Quality competition and reputation of restaurants: the effects of capacity constraints

You-hua Chen, Qinying He & Krishna P. Paudel

To cite this article: You-hua Chen, Qinying He & Krishna P. Paudel (2018) Quality competition and reputation of restaurants: the effects of capacity constraints, Economic Research-Ekonomска Истраživanja, 31:1, 102-118, DOI: 10.1080/1331677X.2017.1421996

To link to this article: https://doi.org/10.1080/1331677X.2017.1421996
Quality competition and reputation of restaurants: the effects of capacity constraints

You-hua Chen\textsuperscript{a, b}, Qinying He\textsuperscript{a} and Krishna P. Paudel\textsuperscript{b}

\textsuperscript{a}College of Economics & Management and Guangdong Center for Rural Economic Studies, South China Agricultural University, Guangzhou, P.R. China; \textsuperscript{b}Department of Agricultural Economics and Agribusiness, Louisiana State University (LSU) and LSU Agricultural Center, Baton Rouge, LA, USA

**ABSTRACT**

Capacity constraints have major effects on restaurant competition. This article captures the impact of capacity constraints on quality and quantity competition of restaurant industry by employing a two-stage asymmetric capacity constraints model. There are several findings from this study. First, results of this study indicate that reputation difference increases total outputs and quality in equilibrium. Second, capacity constraints decrease the consumer surplus (CS) as well as social welfare (SW), but restaurants benefit from capacity constraints sometimes. Third, capacity reduces both the quality and quantity of the competitor. Furthermore, restaurants raise their prices if consumers are quality-sensitive or lower the quality if consumers are price-sensitive under capacity constraints. Fourth, this study compares the total quality, CS, and SW under four cases: no restaurant faces constraints, small restaurant faces constraints, big restaurant faces constraints, and both small and big restaurants face constraints. The total quality investment, CS, and SW are highest if a small restaurant faces capacity constraints while the larger one does not.

1. Introduction

The concept of capacity constraints relates to the economy of scale. Several factors result in capacity constraints in a restaurant sector. First, a restaurant has a limited number of seats. Second, dinner time is restrictive which means customers only visit restaurants during a certain time. Third, chefs can only cook a limited number of food items during the dinner time. The restaurant sector is sufficiently similar to hotel and airline industries as all of them have limited capacity, or they operate with capacity constraints (Heo, Lee, Mattila, & Hu, 2013). Different from hotels and airlines; however, restaurant sector’s capacity is more flexible. Attributes such as location, ambiance, quality and waiting time of consumers, and capacity management are major components in the restaurant industry (Hwang, Gao, & Jang, 2010). Besides, service and food qualities determine the average meal price charged by...
a restaurant (Yim, Lee, & Kim, 2014). Production and service's quality of restaurant involve a significant topic, food safety, which is a major issue all over the world (Chen, Wen, Wang, & Nie, 2017). Therefore, the restaurant industry is unique, and capacity constraints have significant effects on restaurant operation.

Capacity constraints impact industrial competition structure; sometimes even improving the status of disadvantaged firms. For example, Chen, Nie, and Wang (2015) illustrated that capacity constraints offset the second-mover disadvantage of the follower under the Stackelberg competition. Interestingly, customers may experience the following phenomena in a restaurant business. The restaurant offers high-quality food items which are delicious at the beginning, but then the quality will deteriorate, or the price will go up some time later. A similar example can be found in the hospital and banking sectors, where the quality of service goes down if too many customers are waiting behind the yellow (waiting) line. Undoubtedly, all those behaviours are rational. A major reason for these phenomena is that firms are faced with capacity constraints. A restaurant has less incentive to offer high-quality good or service if the demand is more than the restaurant's capacity. To increase profit, restaurants will reduce the quality if most of the customers are price-sensitive or increase the price if most of the customers are quality-sensitive when the demand exceeds its capacity. The major objectives of this study are to model restaurant behaviours and to illustrate why competition strategies are volatile, as well as to offer additional intuitions related to the capacity constraint. Additionally, this study will expand an asymmetric capacity constraints theory by employing an asymmetric capacity constraints competition model.

There are two kinds of capacity constraints: symmetrical and asymmetrical. Symmetrical capacity constraints mean all the firms in the industry are faced with the same capacity constraints. If different firms are faced with different capacity constraints, then it is asymmetrical capacity constraints. Prior studies have employed symmetrical capacity constraints model, or firms of the same industry compete with the same capacity constraints (Chen et al., 2015; Esó, Nocke, & White, 2010; Lester, 2011; Nie & Chen, 2012). These studies are focused on input constraints, which mean the capacity constraints are decided by external factors. Capacity constraints of restaurants are decided by internal factors. In other words, different restaurants have different or asymmetrical constraints. Different from the existing research, this article captures quality and quantity competition by adopting an asymmetrical capacity constraints model. Our results indicate that different restaurants implement different strategies under the condition of capacity constraints, which depend on the preferences of consumers.

Furthermore, this article investigates the relationship between reputation and quality and its effects on restaurants competition. Reputation is defined as ‘a perceptual representation of a company’s past actions and future prospects that describes the firm’s overall appeal to all of its key constituents when compared to other leading rivals’ (Fombrun, 1996). Roberts and Dowling (2002) illustrate that a good reputation is a valuable asset that allows a firm to achieve persistent profitability, or sustained superior financial performance. Good reputation enables a firm to increase its prices or increase its sales. The results of this study show that reputation has a significant influence on restaurants competition and reputation difference increases total quality and quantity of the industry in equilibrium.

The remainder of this article is organised as follows. A literature review is outlined in Section 2, a basic model setup is provided in Section 3, and several propositions are presented after the model section. Concluding remarks and some discussions are provided in
the final section of the article. All the proofs related to the propositions can be found in the appendix section.

2. Literature review

Capacity constraints have attracted much attention recently (e.g. Adilov, 2012; Chen et al., 2015; Chun & Park, 2016; Klein & Kolb, 2015; Mayo & Sappington, 2016; Sena, 2006; van den Berg, Bos, Herings, & Peters, 2012; Whited, 2006). The existing literature on this topic can be classified into four categories: price competition, output competition, merger or collusion, and special industry.

Price and quality competitions are most familiar competition behaviour in industrial organisation. Many researchers have investigated the effects of capacity constraints on price competition. For example, Ishibashi (2008) has analysed collusive price equilibrium with capacity constraints and shown that capacity constraints heighten the collusive leadership price. He argues that a large firm has more incentive to participate in price collusion if it is confronted with capacity constraints. So, a large firm has an incentive to move early to demonstrate its price collusion commitment despite knowing that early price setting is a disadvantage to the firm. More interestingly, Arnold and Saliba (2011) imply capacity constraints yield price dispersion, and a higher capacity constrained firm charges a higher price, while a lower capacity constrained firm charges a lower price. Lester (2011) examines a standard consumer search model with capacity constraints. He comes to the conclusion that as buyers become more able to observe and compare prices ex ante, sellers will set lower prices. Equilibrium may not necessarily hold if a firm plays price competition games with capacity constraints. Moreover, Lester’s study shows that more information for consumers could lead to an increase in restaurant menu prices. Deng and Yano (2006) show that the firm with tight capacity constraints will adopt more aggressive pricing.

Some prior research focuses on the relationship between capacity constraints and output quantity. Nie and Chen (2012) investigate duopoly competition with input constraints. They declare that input constraints reduce market size difference and price difference. To obtain robust conclusions, they discuss their model under different conditions. Surprisingly, the firm-size difference and price difference decrease with total input under capacity constraints under the Stackelberg competition if the stronger firm plays as the leader, which is contrary to the results of the Cournot competition. Esó et al. (2010) captured the effects of capacity constraints on output by employing an upstream–downstream model. Esó et al. (2010) achieved the similar conclusions as Nie and Chen (2012), and they suggest that capacity constraints lead to industry symmetry. These studies imply that capacity constraints are beneficial to disadvantaged firms.

Parts of the existing studies highlight the effects of capacity constraints on other competition strategies such as merger and collusion. Based on scarce input that enhances product quality, Mayo and Sappington (2016) find that auctions will not always ensure the welfare-maximising allocation of scarce inputs. Chen et al. (2015) highlight effects of capacity constraints on innovation competition. Nie (2014) captures effects of capacity constraints on mixed duopoly. Froeb, Tschantz, and Crooke (2003) study the impacts of mergers among firms facing capacity constraints. Their study indicates that capacity constraints attenuated merger effects. Taking dynamic aspects of competition into account, Compte, Jenny, and Rey (2002) research effects of capacity constraints on mergers and collusion. Different from other
research about merger, Compte et al. (2002) captured the effects of asymmetric capacity constraints. van den Berg et al. (2012) discussed capacity constraints under the dynamic Cournot condition and considered the effects of comments on equilibrium while Froeb et al. (2003) and Roy Chowdhury (2009) analyse capacity constraints effects under the Bertrand competition. Those studies also show that capacity constraints induce industrial symmetry.

There are capacity constraints studies that focus specifically on service provider (Burkart, Klein, & Mayer, 2012), retail store (Murray, Gosavi, & Talukdar, 2012), air transportation (Chun & Park, 2016; Evans & Schäfer, 2011; Gelhausen, Berster, & Wilken, 2013) and supply chain (Mayer, Klein, & Seiermann, 2013; Wang & Dargahi, 2013). Genc and Reynolds (2011) identify how capacity constraints affect output equilibrium with wholesale electricity markets, and they find that capacity constraints result in symmetry, in other words, they obtain symmetric supply function equilibrium under capacity constraints. Inderst and Wambach (2001) analyse competitive insurance markets with limited capital. Heo et al. (2013) study the revenue management of the restaurant by highlighting the impacts of capacity constraints.

Capacity constraints is a key issue in industrial organisation theory as capacity constraints impact a firm’s output price, output quantity, output quality, and R&D. Both external and internal factors cause capacity constraints. However, almost all of the prior studies only capture external factors based on symmetric capacity constraints assumption. Many studies on capacity constraints have covered a specific industry, but few of them refer to the restaurant sector. The restaurant industry is unique in the sense that almost all individuals interact with the industry at some point in their life. On the one hand, capacity constraints are evident in the restaurant industry for they have a limited number of seats and business hours. On the other hand, capacity constraints are decided by internal factors and different restaurants have different constraints. Considering these facts, this article highlights the impacts of capacity constraints to the quality competition of restaurant industry by employing an asymmetric capacity constraints model.

3. The model

Because of the low entry barrier, the restaurant industry operates in an almost perfectly competitive market. However, restaurants have horizontal differentiation for the different foods and services supplied by different restaurants and local specificity, which means restaurant competitions are imperfect. Game theory is generally used in imperfect competition (see Chen & Nie, 2014; Chen et al., 2017; D’Aspremont & Jacquemin, 1988; Wang, Nie, Peng, & Li, 2017), so we employ a two-stage Cournot game model.1 We classify consumers into two types, price-sensitive and quality-sensitive, according to their preferences. We assume that there are small and large restaurants, and they compete with asymmetric capacity constraints.

It is possible that both restaurants have the two kinds of consumers, but an acceptable assumption is that consumers of the small restaurant are price-sensitive, while consumers of the larger ones are quality-sensitive.2 A restaurant operating with capacity constraints will be not able to make output decisions by the first order condition of maximising profit. As a rational economic entity, the restaurant operator will make great efforts to minimise loss by modifying the competitive situation. Besides quantity, the two other major endogenous variables are price and quality. Although the easiest way to increase profit is to raise
the price, price strategy has something to do with demand elasticity. Compared to a large restaurant, a small restaurant has larger demand elasticity, which means increasing prices will result in greater loss of consumers and decreases in gross profits. Contrarily, quality compromise will be met by consumer complaints about the large restaurant. Furthermore, a large restaurant has more fixed costs, which also leads the restaurant owner to charge higher prices when faced with capacity constraints. As a result, to increase its profits, a small restaurant will reduce quality, while a large restaurant will raise prices under the condition of capacity constraints.

Denote the small restaurant as restaurant 1 and the large restaurant as restaurant 2. The two restaurants compete with quantity as well as quality, and both quantity and quality impact consumer surplus (CS).

Given the quantity \( q = (q_1, q_2) \) and quality \( x = (x_1, x_2) \), similar to Nie and Chen (2012) and Chen, Wen, and Luo (2016), this study employs consumer utility function as follows:

\[
U(q, x) = (\alpha + \beta x_1)q_1 + \left( \alpha + (\beta + \tau)x_2 \right)q_2 - \frac{1}{2} (q_1^2 + q_2^2) - \gamma q_1 q_2.
\]

Equation (1) shows that consumers’ utility is dependent on both quantity and quality; Consumer utility will increase as the increase of quantity with a decrease speed, while it will increase as the increase of quality with an increase speed. Equation (1) is a common consumer utility function in economics, which is concave in \( q_i \) and has its maximum value. Besides, inverse demand function can be obtained by function easily. \( \alpha \) is a constant. \( \beta \) represent the basic reputation and \( \tau \) is the reputation difference between the two types of restaurant. There are limits for the parameter of benchmark reputation, or \( 0 < \beta < \beta < \beta \). Constrained by the cost function of the firms, \( \beta \) should be not too small. If reputation parameter \( (\beta) \) is too small, then the restaurant has no incentive to make a quality improvement because quality improvement also increases cost. Reputation parameters cannot be too high, as this gives too low or negative equilibrium quantity, since better reputation means higher price. Similar to Wang et al. (2017), a good reputation enables a firm to charge higher prices. To simplify our study, we fix the reputation parameter of the small restaurant to \( 5 \), so we can focus our attention on the reputation difference. Then the CS function can be rewritten as follows:

\[
U(q, x) = (\alpha + \frac{5}{2} x_1)q_1 + \left( \alpha + (\frac{5}{2} + \tau)x_2 \right)q_2 - \frac{1}{2} (q_1^2 + q_2^2) - \gamma q_1 q_2.
\]

Equation (2) shows that a representative CS function is concave in quantity and from (2) we obtain inverse-demand functions of the two restaurants by taking the derivative with respect to \( q_i \):

\[
p_1 = \alpha + \frac{5}{2} x_1 - q_1 - \gamma q_2,
\]

\[
p_2 = \alpha + (\frac{5}{2} + \tau)x_2 - q_2 - \gamma q_1.
\]

The total costs of both small and large restaurants are a function of quantity and quality.

\[
C_i = \frac{1}{2} q_i^2 + \frac{1}{2} x_i^2 + q_i x_i \quad i = (1, 2).
\]
The first and second terms represent the costs resulting from quantity and quality investment, respectively. The last term captures the interaction of quantity and quality investment.\(^7\) Both restaurants operate with capacity constraints, or the products and services a restaurant can supply are less than a certain boundary \(R\) (or \(q \leq R\)). Furthermore, we assume two restaurants compete with asymmetric capacity constraints, or the capacity of the small restaurant is less while the large restaurant has more capacity in general (\(q_1 \leq R_1, q_2 \leq R_2\)). Then, the objective functions of the two restaurants are given as follows:

\[
\begin{align*}
\max_{q_1,x_1} \pi_1 &= (\alpha + \frac{5}{2}x_1 - q_1 - \gamma q_2)q_1 - \frac{1}{2}x_1^2 - q_1x_1, \\
\text{st.} & \quad q_1 \leq R_1 \\
\max_{q_2,x_2} \pi_2 &= [\alpha + (\tau + \frac{5}{2})x_2 - q_2 - \gamma q_1]q_2 - \frac{1}{2}x_2^2 - q_2x_2, \\
\text{st.} & \quad q_2 \leq R_2
\end{align*}
\]

\(R_1\) and \(R_2\) are the capacity constraints of the two restaurants and \(R_1 < R_2\). Equation (5) is a dynamic model, which means two restaurants play a two-stage dynamic game. Both of them make quality-related decisions at the first stage and then choose their optimal quantity at the second stage. All the solutions are obtained by backward induction.

4. Model analysis

Here, we will analyse the base model under four cases: restaurants compete without capacity constraints, a small restaurant with capacity constraints, a large restaurant with capacity constraints, and both of them with capacity constraints. We denote the first order optimum solutions of the two restaurants as \(q_1^*\) and \(q_2^*\).

**Case 1** Assume there are no capacity constraints (or \(R_1 \geq q_1^*, R_2 \geq q_2^*\)). Solving function (5) we get the outputs of the two restaurants, which are the functions of quality investments.

\[
\begin{align*}
q_1 &= \frac{2(3 - \gamma)\alpha + 9x_1 - \gamma(3 + 2\tau)x_2}{2(9 - \gamma^2)}, \\
q_2 &= \frac{2(3 - \gamma)\alpha - 3\gamma x_1 + 3(3 + 2\tau)x_2}{2(9 - \gamma^2)}.
\end{align*}
\]

Substituting Equation (6) to function (5) and differentiating by the quality, we obtain the optimal quality of the two restaurants:

\[
\begin{align*}
x_1^* &= \frac{2\alpha(18 + \gamma^2)[9(9 - 8 - 4\tau^2) - (3 + \gamma)(15 + 2\tau)\gamma + 4\gamma^3]}{729(9 - 8\tau - 4\tau^2) - 27\gamma^2(165 - 24\tau - 20\tau^2) + 9\gamma^4(65 + \tau^2) - 16\gamma^6}, \\
x_2^* &= \frac{2\alpha(18 + 18\tau + \gamma^2)(81 - 45\gamma - 15\gamma^2 + 4\gamma^3)}{729(9 - 8\tau - 4\tau^2) - 27\gamma^2(165 - 24\tau - 20\tau^2) + 9\gamma^4(65 + \tau^2) - 16\gamma^6}.
\end{align*}
\]

And then substituting equations (7) to (6), this study achieves the final representation of quantities of the two restaurants:
Equation (7) implies the following proposition:

**Proposition 1** \( \frac{\partial x^*_1}{\partial \tau} < 0, \frac{\partial x^*_2}{\partial \tau} > 0, \frac{\partial (x^*_1 + x^*_2)}{\partial \tau} > 0; \frac{\partial x^*_1}{\partial \gamma} < 0 \) and \( \frac{\partial x^*_2}{\partial \gamma} < 0. \)

**Proof.** See the appendix. ■

**Remarks:** Reputation difference of the advantaged restaurant stimulates its quality that deters quality improvement by its competitor. That is obvious because reputation advantage means high marginal profits from the quality investment. More importantly, the larger the reputation difference, the more the total quality investment of the industry. In other words, the quality increase of the large restaurant is more than the quality decrease of the small one. The second part of Proposition 1 indicates product substitutability decreases both restaurants' quality. In short, the conclusions of Proposition 1 show that product difference is helpful for quality improvement.

From equation (8), we obtain Proposition 2:

**Proposition 2** \( \frac{\partial q^*_1}{\partial \tau} < 0, \frac{\partial q^*_2}{\partial \tau} > 0 \) and \( \frac{\partial (q^*_1 + q^*_2)}{\partial \tau} > 0. \)

**Proof.** The proof of Proposition 2 is almost the same as Proposition 1. Conclusions are therefore achieved and the proof is complete. ■

**Remarks:** Reputation difference also decreases the outputs of the disadvantaged restaurant. However, just as the reputation difference raises total quality, it increases total outputs at the same time. Reputation difference equals to product difference. Proposition 1 along with Proposition 2 imply that consumers benefit from product differentiation as it increases equilibrium quality and quantity.

**Case 2:** We consider conditions \( q_1 = R_1 < q^*_1 \) and \( q_2 \geq q^*_2 \), which mean only the price-sensitive (or small) restaurant meets capacity constraints. Under this condition, outputs of the constrained restaurant have nothing to do with its quality as it is an exogenous variable. Contrarily, the constrained restaurant makes a quality decision according to its capacity constraints. Equilibrium quality and quantity solutions of the large restaurant can be obtained by backward induction:

\[
x^*_i = \frac{\alpha (45 - 4\gamma^2) [9(9 - 8\tau - 4\tau^2) + (3 + \gamma)(15 + 2\tau - 4\gamma^3)]}{729(9 - 8\tau - 4\tau^2) - 27\gamma^2(165 - 24\tau - 20\tau^2) + 9\gamma^4(65 + \tau^2) - 16\gamma^6},
\]

\[
q^*_i = \frac{\alpha (45 - 4\gamma^2)(81 - 45\gamma - 15\gamma^2 + 4\gamma^3)}{729(9 - 8\tau - 4\tau^2) - 27\gamma^2(165 - 24\tau - 20\tau^2) + 9\gamma^4(65 + \tau^2) - 16\gamma^6}. \tag{8}
\]

**Lemma 1** \( 0 < \tau < \frac{2\sqrt{3} - 3}{2}, \) and \( R_1 < \frac{11}{25} \alpha. \)

**Proof.** See the appendix. ■

**Remarks:** the former part of Lemma 1 implies that reputation difference of the advantage restaurant should not be too large, or it will have no incentive to make a quality investment.
On the other hand, larger $\tau$ requires lower $\gamma$. The latter part of Lemma 1 guarantees capacity constraints of the small restaurant is efficient.

Then we have the following Proposition:

**Proposition 3** $\frac{\partial x^*_1}{\partial R_1} < 0$, $\frac{\partial (x^*_1 + x^*_2)}{\partial R_1} > 0$, $0 < \tau \leq \bar{\tau}$ and $\frac{\partial q^*_1}{\partial R_1} < 0$.

**Proof.** See the appendix. ■

**Remarks:** Capacity increase of the small restaurant reduces the quantity as well as quality of the large restaurant. Interestingly, capacity increase of the small restaurant improves the total quality investment of the industry when reputation advantage of the large restaurant is small. In contrast, capacity increase of the small restaurant decreases the total quality of the industry if the large restaurant has obvious reputation advantage (or $\tau$ is large). The capacity increase has competition effect for it reduces the quality and quantity of the competitor. However, the positive effect of capacity increase of the small restaurant will be offset by the negative effect of reputation disadvantage, which explains why the total quality decreases as the capacity of the small restaurant increases, when the large restaurant competes with large reputation advantage.

**Case 3** If the quality-sensitive (or large) restaurant is faced with capacity constraints or $q_1 = R_1 \geq q^*_1$ and $q_2 = R_2 < q^*_2$, then the output of the large restaurant is an exogenous variable. Resolving function (5) by backward induction, we get the equilibrium quality:

$$x^*_2 = \left( \frac{2(\alpha - \gamma R_2), (3 + 2\tau)R_2}{2} \right).$$

and optimal quantity:

$$q^*_2 = \left( \frac{4(\alpha - \gamma R_2)}{3}, R_2 \right).$$

Using the same reasoning as Lemma 1, we have Lemma 2:

**Lemma 2** $R_2 < \frac{47}{100}\alpha$.

**Proof.** See the appendix. ■

**Remarks:** Similar to Lemma 1, capacity constraints of the large restaurant also has its upper bound, but the upper bound of the large restaurant is larger than the small restaurant.8

**Proposition 4** $\frac{\partial x^*_2}{\partial R_2} < 0$, $\frac{\partial (x^*_1 + 2x^*_2)}{\partial R_2} > 0$ and $\frac{\partial q^*_2}{\partial R_2} < 0$.

**Proof.** See the appendix. ■

**Remarks:** Similar to Proposition 3, Proposition 4 shows the increase in the capacity of the large restaurant decreases the quality and output of the small restaurant. So, the conclusions of Propositions 3 and 4 imply capacity increase of the constrained restaurant reduces the quality and output of the non-constrained competitor. But capacity increase of the large restaurant always increase the quality of the whole industry. In other words, a quality increase of the large restaurant involved by its capacity is larger than the quality decrease of the small restaurant. Furthermore, equations (11) and (12) show the reputation difference has no effect on the quality and quantity of the small restaurant. In contrast to Proposition 3, Proposition 4 illustrates the effects of capacity constraints of the large restaurant have something different from the effects of capacity constraints of the small one.
Case 4 If both restaurants have capacity constraints, or \( q_1 = R_1 < q_1^* \) and \( q_2 = R_2 < q_2^* \), then we have the following solutions of the two restaurants in the equilibrium:

\[
x^*_i = \left( \frac{3R_1}{2}, \frac{(3 + 2\tau)R_2}{2} \right),
\]

\[
q^*_i = (R_1, R_2).
\]

Interestingly, if both restaurants are confronted with capacity constraints, their quality investments are independent of their competitors’ capacity, and only their own capacity impacts their quality, which is distinct from all the other conditions. Comparing the total quality investment of the four cases, we get:

**Proposition 5** Relationships of the total quality investment among different cases are

\[
(x^*_{i,3} + x^*_{2,3}) < (x^*_{i,2} + x^*_{2,2}) < (x^*_{i,1} + x^*_{2,1}) < (x^*_{i,1} + x^*_{2,1}).
\]

**Proof.** Based on equations (7), (9), (11) and (13), we obtain

\[
(x^*_{i,3} + x^*_{2,3}) < (x^*_{i,2} + x^*_{2,2}) < (x^*_{i,1} + x^*_{2,1}),
\]

Conclusions are therefore achieved and the proof is complete. ■

**Remarks:** Proposition 5 provides total quality investment comparisons among the four cases. Quality is the lowest under the condition that both restaurants have capacity constraints. Quality at **Case 3** is more than **Case 4** but less than **Case 1**. Surprisingly, quality investments at **Case 2** are the highest among the four cases. A credible reason is that monopoly is conducive to quality innovation because capacity constraints of the small restaurant enhance the monopoly of the large one, which gives the large restaurant more incentives to make a quality investment.

**Proposition 6** Under the condition that both restaurants are faced with capacity constraints, a price-sensitive restaurant lowers its quality while a quality-sensitive restaurant raises its price.

**Proof.** See the appendix. ■

**Remarks:** A small restaurant cannot increase its prices as most of its consumers are price-sensitive, or the consumers will choose the large restaurant. But the decrease in quality investment lowers its costs. So, the rational choice of the small restaurant is to cut down its quality investment. Because decreasing quality improves profits by reducing costs. The conclusions of Proposition 7 indicate that a new restaurant usually supplies high-quality food and service at the beginning to attract more customers. Once it draws enough loyal customers, it will lower its quality, stealthily. Contrastingly, quality provides larger restaurants with some advantages. The rational choice of the quality-sensitive restaurant is to raise its price for it cannot lower its quality. Interestingly, to make the clients accept a higher price, restaurants will tell them that the reason for the high price is that the prices of the inputs are higher. In other words, restaurants will use higher cost as an excuse for increasing its price under capacity constraints.

Next, we will discuss how capacity constraints impact \( SW \). Let us denote Consumer Surplus and total Social Welfare as \( CS \) and \( SW \), respectively. Then we have:
And function (14) implies the following Proposition:

Proposition 7 The relationships of CS as well as SW among different cases are $CS^{*,3} < CS^{*,2} < CS^{*,1}$ and $SW^{*,3} < SW^{*,2} < SW^{*,1}$.

Proof. See the appendix. ■

Remarks: Capacity constraints impact consumer surplus (CS) and social welfare (SW). Besides, capacity constraints of different restaurants have different effects. CS, as well as SW, is lowest under the condition that both restaurants face capacity constraints. Compared to benchmark, the Cournot competition (or no restaurants have capacity constraints), capacity constraints of the small restaurant decrease CS and SW, while capacity constraints of the large restaurant increase both CS and SW. If we regard capacity constraints of the small restaurant as an increase of the monopoly extent of the large restaurant, while capacity constraints of the large restaurant as an increase of competition, then Proposition 8 illustrates that people benefit from monopoly because monopoly improves CS and SW.

5. Conclusion

Generally speaking, capacity constraints are seen as a disadvantage for the constrained firms. But studies by Esó et al. (2010), Nie and Chen (2012) and Chen et al. (2015) illustrate that capacity constraints are beneficial to disadvantaged restaurants. Following their arguments, this article highlights the effects of capacity constraints on quality and quantity of the restaurant industry by classifying consumers into price-sensitive and quality-sensitive categories. Different from other studies, the capacity constraints of this article are asymmetric, which means different restaurants have different capacity constraints. Based on that assumption, our study shows that the capacity constraints of different restaurants (small restaurants vs large restaurants) have different effects.

We employ a two-stage dynamic Cournot competition model, which is quite common in the industrial organisation literature. The conclusion of this article shows that reputation difference increases total quality investments as well as total outputs at equilibrium. In other words, reputation difference improves CS and SW, which is different from the findings of Tirole (1988) because Tirole’s classical reputation theory shows that less reputation difference means fiercer competition and higher CS, as well as higher SW. Although capacity constraints decrease SW, they give restaurants a chance to improve their profits by increasing the price (quality-sensitive) or reducing the quality (price-sensitive). Therefore, it is not surprising if a restaurant increases price or decreases quality within a short time frame.

Our conclusions are relevant to the health care industry and many other service industries if quality is an integral part of the competition dynamics, which means this study has the potential for broad applications in many industrial sectors, and it also makes a contribution towards an asymmetric capacity constraints theory.
At the same time, this article compares the total quality investment, CS as well as SW among four cases. Results of this study show that total quality investment, CS, and SW are the lowest if both restaurants have capacity constraints. Interestingly, total quality investment, CS, and SW are the highest under the condition when only the small restaurant faces with capacity constraints. In other words, total quality investment, CS, and SW are not the highest if none of the restaurants face capacity constraints.

We set this study to explain restaurant competitions under the industrial organisation framework. A restaurant can cut down its quality or raise its price when confronted with capacity constraints. Both strategies have the obvious consequence of losing consumers. A more suitable choice is to relieve capacity constraints by merger. So, a restaurant may consider merging with another to improve its gross profits when it is confronted with capacity constraints. Some assumptions of this study lack strong economic theories, but they are based on real phenomena. The natural extension of our study is to use empirical data to authenticate our theoretical conclusions. Besides, quality can also cause capacity constraints because high product quality has a high requirement for material inputs. A study focusing on capacity constraints can be expended by quality constraints. Another possibility is that some small restaurants focus on special food and offer high-quality food and service, so any further studies should also pay attention to this phenomenon.

There are three main research limitations for this article: First, we only consider the Cournot competition and ignore other market structure such as the Stackelberg competition; Second, this article only involves quantity input capacity constraints, but quality or both quantity and quality cause constraints. Third, we have not supplied experienced support in this research. All these limitations will be taken into consideration in future research.

Notes

1. Different kinds of oligopoly models such as the Cournot model, the Hotelling model and the Stackelberg competition can also be applied to study duopoly competition, but the Cournot competition is the most suitable one for this study. Because the Stackelberg model is useful for the condition that products difference between different firms are small, or the disadvantaged one will be forced to exit the competition and the Hotelling model has an advantage in costs difference or transportation cost analyses.
2. There are some exceptions, for example, some small restaurants take high quality strategy and they offer high quality food and services. But that are just special cases, so we ignore this type of special small restaurants in this study.
3. There exists the condition that small restaurants have more fixed costs than larger restaurants, but fixed cost increases with firm size is a general conclusion in economics. Besides, we cannot consider all kinds of status or it will be too difficult to capture any valuable conclusions.
4. From firms’ cost functions, it is known that β should be large enough to make \( \beta x_i q_i > \frac{1}{2} x_i^2 + q_i x_i \).
5. \( \beta \) can be equal to 2 or another value, and which has little impact on the conclusions.
6. As we assume two restaurants provide different product (or service), so products substitutability is small in this study. And the existence of \( \tau \) means low products substitutability.
7. This general cost function is widely used in economics studies, see D’Aspremont and Jacquemin (1988), Nie and Chen (2012), Chen et al. (2015, 2017) and Wang and Chen (2017).
8. Our study shows that both of the upper bounds of capacity constraints decrease with substitutability, which means larger product substitutability has more limit to restaurant capacity.
9. The relationship between $x^*_1 + x^*_2$ and $x_{1,1}^* + x_{2,1}^*$ are be obtained by numerical simulation, which are similar to the proof of Proposition 7 and we omit them here. For more details, please see the proof of Proposition 7.

10. Price can be used as a signal for quality, especially in restaurant sector. Higher quality means higher reputation and higher price. So, a restaurant with higher quality has less motivation to lower its price because it operates with more opportunity cost or higher quality restaurant will lose more if it is bankruptcy for quality safety.

Acknowledgements

We sincerely thank the anonymous reviewers for their valuable suggestions, but we take full responsibility for this article.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work is partially supported by Foundation for High-level Talents in Higher Education of Guangdong, GDUPS (2012) and National Natural Science Foundation of PRC (71401057, 71771057, 71573092), Guangdong Social Science Foundation (GD13YLJ02), The Soft Science Project of Guangdong Province (2014A070704008), Collaborative Innovation Center of Scientific Finance & Industry, and Innovative Group Foundation (Humanities and Social Sciences) for Higher Education of Guangdong Province (2015WCXTD009), the key National Social Science Fund of PRC (14AJY020), and the Key Program of National Natural Science Foundation of China (71633002; 71333004).

ORCID

You-hua Chen http://orcid.org/0000-0001-5697-9295

References


**Appendix**

**Proof of Proposition 1**

From equation (7), we have

\[
\frac{\partial x_1^*}{\partial \tau} = \frac{4\alpha(18+\tau^2)(81-45\tau+15\tau^2+4\tau^3)[4\alpha^2+14\alpha\gamma(1+\tau)+\gamma^2]-27(69+60\tau+4\tau^2)}{(729(9-8\tau-4\tau^2)-27\gamma(165-24\tau-20\tau^2)+9\gamma'(65+\tau)+16\gamma'')} < 0
\]

\[
\text{and } x_1^* + x_2^* = \frac{4\alpha(18+\tau^2)(3+9+11\tau+9\tau^2+9\tau^3)+27(30+17\tau^2)+8[18+5\tau+4\tau^2]}{729(9-8\tau-4\tau^2)-27\gamma(165-24\tau-20\tau^2)+9\gamma'(65+\tau)+16\gamma''}.
\]

Then, \(\frac{\partial x_1^* + x_2^*}{\partial \tau} > 0\). For the same reason, we can obtain \(\frac{\partial x_1^*}{\partial Q} > 0, \frac{\partial x_1^*}{\partial \gamma} < 0\) and \(\frac{\partial x_2^*}{\partial \gamma} < 0\) easily. Conclusions are therefore achieved and the proof is complete. ■

**Proof of Lemma 1**

From equations (9)–(10), we have \(3-12\tau-4\tau^2 > 0\) or \(0 < \tau < \frac{2\sqrt{3}-3}{2}\). Capacity constraints mean lower output, so we have \(q_1^* = R_1 < q_3^*\). And min \(\{q_1^* = \frac{a(45-4\tau^2)(9)(9-8\tau-4\tau^2)+(3+\gamma)(15+2\tau-4\gamma)]}{6(9-8\tau-4\tau^2)-27\gamma(165-24\tau-20\tau^2)+9\gamma'(65+\tau)+16\gamma''} \approx \frac{11}{25}a\)

for \(0 < \tau < \frac{2\sqrt{3}-3}{2}\) and \(0 < \gamma < \frac{1}{3}\). So \(R_1 < \frac{11}{25} a\).

Conclusions are therefore achieved and the proof is complete. ■

**Proof of Proposition 3**

\[
\frac{\partial x_1^*}{\partial R_1} = \frac{\partial}{\partial R_1} \left( \frac{a(45-4\tau^2)(9)(9-8\tau-4\tau^2)+(3+\gamma)(15+2\tau-4\gamma)}{6(9-8\tau-4\tau^2)-27\gamma(165-24\tau-20\tau^2)+9\gamma'(65+\tau)+16\gamma''} \right) \approx \frac{47}{100}a
\]

and \(\frac{\partial x_1^* + x_2^*}{\partial R_1} \begin{cases} > 0, & 0 < \tau \leq \overline{\tau} \\ < 0, & \overline{\tau} < \tau < \frac{2\sqrt{3}-3}{2} \end{cases}\). Then we obtain \(\frac{\partial q_2^*}{\partial R_1} = -\frac{4\alpha(18+\tau^2)(9)(9-8\tau-4\tau^2)+(3+\gamma)(15+2\tau-4\gamma)}{6(9-8\tau-4\tau^2)-27\gamma(165-24\tau-20\tau^2)+9\gamma'(65+\tau)+16\gamma''} < 0\) by equation (10).

Conclusions are therefore achieved and the proof is complete. ■

**Proof of Lemma 2**

\[
\min \{q_2^* = \frac{a(45-4\tau^2)(9)(9-8\tau-4\tau^2)+(3+\gamma)(15+2\tau-4\gamma)}{6(9-8\tau-4\tau^2)-27\gamma(165-24\tau-20\tau^2)+9\gamma'(65+\tau)+16\gamma''} \approx \frac{47}{100}a
\]

So \(R_2 < q_2^* = \frac{47}{100}a\) for \(0 < \tau < \frac{2\sqrt{3}-3}{2}\) and \(0 < \gamma < \frac{1}{3}\). Conclusions are therefore achieved and the proof is complete. ■

**Proof of Proposition 4**

Equations 3 and 4 illustrate that \(\frac{\partial x_1^*}{\partial R_2} = -2\gamma < 0, \frac{\partial (x_1^* + x_2^*)}{\partial R_2} = \frac{3+2\tau-4\gamma}{2} > 0\) and \(\frac{\partial q_2^*}{\partial R_2} = -\frac{4\alpha(18+\tau^2)(9)(9-8\tau-4\tau^2)+(3+\gamma)(15+2\tau-4\gamma)}{6(9-8\tau-4\tau^2)-27\gamma(165-24\tau-20\tau^2)+9\gamma'(65+\tau)+16\gamma''} < 0\).

Conclusions are therefore achieved and the proof is complete. ■

**Proof of Proposition 6**

Given the optimal quantity of the two restaurants \(q_1^* = (q_1^*, q_2^*)\) in Case 1 and \(q_1^* = (R_1, R_2)\) in Case 4, we have \(p_1^* = a + \frac{\gamma}{2}x_1^* - R_1 - \gamma R_2 > p_1^* = a + \frac{\gamma}{2}x_1^* - q_1^* - \gamma q_2^*\) if \(x_1^* = x_1^*\). But as a price-sensitive restaurant, the small restaurant cannot raise its price which means the small restaurant should too
keep the price, or $p^{*3}_1 = p^*_1$. So we have $x^{*3}_1 < x^*_1$. For the quality-sensitive restaurant, it should hold its quality, or $x^{*3}_2 = x^*_2$. Then, we have $p^{*3}_2 > p^*_2$.

Conclusions are therefore achieved and the proof is complete. ■

**Proof of Proposition 7**

Proposition 7 is obtained by numerical simulation since the expression is very complex (see the following tables).

**Figure 1.** Numerical simulation of $CS^{*2} - CS^{*3}$. Vertical axis is the value of $CS^{*2} - CS^{*3}, a = 100$ and $R_1 = 44$. Source: Calculated by Mathematica 9.

**Figure 2.** Numerical simulation of $SW^{*2} - SW^{*3}$. Vertical axis is the value of $SW^{*2} - SW^{*3}, a = 100$ and $R_2 = 47$. Source: Calculated by Mathematica 9.
Figure 3. Numerical simulation of $\mathcal{CS}^* - \mathcal{CS}^{+2}$. Vertical axis is the value of $\mathcal{CS}^* - \mathcal{CS}^{+2}$, $\alpha = 100$ and $R_2 = 47$. Source: Calculated by Mathematica 9.

Figure 4. Numerical simulation of $\mathcal{SW}^* - \mathcal{SW}^{+2}$. Vertical axis is the value of $\mathcal{SW}^* - \mathcal{SW}^{+2}$, $\alpha = 100$ and $R_2 = 47$. Source: Calculated by Mathematica 9.

Figure 5. Numerical simulation of $\mathcal{CS}^{+1} - \mathcal{CS}^*$. Vertical axis is the value of $\mathcal{CS}^{+1} - \mathcal{CS}^*$, $\alpha = 100$ and $R_1 = 44$. Source: Calculated by Mathematica 9.
Figure 6. Numerical simulation of $SW^{*,1} - SW^{*}$. Vertical axis is the value of $SW^{*,1} - SW^{*}$, $a = 100$ and $R_i = 44$. Source: Calculated by Mathematica 9.

The results of Figures 1–6 are consistent with the conclusions of proposition 8. Conclusions are therefore achieved and the proof is complete. ■