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Pregledni članak

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OPTION'S VALUE – GREEK MEASURES FLUCTUATIONS AND THEIR CONSEQUENCES

OPTIONS THEORETICAL PRICE

Options are derivatives which are widely used to hedge assets fluctuations (these can be shares, currency, interest rate, etc.). The use of options requires a well understanding of these instruments – both their valuation and ways of hedging them (in the case of sellers of these instruments). Options buyers sell risk which is bought by sellers of these instruments who must mitigate it. A good hedging method to be used by them, requires monitoring the sensitivity of the option's price to changing such parameters as underlying asset prices, time to maturity, risk free interest rate and volatility. It is described by so called Greek measures which will be defined later. The calculations presented in the paper prove that Greeks (Delta, Gamma, Theta, Rho and Vega) fluctuate continuously.

Theoretical value for a European call or put option can be computed using the B-S model:[1]

$$c = SN(d_1) - X e^{-rT} N(d_2)$$

$$p = Xe^{-rT} N(-d_2) - SN(-d_1)$$

where:

$$d_1 = \frac{\ln(\frac{S}{X}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(\frac{S}{X}) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

c – value of a call option
p – value of a put option
S – strike price
σ - volatility of the underlying instrument
X - exercise price
r - risk free interest rate
T - time to expiration given in years
ln – natural logarithm function
e - the base of the natural log function = 2, 71
N (d) – the probability that a random draw from a standard normal distribution will be lower than d

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[1] J.Hull, Kontrakty terminowe i opcje-wprowadzenie, Wig Press, Warszawa 1998, p.299

Some of the important assumptions underlying the formula are the following:[2]

- the stock will pay no dividends until after the option expiration date,
- both the interest rate and variance (volatility) rate of the stock are constant (or in slightly more general versions of the formula, both are known functions of time – any changes are perfectly predictable),
- stock prices are continuous, meaning that sudden extreme jumps such as those in the aftermath of an announcement of a takeover attempt are ruled out.

Those who have short positions in options must hedge them. In order to do so, they are obliged to monitor Greek letters values in order to modify the portfolios that hedge options. However, the paper shows that Greek letters are not fixed during the time of option's life. Their value changes each time when for instance the underlying asset price fluctuates. The point is that the price fluctuates steadily, so new values of Greek letters should be calculated without any delay. The paper proves that it is not only the share price change that influences Greek letters but also fluctuations of risk free interest rate, time to maturity or volatility. Calculations have been done both for a call and a put option. The analysis has been conducted for five Greek letters, i.e. Delta, Gamma and Theta, Rho and Vega.

GREEK MEASURES DEFINITIONS

It should be emphasized that the Greek parameters show only the result of changing just one input data. In fact, if a few factors change, a different measure must be calculated for each of them. The Greeks are defined in the beneath explained way.

- *Delta* measures the sensitivity of the option's price to changing stock prices.[3] For example, if delta=0,325, it means that if the share price increases or decreases by 1 currency unit, the option price change will be equal to 0,325.
- *Gamma* shows the influence of share prices changes on delta, For instance, if gamma=0,090695, it means that if the share price increases or decreases by 1 currency unit, delta will change by 0,090695.
- *Theta* measures the change in the option price when there is a decrease in the time to maturity of 1 day.[4] If theta=-5,137, it means that after one day, which is about 0,27% of a year, the option theoretical value will change by 0,014 [(0,27% × -5,137) : 100].
- *Rho* shows the option price change when the risk free interest rate increases or decreases. If the risk free interest rate changes by 100 basic points and rho=8,255, the option price will change by 0,082 currency rates (0,01 × 8,255).
- *Vega* measures how fast an option price changes with its volatility. Mathematically, an option's vega is the first-order partial derivative of the option price with respect to the volatility of its underlying asset.[5] If vega=5,804 and the volatility changes by 1%, the option price will change by 0,0584 (0,01 × 5,804).

Options valuations in practice

Calculations presented in the following examples prove that Greek letters fluctuate when share prices, volatility, time to maturity or risk free interest rate change. These factors have also a great impact on the option's value.

<u>Example 1</u>
Assumptions:

European Call option

Dividend yield: 0

Maturity: 60 days (0,164 years)

Strike price: 60$

Risk free interest rate: 4,5%

Volatility: 17%

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[2] Z.Bodie, A.Kane, A.J.Marcus, Investments, McGraw-Hill, New York 2002, p.711

[3] R.W. Kolb, Futures, Options and Swaps, Blackwell Publishing, Padstow 2003, p. 472

[4] R.L. Mc Donald, Derivatives Markets, Pearson Education, Boston 2003, p. 373

[5] P.G. Zahng, Exotic Options, World Scientific, Singapore 2001, p. 76-77

Table 1. Correlation between share prices and Greek measures for a call option.

Share price ($)	Option value	Delta	Gamma	Theta	Vega	Rho
57,6	0,814	0,325	0,090695	-5,137	5,804	8,255
60,8	2,343	0,630	0,090105	-6,396	5,804	8,255
64,0	4,760	0,859	0,050704	-5,211	5,804	8,255
67,2	7,703	0,963	0,017552	-3,654	5,804	8,255
70,4	10,844	0,993	0,004003	-2,887	5,804	8,255

Source: own study.

According to the table 1, for a call option, if share prices increase, Delta also grows, Gamma decreases, Theta fluctuates, and Vega and Rho remain unstable. Taking into consideration the fact that share prices fluctuate continuously, one can conclude that Greek letters also change endlessly. As far as Vega and Rho are concerned, these parameters remain unchanged because their values depend on volatility and risk free interest rate. In the example 1 it was assumed that the parameters mentioned earlier are constant. This is the reason why in that case Vega and Rho did not change.

The next example is conducted at the same assumptions, however for a put option. The table 2 presents the results of calculations.

Example 2

Assumptions:

European Put option

Dividend yield: 0

Maturity: 60 days (0,164 years)

Strike price: 60$

Risk free interest rate: 4,5%

Volatility: 17%

Table 2. Correlation between share prices and Greek measures for a put option.

Share price ($)	Option value	Delta	Gamma	Theta	Vega	Rho
57,6	2,782	-0,675	0,090695	-2,515	5,804	-1,537
60,8	1,111	-0,370	0,090105	-3,774	5,804	-1,537
64,0	0,327	-0,141	0,050704	-2,589	5,804	-1,537
67,2	0,071	-0,037	0,017552	-1,032	5,804	-1,537
70,4	0,011	-0,007	0,004003	-0,265	5,804	-1,537

Source: own study.

As the table 2 indicates, the higher share prices are, the lower Delta and Gamma are. Theta fluctuates too. Vega and Rho are stable. The explanations for stability of Vega and Rho parameters are analogous to the previous example.

In examples one and two, it was assumed in advance that volatility and risk free interest rate are constant. In fact, volatility is not the same for the time of option's life and risk free rate may also change. The next two cases show how Greek letters change when volatility changes.

Example 3

Let's assume that volatility parameter is equal to 7% instead of 17%, as it used to be in the previous examples. The other parameters are the same as in examples 1 and 2.

Assumptions:

European Call option

Dividend yield: 0

Maturity: 60 days (0,164 years)

Strike price: 60$

Risk free interest rate: 4,5%

Volatility = 7%

Table 3. The influence of volatility decrease on Greek measures for a call option.

Share price ($)	Option value	Delta	Gamma	Theta	Vega	Rho
57,6	0,096	0,121	0,123054	-1,302	0,405	9,711
60,8	1,466	0,769	0,176538	-3,593	0,405	9,711
64,0	4,435	0,995	0,008614	-2,693	0,405	9,711
67,2	7,632	1,000	0,000023	-2,622	0,405	9,711
70,4	10,832	1,000	0,000000	-2,622	0,405	9,711

Source: own study.

If one compares the example 3 with the example 1, one can notice that the volatility decrease results in changes of Delta, Gamma, Theta, Vega and Rho, i.e. all Greek measures. For example Vega which remained unchanged in spite of share prices fluctuations, changed when the volatility parameter is lower. In the example 1 it was equal to 5,804 and now its value is at the level of 0,405.

Example 4

The same reasoning as in the example 3 will be conducted for a put option.

Assumptions:

European Put option

Dividend yield: 0

Maturity: 60 days (0,164 years)

Strike price: 60$

Risk free interest rate: 4,5%

Volatility = 7%

Table 4. The influence of volatility decrease on Greek measures for a put option.

Share price ($)	Option value	Delta	Gamma	Theta	Vega	Rho
57,6	2,064	-0,879	0,123054	1,320	0,405	-0,058
60,8	0,235	-0,231	0,176538	-0,971	0,405	-0,058
64,0	0,003	-0,005	0,008614	-0,071	0,405	-0,058
67,2	0,000	-0,000	0,000023	-0,000	0,405	-0,058
70,4	0,000	-0,000	0,000000	-0,000	0,405	-0,058

Source: own study.

In the example 4 in comparison with the example 2, all Greek measures have different values. It justifies the thesis that these parameters change together with fluctuations of volatility. The same tendency is observed both for a call and a put option.

The next step is to check if Greek measures also change when volatility grows. It is examined by calculations conducted in examples 5 and 6.

Example 5

Let's assume that volatility parameter is equal to 27%.

Assumptions:

European Call option

Dividend yield: 0

Maturity: 60 days (0,164 years)

Strike price: 60$

Risk free interest rate: 4,5%

Volatility = 27%

Table 5. The influence of volatility increase on Greek measures for a call option.

Share price ($)	Option value	Delta	Gamma	Theta	Vega	Rho
57,6	1,690	0,400	0,061354	-8,360	8,031	7,095
60,8	3,286	0,596	0,058280	-9,302	8,031	7,095
64,0	5,469	0,761	0,044278	-8,515	8,031	7,095
67,2	8,103	0,876	0,027797	-6,811	8,031	7,095
70,4	11,026	0,943	0,014814	-5,114	8,031	7,095

Source: own study.

The example 5 shows that the volatility increase has also had a great impact on Greek measures. If one compares the above table with tables 1 or 3, the conclusion is that all of them have altered much.

Example 6

The same reasoning as in the example 3 will be conducted for a put option.

Assumptions:

European Put option

Dividend yield: 0

Maturity: 60 days (0,164 years)

Strike price: 60$

Risk free interest rate: 4,5%

Volatility = 27%

Table 6. The influence of volatility increase on Greek measures for a put option.

Share price ($)	Option value	Delta	Gamma	Theta	Vega	Rho
57,6	3,658	-0,600	0,061354	-5,738	8,031	-2,674
60,8	2,054	-0,404	0,058280	-6,680	8,031	-2,674
64,0	1,037	-0,239	0,044278	-5,893	8,031	-2,674
67,2	0,472	-0,124	0,027797	-4,189	8,031	-2,674
70,4	0,194	-0,057	0,014814	-2,492	8,031	-2,674

Source: own study.

The table 6 suggests the same reasoning as in the previous example. Thus, it does not depend on the option type that Greek parameters change when volatility grows.

The next step will be calculating the theoretical value for a European call and put option, as well as Greek measures for the risk free interest rate lower and higher than 4,5%, assumed in earlier examinations.

Example 7

Let's suppose that the risk free interest rate is equal to 6,5% and check the value of a put and call option and Greek letters.

Assumptions:

European Call option

Dividend yield: 0

Maturity: 60 days (0,164 years)

Strike price: 60$

Risk free interest rate: 6,5%

Volatility = 17%

Table 7. The influence of risk free interest rate increase on Greek measures for a call option.

Share price ($)	Option value	Delta	Gamma	Theta	Vega	Rho
57,6	0,869	0,341	0,092550	-5,621	5,511	8,315
60,8	2,454	0,647	0,088777	-7,064	5,511	8,315
64,0	4,915	0,869	0,048256	-6,049	5,511	8,315
67,2	7,879	0,966	0,016144	-4,647	5,511	8,315
70,4	11,026	0,994	0,003560	-3,967	5,511	8,315

Source: own study.

In comparison with the table 1, the table 2 depicts Greek measures of a call option for the same parameters but for a higher risk free rate. Therefore, the conclusion is that Greeks change only slightly when a risk free interest rate increases by two percentage points.

Example 8

Assumptions:

European Put option

Dividend yield: 0

Maturity: 60 days (0,164 years)

Strike price: 60$

Risk free interest rate: 6,5%

Volatility = 17%

Table 8. The influence of risk free interest rate increase on Greek measures for a put option.

Share price ($)	Option value	Delta	Gamma	Theta	Vega	Rho
57,6	2,653	-0,659	0,092550	-1,881	5,511	-1,423
60,8	1,038	-0,353	0,088777	-3,325	5,511	-1,423
64,0	0,298	-0,131	0,048256	-2,310	5,511	-1,423
67,2	0,063	-0,034	0,016144	-0,907	5,511	-1,423
70,4	0,010	-0,006	0,003560	-0,227	5,511	-1,423

Source: own study.

The impact of risk free interest rate increase on Greek measures for a put option is analogous to the call option (compare tables: 2, 7 and 8).

The next assumption is the risk free interest rate of 2,5%. The purpose is to check if a decrease in a risk free interest rate results in Delta, Gamma, Theta, Vega and Rho variations.

Example 9

The risk free interest rate is equal to 2,5%. Values of a put and call option, as well as Greek letters are calculated beneath.

Assumptions:

European Call option

Dividend yield: 0

Maturity: 60 days (0,164 years)

Strike price: 60$

Risk free interest rate: 2,5%

Volatility = 17%

Table 9. The influence of risk free interest rate decrease on Greek measures for a call option.

Share price ($)	Option value	Delta	Gamma	Theta	Vega	Rho
57,6	0,757	0,309	0,088796	-4,677	6,079	8,153
60,8	2,228	0,612	0,091500	-5,752	6,079	8,153
64,0	4,600	0,849	0,053233	-4,378	6,079	8,153
67,2	7,521	0,959	0,018997	-2,645	6,079	8,153
70,4	10,655	0,992	0,004455	-1,781	6,079	8,153

Source: own study.

When risk free interest rate diminishes, Greeks slightly change (compare: table 1, 9 and 2, 10), both for a call and for a put option.

Example 10

The same reasoning as in the example 9 will be conducted for a put option.

Assumptions:

European Put option

Dividend yield: 0

Maturity: 60 days (0,164 years)

Strike price: 60$

Risk free interest rate: 2,5%

Volatility = 17%

Table 10. The influence of risk free interest rate decrease on Greek measures for a put option.

Share price ($)	Option value	Delta	Gamma	Theta	Vega	Rho
57,6	2,914	-0,691	0,088796	-3,202	6,079	-1,647
60,8	1,186	-0,388	0,091500	-4,276	6,079	-1,647
64,0	0,357	-0,151	0,053233	-2,903	6,079	-1,647
67,2	0,079	-0,041	0,018997	-1,170	6,079	-1,647
70,4	0,013	-0,008	0,004455	-0,305	6,079	-1,647

Source: own study.

The last but not least factor to be mentioned is time to maturity. Examples 11 and 12 show Greek measures values for the time to maturity equal to 40 days.

Example 11

Time to maturity is equal to 40 days instead of 60 as it has been so far. Values both of a put and call option and Greek letters are calculated beneath.

Assumptions:

European Call option

Dividend yield: 0

Maturity: 40 days (0,082 years)

Strike price: 60$

Risk free interest rate: 4,5%

Volatility: 17%

Table 11. The influence of time to maturity decrease on Greek measures for a call option.

Share price ($)	Option value	Delta	Gamma	Theta	Vega	Rho
57,6	0.366	0.230	0.108153	-5.752	2.660	4.497
60,8	1.750	0.644	0.125706	-8.362	2.660	4.497
64,0	4.328	0.923	0.046481	-5.160	2.660	4.497
67,2	7.425	0.992	0.006455	-3.030	2.660	4.497
70,4	10.617	1.000	0.000386	-2.658	2.660	4.497

Source: own study.

Example 12

The same operations as in the example 11 will be conducted for a put option.

Assumptions:

European Put option

Dividend yield: 0

Maturity: 40 days (0,082 years)

Strike price: 60$

Risk free interest rate: 4,5%

Volatility: 17%

Table 12. The influence of time to maturity decrease on Greek measures for a put option.

Share price ($)	Option value	Delta	Gamma	Theta	Vega	Rho
57,6	2.550	-0.770	0.108153	-3.120	2.660	-0.416
60,8	0.733	-0.356	0.125706	-5.731	2.660	-0.416
64,0	0.111	-0.077	0.046481	-2.528	2.660	-0.416
67,2	0.008	-0.008	0.006455	-0.398	2.660	-0.416
70,4	0.000	-0.000	0.000386	-0.027	2.660	-0.416

Source: own study.

When one compares the table 11 with the table 1 and the table 12 with the table 2, it can be noticed that the time to maturity change from 60 to 40 days resulted in all Greeks fluctuations.

Final conclusions

The calculations presented in the paper prove that Greek measures are liable to changes, both in the case of a call and a put option. It derives from the fact that the option's price is influenced by such factors as: underlying asset price, time to maturity, risk free interest rate and volatility.

Therefore, options sellers are obliged to follow these fluctuations and modify their hedging portfolios without any delay by buying or selling more options.

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Abstract

Options are financial instruments that can be applied in many situations. Options buyers sell risk which is bought by their sellers who are obliged to reduce it as much as possible. It can be done by using hedging strategies based on Greek letters and options values analysis. The author proves that Greek letters are not constant during the time of option's life. The paper shows to what extent they are influenced by such factors as risk free interest rate, volatility, underlying asset price and time to maturity. The conclusion is that options sellers must play an active role, i.e. follow fluctuations of all these parameters and modify their hedging portfolios regularly.