r_n(1), r_n(2), \ldots, r_n(T)\} \). If \( (T+1)p \) is not an integer value then the two adjacent observations can be interpolated to calculate the VaR. Historical simulation has some serious problems, which have been well-documented. Perhaps most importantly, it does not properly incorporate conditionality into the VaR forecast. The only source of dynamics in the HS VaR is the fact that the sample window is updated with the passing of time. However, this source of conditionality is minor in practice. Historical simulation method assigns equal probability weight of \( \frac{1}{N} \) to each observation. This means that the historical simulation estimate of VaR at the \( cl \) confidence level corresponds to the \( N(1-cl) \) lowest return in the N period rolling sample. Because the crash is the lowest return in the N period sample, the \( N(1-cl) \) lowest return after the crash, turns out to be the \( (N(1-cl)-1) \) lowest return before the crash. If the \( N(1-cl) \) and \( (N(1-cl)-1) \) lowest returns happen to be very close in magnitude, the crash actually has almost no impact on the historical simulation estimate of VaR for the long positions in a portfolio of securities. From the equation for historical simulation it can be seen that HS VaR changes significantly only if the observations around the order statistic \( r_w(T+1)p \) change significantly (Dowd, 2002). For instance, when using a 250-day moving window for a 1% HS VaR, only the second and third smallest returns will matter for the calculation. Including a crash in the sample, which now becomes the smallest return, may therefore not change the HS VaR very much if the new second smallest return is similar to the previous one.

Moreover, the lack of a properly defined conditional model in the HS methodology implies that it does not allow for the construction of a term structure of VaR. Calculating a 1% 1-day HS-VaR may be possible on a window of 250 observations, but calculating a 10-day 1% VaR on 250 daily returns is not. Often the 1-day VaR is simply scaled by the square root of time, but this extrapolation is only valid under the assumption of IID daily returns, which is, as proven in the previous section, not valid (Alexander, 2000).

6. TESTING THE HISTORICAL SIMULATION WITH DIFFERENT OBSERVATION PERIODS ON STOCK INDEXES FROM EU NEW MEMBER AND CANDIDATE STATES

The simplest way to verify the accuracy of a particular VaR model is to record the failure rate, which gives the proportion of times VaR is exceeded in the analyzed time sample. The historical simulation is tested by Kupiec test, a simple expansion of the failure rate, which is also prescribed by Basel Committee on Banking Supervision. The setup for this test is the classic framework for a sequence of successes and failures, also known as Bernoulli trials. The number of exceptions (\( x \)) follows a binomial probability distribution (Jorion, 2001):

\[ f(x) = \binom{T}{x} p^x (1 - p)^{T-x} \]

\( T \) - sample size
\( p = 1 - \text{confidence level} \)
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The variable $x$ has the expected value of $E(X) = pT$ and variance $V(X) = p(1-p)T$. The binomial distribution can be used to test if the number of exceptions is acceptably small for a model to be accepted as correct.

For the period of 500 days realized daily returns of CROBEX, SOFIX, BBETTNRM and XU100 index are compared to the VaR forecasts obtained by using historical simulation at 95% and 99% confidence level and different rolling windows (50, 100, 250 and 500 days), and are shown in figures 13 to 20. Backtesting results are given in tables 4 to 7.

**Figure 13** – Historical simulation VaR at 5 percent with 50, 100, 250 and 500 days observation period for CROBEX index in the period 22.11.2004 - 02.01.2007

**Figure 14** – Historical simulation VaR at 1 percent with 50, 100, 250 and 500 days observation period for CROBEX index in the period 22.11.2004 - 02.01.2007

**Figure 15** – Historical simulation VaR at 5 percent with 50, 100, 250 and 500 days observation period for SOFIX index in the period 23.12.2004 - 02.01.2007
Figure 16 – Historical simulation VaR at 1 percent with 50, 100, 250 and 500 days observation period for SOFIX index in the period 23.12.2004 - 02.01.2007

Figure 17 – Historical simulation VaR at 5 percent with 50, 100, 250 and 500 days observation period for BBETINRM index in the period 08.12.2004 - 03.01.2007

Figure 18 – Historical simulation VaR at 1 percent with 50, 100, 250 and 500 days observation period for BBETINRM index in the period 08.12.2004 - 03.01.2007
Figure 19 - Historical simulation VaR at 5 percent with 50, 100, 250 and 500 days observation period for XU100 index in the period 07.01.2005 - 04.01.2007

![Figure 19](image)

Figure 20 - Historical simulation VaR at 1 percent with 50, 100, 250 and 500 days observation period for XU100 index in the period 07.01.2005 - 04.01.2007

![Figure 20](image)

Table 4 - Backtesting results for 500 HS VaR forecasts for CROBEX index

<table>
<thead>
<tr>
<th>Model</th>
<th>HS 50 (95%)</th>
<th>HS 50 (99%)</th>
<th>HS 100 (95%)</th>
<th>HS 100 (99%)</th>
<th>HS 250 (95%)</th>
<th>HS 250 (99%)</th>
<th>HS 500 (95%)</th>
<th>HS 500 (99%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of failures</td>
<td>35</td>
<td>11</td>
<td>29</td>
<td>13</td>
<td>21</td>
<td>8</td>
<td>17</td>
<td>2</td>
</tr>
<tr>
<td>Frequency of failures</td>
<td>0.07</td>
<td>0.022</td>
<td>0.058</td>
<td>0.026</td>
<td>0.042</td>
<td>0.016</td>
<td>0.034</td>
<td>0.004</td>
</tr>
<tr>
<td>Kupiec test</td>
<td>0.019643</td>
<td>0.005208</td>
<td>0.17647</td>
<td>0.000646</td>
<td>0.75905</td>
<td>0.06711</td>
<td>0.94408</td>
<td>0.87661</td>
</tr>
</tbody>
</table>

Table 5 - Backtesting results for 500 HS VaR forecasts for SOFIX index

<table>
<thead>
<tr>
<th>Model</th>
<th>HS 50 (95%)</th>
<th>HS 50 (99%)</th>
<th>HS 100 (95%)</th>
<th>HS 100 (99%)</th>
<th>HS 250 (95%)</th>
<th>HS 250 (99%)</th>
<th>HS 500 (95%)</th>
<th>HS 500 (99%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of failures</td>
<td>34</td>
<td>12</td>
<td>24</td>
<td>5</td>
<td>26</td>
<td>4</td>
<td>20</td>
<td>4</td>
</tr>
<tr>
<td>Frequency of failures</td>
<td>0.068</td>
<td>0.024</td>
<td>0.048</td>
<td>0.01</td>
<td>0.052</td>
<td>0.008</td>
<td>0.04</td>
<td>0.008</td>
</tr>
<tr>
<td>Kupiec test</td>
<td>0.03026</td>
<td>0.001901</td>
<td>0.52865</td>
<td>0.38404</td>
<td>0.36861</td>
<td>0.56039</td>
<td>0.82115</td>
<td>0.56039</td>
</tr>
</tbody>
</table>

Table 6 - Backtesting results for 500 HS VaR forecasts for BBETINRM index

<table>
<thead>
<tr>
<th>Model</th>
<th>HS 50 (95%)</th>
<th>HS 50 (99%)</th>
<th>HS 100 (95%)</th>
<th>HS 100 (99%)</th>
<th>HS 250 (95%)</th>
<th>HS 250 (99%)</th>
<th>HS 500 (95%)</th>
<th>HS 500 (99%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of failures</td>
<td>37</td>
<td>16</td>
<td>33</td>
<td>14</td>
<td>31</td>
<td>7</td>
<td>39</td>
<td>11</td>
</tr>
<tr>
<td>Frequency of failures</td>
<td>0.074</td>
<td>0.032</td>
<td>0.066</td>
<td>0.028</td>
<td>0.062</td>
<td>0.014</td>
<td>0.078</td>
<td>0.022</td>
</tr>
<tr>
<td>Kupiec test</td>
<td>0.007661</td>
<td>1.73E-05</td>
<td>0.045412</td>
<td>0.000206</td>
<td>0.09445</td>
<td>0.13232</td>
<td>0.002701</td>
<td>0.005208</td>
</tr>
</tbody>
</table>
Table 7 – Backtesting results for 500 HS VaR forecasts for XU100 index

<table>
<thead>
<tr>
<th>Model</th>
<th>HS 50 (95%)</th>
<th>HS 50 (99%)</th>
<th>HS 100 (95%)</th>
<th>HS 100 (99%)</th>
<th>HS 250 (95%)</th>
<th>HS 250 (99%)</th>
<th>HS 500 (95%)</th>
<th>HS 500 (99%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of failures</td>
<td>39</td>
<td>11</td>
<td>34</td>
<td>13</td>
<td>32</td>
<td>8</td>
<td>31</td>
<td>7</td>
</tr>
<tr>
<td>Frequency of failures</td>
<td>0.078</td>
<td>0.022</td>
<td>0.068</td>
<td>0.026</td>
<td>0.064</td>
<td>0.016</td>
<td>0.062</td>
<td>0.014</td>
</tr>
<tr>
<td>Kupiec test</td>
<td>0.002701</td>
<td>0.005208</td>
<td>0.03026</td>
<td>0.00646</td>
<td>0.066371</td>
<td>0.06711</td>
<td>0.09445</td>
<td>0.13232</td>
</tr>
</tbody>
</table>

Backtesting indicates that historical simulation based on shorter rolling windows (50 and 100 days) performs poorly at both 95% and at 99% confidence level. Historical simulation based on longer observation periods (250 and 500 days) was accepted at 5% significance level, as being unconditionally correct for all the tested indexes except BBETINRM index, where historical simulation with 500 day rolling window failed completely based on all the tested observation periods and both confidence levels. The fact that the historical simulation based on longer observation periods was unconditionally correct for three out of four tested indexes can be attributed to the fact that more extreme losses were present in the observation period, and that is why the binomial test accepted these models as being unconditionally correct. Under the assumption that the historical simulation based on 250 and 500 days rolling windows provided correct unconditional coverage for three out of four indexes, historical simulation based on 250 days rolling window was the best performer, since it resulted in smallest estimates of VaR, resulting also in the lowest level of capital requirements that have to be held by the bank for market risk purposes. The obtained results point to the conclusion that even though historical simulation provided correct unconditional coverage for most of the tested indexes, investors should be very careful when employing it. In the financial markets of the EU new member and candidate states where the assumption that the returns are IID should be rejected, the use of historical simulation based on shorter observation periods for risk management purposes is highly questionable, and not recommendable.

7. CONCLUSION

All of the stock indexes from EU new member and candidate states exhibit some common properties. Due to the various influences and process that these economies as going through the, CROBEX (Croatia), SOFIX (Bulgaria), BBETINRM (Romania) and XU100 (Turkey) index all show clear positive trend in a longer time period. A significant difference between CROBEX index and other indexes from EU new member and candidate states is the strong positive skewness of the returns as opposed to the negative skewness of other indexes. This fact is very important for all the investors meaning that the probability of positive returns is far greater on CROBEX index than it is on other tested indexes. The high value of kurtosis for these indexes indicates that banks and other investors investing in these markets can expect unusually high both positive and negative returns on their portfolios. All of the analysed indexes exhibit asymmetry, leptokurtosis and based on performed tests of normality, it can be said with great certainty that their returns are not normally distributed. Employed tests show significant autocorrelation and ARCH effects in the squared returns of all the analysed indexes. These phenomena violate the IID assumption that is a necessary requirement for the proper implementation of historical simulation. This discovery is very indicative for risk managers in banks and other financial institutions, meaning that because the elementary assumption of historical simulation is not satisfied, the VaR figures obtained via historical simulation are not to be completely trusted and at best provide only unconditional coverage. The obtained results point to the conclusion that even though historical simulation provided correct unconditional coverage for most of the tested indexes, investors should be careful when employing it. In the financial markets of EU new member and candidate states where, as the results show, the assumption that the returns are IID is not valid, the use of historical simulation based on shorter observation periods for risk management purposes is not recommendable.
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