ON THE CONSTRUCTION OF A PROBLEM SEQUENCE

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This article is a complementary article to three earlier published articles (Burman & Wallin, 2014; Burman, 2014; Burman, 2016) about the use of problem sequences in mathematics instruction in the grades seven to nine in Finland. The pupils work with problems using the same strategy in different contexts, or with only one problem where they gradually proceed towards the solution. In both cases, the problems are solved in steps under the guidance of the teacher. The article focuses on the considerations designer of a problem sequence has, as the design of the sequence is accomplished. In general, the pupils are supposed to be provided with the possibility to think creatively, to work mostly in groups but also individually, and to be inspired by tasks related to real-world situations.

Key words: *problem solving, problem sequences, design principles*

ABOUT PROBLEM SOLVING IN FINLAND

For over 30 years, problem solving has been a general overall goal in the Finnish curricula, and the emphasis on mathematical problem solving and thinking skills is repeatedly present in the current curriculum (Laine & Pehkonen, 2016; National Board of Education 1985, 2014). Although ideas about how to teach using problem solving have been widely spread, an implementation of problems that are open and require creativity is not very common (Laine & Pehkonen, 2016). In addition, pupils often find that they receive problem tasks only when the ordinary tasks have been successfully solved. They also learn that problem tasks may be found at the end of (a chapter in) the textbook, or that problem tasks are handed over by the teacher when there is time left at the end of a lesson.

A PROJECT USING PROBLEM SEQUENCES

The work behind this article started in 2013 as a project with the aim to develop the pupils' skills in mathematical thinking and reasoning as well as in problem solving and readiness to work with modelling and applications. Briefly, the aim was to develop the pupils' skills in thinking creatively. In order to achieve this aim, it seemed necessary to make some kind of supplement to the courses (or textbooks), as well as to inspire teachers to use problems in a more systematic way. The result of the design process was called a problem sequence, which was designed to be solved in steps. A problem sequence is a series of often four or five tasks, which together practise the use of a specific problem-solving method in different contexts, or solving one (more extensive) problem stepwise. Later, in 2014, the project was accepted as a part of the national developmental programme LUMA Finland (Pollari, 2014), and in 2016, the project was granted an additional period of three years, 2017-2019, in the LUMA Finland programme.

The present article is the fourth one in a series of four articles. The first article presented the emergence of the problem sequences, partially based on pilot-tests (Burman & Wallin, 2014). The theoretical path to the problem sequences is presented in the next paragraph. One of the basic ideas in the actual project was to design and use tasks not only for exceptionally talented pupils, but for all pupils. Thus, the second article focused on working with problem sequences in heterogeneous classes (Burman, 2014), where e.g. the work with problems in small groups was considered important. However, the original problem sequences were also evaluated (Burman & Wallin, 2014), and one of the crucial aspects as concerns understanding and developing higher-order thinking was the building of new cognitive schemes. Thus, some of the problem sequences are designed to use a new cognitive scheme and to organize the given information in a way that is appropriate for acquiring a new cognitive scheme. Hence, the third article addressed the use of Venn diagrams, which the pupils were invited to apply as they were working (mostly in groups, but also individually) with tasks related to real-world situations (Burman, 2016). Finally, the current article will examine the considerations a designer of a problem sequence has, as well as the matters the users of problem sequences will have to consider, as they are either designing or using problem sequences.

THE THEORETICAL PATH TO THE PROBLEM SEQUENCES

In an article addressing possible methods for educational change, Pehkonen (1997) uses the concept problem field as a set of connected problems, which form a sequence of problems. Furthermore, he notes that in a problem field, the difficulty of the problems may range from very simple ones that can be solved by the whole class, to more difficult problems that only the more advanced students might be able to solve (Pehkonen, 1997). The concept problem sequence is related to his definition of problem fields, but for the present purpose, it is preferable to use the concept problem sequence. The necessary focus is on problems formed by a sequence of subproblems, or on a problem solved step by step, rather than on separate problems taken from a certain field of problems or problems as separate extensions from a certain problem (Burman & Wallin, 2014).

Based on the theoretical background, it seemed relevant to improve the mathematical thinking and the quality of reflection and thought by offering the pupils challenges and possibilities to work with problems in steps, completed with discussions in the class and with teacher involvement between the steps. Additionally, the use of real-world problems and elements from modeling in mathematics were considered appropriate. Finally, the creation of a supportive environment, where working in groups and good teamwork were important, was highly desirable (Burman & Wallin, 2014).

The problem sequences are also to a great extent characterized by the teacher using problem sequences. As a problem sequence is used, the teacher's role is at least twofold. The problem solving is designed to take place in several steps, and between the steps, the teacher and the pupils can discuss results and possible extensions. The teacher can also give new information and a new orientation for the work (Burman & Wallin, 2014). If the problem sequence is used to train a certain problem-solving method, the context for the following steps and the level of difficulty must be decided. Consequently, the teacher's role is crucial for the success of a problem sequence and the quality of the work in the class. Thus, the attribute "teacher-guided" in front of the concept "problem sequence" is highly motivated and more evidence for this opinion can be found further on in this article.

ABOUT THE DESIGN OF PROBLEM-SOLVING TASKS

Before proceeding to the main aim of this article, some general aspects of designing problem-solving tasks deserve some attention. The design of tasks in-

tended for problem solving in classes shows similarities with design-based research, as described by the Design-Based Research Collective (The Design-Based Research Collective, 2003). The Design-Based Research Collective proposes five characteristics for good design-based research, and one of these characteristics is that development and research take place through continuous cycles of design, enactment, analysis, and redesign (The Design-Based Research Collective, 2003; Cobb et al., 2001). Similar cycles could also serve as a description of the method used for the design of the problem-solving tasks, which this article will examine. The design of a problem sequence is based on ideas and experience from numerous sources; the sequences are tested, the results are analysed, and necessary steps of redesign are performed. Analyses of results provide the designer with insight in possible aspects of success and may also lead to an improvement of future designs, but still, claiming success for educational interventions in a design, or educational interventions in general, is very problematic (cf. The Design-Based Research Collective, 2003).

As ten Berge, Ramaekers, and Pilot (2004) wished to support teachers to choose and design cases and tasks that are effective for the students' learning, they established guidelines with four design principles: authenticity, complexity, structure, and challenge. These four principles can serve as the main points of the general aspects connected to the design of problem-solving tasks. The four principles can also serve as the frame for this short survey of general aspects.

Many authors in the literature have underlined the importance of using authentic (or realistic) cases and tasks, often mentioned in order to provoke higher-order thinking (Gravemeijer, 2008; ten Berge et al., 2004). Awareness of the features that are crucial to authenticity seems important and the design of realistic tasks that can provoke higher-order thinking is considered difficult for teachers (ten Berge et al., 2004). By working through problematic situations, pupils learn how to construct and adjust strategies to solve (new kinds of) problems, which is highly desirable if the goal is to receive a residue of their work in problem solving (Hiebert et al., 1996). Ten Berge et al. (2004) describe the importance of involving the students in the problems and engaging them in meaningful thinking and interaction. Authentic tasks are also considered challenging to most students as the underlying problems often are open-ended, realistic, demanding, and meaningful (ten Berge et al., 2004).

The second principle, complexity, can be addressed from at least two points of view. On the one hand, ten Berge et al. (2004) emphasize that reducing the com-

plexity of real life situations and problems to a level that students can handle is a key issue of education, and furthermore, matching the complexity in authentic tasks to the level students can cope with seems important. On the other hand, Palm and Burman (2004) have analysed tasks-reality concordance in Finnish and Swedish national assessments and focussed on the quality of real life simulations. They found a large difference in the simulation of different aspects in real-life tasks regarding the proportion of tasks simulating the aspects with a reasonable fidelity.

The structure principle seems to have a considerable significance for constructing problem-solving tasks in the form of teacher-guided problem sequences. Gravemeijer (2008) states that students have to reinvent mathematics and that they have to be supported. Thus, he speaks about a guided reinvention. Clearly, the role of the teacher is widely underlined (ten Berge et al., 2004; Hiebert et al., 1996). The teacher will need to take an active role in selecting and presenting tasks (Hiebert et al., 1996), supply the pupils with part-task practice and supportive information just-in-time (ten Berge et al., 2004) and bear the responsibility for developing a social community that shares knowledge in searching for solutions (Hiebert et al., 1996). In spite of many specifications, the role of the teacher is suggested to be summarized in providing information and setting tasks (Hiebert et al., 1996).

The challenge principle is crucial, as challenge is one of the aspects that can promote intrinsic motivation to get started with a problem and to proceed until the problem is solved; thus, to make progress in learning problem solving. In this respect, dilemma-focussed cases are considered to bring participants into an inquiry mode (ten Berge et al., 2004). Moreover, the structure of the task is also important for keeping up the motivation. Preferably, students could work on carefully designed sequences of case studies. The cases may develop from simple to complex, with diminishing guidance by the teacher (ten Berge et al., 2004). Thus, the role of the teacher is again stressed with respect to the challenge. After having discussed solutions offered by the students, the teacher may present the next step, and if the students have found no correct solutions, the discussion still has a function of providing the basis for a sensible next step (Gravemeijer, 2008). Finally, the sequence of case studies can be followed by some kind of summative assessment, and in that case, the challenge of a coming test continues to maintain the motivation of the students.

In Table 1, the main aspects of interest are listed in connection to the four design principles.

Design principles	Main aspects of interest
Authenticity	Authentic (or realistic) cases and tasks Meaningful thinking and interaction
Complexity	Reduced complexity of real life situations and problems Quality of real life simulations
Structure	Guided reinvention of mathematics The teacher's role: providing information and setting tasks
Challenge	Carefully designed sequences of case studies Motivation and concluding assessment

TABLE 1: Main aspects of interest

THE DESIGN OF A TEACHER-GUIDED PROBLEM SEQUENCE

A teacher-guided problem sequence can be divided into three phases: the initial phase, the problem-solving phase, and the final or evaluating phase. In the following, these three phases are generally examined, and then more specifically illustrated in two examples with considerations based on experiences from the author's own class.

The initial phase

The problem sequence may consist of several problems from different contexts to be solved using the same specific strategy (Type 1), or one problem where the pupils gradually proceed towards the solution (Type 2). Thus, one of the first two decisions that must be made concerns the type of problem sequence. Of course, the topic of the sequence should also be decided. Sometimes the strategy or method may be more important than the topic, and in that case, several different contexts can be chosen instead of only one topic.

There are also other aspects to consider. The pupils may lack some necessary prior knowledge, both concerning problem-solving strategies and with respect to background information from the actual contexts. As a consequence, the pupils should be introduced to some mathematics, or to some contexts from real life. Furthermore, the contexts or the problem itself should also be attractive enough to bring the pupils into an "inquiry mode". Finally, it might also be possible to connect the sequence to something actual or something important in the pupils' lives. Hence, designers of problem sequences may be characterized as persons (teachers) who try to figure out how to connect a sequence to actual interesting

real-life situations and cases that the teacher meets or that the pupils frequently talk about.

The problem-solving phase

In the problem-solving phase, decisions about the pupils' activities in the actual problem-solving phase must be made. For a problem of type 1, it is essential to choose suitable contexts, and at the same time ensure the possible relations to real life. Moreover, the number of the steps is also important. For a problem of type 2, it is even more necessary to decide how many steps there should be. To proceed from one step to the next should not be too demanding. Simultaneously, every step needs some kind of challenge and some kind of progression. Finally, it is important to decide if the pupils should work in groups, or individually. Certainly, both ways are possible, but in most of the problem sequences, work in groups has been tested. Obviously, it is easier for the teacher to discuss with a few groups than with three or four times as many individuals. Another reason for using groups is that low achievers in the classes are not left alone when working in groups. Consequently, after each step the designer (teacher) decides what the next step should be, and it is necessary to be very flexible and to take into account the results in the previous step and the feed-back from the pupils. It is not always preferable to proceed in the same way in two different classes.

The final phase

The final phase contains the evaluation of the pupils' problem-solving abilities in relation to the sequence. However, the pupils might also ask for an additional step before any form of assessment. Assessing the contribution from different pupils, when a group successfully has solved a problem, may easily lead to scores and marks that are unfair. Furthermore, the process of dividing the pupils in a class into groups of three or four pupils will not always result in a situation, where all groups are working well. Assessing each pupil's individual work seems more reliable, but obviously, the problem-solving process is not the same when the work is done individually and not in a group. However, there are at least two possibilities left to consider. The ability to solve a problem of the same kind as the problems in the previous steps may be assessed by giving an individual task to all pupils. It is also possible to include a task or elements of the sequence in the next

regular course examination. In both cases, the teacher also has to decide in what way the scores or the success in a problem sequence will be taken into account. In the tests in the author's classes, a successful problem sequence most often had an influence of the same kind as good lesson activity, but it is equally possible to e.g. count a weighted average, where evidence from different sources are assembled.

EXAMPLE 1. Fractions and percentages (a sequence of type 1)

The first example shows a problem sequence of type 1 and the problems are supposed to be solved with the use of Venn diagrams. As the pupils may not be familiar with Venn diagrams, such diagrams must be introduced, but preferably at first as an alternative tool to solve problems that can be solved without Venn diagrams. It is also very natural to include fractions and percentages in the tasks and connect the sequence about Venn diagrams to a course where fractions and percentages are used.

The problem in step 1:

Of the pupils in a class, 70 % like mathematics, 60 % like English, and 50 % like both. How many of the 30 pupils in the class do not like any of the subjects?

The following steps are part of the problem-solving phase and suitable contexts were decided to be subjects in the school, pets, and travelling. In order to raise the motivation, two or even three of the problems could be based on surveys in the class. If there is not time enough to implement all of these surveys in the class, corresponding data must be collected in advance, or, if it is not possible to collect data at all, trustworthy data could be fabricated. In this sequence, as in most of the sequences, the problems in the problem-solving phase are solved in groups with four, or sometimes three pupils.

The problem in step 2: *A survey was conducted in grade 8 in a school. One result was that $\frac{4}{5}$ of the pupils liked to work with computers. Of those who liked to work with computers, 75 % also liked mathematics. Of those who did not like mathematics, twice as many liked to work with computers, compared to those who did not like to work with computers. How many pupils could have liked mathematics in grade 8 if we also know that there were more than 50, but not as many as 100 pupils?*

The problem in step 3:

The teacher asks for a show of hands of those who have a cat (C), a dog (D), or some other pet (P) at home. The teacher writes the numbers in the three categories C , D and O on the board. Then, the teacher continues to ask for those who belonged to the categories C & D , C & O , D & O, and also the category C & D & O. The results are written on the board in rows beneath each other.

Finally, the pupils are given the central task in this step: with the help of circles, calculate how many of you have no pet at home. Using the numbers written on the board and circles, the groups try to find the correct answer. After a while, the groups hand in their results, and immediately after that, the actual number of pupils with no pet at home is determined by calculations on the board. Of course, it is possible to check if the answer is correct by asking the class who has no pet at home.

The problem in step 4:

A survey including 78 pupils gave the result that the following number of pupils had visited Sweden (S), Norway (N) and Denmark (D) during the summer holiday:

S	49	*S and N*	12
N	15	*S and D*	18
D	22	*N and D*	6
		S , N and D	5

1. *How many of the pupils had visited no other Nordic country?*
   The pupils were guided to use a circle for each of the three countries.
2. *What fraction was the number of those who had visited Sweden and Denmark, but not Norway, compared to all pupils included in the survey?*

The progression aspect in the problem-solving phase is satisfied by the shift from two circles in step 2 to three circles in step 3 and 4. Furthermore, the pupils also gradually understand that the use of circles becomes more necessary, as the numbers and facts in the data increase. The motivation should be high for several pupils, as they are quite deeply involved in the numbers they are working with. Especially the possibility to check the number of pupils with no pets at home should be of interest to most pupils. On the contrary, the motivation may decrease to some extent every time true numbers from the actual class are replaced with number data from outside the class, or from the teacher's fabrication.

If there is no indication that another step should be added, it is time to make an

individual examination. Of course, the degree of difficulty should be decided, as well as if the context should be a new one or an already used one. The natural end of a problem sequence is an individual task as the fifth step, but the alternative to include a problem similar to those in the sequence in the next regular course examination may also be considered.

The problem in step 5:
In a class with 25 pupils, a survey is conducted. The survey shows that the following percentages of the pupils would like to have bread (B) , fruit (F) and yoghurt (Y) as a healthy meal in the afternoons:

B	*48 %*
F	*64 %*
Y	*32 %*

It is possible to vote for two alternatives or even all three alternatives. In addition, the following results are known:

B and F	*32 %*
B and Y	*24 %*
F and Y	*28 %*

It is also a fact that one fourth of the pupils who has voted for both bread and fruit has not voted for yoghurt. Then, the question is: How many percent of the pupils has voted for none of the three alternatives?

In the real class situation, the teacher has to decide in which lessons the steps should be included, and how often the steps should be implemented. Very seldom a whole lesson is needed and it might be preferable to take one step every week. Often the answers to these questions may come from the regular program in the course and where the necessary time most easily can be found. It is also important to reserve the amount of minutes needed in the lesson plans and to be prepared to be flexible. Hopefully, the pupils who have been members of a successful group also succeed to solve a problem individually, but that is not always the case.

EXAMPLE 2. How many trees are there in the forest? (a sequence of type 2)
In this example, a problem sequence of type 2 is in focus. The pupils are invited to solve an open problem, and on the way to the solution suggest different ways of action. Before presenting the actual problem, it is reasonable to have a step, where the pupils answer a battery of questions, which are intended to

reveal how familiar each pupil is with the forest and with being in the forest. It is expected that the results may differ very much in urban schools, compared to schools in the countryside, but unexpected individual differences may also occur. The results can be used when dividing the pupils into groups. Between the first and the second step, converting area units can be repeated. In the second step, the problem is presented and the pupils are told that they will work in groups. The groups will make a plan how to solve the problem. As the goal for the whole sequence is to make a good estimate, the result of the discussion in the second step must be that the pupils realize the necessity of taking some kind of a sample. This sequence can be connected to a course in geometry.

The questionnaire in step 1:
How familiar are you with being in the forest?
Mark the statements that suit you best!
(A few of the statements are listed below.)
I have been in the forest picking berries.
I have been in the forest planting trees.
I have been in the forest walking the dog.
I have been in the forest at least once a week.
I have been in the forest in only one of the seasons.

The problem in step 2:
Suppose that you will have to estimate the number of trees in a forest parcel. It is not possible to count the trees, as the parcel is 6.0 hectare. In the first part of the problem, the task is to make a plan in order to find out the number, or at least a sufficiently good estimate of the number.

In the following two steps of the problem-solving phase, the pupils are supposed to suggest how to take the samples and to solve the problem why it is recommended to use a radius of 4 m when the samples are taken from a number of circles. In both steps the pupils work in small groups. In the former step, the pupils may not be familiar with a design task in mathematics, but they might learn something about geometric figures. However, the real challenge comes in step 4, where a deeper thinking is needed. The teacher can decide if and when some of the groups need a hint to be able to proceed in their thinking. Obviously, the teacher now uses the possibility to give the sequence a partly new orientation and

chooses to proceed with circles in step 4, although many different suggestions may have occurred in step 3.

The problem in step 3:
Suppose that you want to estimate the number of trees by taking some samples. Describe how you will accomplish the task.

The problem in step 4:
Suppose that you have decided to use circles. Why do you think that the Finnish Forest Centre suggests that the radius should be 4 m if the samples are taken from circles?

After step 4, the main problem is solved, but the counting of an estimated number of trees in a real situation is not yet fulfilled. Thus, one possibility is to proceed with an evaluating and individual task as step 5. The alternative is to use step 5 to practise the counting in groups, and then include a task of that kind in the course examination. Feedback from the pupils, available time in the course, and considerations about the course examination may determine the decision. The final phase in this example is accomplished as an individual task. The pupils should be prepared for such a task after the discussion that must follow step 4.

The problem in step 5:
Some samples from a forest parcel show that there are on an average 13 trees in circles with the radius 4 m. Estimate the number of trees in the whole parcel, which is 6.0 hectare.

In this problem sequence, it is possible to accomplish the steps 1 and 2, or the steps 2 and 3 in the same lesson. However, there are advantages of implementing only one step each lesson and working with a problem sequence for a longer time than one or two weeks. In a sequence of type 2, the interest to make efforts to reach the solution may increase for some pupils, if the goal is challenging. For a sequence of type 1, the problem-solving method becomes more familiar to the pupils, as the method is used several times in different contexts.

CONCLUDING REMARKS

The focus in this article has been on the matters the designer or user of teacher-guided problem sequences has to consider. There are steps of many kinds, and the different steps need different length of time. The quality of the problems in the steps varies, as the examination and description of the designer's considerations have been central. When carrying out a sequence, it is essential to reach an atmosphere in the class where creative reasoning and higher-order thinking frequently occur. From the tests in classes, such an atmosphere includes e.g. making the pupils understand that an incorrect suggestion can be made without any condescending remarks from other pupils, and that the process to reach the correct answer often matters more than the correct answer itself. Obviously, a general conclusion is that teachers who want to use problem sequences should be able to design problem sequences, or at least modify the ideas from given sequences, as there is always a need to make adjustments to the actual group, to the course, to the schedule, and to the situation.

The four design principles authenticity, complexity, structure, and challenge seem to be important for a successful design and implementation of problem sequences. Authenticity implies that the pupils are engaged in meaningful thinking and interaction, where the real world outside the classroom is simulated to an extent that the pupils understand the usefulness in solving the actual problem. Reducing the complexity of real-life situations and problems to a level that pupils can handle is necessary, because the pupils must consider the problems possible to solve. The teacher will need to take an active role throughout the whole problem sequence, starting from the design of the structure, including the two types of problem sequences, and finishing with the assessment phase. Last but not least, the problems should have appropriate elements of challenge in order to provide the pupils possibilities to exercise the desirable higher-order thinking.

Although the tests using problem sequences have been performed in Swedish-speaking classes in Finland, problem sequences can be tested and used in other languages and other countries, as well. Obviously, the need for modifications and adjustments is always present regardless of languages and countries. The description of considerations a designer or teacher faces by only two examples does not attempt to be complete. Of course, this report from an ongoing project leaves many open questions about how to implement problem sequences in the best possible way, but the article might still give inspiration to use problem sequences.

For researchers there are also many unanswered questions, concerning e.g. the consequences of different ways to divide the pupils into groups, and above all, the assessment of pupils' achievements when they have been working in groups. The assessment question is far from specific for problem sequences but challenges as regards assessment of groups should not prevent teachers from using group work as there are other circumstances supporting problem solving in groups.

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