

# ABOUT THE KNOWLEDGE OF TEACHER STUDENTS IN RESPECT TO PROBLEM ORIENTATION IN MATHEMATICS EDUCATION

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UDK 37-051:<37.091.33:51>=111  
DOI: 10.15291/magistra.1490

Received: 4. 1. 2017.

*Problem orientation as a part of mathematics education is required by mathematicians already for a long time and since the beginning of ProMath meetings the situation in school has been improved a little bit, but it is still practiced not sufficiently. Resistors among other enhancements are special beliefs about mathematics education and insufficient knowledge about problem orientation. This is the reason I offered again a seminar for teacher students about problem orientation in mathematics education in summer 2016. In the following first I will briefly discuss different type of problems and then as the main part report and discuss results of a written survey on knowledge to problem orientation at the beginning of the seminar as well as personal comments the students have made during the seminar sessions and in protocols on the seminar sessions.*

KEY WORDS: *problem solving, problem orientation, preservice teachers, teacher knowledge*

For about two years I have noticed that most of the examples used for experimental research of problem solving are very limited tasks. I would say tasks which are very similar to the traditional word problems.

But for more than twenty-five years in the didactic community of problem solving in mathematics education various types of tasks have been discussed especially with the intention of opening problem solving activities for different goals (see e.g. Nohda 1991, Silver 1995 and 1997, Graumann & Pehkonen 2007, Singer et al. 2015 and Felmer et al 2016). And also in this conference in Zadar happily different types have been discussed.

Contrariwise in the German “Bildungsstandards” (standards for mathematics

education) we only find the term “Problemlösen” and internationally mostly one uses the term “problem solving” though especially in open problems the solving of a problem is not the main focus. To illustrate this wide field of working with problems in mathematics education I prefer the term “problem orientation”.

This conspicuity gave me the impetus to a questionnaire for teacher students in a didactical seminar as well as the presentation here. In the following I will recall several types of tasks concerning problem orientation in mathematics education,

- name the background of the questionnaire and quote the questions,
- discuss responses of the students and
- describe general remarks made by the students during the seminar.

### TYPES OF PROBLEMS

A problem may initially be divided into three aspects: the *problem statement*, the *handling of the problem* in respect to a solution and the *final situation* with retrospective.

In Getzels & Csikszentmihlyi, M. (1975, p. 102) where these three aspects are differentiated whether they are known or not (+ or -) by the teacher and the student we find a list of different types of tasks. Owing to combinatorics we can build 64 types (26), but not all have a realistic meaning. But this shows big amount of possibilities to differentiate. I will mention only some of them.

	Problem statement		Method for solving the problem		Solution	
	Teacher	Student	Teacher	Student	Teacher	Student
1 (Routine task.)	+	+	+	+	+	-
2 (class. problem)	+	+	+	-	+	-
3 (open problem)	+	-	+	-	+	-
3a (open problem)	-	-	+	-	-	-
4 (investigation)	+	-	+	-	-	-
...	...	...	...	...	...	...
(totally open situation)	-	-	-	-	-	-

In line 3a (open problem) also the teacher does not know the exact problem statement or/and the precise solution. The last case with “ - “ in all six columns could be seen as “totally open situation” as we e.g. know it from some starting situations in a project week.

Let me now recall shortly special types of problems which became common in recent years in the didactical literature. As already mentioned above one speaks of **open problems** if the problem statement or/and the final situation is not clearly defined. Depending on whether the problem statement or the final situation or both has ways of interpreting open we have the subcategories *open-ended problems*, *open approach problems* and *both-sided open problems*.

Special open problems are the so-called “**Fermi problems**” at which no clear answer is expected. One can only give approximate answers. The purpose of such problems is to find possibilities of estimation and appropriate further calculations of a given object or situation, e.g. when the number of atoms in the universe or the number of piano tuners in Chicago in the 1920s (such as the nuclear physicist Fermi asked his students) is to estimate.

Another type of open problems is called **investigation**. For this the students first have to look out for data or relationships or make experiences with special cases etc. and after that clear up their specific task.

I would not leave unmentioned the so-called **problem fields** because I myself prefer these types. A problem field contains some closely related problems mostly generated by one prime problem. By working with such a problem field students can find out connections or similarities/analogies between different mathematical objects or between mathematical objects and real situations. Sometimes one can even find a little theorem or general structure. The students are also asked to find related problems or to variate the given problem.

But not only in a problem field students are asked to find problems by themselves. In the didactical discussion referring to problem solving this is a demand too. The term **problem posing** is usual in this connection.

I think without describing more types – like e.g. done in the discussion about word problems in the 1980<sup>th</sup> or in the presentation of Cadez & Kolar here in Zadar – the named terms give an idea about the various field of different types of problems.

In the following I will report on a questionnaire with which I wanted to explore the knowledge and beliefs of preservice teacher students. The questionnaire contained the question about different types of problems but was designed for a broader profile.

## BACKGROUND OF THE QUESTIONNAIRE

In the last summer semester I carried out a seminar for 15 preservice teacher students on the topic of problem orientation in mathematics education with special respect to geometry teaching in secondary school (grades 5 to 10). The seminar lasted 13 times per 1½ hour. During the seminar several theoretical papers have been mixed up with experiences by working on problems – especially problem fields concerning geometry teaching.

In the first session – before the students have been influenced of working with problems or discussing about problem orientation in the seminar – I gave my questionnaire to them. The students have visited already two to three semesters of mathematical and didactical seminars/lectures. So they have had some previous knowledge but not from a seminar concerning problem solving.

The questions of my questionnaire are the following:

- a) What do you associate with problem orientation?  
Describe shortly your belief/idea and/or notice keywords or examples!
- b) Do you have so far experiences (in school or at university) with problem orientation in mathematics education? Please give a short description!
- c) Give reasons for the role of problem orientation (besides traditional methods) in mathematics education.
- d) Do you know different types of tasks that are suitable for problem orientation in mathematics education? If yes, which ones?
- e) Do you know reasons why it is so difficult to carry out problem orientation? Do you know drawbacks of teaching problem orientation?
- f) Notice other remarks which come in your mind concerning problem orientation in mathematics education!

## RESPONSES OF THE STUDENTS TO THE QUESTIONNAIRE

I got back the paper with the answers from all 15 participants but not all of them gave respectively meaningful answers to each of the six questions. There have been a lot of different responses which I tried to bundle as far as possible. In the following I will present – according to the questions and structured by me – the answers and comments of the students.

Question a): All students gave comments, some of them more than two and some of them gave only a hint with loose reference to problem orientation. But all answers showed that problem orientation was a term they at least had heard of. The following keywords (bundled by me) may illustrate that.

- Deal with problems, research-based learning, self-sufficient learning, active learning, learning by doing (especially with open tasks).
- Processes to find solutions, heuristics, Polya, ways for solution to orientate on, develop strategies to solve real problems, finding approaches for a solution, problems tackle cleverly.
- Tasks which are more difficult and on which you have to contemplate, series of tasks you have to deal with in one lesson, area of themes that bring understanding problems.
- Support students optimal while working with problems that enter to a theme.
- No dull memorizing, cognitive mental impetus, gain mathematical competences on a new way.

Not considering the three last indentation points which do not meet the issue properly one can find out that a larger group of students focuses on the self-activity while a smaller group highlights the heuristic.

Question b): The experiences of the students with problem orientation have been very different. The locations (with the number of students that named the location in brackets) are the following.

- Seen in a school book (1).
- In the practical training in school before start of the study (1).
- In the practical training while studying (4).
- In a seminar concerning themes of didactic of mathematics (7).
- On the side in a seminar concerning themes of mathematics (2).
- No experiences about problem orientation so far (2).

Question c): All students specified at least two keywords which I will describe bundled in the following way.

- Opportunity to deepen, to deal more accurately with, to explore mathematical contents and to see connections with other contents, deepen basic ideas.
- Learn to know different methods to act with problems, know open

ways of solution, learning mathematising of real world problems, relevance for application-specific contexts, later training and lifelong learning.

- Self-activity while dealing with problems, independent search for methods and mistakes, bigger cognitive participation, stimulation of own considerations, no dull learning by heart, stimulation of critical thinking.
- Take away fear in relation to “mathe”, recognize and address own problems with mathematics, psychological background, more motivation (because of meaningfulness), more alternation.

When I try to bundle in respect to the persons first we find only a few students who named keywords concerning to different aspects. Several students rely on their statements to question a), i.e. the training of self-activity or/and critical thinking as well as to reduce the fear of difficult tasks. A larger group of students have their main focus on the application of mathematics.

Question d): Some students gave suitable information but most of them have no correctly knowledge about different types of problems. The mentioned terms (with the named number in brackets) are:

- Closed problems (1)
- Open tasks (3)
- Fermi-tasks (1)
- Tasks for finding reasons/argumentations and finding problems (1)
- Bloom-task [nice tasks with different branches] (2)
- Word problems or tasks for modelling (5)
- No mentioned term or false terms (9)

Looking out for the number of persons who mentioned at least one of the first four named indentation points we find two students who mentioned only the term “open task” and one student who named “Fermi-task” and “bloom-task” and only one student who named three different types namely “closed tasks”, “open tasks” and “tasks for finding reasons/argumentations and finding problems”. Thus summarized one can say that the knowledge about different types of problems substantially is equal to zero among the students I have questioned.

Question e): All students could mention obstacles for teaching problem orientation. The named terms (number in brackets) are:

- Difficulties of problems, no algorithm, no helping frame, no scheme “F” (7)

- Time consuming /missing time (6)
- Need of high competences, required creativity and staying power (3)
- Difficulties of assessment, no uniform results, phases of backup in detail necessary (3)
- No motivation with all students, heterogeneous groups (2)
- Teacher training is not so good focused on problem orientation (2)
- Missing of adequate material (1)
- No money (1)

Question f): Only one half of the students did make further remarks. The named ones (number in brackets) are the following.

- Different remarks deepening reasons for problem orientation like “activity”, “openness”, “relation to reality” (8).
- Better teacher training with more focus on problem orientation (2)
- Problem of theory and school routine (2)

### **SOME ADDITIONAL REMARKS FROM THE STUDENTS DURING THE WHOLE SEMINAR**

While working with two single classical problems (see e.g. Graumann 2010) in the first session after processing the questionnaire some students did have *difficulties by holding on* in finding a solution.

The later on presented solutions concerned trying out, using tables and working with algebra. A question come up whether “*trying out*” is an *accepted mathematical method* at all. We came up that even mathematicians sometime use that method and in respect to our problem all possible cases can be caught if the trying out method is accompanied with suitable reflections. A correct mathematical proof must not be all the time formalistic. In a seminar session a little bit later it came up a question of *looking out for all* triangles with a special condition. For this of course it is necessary to find a suitable *systematic search*. This was a new experience for most of the students. In the discussion we mentioned other examples where a systematic search is necessary. Besides I could show that sometimes the question “find all” can lead to real mathematical thinking too.

Later on one student was *asking for an algorithm* to find all these searched triangles. In the following discussion it came out that many mathematical problems cannot be solved with an algorithm.

## CONCLUSION

The results of the questionnaire and the additional remarks showed us that my preservice teacher students have a basic knowledge about problem orientation in mathematics education but that there are still a lot of gaps. Besides one must consider that these students enrolled for a seminar on problem orientation as desired by their own wish. So I cannot generalize this result to all preservice teacher students in Bielefeld. One must assume that the knowledge about problem orientation in mathematics education in general is rather less.

But in any case one can make the statement that aspects of problem orientation in mathematics education must have a greater extend in teacher training in different manners. This concerns different types of problems as well as general aims of problem orientation and working with problems. Important are experiences of the students with working on problems combined with theoretical reflections.

As stimulation finally I present the titles of theoretical parts of my seminar: “General introduction to problem orientation in mathematics education”, “Discovery learning and education”, “Active-discovering learning in mathematics teaching in respect to teacher education”, “Polya and his school of thinking”, “Heuristics in mathematics education”, “Self-regulated learning – theory and methodical instructions”, “Aims of problem orientation with concretizations in mathematics education”, “Constructivism and mathematics education”, “Logic of failure”.

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