GERMAN TEACHER EDUCATORS' CONCEPTIONS ABOUT TEACHING PROBLEM SOLVING IN MATHEMATICS CLASSROOM – AN OBSTACLE TO A LARGE-SCALE DISSEMINATION?!

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Problem solving in Germany has roots in mathematics and psychology but it found its way to schools and classrooms, especially through German Kultusministerkonferenz, which represents all government departments of education. For the problem solving standard to get implemented in schools, a large scale dissemination through continuous professional development is very much needed, as the current mathematics teachers are not qualified to do so. As a consequence, one organ in Germany focuses on setting up courses for teacher educators who can "multiply" what they have learned and set up their own professional development courses for teachers. However, before attaining to this work, it is crucial to have an understanding what conceptions about teaching problem solving in mathematics classroom mathematics teacher educators hold. In this research report, I focus on mathematics teacher educators' conceptions about problem solving standard and their effects regarding a large-scale dissemination.

Keywords: *mathematics teacher educator, conceptions, problem solving, teaching problem solving, large-scale dissemination.*

INTRODUCTION

Mathematical problems have been central in the mathematics since antiquity, but since the 1980s, mathematics educators have agreed upon the idea of developing problem-solving ability. Since then problem solving has become a focus of mathematics education as a means of teaching curricular material and seeking the goals of education (Stanic & Kilpatrick, 1989). For instance, the (inter-)national educational standards (Kultusministerkonferenz [KMK], National Council of

Teachers of Mathematics [NCTM]) have strongly endorsed the inclusion of problem solving in school mathematics. Problem solving is fundamental not only to *doing* mathematics but also to *teaching* and *learning* mathematics (e.g., NCTM, 1980, 1989, 2000; Schoen, 2003).

However, in order for this new standard to get implemented in mathematics lessons, working with both pre- and inservice teachers is crucial. In the last few years focus is, therefore, on training those who will disseminate change on a larger scale, here mathematics teacher educators. If problem solving is what mathematics is all about, then mathematics teacher educators should be in the business of helping teachers implement this curricular goal. And if they pass on new curricular developments, then their understanding, conceptions and knowledge about problem solving and teaching problem solving, are crucial in spreading ideas about problem solving. In figuring out how this process standard will be implemented in professional development courses and what the future of problem solving in mathematics classroom will be, knowing mathematics teachers' conceptions about problem solving and teaching problem solving is paramount (Pehkonen, 1993). In this paper – similar to the study of Pehkonen (1993) – I focus on German teacher educators' conceptions about problem solving, the teaching of problem solving and the prerequisites they feel teaching problem solving requires. On this basis, I discuss to what extent their conceptions contribute and/or withhold a large scale dissemination of problem solving in mathematics classroom.

PROBLEM SOLVING IN GERMANY AND THE ROLE OF BELIEFS

Problem solving in Germany has roots in mathematics and psychology but it found its way to schools and classrooms, especially through German Kultusministerkonferenz which represents all government departments of education (KMK, 2003, 2005). According to them problem solving encompasses working on given and individually posed problems, choosing and using heuristic strategies (heuristic auxiliary tools, heuristic strategies, heuristic principles), and checking results for plausibility, reflecting on the problem solving process and generating ideas for problem solving. That said, problem solving competence relates to *cognitive* (here heuristic), *motivational* and *volitional* knowledge, skills and actions of an individual required for independent, effective and successful coping with mathematical problems (Besuden, 1985; Bruder, 2002; Heinze, 2007; Kuzle & Bruder, 2016; Kuzle & Gebel, 2016, accepted; Reiss & Törner, 2007; Zimmermann, 1983). Accordingly, students should

a) learn approaches (heuristics) for solving mathematical problems and learn to apply them appropriately in a given situation,
b) develop reflectivity for own actions, and
c) develop willingness to work hard (cf. Bruder, 2002).

In addition, the problem solving standard is neither introduced as an overreaching practice restricted to a specific mathematical topic nor as an isolated topic of the mathematics classroom. Thus, there is a curricular basis for problem solving as an integral part of school mathematics. As the problem solving encompasses so many different abilities, problem solving is not an easy task for many students who have to become good problem solvers as well as for teachers who ought to help their students develop these abilities. The empirical studies, the PISA and TIMSS studies, portray this picture: students are often unable to solve problem tasks and teachers lack knowledge to foster students' problem solving abilities, and at the same time to consolidate their own competences in the area. Moreover, quality analyses in German school system evaluate the compliance of teaching practices with the educational state standards. For example, in autumn of 2014 one such quality analysis was conducted in one urban school in the federal state of North-Rhine-Westphalia, in which different lessons were observed and rated in 36 areas. The results in the area of problem solving portrayed a rather poor image with respect to implementation of the state's standards: problem solving tasks were rarely introduced, when so, they were primarily done by the teacher; routine tasks dominated the lessons, and in only 1/3 of examples problem solving strategies were explicitly applied (Kuzle, 2016; Kuzle & Gebel, 2016, accepted).

In the context of this reform agenda in order for this new standard to be implemented in school, working with both pre- and inservice teachers is crucial. By working with preservice teachers only, we cannot reach a big mass. According to Gal's rule (2013) only 2.5% of teachers are novice teachers coming from the university. Thus, this small group might receive "up to date" quality teaching and learning ideas as advocated by current professional organisations and new curricula. Gal further warned that, when we train as many inservice as preservice teachers, a quota of barely 5% can be achieved. In the last few years focus is, therefore, on training those who will disseminate change on a larger scale, here mathematics teacher educators, through the German Center for Mathematics Teacher Education (DZLM[1]).

[1] DZLM was founded in 2010 as a cooperation of seven German universities.

Whom can we consider to be a "mathematics teacher educator"? In this paper, the phrase will be used as a general term for those teachers who have a role in teacher education outside universities as part of their job, in addition to teaching in schools. In Germany, for instance, mathematics teacher educators have different roles depending on the federal state; these may include mathematics coaches, expert advisors, and regional professional development (PD) organizers. They are central for providing opportunities for teachers' professional development and are responsible for strengthening classroom teachers' understanding of mathematics content, and helping teachers develop more effective mathematics teaching practices, and with that influence also student learning (Kuzle & Biehler, 2015).

Beliefs refer to the one's view about self, about the environment, about the topic, and about mathematics (Schoenfeld, 1985). Beliefs are more cognitive than attitudes and emotions, and are felt less intensively. Conception refers to "a general notion or mental structure encompassing beliefs, meanings, concepts, propositions, rules, mental images, and preferences" (Philipp, 2007, p. 259). Thus, conceptions represent a larger category than just beliefs, somewhat higher-level beliefs. As such they are considered to be deliberate and justified (Philipp, 2007). Teachers' personal beliefs and theories about mathematics, learners and learning, teaching, subjects or curriculum, learning to teach, interpretation of content, and about the self are widely considered to play a significant role in teaching practices (Pajares, 1992; Thompson, 1992; Wilson & Cooney, 2002). Pajares (1992) explained that "the beliefs teachers hold influence their perceptions and judgments, which, in turn, affect their behaviour in the classroom" (p. 307).

Pehkonen (1993) reported on Finnish teacher educators' conceptions about the problem solving, namely importance of problem solving, teaching of problem solving, existing preconditions for teaching problem solving and teacher's prerequisites of problem solving. His study was carried out in connection with a two-part seminar on problem solving (5 days per seminar part). In the seminar the mathematics teacher educators (N = 44) shared their experiences and ideas about problem solving, received inputs on teaching problem solving (prerequisites, implementation, reasons for teaching problem solving, practical exercises), and finally shared their experience on teaching problem solving after having participated in the seminar. The study results which were based on a questionnaire with respect to the above listed four aspects, have shown that despite the seminar, the term problem solving has not been fully understood as well as how problem solving can be learnt and taught. Moreover, the teacher educators conceived problem solving being best done by using good problems. Thus, solving many good problems ensures developing problem solving ability. Nevertheless, teachers' experience (both practical and theoretical), mind-openness and self-confidence were viewed as important prerequisites for implementing problem solving in teaching practices. The author concluded that in Finland there is still a long way to go before teacher educators' perceptions about the teaching of problem solving in mathematics align with those advocated by the mathematics educators. Since then there has been very little research in this area. Elliot et al. (2009) contend that this area has been under-defined and under-studied – we know very little about what CPD leaders need to know and be able to do – but is slowly growing in its importance.

If problem solving is what mathematics is all about, then mathematics teacher educators should be in the business of helping teachers implement this curricular goal. And if they pass on new curricular developments and that on a large-scale, then their understanding, conceptions and knowledge about problem solving are crucial in spreading ideas about problem solving. In figuring out how this process standard will be implemented in professional development and what will be the future of problem solving in mathematics classroom, knowing mathematics teachers' conceptions about problem solving, similar to the study of Pehkonen (1993), is paramount. The aim of this study is threefold:

1) What are German teacher educators' conceptions on the importance of integrating problems solving in school mathematics?
2) What are German teacher educators' conceptions on how problem solving should be taught in school mathematics?
3) What are German teacher educators' conceptions on teachers' prerequisites for integrating problem solving in school mathematics?

METHOD

The study participants were 64 mathematics teacher educators, who agreed to participate in the study. They were all secondary mathematics teachers with more than 5 years of teaching experience. Their experiences as mathematics teacher educators, however, varied: some had over 10 years' experience as a mathematics teacher educator, some close to 5 years, whereas some were new in their role. Their experience with respect to problem solving varied from having already implemented problem solving practices in their teaching and having implemented a professional development on problem solving to having developed materials for one to theoretical experience only. Data was gained through quantitative methods. One questionnaire was administered electronically and returned to the author by all participants within 2 weeks. The questionnaire consisted of 4 item categories, one of them being a free open-ended question and 3 item categories pertaining to the three research questions containing diverse statements with respect to them. Here an adapted version of the questionnaire of Pehkonen (1993) on the study of Finish teacher educators' conceptions about different dimensions of problem solving was used.[2] With respect to the first research question 9 statements were given, with respect to the second research question 13 statements, and with respect to the third research question 12 statements. The respondents indicated their replies on a five Liker scale (1 = I strongly disagree, 5 = I strongly agree) by circling the number on the scale that best reflected their conceptions (see Figure 1).

	I strongly disagree	I somewhat disagree	I neither agree nor disagree	I somewhat agree	I strongly agree	N/A
Problem solving develops mathematical thinking.	☐	☐	☐	☐	☐	☐
Problem solving can be learned only by solving problems.	☐	☐	☐	☐	☐	☐
The teacher must be familiar with problem solving theory.	☐	☐	☐	☐	☐	☐

FIGURE 1. Sample items from the questionnaire.

The reliability of questionnaire was calculated, resulting in the reliability of 0.71 which is an acceptable score. Afterwards descriptive statistics were calculated for all quantitative data from the questionnaire. In addition, the percentage of

[2] In Pehkonen's (1993) study, the first category consisted of 7, the second category of 9, and the fourth category of 13 items. The third category was not used. Items 1-7, 10-11, 23-34 were taken from Pehkonen (1993). Item 12 was adapted, whereas items 8-9 and 15-22 were new.

agreement was calculated. In order to so do, the percentage of the response 4 and 5 on the Likert scale were calculated. Lastly, the consensus level as a percentage was calculated similarly to as defined in Pehkonen[3] (1993):

- *complete consensus*, if at least 95% of the test subjects' views were on the same extreme end of the scale;
- *consensus*, if at least 85% but less than 95% of the test subjects' views were on the same extreme end of the scale;
- *near consensus*, if at least 75% but less than 85% of the test subjects' views were on the same extreme end of the scale;
- *lack of consensus*, if at least 60% but less than 75% of the test subjects' views were on the same extreme end of the scale;
- *no consensus*, if less than 60% of the test subjects' views were on the same extreme end of the scale.

The percentage of consensus was used to describe the consensus level of the test subjects' views about the statements.

RESULTS

I structure the results section in four parts. In the first three parts I focus on the three research questions on the basis of the questionnaire. In the last part I focus on the consensus level of the responses across all items, which is used to summarize the results pertaining to the three research questions.

German teacher educators' conceptions on the importance of integrating problems solving in school mathematics

Nine statements dealt with the first research question, namely why is problem solving in school mathematics important (see Table 1). The consensus level for three statements (1, 2, 4) reached at least 75%. Statement 1 reached a complete consensus (98%), whereas statements 2 and 4 (each 94%) a consensus. It can thus be said that the participants agreed that problem solving develops mathematical thinking and creativity, and that problem solving is of every day value. On all other statements, no consensus was reached. Here the participants' conceptions on items 5, 7, and 8 were interesting. Roughly one third of the participants took a neutral

[3] Other than in Pehkonen (1993), the consensus of agreement less than 75% was broken into two categories.

position with respect to item 5, whereas almost one half agreed to it. On item 7 roughly one-third of participants did not agree with the statement or did not provide an answer, whereas more than one half of them agree with it. Similarly, on item 9, roughly one third of the participants had a neutral position, and more than one half of them agreed with it. The connection between problem solving and using problems to learn and understand mathematical concepts and ideas seems unclear to some of the teacher educators. Item 3 had the smallest agreement percentage of only 20%, and revealed that one half of the participants did not perceive problem solving as a fun activity for pupils. Thus, the teacher educators primarily perceived problem solving as a means to develop cognitive skills such as creativity, and mathematical thinking, and to show application of mathematics. However, the perspective on problem solving as a means for students to learn mathematics, and to grow in their understanding of mathematics was not viewed as important.

Item	N/A	I disagree			I agree				
		1	2	3	4	5	Mean	SD	Agreement (%)
(1)	Problem solving develops mathematical thinking.								
		0	0	1	36	27	4.41	0.52	98
(2)	The application of mathematics in everyday life is problem solving.								
		0	1	3	29	31	4.41	0.65	94
(3)	Pupils think problem solving is fun.								
		3	29	19	12	1	2.67	0.88	20
(4)	Problem solving develops creativity.								
		0	1	3	34	26	4.33	0.64	94
(5)	New things are learned better through problem solving.								
		1	4	20	34	5	3.59	0.78	61
(6)	Problem solving shows the limitedness of algorithms.								
	14	2	6	23	17	2	2.52	1.53	30
(7)	Problem solving emphasizes the process nature of mathematics.								
	10	0	1	12	28	13	3.36	1.59	64
(8)	Problem solving helps pupils understand mathematics.								
		1	7	18	29	9	3.59	0.91	59

(9)	Problem solving in mathematics makes mathematics more interesting to pupils.							
	0	12	27	20	5	3.28	0.86	39

Table 1. Results of the questionnaire regarding the first research question.

German teacher educators' conceptions on how problem solving should be taught

Thirteen statements dealt with the second research question, namely how problem solving should be taught in school mathematics (see Table 2). No statement reached a complete consensus. The consensus level for six statements (12, 13, 14, 16, 18, 22) reached at least 75%. The responses to statement 14 showed consensus, the percentage of consensus being 94%. Thus, the respondents attributed the context of the problem being close to students' environment to be important. A high percentage of consensus (92%) was reached for the statement 13. It seems that teaching heuristics has an appeal to a majority of mathematics teacher educators, as it may be viewed as being easily taught. The responses to statements 16 and 22, that reached a consensus (91% and 89% respectively), indicate that the respondents view students having to struggle and see many different solutions to a given mathematical problem as a part of learning problem solving. In addition, the respondents viewed teacher role as being important; they should teach problem solving creatively and in such way to motivate their students (consensus of 88% and 86% respectively). Near consensus on statements 11 and 15, the percentage of near consensus being 83% and 81% respectively, is worrying. This implies, that the mathematics teacher educators attribute solving many problems as the only important practice when teaching problem solving. However, research (e.g., Kilpatrick, 1985) commented that solving lots of problems is necessary, but probably not sufficient, for becoming a better problem solver. This is also in contradiction to their responses to statement 13. It may be that the word "only" was overlooked. Moreover, they responses show that they value getting the answer same to knowing how to reach the problem solution. Both statements (19 und 20) with respect to the role of technology when teaching problem solving reached lack of consensus (69%). Here, 22% or rather 28% of respondents chose the neutral alternative. Though the use of technology such as dynamic geometry software supports a plethora of mathematical behaviors and for that reason has become almost synonymous with mathematical problem solving (e.g., Wilson, Fernandez, & Hadaway, 1993), it may be that the mathematics teacher educators had limited knowledge of technology use in mathematics classroom, and for that reason lack of consensus/neutral responses were given. The statements 10, 17 and 21 had the smallest consensus percentage, respectively, was 48%, 45% and 31%. It seems that, the role of the teacher when teaching problem solving is not clear to the majority of mathematics teacher educators. Statement 21 (pupils should also practice problem solutions demonstrated by an expert) was the only statement where the proportion of disagreement (38% of the responses) exceeded that of other alternatives. Again, this reveals that modeling problem solving for students (so called imitation) is not familiar to the mathematics teacher educators as a practice for problem solving instruction (Kilpatrick, 1985) and that they have a clear view that students imitating teachers' problem solving behaviors is not an important practice in teaching problem solving.

Thus, the teacher educators primarily perceived problem solving should be taught creatively and by giving students lots of problems from their environment, letting them struggle and by teaching heuristic strategies. The role of the teacher practices (e.g., imitation) and means to do (e.g., using technology) was not part of majority of mathematics teacher educators' conceptions on teaching problem solving.

Item	N/A	I disagree			I agree				
		1	2	3	4	5	Mean	SD	Agreement (%)
(10)	Problem solving should be taught by focusing on problems in the teaching of mathematics.								
	3	2	15	13	27	4	3.11	1.20	48
(11)	Problem solving can be learned only by solving problems.								
	2	0	4	5	29	24	4.05	1.10	83
(12)	Problem solving should be taught creatively.								
	3	0	0	6	34	21	4.05	1.08	86
(13)	The teaching of problem solving should also include heuristic strategies.								
	1	0	0	4	30	29	4.33	0.81	92
(14)	Problem familiar to the pupils from their environment should be dealt with during problem solving.								
		0	3	1	25	35	4.44	0.75	94
(15)	Knowing how to solve a mathematics problem is as important as getting the correct answer.								
		0	9	3	22	30	4.14	1.029	81

(16)	Allowing a pupil to struggle with a mathematical problem, even feel a little tension, can be necessary for learning to occur.								
	5	0	0	1	29	29	4.13	1.31	91
(17)	It is the teacher's responsibility to provide the pupils with clear and concise solution methods for mathematical problems.								
	1	4	12	18	24	5	3.17	1.11	45
(18)	A vital task for the teacher is motivating pupils to resolve their own mathematical problems.								
		0	1	7	29	27	4.28	0.72	88
(19)	Technology should be used to bring new ideas for solving a problem.								
	1	0	1	18	35	9	3.77	0.82	69
(20)	Using technology increases pupils' interest when problem solving.								
	2	0	4	14	30	14	3.75	1.06	69
(21)	Pupils should also practice problem solutions demonstrated by an expert.								
	2	4	20	18	17	3	2.83	1.13	31
(22)	Pupils should see different solutions to a problem to help them understand better included mathematical concepts.								
	1	0	2	4	33	24	4.19	0.88	89

Table 2. Results of the questionnaire regarding the second research question.

German teacher educators' conceptions on teachers' prerequisites for integrating problem solving in school mathematics

Twelve statements dealt with the third research question, namely what prerequisites does the teaching of problem solving set for teachers (see Table 3). The consensus level for 10 of these statements (23-26, 29-34) reached at least 75%. There was complete consensus among the respondents regarding statements 24, 25, 30 and 31, the respective consensus being 98%, 98%, 97%, and 98%. Thus, the mathematics teacher educators agreed unanimously that the following prerequisites are needed for integrating problem solving in school mathematics: the teacher must be familiar with problem solving on the level of practical implementation; the teacher must have the courage to face problem-solving situations; the teacher should have enough time to prepare the teaching of problem solving; and the teacher should have the ability to make pupils participate in teaching. There was consensus among the respondents regarding statements 23, 26, 29, 32, and

34, the respective consensus being 90%, 88%, 92%, 86% and 89%. Thus, the mathematics teacher educators agreed that the following prerequisites are needed for integrating problem solving in school mathematics: the teacher must be familiar with problem solving theory; the teacher must be enthusiastic about teaching problem solving; the teacher must be patient enough not to give pupils too much help with their problem solving; the teacher must be given sufficient training in the teaching problem solving; and the teacher must think that problem solving is important. Near consensus was reached regarding statement 33 with 80% of agreement. Thus, the respondents almost agreed that teacher's personal experience of successful problem solving is a prerequisite for implementing problem solving practices in school mathematics. Lack of consensus was reached regarding statements 27 (deviation from textbook) and 28 (teacher's open mindedness) with 64% of agreement for each statement.

Thus, the mathematics teacher educators viewed particular cognitive and affective characteristics as a requirement for integrating problem solving in school mathematics. With respect to the former they viewed teachers' experience as a problem solver and having received training on problem solving containing both theoretical and practical inputs as a necessary objective prerequisite. Time to prepare for problem solving lessons was another factor that was unanimously agreed upon. With respect to the latter teacher's personality characteristics such as courage to face something unexpected, enthusiasm about problem solving and patience were viewed as a necessary subjective prerequisite.

Item	N/A	I disagree			I agree				
		1	**2**	**3**	**4**	**5**	**Mean**	**SD**	**Agreement (%)**
(23)	The teacher must be familiar with problem solving theory.								
	2	0	2	2	29	29	4.23	1.03	90
(24)	The teacher must be familiar with problem solving on the level of practical implementation.								
		0	0	1	25	38	4.58	0.52	98
(25)	To be able to teach problem solving, the teacher must have the courage to face problem-solving situations.								
		0	0	1	20	43	4.66	0.51	98
(26)	For problem solving to be taught successfully, teachers must be enthusiastic about teaching it.								
		0	1	7	26	30	4.33	0.73	88

(27)	A teacher with the ability to deviate from the order set out in textbooks will teach problem solving successfully.								
	4	0	2	17	25	16	3.67	1.24	64
(28)	The teaching of problem solving requires that the teacher has an open mind.								
	4	0	3	16	24	17	3.67	1.26	64
(29)	The teacher must be patient enough not to give pupils too much help with their problem solving.								
		0	0	5	33	26	4.33	0.61	92
(30)	The teacher should have enough time to prepare the teaching of problem solving.								
		0	0	2	25	37	4.55	0.56	97
(31)	The teacher should have the ability to make pupils participate in teaching.								
		0	0	1	22	41	4.63	0.52	98
(32)	Teachers must be given sufficient training in the teaching problem solving.								
		0	1	7	33	23	4.22	0.70	86
(33)	To be able to teach problem solving, the teacher must have personal experience of successful problem solving.								
		0	3	10	32	19	4.05	0.80	80
(34)	To teach problem solving successfully, the teacher must think that problem solving is important.								
		0	1	6	31	26	4.28	0.70	89

Table 3. Results of the questionnaire regarding the third research question.

Summary of the results: Consensus level of the responses

Table 4 shows the items for which at least near consensus level of agreement (higher than 75%) was reached. The reported results demonstrated a positive picture of German mathematics teacher educators' conceptions about problem solving. Four items received the highest percentage of the agreement (98%). One item focused on the importance of problem solving as a way to support development of mathematical thinking. Other three items came from the third category that emphasized teachers' familiarity with problem solving, courage to face problem solving situations and ability to make pupils participate in teaching as needed prerequisites for implementing problem solving in school mathematics. This was then followed by allotted time to prepare for teaching problem solving (97% of agreement). On little more than 1/3 of the statements lack of consensus or no consensus was reached.

		Statement	Percentage of consensus (4 or 5)
Statements about which the test subjects' conceptions showed complete consensus (95-100%)			
I	(1)	Problem solving develops mathematical thinking.	98
III	(24)	The teacher must be familiar with problem solving on the level of practical implementation.	98
III	(25)	To be able to teach problem solving, the teacher must have the courage to face problem-solving situations.	98
III	(31)	The teacher should have the ability to make pupils participate in teaching.	98
III	(30)	The teacher should have enough time to prepare the teaching of problem solving.	97
Statements about which the test subjects' conceptions showed consensus (85-94%)			
I	(2)	The application of mathematics in everyday life is problem solving.	94
I	(4)	Problem solving develops creativity.	94
II	(14)	Problem familiar to the pupils from their environment should be dealt with during problem solving.	94
II	(13)	The teaching of problem solving should also include heuristic strategies.	92
III	(29)	The teacher must be patient enough not to give pupils too much help with their problem solving.	92
II	(16)	Allowing a pupil to struggle with a mathematical problem, even feel a little tension, can be necessary for learning to occur.	91
III	(23)	The teacher must be familiar with problem solving theory.	90
II	(22)	Pupils should see different solutions to a problem to help them understand better included mathematical concepts.	89
III	(34)	To teach problem solving successfully, the teacher must think that problem solving is important.	89
II	(18)	A vital task for the teacher is motivating pupils to resolve their own mathematical problems.	88
III	(26)	For problem solving to be taught successfully, teachers must be enthusiastic about teaching it.	88
II	(12)	Problem solving should be taught creatively.	86
III	(32)	Teachers must be given sufficient training in the teaching problem solving.	86
Statements about which the test subjects' conceptions were near consensus (75-84%)			
II	(11)	Problem solving can be learned only by solving problems.	83
II	(15)	Knowing how to solve a mathematics problem is as important as getting the correct answer.	81

III	(33)	To be able to teach problem solving, the teacher must have personal experience of successful problem solving.	80
Statements about which the test subjects' conceptions showed lack of consensus (60-74%)			
II	(19)	Technology should be used to bring new ideas for solving a problem.	69
II	(20)	Using technology increases pupils' interest when problem solving.	69
I	(7)	Problem solving emphasizes the process nature of mathematics.	64
III	(27)	A teacher with the ability to deviate from the order set out in textbooks will teach problem solving successfully.	64
III	(28)	The teaching of problem solving requires that the teacher has an open mind.	64
I	(5)	New things are learned better through problem solving.	61
Statements about which the test subjects' conceptions showed no consensus (0-59%)			
I	(8)	Problem solving helps pupils understand mathematics.	59
II	(10)	Problem solving should be taught by focusing on problems in the teaching of mathematics.	48
II	(17)	It is the teacher's responsibility to provide the pupils with clear and concise solution methods for mathematical problems.	45
I	(9)	Problem solving in mathematics makes mathematics more interesting to pupils.	39
II	(21)	Pupils should also practice problem solutions demonstrated by an expert.	31
I	(6)	Problem solving shows the limitedness of algorithms.	30
I	(3)	Pupils think problem solving is fun.	20

Table 4. Distribution of statements regarding the consensus level of at least 75%.

Table 5 offers a bidirectional view regarding the consensus level and the research questions. The highest number of complete consensus was found for the statements pertaining to the third research question, whereas none with respect to the second research question. On the other hand, the highest number of consensus was found for the statements pertaining to the second research question. Most of the 7 statements for which no consensus was reached (less than 60%) pertained to research questions 1 and 2. This trend does not change, when we look at the statements for which the consensus level did not reach more than 75% of agreement. Such distribution is understandable because what should be taught is outlined in the curriculum, which do not give advice how to implement problem solving practices. Thus, the teachers can assess the situation and are able to say what prerequisites are needed to implement problem solving, but lack the knowledge of effective instructional practices to achieve this goal or are not sure what these are. Moreover, the teacher educators experience' in teaching problem solving varied which can elaborate for very little consensus with respect to the importance of integrating problem solving in school mathematics.

	Research question 1: importance of problem solving	**Research question 2: teaching of problem solving**	**Research question 3: prerequisites needed for teaching of problem solving**
Complete consensus	1	–	24, 25, 30, 31
Consensus	2, 4	12, 13, 14, 16, 18, 22	23, 26, 29, 32, 34
Near consensus	–	11, 15	33
Lack of consensus	5, 7	19, 20	27, 28
No consensus	3, 6, 8, 9	10, 17, 21	–

TABLE 5. Overview of all statements with respect to the consensus level.

DISCUSSION

There is consensus among mathematics educators that problem solving is fundamental not only to *doing* mathematics but also to *teaching* and *learning* mathematics (e.g., KMK, 2003, 2005; NCTM, 2000; Schoen, 2003; Wilson et al., 1993). While many countries redesigned their curriculum to fit the PISA and NCTM recommendations, the integration of problem solving into mathematics classes is only present to a limited extent, if at all (Reiss & Törner, 2007). For the problem solving standard to get implemented in schools, a large scale dissemination through CPD is very much needed, as the current mathematics teachers are not qualified to do so. In Germany, this task is done by mathematics teacher educators, which themselves have rather limited experience in problem solving, both as problem solvers and as teachers. Moreover, understanding their conceptions about problem solving and the role of the teacher in a problem solving classroom, and how problem solving should be taught can shed light how the problem solving standard will be implemented in professional development courses.

Although the results showed that the mathematics teacher educators had positive conceptions about problem solving, several items revealed that problem solving and what it means to "teach" problem solving has not fully be understood by them. Some associated teaching problem solving with teaching heuristic strategies and giving students lots of isolated tasks to solve. This is a rather limited picture of what and how problem solving should be taught. Such conception may be related to their calculation and instrumentalist view of teaching mathematics. Moreover, majority viewed problem solving as a skill or goal or in another words as a high-level skill that students practice only after they have mastered lower-level skills such as solving routine exercises (Stanic & Kilpatrick, 1989). Only some viewed problem solving as means of teaching and learning mathematics. Their conception of problem solving allowing students to simultaneously grow in their mathematical understanding and to become better problem solvers was rather limited. Thus, problem solving was more conceptualized as an add-on to mathematics lessons instead of its integrative part. Thus, developing of connected knowledge of problem solving may help develop their understanding of what it means to problem solve and to teach problem solving.

The situation could be improved by providing CPD courses for mathematics teacher educators focusing on problem solving before taking this role open themselves. A learning sequence can be structured using a three-phase model – *learning-off-job*, *learning-by-job*, and *learning-on-job* – as suggested by Müller (2003) and Wahl et al. (1991). This model is embedded in a so-called "sandwich principle" with three different types of learning phase.

Learning-off-job (phase I) consists of active learning of important and fundamental knowledge instances for designing PD courses.

Learning-by-job (phase II) consists of planning and implementing a PD, followed by reflection. This is done on the basis of knowledge acquired in the first phase and through an expert support-system (e.g., supervision, counseling).

Learning-on-job (phase III) consists of further autonomous education. This includes activities such as clarification of own mentor role, self-expectations, and needs for further development.

With respect to *learning-off-job* developing integrated knowledge of problem solving (Chapman, 2015) (both vertically and horizontally) should be stressed rather than learning different aspects of problem solving. With respect to the former investigating the wide range of teaching practices of those who have successfully implemented problem solving into their mathematics lessons may provide an overview of the possibilities for own possible teaching practices, get better ideas what teaching of problem solving looks like and through it develop a better understanding of problem solving and what teaching problem solving is. With respect to the latter a role change (student-teacher-teacher educator) would allow them to slowly grow in their competence development, gain teaching experience, and gather advice from other colleagues. Thus, they would build integratively on their problem solving competences through alternation of theory, teaching, and training practices. *Learning-by-job* could be organized through collaborative lesson studies with other mathematics teacher educators, which has been proved to support teachers' competence development effectively.

The conceptions of German mathematics teacher educators are not completely aligned with those of mathematics educators. There are still some limited conceptions of teaching problem solving. Nevertheless, carefully conceptualized work with mathematics teacher educators may close this gap and allow for proper large-scale dissemination in school mathematics.

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