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The Influence of Heuristic Pre-Arithmetic Games on the Formation of Mental Representations of Arithmetic Concepts

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Abstract

Considering the importance of visual representations (pictograms and ideograms) for the understanding of mathematical concepts, we have developed a system of heuristic mathematical games where emphasis is placed on the development of meaning of arithmetic concepts (mental representations and symbols). The paper shows the results of an experimental research carried out for the purpose of examining the influence of application of a system of didactic games on the development and continuity of mental representations on arithmetic concepts and on understanding the meaning of corresponding mathematical symbols. The research was conducted on a group sample which involved 143 children, between six and seven years of age from three preschool institutions in Serbia. The results obtained through measures of descriptive statistics and t-test for independent variables, showed that the systematic application of didactic games, founded on the principle of discovery of mathematical characteristics, can influence the development of mental images on arithmetic concepts and images, and that schemes formed as such continue to participate in understanding further corresponding mathematical symbols and records. After retesting, through correlation analysis, the results showed that the acquired mental images remained present among children in the experimental group even after the activities of the experimental factor.

Key words: *development of symbolic thought; learning through discovery; preschool age; representation of mathematical concepts.*

Introduction

From the perspective of mathematics teaching, the ability to identify and use relations between numbers, from counting to calculation, is marked as the central aim of preschool education, which should continue its development in primary school (Wittmann, 2001). One of the dominant theoretical and methodological approaches to researching the concept of number is Piaget's theory (Pitta-Pantazi, 2014), which is based on the formation of integers through the ability of logical reasoning (number conservation, seriation, class inclusion, transitivity). Piaget's tradition (also) tends to disregard the importance of counting and direct (intuitive) recognition of s*ubitizing*. Direct recognition of the subitizing is a basic skill that young children should develop (Baroodi, 1987; Marjanović & Mandić, 2009). That is a complex skill which should be developed and practiced through experiences with structured material and representations.

Recent research indicates that a child's understanding of numbers and arithmetic operations increases successively into higher abstract, complex and conceptual structures (Fuson, 1992, 2012). According to Feigenson et al. (2004), approximate images of large numbers and precise images of small numbers make the foundation of numeric intuitions and serve as a basis for sophisticated arithmetic concepts. Over the last few years curricular reforms intensively used the concept *number sense* as an essential outcome of preschool and school programs. However, there is no generally accepted definition of the mentioned phrase. Howden (1989) described the phrase *number sense* as good intuition about numbers and their relationships. That ability gradually develops as a result of research of quantity and its visualization in different contexts. *Number sense* implies the following assumptions (Verschaffel, Greer, & De Corte, 2007): (a) using various representations of numbers, (b) identifying relative and absolute numbers, (c) use of several different units of counting, (g) place value, (d) conceptual understanding of operations, (e) estimates of size, (g) mental calculation, and (h) evaluation of result accuracy.

In terms of early mathematics education, the development of number sense is of key significance: processes of early learning related to number sense have a permanent effect on learning mathematics in school (Jakimovik, Trajanovska, Gogovska, & Atanasova Pachemska, 2013; Krajewski & Schneider, 2009). Tall (2004) refers to the present structure of knowledge, which is the result of previous experience, as "met before". Some of the "met befores" – usually found in well-thought out teaching programs – can be a positive foundation for successful development, while others can lead to conflict in a new context and negatively affect learning. The "met before" theory therefore represents positive and negative aspects in one theory.

Some of the documented issues that students have with arithmetic and algebraic content have their roots in early mathematics education. Research shows that students in primary school, and even in high school, interpret the equal sign incorrectly

(Knuth, Stephens, McNeil, & Alibali, 2006; Zeljić, 2015). The equal sign is often interpreted by students as the "do something sign", rather than the symbol of equality between the left and the right sign of the equation which results in their acceptance of equality such as 4 + 3 = 6 + 1. Researchers conclude that the cause is that children from preschool education encounter the meaning of the equal sign as a command "calculate the value of the phrase" (Falkner, Levi, & Carpenter, 1999; McNeil, Grandau, Knuth, Alibali, Stephens, Hattikudur, & Krill, 2006). After several research projects in which she wanted to examine the problem of interpreting mathematical expressions, Kieran (1992, 1996) speaks of two concepts of mathematical expressions: *procedural* (refers to work with concrete numbers, work focused on the result) and *structural* (operations on expressions as mathematical objects). Kieran (1996) indicates that research showed that students, for example, do not see the point of expressing the sum 8+3, but want to transform that sum into number 11. This way of thinking leads to the conclusion that the expression $x + 3$ has no meaning for students. They think that they should do something with it, but do not know what. Students are not capable of thinking about operations as subject focus. In early arithmetic the expression 4 + 3 is interpreted exclusively as a problem and it is understood as "add number 3 to number 4". Expressed like that, students see the expression $x + 3$ as a process of "adding three" and not as a result in itself. For developing conceptual meaning, the activity of *visual representation of quantitative relations is essential*. That activity, along with the use of various representations for showing problem situations, is seen by many authors as a significant component of early algebraic thought. In that sense, Kieran (1996, p. 275) defines early algebraic thought as "the use of any of various representations which cover the quantitative situation in a relational way". Recognizing and describing quantitative relations using various representations contributes to the understanding of conceptual meaning of an expression as mathematical objects, and not only processes.

Many children have problems with mathematical reasoning, which are usually manifested at the age of 7 or 8, as that point is characterized by a qualitative change of activities for children and in the manner of learning (Carruthers & Worthington, 2003). The authors further emphasize the teacher's tendency to make learning procedural as children become older (around the age of 8, i.e. third grade in the English educational system), so there is an assumption that they can deal with mathematical notations without much difficulty. This tendency neglects the fact that there is a great gap between informal and formal learning and between concrete and abstract thought.

Empirical research showed that the transition from concrete to abstract representations is not easy. Concrete materials ensure a practical context which can activate the real world of knowledge while learning (Schliemann & Carraher, 2002); they can be induced physically and action can be imagined which encourages memory and understanding (Glenberg, Gutierrez, Levin, Japuntich, & Kaschak, 2004). Concrete materials also enable students to develop their knowledge of abstract concepts (Brown, McNeil, & Glenberg, 2009). Despite these advantages, there is a reason for caution as relying solely on concrete objects can lead to the inability to avoid noise, and thus reach one level of abstraction. In other words, that a child is "attached" to concrete materials making it difficult to move towards abstract forms of thinking.

Mathematical concepts are not concrete, but they are characterized by the relationship between *concrete and abstract objects* (Nührenbörger & Steinbring, 2009). We can say that teaching mathematics is founded on the consensus that mathematical ideas are expressed through "external" ways of representation (concrete materials, images/diagrams, spoken words, written symbols) and that in the latter process of learning they are interiorized: mental models or cognitive representations (Cooper & Warren, 2011). Dreyfus (1991) extracts two phases of learning (with respect to using and use of representations): learning begins with using one representation in the first phase towards the ability of flexible use of representations in the final phase. Dreyfus and Eisenberg (1996) claim that smooth and flexible use of "structurally same", multiple representations of mathematical concepts, is related with conceptual understanding of those concepts. Mathematical relations, principles and ideas can be expressed in several structurally equal representations including: *a) objects from the real environment, b) visual representations (i.e., images, diagrams), c) verbal representations (written and spoken language) and d) symbolic representations (numbers, letters).* In the process of developing abstraction and generalization, Cooper and Warren (2011) advocate the use of various representations where representations develop from presenting situations in the real world to abstract diagrams. In the process, the order of representation use is very important as "each subsequent should compensate the limitations of the previous" (Cooper & Warren, 2011, p. 210). Goldstone and Son (2005) developed a theory of "concreteness fading" as a "process in which concreteness of representation progressively decreases and a relatively idealized and de-contextual representation emerges which is still clearly related to physical situations of the model". Representations which contain many details of original experiences are referred to as *perception-based representations* (Van Oers, 2010). Expressive and communicative representation implies emphasis of what is significant (Terwel, Van Oers, Van Dijk, & Van den Eeden, 2009). Authors point out that *representations founded on meaning* are more abstract than mere presentation of reality.

Recognizing the influence of semiotic representations on cognitive activities represents the core of the implementation of representation theories (Duval, 1999). That is the most interesting point of analysis of representation characteristics. When children are actively involved in the development and evaluation of representations, they develop mathematical knowledge, which enables them to generate new processes of problem solving (Terwel et al., 2009). Different representations enable children to create relationships with their everyday environment. They represent mathematical tools which enable children to structure the physical world and understand the realistic world which surrounds them. The sense of a number can be meaningful in some contexts and can direct children towards numerical sense (Mix, Huttenlocher, & Levine, 2002). In short, a child first learns the meaning of the concept of number as contextually dependent concepts, which later become interrelated and finally result in a mature set of meaning (Fuson & Hall, 1982).

At the preschool level, children develop basic mathematical skills through play which supports individual strategies in mathematical problem solving (Schuler, 2011; Slunjski & Ljubetić, 2014; Stebler, Vogt, Wolf, Hauser, & Rechsteiner, 2013). "Guided play" implies not only organization of the play activity – e.g., selection of appropriate tools and games – but also the stimulation of discussion and mathematical thinking and reasoning (Hirsh-Pasek, Golinkoff, Berk, & Singer, 2009), as well as the development of children's ability for the process of game (Samuelsson & Carlsson, 2008; Van Oers, 2010). Several authors cite results which speak in favor of using games for improving children's basic numeric skills and for developing motivation during math learning (Cutler, Gilkerson, Parrott, & Bowne, 2003; Gerdes, 2001; Young-Loveridge, 2004). In a longitudinal research, Poland and associates (Poland, van Oers, & Terwel, 2009) researched ways in which children (5-year-olds) learn to structure and express relationships in the real world in the context of play, and how teachers in early education support finding means of communication in such relationships. The research (also) showed that teachers can actively help little children to create structural symbolic representations (such as schemes, diagrams, pictograms, etc.) which can be used for communicative purposes and for organizing activities in the real world.

The mentioned assumptions on developing meaning for initial arithmetic concepts, the role of well-structured representative systems in the process, the significance of play as a context for developing arithmetic concepts, served as a foundation for developing a system of pre-arithmetic games whose didactical value was experimentally tested.

Methodology

Starting with the fact that the preschool period should be understood as traditional and where conditions should be ensured for fine didactic transitions from concrete to abstract manner of thinking using iconic representations, our research was directed towards improving strategies for developing arithmetic concepts at the preschool level. The general foundation of the preschool program in the Republic of Serbia recommends that each preschool institution creates its own preschool program. The content relating to the concept of number are the following: counting, unconventional and conventional symbols for numbers to the number 10, number sequence, comparison of number sets, mathematical symbols for expressing equality and inequality of sets, mathematical operations of addition and subtraction within the first 10, and ordinal numbers.

In searching for adequate methods, procedures, techniques and instruments through which content of preschool math would best be put into the function of child development, and forming representations of initial mathematical concepts, we have opted for the *heuristic approach* and referred to the developed games as *heuristic mathematical games*. The foundation for developing the new system of didactic games were Equation games by academic Marjanović (2001). The phrase *heuristic mathematical games* comprises three characteristics: a) *discovery* – as a method or manner in which concepts can be formed at this age level, b) *mathematical characteristics of objects* and the situation of surrounding reality through which we influence the formation of representations of mathematical concepts and c) *play* as a procedure or activity through which children develop mathematical concepts. When we say *discovery*, we primarily refer to the revelation of mathematical characteristics of objects from the real environment, and then on the discovery of relationships between mathematical concepts. Through careful selection of examples from the surroundings, familiar to the preschool child we ensure conditions for isolating important mathematical characteristics which all representatives of a class have. That "discovery" participates in further classification of the concept, equation with other examples of the same class or differentiation of the example from the non-example or counterexample of the given concept. Techniques of discovery in heuristic games are founded on: observation and comparison by similarity (equation games), *extraction founded on differences* (games of visual discrimination), assessment and evaluation (experimental games). Some games are founded on principles of combinatorics (combinatorial games). Representation games are dedicated to *discovery of practical meaning of mathematical concepts*. As the mathematical characteristic became a criterion for classifying other examples, we enable children to discover *new examples* for the formed concepts. *Creation games* serve that purpose and are founded on a preschool child's imagination.

Learning through playing is founded on the observational and manipulative experience of children in contact with objects, representations of objects and real-world situations. Mathematical characteristics, which are recognized in contact with examples from the surroundings represent the *basic content of the game* for the formation of mental representations of mathematical concepts, while formed mental representations and concepts are seen as final content ("products") of children's thought efforts. At the preschool age, insight into mental representations of mathematical concepts is established through activities, drawings or models. Children's drawings correspond very well with mental representations which are formed, while symbolic coding is characterized by use of language.

The issue at question is the nature of iconic tools which stimulate the development of appropriate mental representations which are an adequate foundation for further learning and development of arithmetic concepts (Fuson 1992; Krajewski & Schneider, 2009; Tall, 2004). The aim of early arithmetic education is to develop number sense (Howden 1989; Verschaffel et al., 2007), which implies the development of good intuition on numbers (Howden, 1989) and the formation of adequate mental representation of numbers (Marjanović & Mandić, 2009; Verschaffel et al., 2007) and their structure. Realistic situations are observed as an example for the concept, iconic symbol for carrier of meaning for the concept, and symbols as a label for the concept. Accordingly, through the system of games children are offered various images of realistic situations, abstract iconic representations and corresponding symbols (Figure 1). In presenting numbers, the structure of the number is emphasized (Verschaffel et al., 2007), for example 4 = 2 + 2 and 4 = 3 + 1.

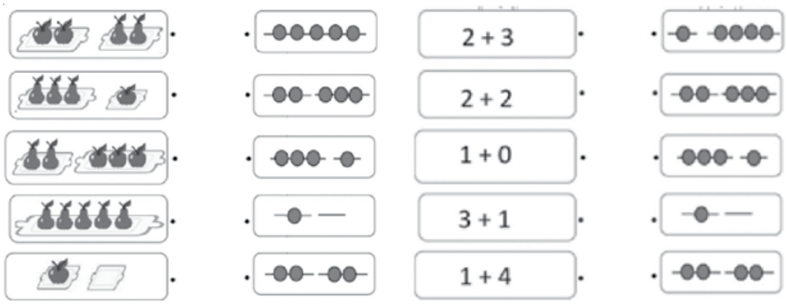

Figure 1. Equation of realistic representation and abstract representations of numbers. Equation of abstract representations and symbols

Aim and Tasks

The aim of the research was to examine the influence of a systematic application of heuristic pre-arithmetic games on the formation and duration of mental representations of arithmetic concepts and understanding of meaning of corresponding mathematical symbols among six and seven-year-old children.

The particular research tasks can be defined in the following way:

- examine the degree of development of iconic representation of arithmetic concepts among six and seven-year-old children prior to the application of the heuristic pre-arithmetic games;
- examine the correlation between the development of iconic representations and understanding of meaning of corresponding mathematical symbols and notations;
- examine the influence of the system of heuristic games on the duration of iconic representations of arithmetic concepts.

Sample of Participants

The research sample was a group or cluster made up of eight preschool groups, i.e. 143 children between the ages of six and seven from preschool institutions in Serbia. In line with the technical and organizational conditions and abilities of the researchers, the research included six and seven-year-olds from preschool institutions in Vršac, Belgrade and Aranđelovac. The selection of groups was nonrandom: the research requires cooperation with teachers and their long-term engagement, and it was realized in cooperation with teachers who showed willingness to participate.

Methods, Procedures, Techniques and Research Instruments

According to the aim and research tasks, we opted for the *experimental research method*. For examining the influence of heuristic games on the development of mathematical concepts among children we used an experiment with several parallel groups, i.e. four experimental (seventy-one participants) and four control groups (seventy-two participants). The *experimental factor* consisted of a system of heuristic pre-arithmetic games which we have created, while using careful control of hindering factors the majority of organizational elements of teaching and education was aligned in both the control and experimental groups. The *independent variables* of the experimental research were the system of heuristic didactic games, treated as the experimental factor, and according to the type of data, they were a category variable, operationally defined in two groups: *Experimental group* – the group in which the experimental factor is introduced (system of heuristic pre-arithmetic games); *Control group* – the group with an absent experimental factor. The *dependent variables* were: 1) Illustrations of iconic representations of arithmetic concepts (numeric variable, operationally defined with the test score: I – initial test and F1 – the first final test); 2) Understanding the meaning of corresponding mathematical symbols and notations (numeric variable, operationally defined with the test score F2 – second final test); 3) Duration of the formed representations of arithmetic concepts (numeric variable, operationally defined with the score on the questionnaire R – retest).

After preparing the research, we conducted a pilot test on the entire sample of children in preschool institutions. After the analysis of data, we approached the organization and control of the so called hindering factors which could have an influence on research results. Equation of groups was carried out according to the following criteria: *children's achievement on the initial test, time spent in kindergarten, teacher's estimate on the degree of development of mathematical concepts among children, family type and parents' education,* considering that the control and experimental groups were primarily equated by *age, work plan and program* anticipated for the school preparation period and most organizational elements of teaching and education. Prior to introducing the experimental factor, we conducted a three-day training session with teachers on the application of selected heuristic mathematical games. For the standardization of groups we used the *guided interview* technique with teachers in the control and experimental groups, *technique of scaling* for standardizing participants according to teachers' estimates, length of stay in a kindergarten, type of family and parents' academic degrees. For testing the initial state and verification of research results the *testing* technique was applied. In addition to the *guided interview protocol* and *estimate scale,* we applied specifically designed *picture tests* created by the author Aleksandra Mandić, first published in the doctoral dissertation (Mandić, 2013). *Test P* (pilot test) was intended for testing children's ability to: *count and represent numbers to 5*; *count and represent numbers to 10; structure of numbers to 10; conservation of initial numbers; understanding of notions of more, less, equal.* Tests I, F1, F2 and R examined children's achievement in recognizing: *additive schemes for adding and subtracting, adding three numbers, intuitive adding of two numbers, intuitive subtraction of two numbers; understanding the meaning of the word; representation of growing and falling sequence of numbers up to 5.*

The objective of the initial test was to indicate the *initial state and degree of development of mental representations of arithmetic concepts* among preschool age children, which would be learned more formally during the first grade of primary school. The first final test was used to establish the *final state and influence of heuristic mathematical games on the formation of mental representations of arithmetic concepts*. The second final test was used to indicate the *correlation between formed representation and achievement in equation with corresponding mathematical symbols and notations.* The retest was used to provide *information on the duration of formed representations and understanding the meaning of corresponding mathematical notations 30 days after the experimental intervention*.

Within the framework of *statistical data analysis*, *metric characteristics of applied tests* were analyzed first (calibration of measuring instruments). For that purpose, the following was calculated: *reliability of tests* through Cronbach's α coefficient; *influences of each individual task on the reliability of the entire scale (test)* to which it belongs (α without the given task) and *indexes of discrimination* (corrected item-total correlation) which indicates the degree in which each independent task is able to differentiate participants who achieve higher scores on the task from participants who achieve lower scores on the same task. For the assessment of achievement on the tests *measures of descriptive statistics* were calculated and considered: a) *Measures of central tendency* (arithmetic mean, median and mode); b) *Measures of deviation* (standard deviation, variance, range, skewness, kurtosis, minimal and maximum value, Kolmogorov-Smirnov test). In the comparison of the effects of the experimental and control group the following *methods of establishing statistically significant differences in* arithmetic means were used: a) *t-test for independent samples,* that compared achievements of the experimental and control group on each individual test; b) t-*test for paired samples*, which compared achievements separately for the experimental and control group in order to determine tendencies in both groups between two measures (tests); c) c*orrelation coefficient* between two measures in the same group in order to observe the trend in scores within one group. In testing the consistency of developed mental images, we checked the correlation between final tests and the retests. For that purpose, the *Pearson correlation coefficient* was calculated. Finally, the correlation of results on all of the respective tests was calculated with respect to the socio-demographic variables as external (criterion) variables. For that purpose, the *point-biserial correlation coefficient* was applied.

Results

Table 1 shows the results of correlation of external socio-demographic factors on children's test achievements.

Table 1
Correlation of socio-demographic factors and children's achievements on tests

Group		P_score	I_ score	F1_ score	F2_ score	R_ score
Experimental group	Length of stay in kindergarten	.118	.103	.049	-.085	-.016
	Mother's education	.107	.305**	.154	.121	.221
	Father's education	.145	.206	.075	.117	.102
	Teacher's estimate	.386**	.242*	.299**	.360**	.453**
Control group	Length of stay in kindergarten	-.079	.150	.141	.110	.141
	Mother's education	.130	.503**	.313**	.295*	.313**
	Father's education	.157	.258*	.329**	.234	.329**
	Teacher's estimate	.660**	.484**	.567**	.435**	.567**

** - statistically significant correlation at the .01 level
* - statistically significant correlation at the .05 level

In testing the *degree of development of mental images on arithmetic concepts prior to the application of a new game system,* measures of descriptive statistics for the P scale (i.e., results which participants in the control and experimental group achieved in the pilot test) are within the range of .00 to 14.00 for the experimental and 1.00 to 17 for the control group. The values of arithmetic means M= 10.95 (experimental group) and M= 10.88 (control group) show that children successfully solved more than half of the tasks which the P scale contains. That is indicated by the skewness values, which is negative, thus showing that a greater number of participants is placed on the part of the scale which is higher than the arithmetic mean (skewness= -1.22 in the experimental and skewness= -.95 in the control group). Kurtosis, on the other hand, shows that within the experimental group (kurtosis=1.77) there is a leptokurtic distribution, while it is minimally flat or platykurtic in the control group (kurtosis=.03). The Kolmogorov–Smirnov test (K-S = 1.38 in the experimental and K-S = 1.55 in the control group) for both groups is statistically significant at the .01 level and shows that the results are grouped on a smaller part of the scale and therefore we cannot expect a normal distribution (as is the case in almost all mathematical tests which are given to children in the transition period when they are only developing particular mathematical concepts such as number, its consistency and relation between values). The results show that none of the items stand out with respect to their values in the groups tested. Differences obtained suggest that a statistically significant difference is not present at any of the significance levels. In other words, we can conclude that the *groups are equal in the sense of achievement on tasks on the pilot test. Descriptive statistics of results for scale I* show that their achievement scores are in the range from .00 to 35.00 in the experimental group and .00 to 36.00 in the control group. Values of arithmetic means M = 6.05 (control group) and M = 3.91 (experimental group) show that children successfully resolved most tasks. Still, the distribution, according to the positive sign of skewness, moved towards higher scores, i.e. that there is a number of children who contribute to a positive deviation of the scale in relation to the arithmetic mean of both groups (skewness= 1.87 in the experimental and skewness= 2.88 in the control group). We emphasize that the deviation is somewhat higher in the control group. In addition, kurtosis shows that for both groups participants are placed in a narrow range of scores which leads to a leptokurtic distribution (kurtosis= 3.49 in the experimental and kurtosis= 1.41 in the control group). The Kolmogorov–Smirnov test (K-S = 1.85 in the experimental group and K-S = 36.00 in the control group) is statistically significant at the .01 level in both groups and shows that the results are grouped on the smaller part of the scale and that a normal distribution cannot be expected (as is the case in almost all mathematical tests which are given to children in the transition period when particular mathematical concepts such as number, its consistency and relation between values are being developed). Values of the arithmetic mean are rather low with respect to the total possible range, in both the control and experimental group. The general conclusion regarding the comparison of the two groups is that children in the experimental group achieved somewhat better scores, but not significantly. Taking into consideration all of the results of the initial test, we can say that *the achievements of children in the initial test in both groups are rather weak. The results confirm that children did not sufficiently develop the notion of number in the sense of formed mental images nor do they have developed mental schemes on which adding and subtracting operations are based.*

The influence of games on the degree of development of iconic representations, and the understanding of meaning of mathematical symbols and notations was tested based on the results of final tests F1 and F2, which should show the influence of the experimental factor on better achievement on final tests. Measures of descriptive statistics for the F1 scale are in the range of .11 to 34.00 for the experimental group and .00 to 34.00 for the control group. Values of arithmetic means M = 32.46 (experimental group) and M= 8.00 show that participants in the experimental group are far better in solving tasks in this test. Skewness shows that the results of the experimental group are turning towards scores higher than the arithmetic mean (skewness = -3.98), and towards lower scores within the control group skewness = 1.22. Furthermore, kurtosis indicates that within the experimental group (kurtosis = 19.02) there is a tall or leptokurtic distribution which is significantly more elongated than for the control group. The control group also has a leptokurtic distribution, but of a weaker intensity (kurtosis = 1.43). The Kolmogorov–Smirnov test (K-S = 2.93 in the experimental and K-S = 34.00 in the control group) is statistically significant in both groups at the .01 level and shows that the results are grouped on a smaller part of the scale and therefore a normal distribution cannot be expected. The value of the *t*-test for independent samples shows that participants in the experimental group are far more successful in solving tasks in the F1 test. *Participants in the experimental group were statistically significantly more successful (at the level $p<.01$) at all tasks on the F1 test, which directly speaks of the efficiency of the applied system of games.*

Measures of descriptive statistics for the F2 scale are in the .11 – .37 range for both groups of participants. Values of arithmetic means M = 33.54 (experimental group) and M= 5.09 (control group) show that *children in the experimental group were far more successful in solving tasks in the second final test.* Skewness indicates that the scores of the experimental group are moving toward scores higher than the arithmetic mean (skewness = -3.55), and towards lower scores within the control group skewness = 2.23. Kurtosis indicates that within the experimental group (kurtosis = 13.63) the distribution is rather tall or leptokurtic, which is significantly taller than in the control group. The control group also has a leptokurtic distribution. It is of a smaller intensity, but does not lead to a significant deviation from the normal distribution, even in the group with far lower scores (kurtosis = 4.43). The Kolmogorov–Smirnov test (K-S = 2.58 in the experimental and K-S = 148.00 in the control group) in both groups is statistically significant at the .01 level and shows that the results are grouped on a smaller part of the scale and therefore a normal distribution cannot be expected. The value of the t-test for independent samples indicates that *children from the experimental group were more successful in solving tasks from the final test (F2).*

The mentioned conclusions are confirmed through comparison of achievements of participants in the experimental and control group in all items of the F2 test. Participants in the experimental group are statistically more significant (at the level of p<.01) in all tasks of the second final test F2.

For testing continuity of formed images, a correlation analysis was applied. If there is a relatively high correlation between the results of final tests which are applied in the group with the experimental factor, a stable relationship between the results of the final test and results on the retest should be established and should indicate that the formed mental images are not of a momentary character, but persist in children's cognitive structures.

Table 2

Correlation between the results on applied tests

Group	P	I	F1	F2	R
Experimental group	1	.418**	.466**	.571**	.548**
		1	.184	.220	.151
			1	.459**	.837**
				1	.724**
					1
Control group	1	.486**	.535**	.270*	.535**
		1	.838**	.692**	.838**
			1	.773**	1.000**
				1	.773**
					1

** - statistically significant correlation at the *.01* level
* - statistically significant correlation at the *.05* level

Table 2 shows the correlations between results on the applied tests. It can be seen that almost all of the correlations are statistically significant at the .01 level of statistical significance. In other words, participants' achievements on all tests are highly correlated for children in both groups (control and experimental). This is not the case for the results achieved on test I (initial) within the experimental group. Within the control group, all results are aligned and highly correlated with each other (except for correlations F1a and P), which may indicate that children in the control group advance more in a certain degree, and with the increase in their knowledge there is an increase in scores in the final tests, and thus a decrease of alignment with previous testing and results). As for the experimental group, the results on test I are only statistically significantly correlated with the result on the P test. These two tests were applied prior to introducing the experimental factor when the system of existing mental images in children was approximately identical. The other three correlations in the experimental group of results on the I scale (with results on scales, F1, F2, and R) are statistically not significant at any level while they are very high and statistically significant at the .01 level between final tests (F1 and F2) and the retest (R).

Discussion

The results which show a correlation between socio-demographic factors with achievements of children indicate primarily that *teachers' estimates are a valid external criterion for estimating results* that children achieve on tests regardless of their exposure to the experimental factor (in form of particular preparation material and practice). That means that teachers are very well informed about the development of mathematical concepts of children with which they work. On the other hand, in the experimental group, all variables are not correlated with test achievement (*except with mother's education*) while in the control group all variables are statistically significantly correlated. Therefore, children whose parents have higher level of education have better developed images and understand the meaning of elementary mathematical concepts than children whose parents are of a lower academic level. The only variable which is *not a predictor of success* on all the tests is *length of stay in the kindergarten*, based on which we can conclude that time spent in kindergarten is mostly filled with basic care of children. Education and nurturing are inseparable in preschool education and exclusion of either is not conducive to the quality of child development. Emphasis on the care component leads to the situation where time spent in kindergarten does not influence the development of mathematical thinking and skill, however, we must stress that the aim of learning mathematical content should not be focused on the development of procedural skills (the final result in the sense of correct and incorrect answers), but on the development of logical-mathematical thinking of children through the process of integrated learning based on games or spontaneous learning and discovery, which enriches a child's overall potential.

The results obtained through the application of the pilot test show that participants *successfully count to 5, that they have developed the ability to conserve the number and correctly compare numbers to 5.* The results of the initial test show that children: *do not differentiate between manners of grouping elements, do not have formed adding schemes* *based on which operations of addition and subtraction are based, and that they do not notice complex ways of grouping, which correspond to adding three numbers.* The picture test tested the ability of iconic addition and subtraction where children, regardless of group belonging, *showed rather weak results.* The pilot and initial test also showed a high degree of lack of understanding the meaning of words with which we present ordinal numbers and the notion of pair. Children at the age of 7 *have not developed images on growing and falling sequence of numbers to 5, but relate their images with the notion of height. If we take into consideration that the majority of children at an early age learn to count within their family environment and learn to compare numbers to 5, the issue that arises is the content of mathematical activities within preschool institutions.*

The qualitative analysis confirms and shows that children have not adequately developed *number sense.* Association with exclusively concrete representation which contains many unimportant details (Van Oers, 2001) leads to mistakes in new problem situations, as children have not formed adequate mental images (Terwel et al., 2009). This claim is supported by results on the task where we ask that children circle the picture with fewer fruits. The majority of children, in both groups separated the tomato fruit associating image of number on the dimension and size of the object shown. Figure 2 shows a typical example of a correctly observed set with a fewer number of elements, neglecting the types of objects whose numbers are compared, and an example of not understanding the meaning of the word *less*.

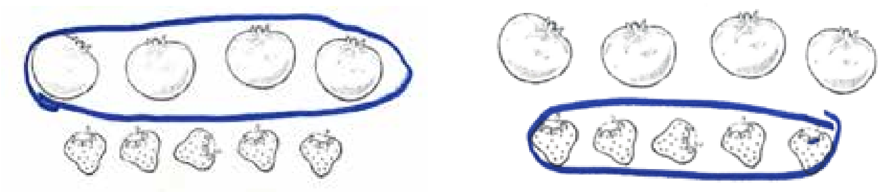

Figure 2. Comparison of sets in terms of number

Analysis of results where we asked that they *color as many circles as there are pearls on the bracelet* showed that children reacted in two ways. Most children colored the circles from left to right, without skipping circles, and a smaller number of children colored in by skipping circles in the sequence (Figure 3). This occurrence can be associated with the image itself where the pearls are grouped with two gaps in the middle. That shows that *some participants find that the manner of grouping is important for the total number of elements in a set* (Freudenthal, 1991; Marjanović & Mandić, 2009; Verschaffel et al., 2007). We hold that during this period, children should be offered activities and didactic games through which the manner of grouping as a noise will be abstracted, and clearer mental images of numbers developed (Feigenson et al., 2004; Howden, 1989; Verschaffel et al., 2007). Such representations cannot be a foundation for introducing and using structured didactic material (Freudenthal, 1973), and do not represent a solid foundation for further learning (Fuson 1992; Krajewski & Schneider, 2009; Tall, 2004). *Representations of numbers to children contain redundant details* (Van Oers, 2001) such as space between two objects.

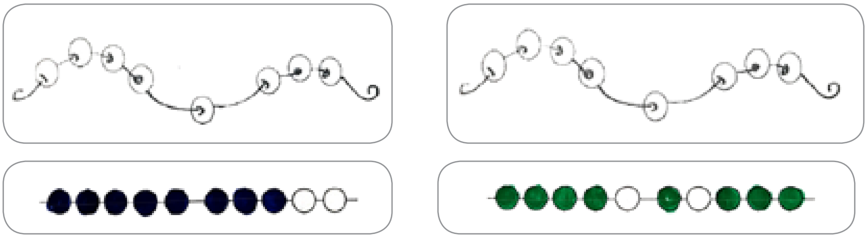

Figure 3. Representation of number 8

The last task in the initial test showed the extent to which there is a developed representation on the ascending and descending sequence of numbers to 5. The majority of children when asked to *add extra or fewer circles in each succeeding jigsaw* reacted by drawing only one circle, moving by one cell up or down (Figure 4). It is evident that *the participants associated the concept of one more or less with spatial dimensions and the meaning of the word higher and lower,* i.e. representation of quantity is observed as object that is arranged by length or height. Such wrongly formed representations (Howden 1989; Verschaffel et al., 2007) are contributed by teachers' inappropriate examples, but also in examples of materials for preschool age children. We frequently encounter two trees of various height between which is a relational symbol for *greater* or *smaller.* The consequences of such an approach have also occurred in the initial testing (Tall, 2004). The following picture shows our observation.

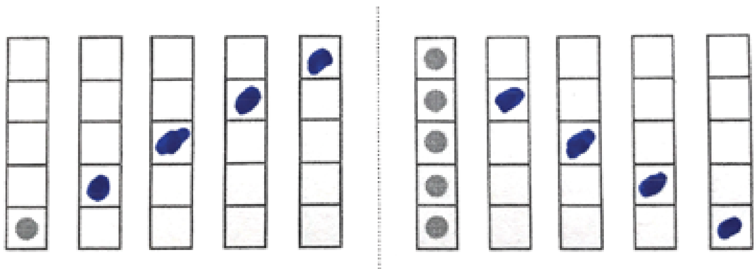

Figure 4. Incorrect representation of a growing and falling number sequence to number 5

The results of the first final test showed that systematic application of heuristic pre-arithmetic games significantly contributes to the development of mental images of arithmetic concepts. Participants in the experimental group, within which they had the opportunity to become acquainted with content of created games over a period of three months, significantly *developed their mental representations of additive schemes, developed the ability to add and subtract at an iconic level, entirely understood the meaning of ordinal numbers and developed an idea of ascending and descending sequence of numbers to 5*. The results of the first final test show that children between the ages of six and seven are *entirely ready for developing arithmetic concepts that were mentioned, and that their achievement is not defined with the family type, length of stay at kindergarten nor parents' level of education.* On the other hand, the results of the second final test show that *formed iconic representations contribute to the understanding of corresponding mathematical symbols and notations.* Essentially, we can conclude that a *child's success is mostly defined by adequate teaching and educational influences, methods and techniques as well as representative examples on which the formation of mathematical concepts will be based.*

The qualitative analysis of results from the control groups showed that *wrong representations of elementary arithmetic concepts included in the tests are permanently contained and transferred to all tasks of the same type* (Tall, 2004). For clearer understanding, we show the most frequent incorrect answers related to the meaning of the word *pair* (Figure 5) and for one more (less) (Figure 6).

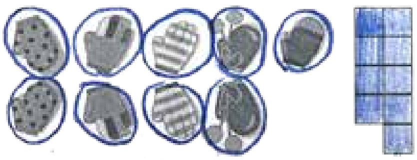

Figure 5. Concept of pair

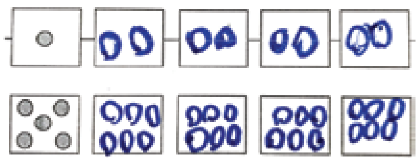

Figure 6. Ascending and descending sequence of numbers to 5

The achievement of children in the control group shows that the *systematic approach can form representations based on meaning* (Cooper & Warren, 2011; Feigenson et al., 2004; Goldstone & Son, 2005; Kieran, 1992; Schliemann & Carraher, 2002), *and children can be lead to understand mathematical symbols correctly.* Presenting quantity through various representation systems (Cooper & Warren, 2011; Dreyfus, 1991; Goldstone & Son, 2005; Nührenbörger & Steinbring, 2009) by associating structurally equal representations, through emphasizing a characteristic example, *leads to the development of meaning of initial arithmetic concepts and formation of adequate mental representations* (Baroodi, 1987; Feigenson et al., 2004; Marjanović & Mandić, 2009; Terwel et al., 2009; Verschaffel et al. 2007). The activity of recognizing structurally equal representations with abstracting mathematical relations proved to be an important activity for developing mathematical thinking.

Achievements of children in the experimental group in the second final test also show that *children between the ages of six and seven are ready for reading and understanding mathematical symbols through which we present numbers, sums, differences, sums of three numbers, mental addition and subtraction, and understanding mathematical notations of arithmetic operations of addition and subtraction.* That can be achieved through careful development of representative systems and systems of children's mental representations successively into higher abstract, complex and conceptual structures (Fuson, 1992). Although it is often believed that at this age much cannot be achieved in the area of preschool mathematics, the results obtained through this research speak that *children prior to attending school are ready for establishing and understanding the meaning of basic arithmetic rules such as exchange of addends or understanding zero as a neutral element.* Recognition of appropriate schemes, as in the following task (Figure 7) shows that children accepted the notation as an independent object, which can represent a problem even to much older children (Falkner et al., 1999; Kieran, 1992; Knuth et al., 2006; McNeil et al., 2006). The meaning of these notations is presented through abstract representations which are not based on images of real-life situations in previous phases of learning (Cooper & Warren, 2011; Feigenson et al., 2004; Goldstone & Son, 2005; Nührenbörger & Steinbring, 2009; Zeljić, 2015). We note that *children were equally successful in independent symbolic expression, i.e. writing notations.*

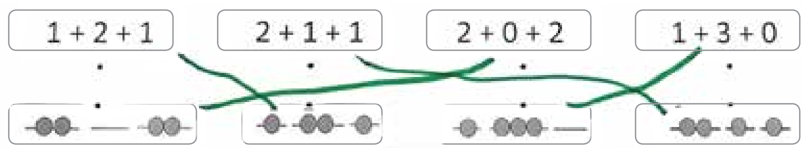

Figure 7. The meaning of notation through reliance on additive schemas

After retesting and statistical analysis of data we obtained results which indicate that *the developed and acquired mental representations remained present with children in the experimental group and after implementing the experiment.* These findings are to the advantage of the idea of forming mathematical concepts which are founded on meaning and visualization of instances in surrounding reality, but also confirm the advantage of heuristic methods and learning through discovery over traditional methods of teaching and education which is mostly based on relating facts and set knowledge. *Based on these findings we can conclude that formed mental representations remain permanently in a child's memory and represent a foundation for further learning and development of arithmetic concepts and arithmetic thought* (Fuson 1992; Krajewski & Schneider, 2009; Tall, 2004).

Conclusion

Preschool and primary school education are characterized by different conditions and organization of the learning processes. These differences presuppose different demands regarding the system of related and coherent mathematical content and activities in the transition from preschool to primary school. Relational understanding of numbers and arithmetic operations is an important aim of mathematics education during the mentioned transition. The results of our research indicate that systematic application of heuristic pre-arithmetic games influences the formation and continuity of iconic representations of arithmetic concepts, contributes to the understanding of meaning of corresponding mathematical symbols and notations, and also develops a child's ability to use new techniques of learning through discovery. Through the application of the new system of didactic games it is possible to alleviate the "semantic step" which children are exposed to with starting primary school. On the other hand, the results obtained confirm the practical, didactic value of the developed games, research instruments and point to possibilities of innovation and improvement of teaching in the process of preparing children for school. Significant success of children in the application of the new technique of learning through discovery and the continuity of mental representations once again indicate the advantage of the heuristic approach in forming mathematical concepts over traditional methods of work. We are certain that the conclusions we bring based on theoretical and experimental research open possibilities for further development and interpretations of arithmetic and geometry content for preschool children. The boundaries and possibilities of a preschool child have not been researched to date. The results we show indicate the ability to understand mathematical symbols and elementary mathematical notations, however, it would be of importance to research the extent to which children are capable of doing arithmetic with numbers from 10 to 20. In most countries children start school at the age of six, so the basic question is what children gain or lose with the introduction of arithmetic during preschool education. New technologies offer possibilities of implementing educational games for everyday activities of children, so the results of this research could serve as a foundation for creating educational software which can be used at the preparatory preschool age. In that way the process of forming mathematical concepts and the process of assessing their development would be entirely individualized.

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Utjecaj heurističkih predaritmetičkih igara na oblikovanje mentalnih reprezentacija o aritmetičkim pojmovima

Sažetak

Imajući u vidu značaj slikovnih reprezentacija (piktograma i ideograma) za razumijevanje značenja matematičkih pojmova, razvili smo sustav heurističkih matematičkih igara u kome je naglasak na razvijanju značenja aritmetičkih pojmova (mentalnih predstava i simbola). U ovom radu izloženi su rezultati eksperimentalnog istraživanja provedenog s ciljem da se ispita utjecaj primjene kreiranog sustava didaktičkih igara na razvoj i trajnost mentalnih reprezentacija o aritmetičkim pojmovima, kao i na razumijevanje značenja korespondentnih matematičkih simbola. Istraživanje je provedeno na grupnom uzorku u koji je bilo uključeno sto četrdeset troje djece uzrasta od šest do sedam godina iz tri predškolske ustanove u Srbiji. Rezultati istraživanja, dobiveni primjenom mjera deskriptivne statistike i t-testa za nezavisne uzorke, pokazali su da se sustavnom primjenom didaktičkih igara zasnovanih na otkriću matematičkih svojstava može utjecati na oblikovanje mentalnih predstava o aritmetičkim pojmovima, te da tako oblikovane predstave i sheme dalje sudjeluju u razumijevanju korespondentnih matematičkih simbola i zapisa. Nakon ponovljenog testiranja, primjenom korelacijske analize, dobiveni su rezultati koji pokazuju da su stečene mentalne predstave ostale prisutne kod djece eksperimentalne grupe i nakon djelovanja eksperimentalnog faktora.

Ključne riječi: *razvoj simboličkog mišljenja; reprezentacije matematičkih pojmova; predškolski uzrast; učenje putem otkrića.*