

THE EFFECT OF PROBLEM SOLVING COURSE ON PRE-SERVICE TEACHERS' BELIEFS ABOUT PROBLEM SOLVING IN SCHOOL MATHEMATICS AND THEMSELVES AS PROBLEM SOLVERS

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ABSTRACT

Problem solving in schools begins with mathematics teachers. The degree to which mathematics teachers are prepared to teach for, about and through problem solving influences on their implementation of problem solving in school. We conducted a small scale study where we examined the effect of implementation of heuristic strategies and Polya's steps in mathematics method course. We assessed pre-service teachers' knowledge and attitudes about them as problem solvers before and after the course. Moreover we assessed their beliefs of problem solving in school mathematics. Those beliefs were assessed in two occasions: right after the course and after finished teaching practice. Although students' knowledge on problem solving was improved, the results of students' beliefs show that it is important that pre-service teachers, and consequently in-service teachers, are constantly reminded on the positive effect of constructivist and inquiry-based approach on teaching mathematics.

KEYWORDS: *pre-service teachers, explicit teaching, problem solving, beliefs and attitudes*

1 INTRODUCTION

Research on mathematical problem solving has a long history dealing with the fundamental question regarding the teaching and learning of it. In mathematics education, the research on problem solving reached its peak two decades ago. However, problem solving still remains as important area, especially in countries

dealing with mathematics curriculum reform. One of such countries is Croatia. Mathematics curriculum in Croatia is in the process of transition from traditional curriculum (Glasnović Gracin, 2011), where emphasis is placed on algorithms and the view of mathematics as a tool, to the reformed curriculum where mathematics is conceived as the medium of communication (MZOS, 2010). New curricular document emphasizes a role of problem solving in school mathematics in all educational cycles as one of several important mathematical processes describing that “doing mathematics” means being actively involved in a wide variety of physical and mental actions.

Problem solving in schools begins with mathematics teachers. The degree to which mathematics teachers are prepared to teach for, about and through problem solving influences on how they implement problem solving in school (Chapman, 2015). In order to change teachers’ practices in school, teachers’ beliefs need to be considered. When entering universities, future teachers already have ideas what it takes to be an effective teacher and they bring those beliefs to their teacher preparation program (Pajares, 1992). It is very important that teacher education programs assess how well they nurture beliefs that are consistent with the philosophy of learning and teaching (Hart, 2002). According to above mentioned considerations, we implemented problem solving strategies and Polya’s steps in mathematics method course for pre-service teachers. We examined pre-service teachers’ beliefs on problem solving in school mathematics and their attitudes about themselves as problem solvers as well as their knowledge on problem solving after the course and after teaching practice in teacher preparation program.

2 THEORETICAL BACKGROUND

Problem solving knowledge

Genuine problem solving involves engaging in a task for which the solution method is not known in advance. Polya (1945) suggested general strategies of solving problems based on questions like: “What is the unknown? What are the data? What are the conditions? Do you know a related problem that has already been solved? Prepare a plan for the solution. Verify the gained results.” Some of those strategies, commonly known as heuristics, are: Draw a diagram, Guess and check, Look for a pattern, Make a systematic list, Use before after conception, Trial and error strategy, Working backwards.

According to Schoenfeld (1985), for successful problem solving one must be equipped with and competently use appropriate resources (e.g. mathematics concepts and procedures), heuristic strategies (specific and general heuristics), meta-cognitive control (monitoring and overseeing the entire problem solving process), and appropriate beliefs (one's perspective, motivation, and confidence). Contrary to a frequently held assumption, the resources are not the primary determinant of success in problem solving. Schoenfeld (1992) pointed that mathematicians with powerful heuristics and control are likely to be able to solve problems even when their resources are severely lacking, and that students, who possess the necessary resources, may be unable to solve problems because their belief systems do not allow the connections to be made. When studied students in the fifth grade during word problem sessions, Resnick (1988) found that students' insecurity caused blockage in successful problem solving. Moreover, whether a person will be able to learn and use a selected heuristic strategy does not depend only on the learning environment, but also on his attitude to problem solving (Eisenmann et. al., 2015). Therefore, mathematical knowledge resources may be a necessary, though not sufficient, condition for successful problem solving.

Attitudes and beliefs influencing problem solving

There are several possible factors that might contribute to the difficulty of moving mathematics instruction in a school away from more algorithmic-oriented activities and exercise toward more problem-oriented activities. One of such difficulty is related with teachers' beliefs about themselves as problem solvers and their attitudes toward problem solving. Before they come to school as licensed teachers and take over their classrooms, teachers have experienced many hours of instruction during their schooling. It is very likely that those instructions were in a traditional style. Thus, their beliefs about teaching are developed over the years when teachers were school students (Lortie, 1975) and are well established by the time they entered university. When entering the classroom, those teachers have to cope with their prior experiences and new requirements of contemporary teaching which usually clash. It is likely that those teachers will view mathematics as a collection of static facts and procedures rather than a process of investigation (Felbrich, Muller, & Blomeke, 2008). The change in their attitudes and beliefs about teaching is possible, but changing teachers' beliefs takes time (Richardson, 1996). It is possible that teachers who are not comfortable with problem solving

or who had poor experiences with problem solving have hesitation or push back when asked to teach about or through problem solving. Therefore, engaging such teachers as problem solvers can help in changing their attitudes about problem solving. Pajares and Kranzler (1995) claim that self-efficacy in problem solving has a positive impact on person's ability to solve the problem and also helps in decreasing mathematics anxiety. On the other hand, successful problem solving can be reflected in person's increased self-efficacy (Babakhani, 2011).

3 RESEARCH FOCI

Elia, Van den Heuvel-Panhuizen and Kolovou (2009) proposed that future studies should investigate if the pattern between heuristic strategies and problem solving success changes when students receive systematic strategy training in non-routine problem solving. Therefore we conducted a study with following research questions:

- What characterizes pre-service teachers' knowledge in solving non-routine tasks/problems before and after systematic training in non-routine problem solving?
- What do pre-service teachers believe about themselves as problem solvers after systematic training in non-routine problem solving?
- Does the teaching practice influence on pre-service teacher beliefs on problem solving and to what extent?

4 DESIGN OF THE MATHEMATICS METHOD COURSE

This study took part in the context of mathematics method course for secondary school teachers. The mathematics method course was divided in three parts; the first part consisted of pre-service teaching through two semesters; the second part encompassed writing a professional paper for teacher journal and the third part dealt with the problem solving and geometric constructions. The course allocated 15 hours for problem solving through one semester. In the course, we used explicit teaching to teach problem solving strategies. Explicit teaching encompasses a structured, systematic and effective teaching methodology for raising students' achievements (Archer & Hughes, 2011). It is called explicit because it contains a direct approach which includes the development of guidance and an explanation of processes. Such teaching approach is used mainly in the areas of

reading and mathematics (Ellis, 2005). According to Tetzlaff (2009), explicit teaching should consist of five steps: orientation; presentation; structured practice; guided practice; independent practice.

Here we will briefly describe the explicit teaching method. In the orientation phase, the instructor provides an overview of what will be taught, but also places the lesson in a context that learners can relate to. Part of the orientation phase also involves providing examples of the completed task so that learners have a model of what their final product can look like. In the presentation phase, the instructor breaks the overall objective of the lesson into smaller and easy-to-follow steps. Here the instructor demonstrates the process of completing the task, modelling the type of thinking the instructor wants learners to mimic by thinking-out-loud as he works through the steps. After presenting and demonstrating the process used to achieve the lesson's goal, the instructor goes to structured practice phase, where he works through the process again, and this time each learner is practicing along with the instructor. During this phase it is critical that the instructor asks learners questions to check and assess their understanding to clarify any confusion. The questions *what* and *how to do tasks* are not enough in this phase. More important is the question *why* those actions are necessary for the task, so that learners understand the importance of each step in a process.

In the phase of guided practice, we used Mathematics Practical Worksheet, similar as presented by Toh, Quek and Tay (2009). Their worksheet contains sections that guides the problem solver through the four stages of Polya's model and also incorporates Schoenfeld's model, highlighting the cognitive resources, use of heuristics, control, and belief systems of the problem solver. The Mathematics Practical worksheets consists of following sections:

1. Understand the Problem (You may have to return to this section a few times. Number each attempt to understand the problem accordingly as Attempt 1, Attempt 2, etc.)

(a) Write down your feelings about the problem. Does it bore you? Scare you? Challenge you? (b) Write down the parts you do not understand now or that you misunderstood in your previous attempt. (c) Write down your attempt to understand the problem; and state the heuristics you used.

2. Devise a Plan (You may have to return to this section a few times. Number each new plan accordingly as Plan1, Plan 2, etc.) (a) Write down the key concepts that might be involved in solving this problem. (b) Do you think you have the required resources to implement the plan? (c) Write down each plan concisely

and clearly.

3. Carry out the plan (You may have to return to this section a few times. Number each implementation accordingly as Plan 1, Plan 2, etc.) (a) Write down in the Control column the key points where you make a decision or observation, e.g. go back to check, try something else, look for resources, or totally abandon the plan. (b) Write out each implementation in detail under the Detailed Mathematical Steps column

4. Check and extend (a) Write down how you checked your solution. (b) Write down your level of satisfaction with your solution. Write down a sketch of any alternative solution(s) that you can think of. (c) Give one or two adaptations, extensions or generalizations of the problem. Explain succinctly whether your solution structure will work on them.

Students were given assignments to solve also as a homework, what can be considered as a part of the independent practice. The problem solving strategies that were introduced in the course were: working backwards, finding a pattern, adopting different point of view, solving simple analogues problem, considering extreme case, making a drawing and making a list.

5 METHODOLOGY

The entire cohort of final year pre-service mathematics teachers was required to participate in this study. There were 20 students enrolled in mathematics method course in Department of Mathematics, University of Osijek, therefore, the study presented in this paper belongs to small scale study. We assessed students' knowledge in problem solving at the beginning of the mathematics method course and after the part of the course that dealt with problem solving. The initial testing contained two questions:

IP 1. A snail climbs the pole 10 meters high. During the day, it climbs 5 m, and at night it descends 4 m. How many days does it need to climb to the top of the pole?

IP 2. A greyhound chases a fox which is 30 m ahead of him. Greyhound's jump is 2 m long, and fox's jump is 1 m long. While the greyhound does 2 jumps the fox does 3 jumps. Can the greyhound catch up with the fox? How many meters does the greyhound have to run in order to catch up the fox?

We have chosen items IP1 and IP2 for the initial testing because we considered IP1 to be easy to solve, but contained problematic and no obvious con-

dition, and IP 2 was moderately difficult item where students should employ non-algorithmic knowledge. Final testing contained 4 questions, and students were supposed to use appropriate strategy and solve the problem. The following problems were given to students:

PP 1. What is larger $\sqrt[9]{9}$ or $\sqrt[10]{10}$?

PP 2. Find the units digit of $7^{2^4} \dots 7^{2^{16}} + 3^{1+2+6+7+8+\dots+2016}$

PP 3. Marko works for an art gallery. He is designing a large wall covering for a client. The entire design is made up of 50 concentric squares. Figure 1 shows the first four squares of his design and gives the length of one side of each square. Marko is going to outline the perimeter of each square with wool. How many meters of wool does he need to outline all 50 squares?

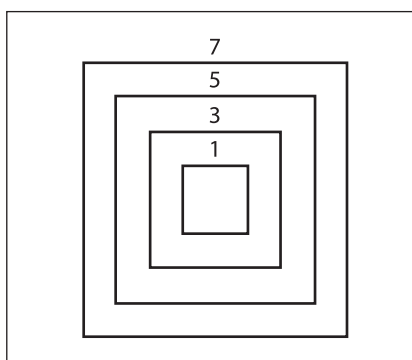


FIGURE 1. Marko's creation

PP 4. If now is 10:45h, what time will it be in 143 999 999 995 minutes from now?

Also, we examined what student think about themselves as the problem solvers before and after the problems solving course. We used a set of statements from Yusof and Tall (1994). Here we used Likert scale items to be rated on 4-point scale (1-strongly disagree, 2-disagree, 3 -agree, 4 – strongly agree). Unlike Yusof and Tall (ibid), we omitted the neutral statement because we wanted students to take a stand.

Students' beliefs about problem solving and school mathematics were examined in two occasions, after the part of course on problem solving and after they

finished teaching practice in school. Those beliefs were examined using Standards Belief Instrument (SBI) (Zollman & Mason, 1992) which determines how consistent an individual's beliefs are with the philosophy of NTCM Standards of teaching mathematics (NTCM, 2000). Standards promote contemporary teaching of mathematics where mathematics is conceived as the medium of communication, not only a tool. New Croatian curriculum, called Nacionalni okvirni kurikulum [NOK] (MZOS, 2010), refers to mathematics in a similar way, therefore we decided to use SBI. For instance, NOK promotes an importance of problem solving in all educational cycles, as well NTCM Standards which emphasize that problem solving is not a distinct topic but process that should permeate the entire program. The SBI consists of 16 Likert scale items to be rated on 4-point scale (1-strongly disagree, 2-disagree, 3 -agree, 4 – strongly agree). Eight of those items were consistent with the contemporary, constructivist learning approach of mathematics and eight of those items were inconsistent with aforementioned approach. The statements in the instrument were slightly modified to fit our situation; the original instrument addresses K-4 teachers, therefore we omitted the term *K-4 mathematics* from the statements and last statement from the instrument that deals with kindergarten children. All statements can be seen in the Appendix. After the data were collected, the responses for eight negative valence items of the SBI were re-aligned (response subtracted from 5.0). Thus the rating of positive and negative valence items would be in a consistent direction ranging from 1.0 to 4.0. The closer the re-aligned response number to 4.0 the stronger is the agreement with the contemporary approach of teaching.

Due to the small number of participants, we used Wilcoxon signed rank test to compare students' answers on beliefs and attitudes items.

6 RESULTS

Knowledge in solving non-routine problems

Here we will describe students' solutions in the initial testing, before they had learnt any problem solving strategies. In the initial test on problem solving, not all students were able to solve item IP 1. More than half gave the wrong answer 10 days, concentrating on the fact when the snail reaches top of the pole and does not descend below 10 m in its climbing up and climbing down. Only students who drew the pole and traced its journey up and down gave a correct answer,

but several of them also had wrong calculations as the students who did not use drawing in their solution. When it comes to item IP 2, students used drawings and made systematic lists as an attempt to solve the problem. In Figure 2, one can see example of one student's attempt to solve a problem. Only three students reached final solution.

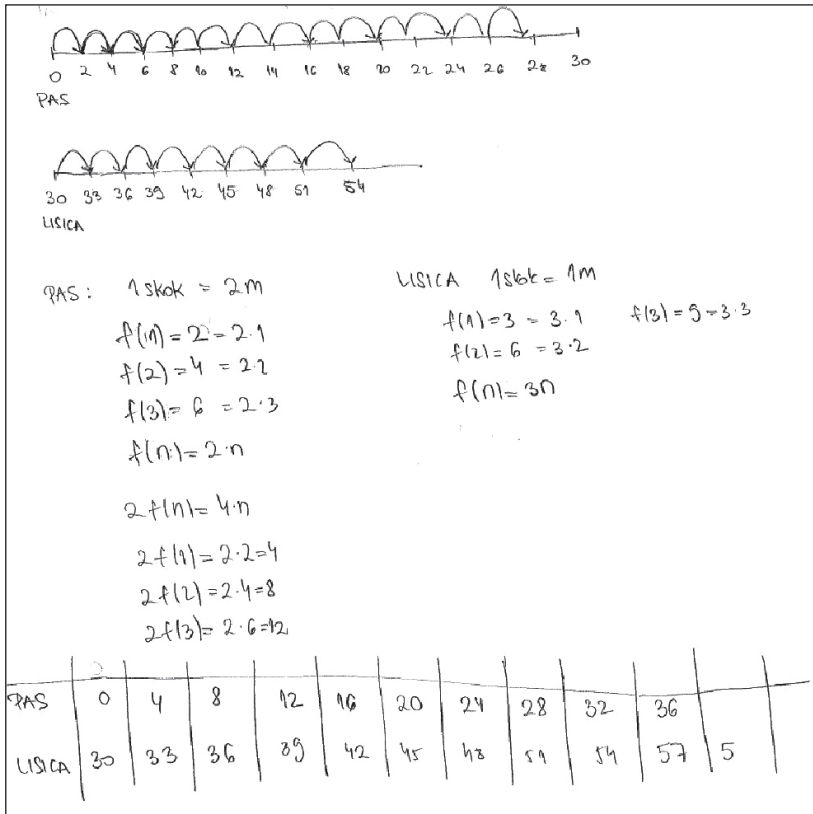


FIGURE 2. Student's solution to the snail problem

In the final exam, students had to solve four problems/non-routine tasks, and their results can be found in Table 1. From the table it can be seen that students used some of the learnt problem solving strategies, although not all students solved correctly given problems. Mistakes they made were connected with small errors in calculation and in some aspects of reasoning, but not with erroneous reasoning. However, no question was left without any attempt of solving. The most

difficult item was PP 4, where the number of correct solutions was the smallest. Students recognized that the number of given minutes will be exactly divisible by 60 (to obtain hours) and then 24 (to obtain days) if they add 5 minutes. But at the end they forgot to subtract the 5 minutes from the obtained time. Other items contained also minor errors in students' reasoning, such as forgetting to multiply by four for perimeter in PP3 or wrong conclusion about unit digit in sum $2 + 4 + 6 + \dots + 216$ in PP2.

Items	Strategy		No strategy	Correct
PP 1	13 systematic list	4 working backwards	3	14
PP 2	20 finding a pattern		0	10
PP3	10 finding a pattern	10 systematic list	0	14
PP 4	17 different point of view		17	3

TABLE 1. Result of the final exam on problem solving

Students' attitudes about themselves as problem solvers

Results on students' attitudes about themselves as the problem solvers can be seen in Table 2. If we look at the results obtained before problem solving part of the course, we can see that students express mainly positive attitudes toward problem solving, feeling confident in problem solving (Item 1) and experiencing pleasure when solving problems (Item 2). Moreover, they are willing to try new approach if necessary (Item 6). Although they expressed themselves negatively about being anxious (Item 3) or giving up when problem is difficult (Item 7), they show that they fear of unexpected problems to certain degree (Item 4) and that mathematics is about getting correct answers (Item 5).

Students' attitudes toward problem solving after experiencing systematic training in problem solving were quite similar (see Table 2). Only two items showed change that was statistically significant on the level of 0.10. The level of anxiety for solving problems decreased (Item 3). The number of students who agree with this statement decreased from 30% to 15%, and the number of students who strongly disagree increases from 5% to 30%. Also, students expressed themselves as being more persistent when encountering difficult problem (Item 7). The number of students who strongly disagree with the statement increased from 25% to 45%.

<i>Items</i>	<i>Mean Before</i>	<i>Mean After</i>	<i>p-level</i>
1. I feel confident in my ability to solve mathematics problems.	2.95	3	0.7532
2. Solving mathematics problem is a great pleasure for me.	3.45	3.4	0.7532
3. I feel anxious when I am asked to solve mathematics problems.	2.35	1.65	0.0029
4. I often fear unexpected mathematics problems.	2.15	2.15	1
5. I feel the most important thing in mathematics is to get correct answers.	2.35	2.25	0.5939
6. I am willing to try a different approach when my attempt fails.	3.55	3.65	0.4631
7. I give up fairly easily when the problem is difficult.	2	1.65	0.0630

TABLE 2. Students' attitudes

Students' beliefs about mathematics

The Table 3 contains results of beliefs questionnaire about problem solving and school mathematics. The first set of results are the results of beliefs questionnaire given after the problem solving part of the course. In six items that have positive valence, students' mean scores are above 3. This suggests that students see school mathematics as active process (Item 15), where children's reasoning has higher value than finding correct answer (Item 13). Moreover, they value sharing thinking (Item 2) and communication within classroom (Item 3) and correlating mathematics with other school subjects (Item 6). Also they consider that the goal of mathematics education is to help children in increasing their self-efficacy (Item 4). However, their beliefs on traditional aspects of mathematics are prevailing what can be seen from mean scores in other items. For instance, students consider that mathematics should be learnt as absorbing non-connected pieces of information (Item 11) or that there is only one correct way in which children should think or justify their (Item 5). Examining students' results individually, we found that only two students had mean scores above 3, while mean scores for other 17 students was between 2.53 and 2.86.

The second set of results are the results from the beliefs questionnaire given after finished teaching practice in schools. Those results are similar to the results obtained after the problem solving part of the course. The same six items (Items 2, 3 4, 6, 13 and 15) have mean scores above 3, while nine other items have mean

scores below 3. However, we detected change in mean scores in several items. In some items, mean score increased, but this change is statistically significant (at the level of 0.10) only for Item 5 where it is suggested that children should justify and conjecture in various ways. The significant change happened because the number of students who strongly disagreed decreased from 50 % to 35%, and the number of students who strongly agree increased from 5% to 20%. Increasing mean score indicates that students' beliefs shifted slightly away from traditional approach to teach and learn mathematics.

Besides increase, we detected decrease in mean scores; this decrease is statistically significant (at the level of 0.10) only for Item 12 where mathematics is perceived as collection as concepts, skills and algorithms. The number of students who strongly disagree changed from 20% to 45 %. This decrease indicates the influence of teaching practice in school, where mathematics is still presented as collection of various algorithms and concepts, usually not connected.

Examining students' results individually, we found that five students had mean scores for above 3, while mean scores for other 15 students was between 2.53 and 2.86.

<i>Item</i>	<i>Valence</i>	<i>Mean After course</i>	<i>Mean After teaching practice</i>	<i>p-value</i>
1.	-	2.8	2.4	0.1095
2.	+	3.7	3.9	0.1422
3.	+	3.35	3.4	0.7353
4.	+	3.45	3.45	1
5.	-	1.85	2.35	0.0831**
6.	+	3.65	3.55	0.5286
7.	-	2.5	2.9	0.1000
8.	-	2.55	2.3	0.2393
9.	-	1.85	2.05	0.2367
10.	-	2.05	2.1	0.8139
11.	-	1.95	1.7	0.1823
12.	-	2	1.7	0.0587**

13.	+	3.25	3.2	0.7353
14.	+	2.65	2.4	0.3078
15.	+	3.85	3.9	1

TABLE 3. Students' beliefs about mathematics

7 DISCUSSION AND CONCLUSION

Pre-service teachers' knowledge and attitude in solving non-routine problems

Even though the results of the final exam do not show impressive results on students' knowledge in non-routine tasks, there is a significant shift in students' written solutions when we compare their work before and after teaching in problem solving. In the initial exam, in item IP 2, which proved to be challenging and difficult, students used drawings and made systematics list, but their written solution contained ill logic and erroneous reasoning. They were not able to connect given data. In the final exam, their written solutions had more meaning. Although students made mistakes in their calculation and in some aspect of reasoning, one could follow their line of thought, and detect particular strategy the used for problem solving. If we were to assess their overall reasoning, not just their correct solution, we can conclude that they improved knowledge to tackle non-routine problem. On the other hand, if we take into account the design of course where we introduced and taught problem solving strategies in systematic way, we can argue that students failed in their knowledge improvement, and that such design of the course did not achieve desired outcome. However, there is a third component which influenced on students' results such as students' emotional condition. The final exam is not surrounding where students' feel comfortable, but pressured and it could be argued that final exam belongs to high stake exams. Therefore, we believe, that this aspect should be taken into account when one makes conclusion of the overall pre-service teachers' knowledge in solving non-routine tasks.

In all, pre-service teachers became skilled in utilization of heuristic strategies. We cannot obtain valid conclusion if they gained metacognitive control (monitoring and overseeing the entire problem solving process) from the results. For such conclusion, we should collect data in another form like examining their solutions when they used Mathematics Practical Worksheet. Those worksheets proved to

be a valuable asset in several studies on teaching problem solving to pre-service teachers in courses on number theory (Toh et al., 2104) and differential equations (Toh et al., 2013). Authors of those studies reported that Mathematics Practical Worksheets allowed assessing components of metacognitive control and that pre-service teachers changed their approach in tackling mathematics problem in positive direction.

If we continue along Schoenfeld's framework for analyzing mathematical behavior, the next component is concerned with appropriate beliefs or attitudes. When it comes to students' beliefs (perspective, motivation, and confidence), we can see the change after experiencing the training in problem solving. It is important to notice that students' anxiety level decreased as well as their tendency to easily give up from solving problems. Therefore, we believe that students' confidence increased and that design of the course with explicit teaching had positive effect on students' attitudes about them as problem solvers. Also we believe that this design of the course, with specific steps in teaching, as well as the utilization of Mathematics Practical Worksheet were very beneficial. Jonassen (2011) proposed that problem-solving should be learnt in environments in which problems are precisely classified and linked to explicit heuristic strategies. Our design of the course enabled students to engage with problem solving in environment that on the first sight did not significantly differ from their prior experiences where their learning was "guided". Portnov – Neeman and Amit (2015) also used explicit teaching of problem solving strategies. Their primary school students became more active problem solvers who understood the purpose of the strategies and their solution stages, developing those strategies as they saw fit, and freeing themselves from the restraints of the strategy when necessary.

Pre-service teachers' beliefs about problem solving and school mathematics

The results on students' beliefs about problem solving and school mathematics indicate that students still have traditional view of teaching mathematics, although they highly value some aspects of the constructivist approach. Their beliefs are stable and consistent, and the teaching practice in school did not highly influence on those beliefs. Moreover, it seems that the teaching practice in school did not support mathematics as medium of communication, reasoning and problem solving, but uphold the perspective of mathematics as collection of procedures, facts and algorithms.

Zollman and Mason (1992) claimed that beliefs regarding mathematics education should be examined when mathematics education is being reformed, and that their questionnaire can be utilized as a basis for evaluating teachers' perspective on the real purpose of mathematics education. Even though their claim was made more than 20 years ago, it is still applicable in the context of Croatian educational system where the education reform is trying to change the approach to school mathematics from traditional, where students are passive observers, who absorb facts and procedures without application in real world, to constructivist, inquiry based teaching, where students are given opportunities to construct understandings of mathematical concepts in social groups and through interaction with the teacher as a facilitator.

Hart (2002) claimed that there is substantial evidence that teachers' beliefs about mathematics impact their teaching of mathematics. She also suggested that teacher education programs should assess their effectiveness, whether beliefs they advocate correspond to their philosophy of learning and teaching. Hart (ibid) suggested that pre-service teachers' beliefs should be evaluated also after experiencing teaching in classroom. In our study, we took that step further, and in that light, our results also suggest another finding. To certain extent, the education courses, and especially mathematics education courses failed to portray how reformed mathematics teaching should look like and to promote the benefits of such approach. Implicitly, our findings also suggest that schools, where students had their teaching practice, still have traditional approach to mathematics education. For several years now, Croatian mathematics in-service teachers have been educated and informed on the reformed teaching of mathematics. The students in our study had teaching practice in more than 30 primary and secondary schools. The question that naturally arises from our results is: How can we expect that these new teachers discard traditional approach of teaching mathematics if the educational system (schools and university) failed in empowering them in this direction?

Limitations of the study

One of the frequent limitations of studies in mathematical problem solving is the size of the sample (Eisenmann et al., 2015). The study reported in this paper is small scale study where we were limited with the number of student enrolled in the course. We cannot influence on the number of enrolled students but we could

expand problem solving on another mathematics courses. Also another limitation is the length of the study. This limitation can be overcome by expanding the problem solving through two semesters, not just one. We believe that long-term strategic teaching of problem solving would have greater impact on pre-service teachers' mathematical thinking.

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APPENDIX

Items in the beliefs questionnaire, adapted from the SBI:

1. Problem solving should be a SEPARATE, DISTINCT part of the mathematics curriculum.
2. Students should share their problem-solving thinking and approaches WITH OTHER STUDENTS.
3. Mathematics can be thought of as a language that must be MEANINGFUL if students are to communicate and apply mathematics productively.
4. A major goal of mathematics instruction is to help children develop the belief that THEY HAVE THE POWER to control their own success in mathematics.
5. Children should be encouraged to justify their solutions, thinking, and conjectures in a SINGLE way.
6. The study of mathematics should include opportunities of using mathematics in OTHER CURRICULUM AREAS.

7. The mathematics curriculum consists of several discrete strands such as computation, geometry, and measurement which can best be taught in ISOLATION.
8. INCREASED emphasis should be given to reading and writing numbers SYMBOLICALLY.
9. In primary school, INCREASED emphasis should be given to use of CLUE WORDS (key words) to determine which operation to use in problem solving.
10. In primary school, skill in computation should PRECEDE word problems.
11. Learning mathematics is a process in which students ABSORB INFORMATION, storing it in easily retrievable fragments as a result of repeated practice and reinforcement.
12. Mathematics SHOULD be taught as a COLLECTION of concepts, skills and algorithms.
13. A demonstration of good reasoning should be regarded EVEN MORE THAN students' ability to find correct answers.
14. Appropriate calculators should be available to ALL STUDENTS at ALL TIMES.
15. Learning mathematics must be an ACTIVE PROCESS.

