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Manufacturing lot size and product distribution problem with rework, outsourcing and discontinuous inventory distribution policy

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SUMMARY

Product quality, timely delivery, and lower cost are critical operational goals to nowadays manufacturers, and company managements constantly seek different approaches to achieve these goals in order to stay competitive in turbulent global markets. This study investigates a practical manufacturing lot size and distribution problem with rework, outsourcing, and discontinuous inventory distribution policy. In real manufacturing environments, due to different controllable and/or uncontrollable factors, production of the nonconforming products is inevitable. Careful inspection into identifying nonconforming items and instant correction of the defects are considered in the proposed study. In additions, due to the limited production capacity in real manufacturing environments, sometimes, outsourcing can be used to cope with occasional unsteady demands, or running short of in-house capacity, to allow the management to maintain a smooth operation and/or shorten the production cycle length. Furthermore, in vendor-buyer integrated supply chains, multi-delivery policy is often considered for distributing finished products to customers. Motivated by the aforementioned practical situations, this study develops a mathematical model to explicitly investigate such a manufacturing lot-size and product distribution problem. Optimization techniques are employed to solve the problem and a numerical example is provided to show the applicability of our research results.

KEY WORDS: *mathematical modelling, manufacturing lot size, optimization, outsourcing, rework, product distribution.*

1. INTRODUCTION

This study investigates a practical manufacturing lot size and distribution problem with rework, outsourcing, and discontinuous inventory distribution policy. The classic economic manufacturing quantity (EMQ) model [1] used mathematical modelling to balance production setup and holding costs, and determine the most economic manufacturing lot size that minimizes total system relevant costs. It presumes that all fabricated products are of perfect quality. However, in real manufacturing environments, due to different controllable and/or uncontrollable factors, production of nonconforming items is inevitable. These imperfect quality products, sometimes, can be reworked and repaired with additional cost. Agnihothri and Kenett [2] studied the impact of defects on several performance measures of a manufacturing process with *100%* inspection followed by rework. Management guidelines for short-term control decisions, such as identifying potential bottlenecks under increased workloads and allocating additional resources to release bottlenecks, were provided. A budget allocation method for process improvement projects was proposed in their study to meet the long-term goal of continuously decreasing defect levels. Flapper and Teunter [3] considered that the rework process is the transformation of nonconforming items into usable products that maybe the same or lower quality, and claimed that rework can be profitable, especially if disposal costs are high and if materials are expensive and limited in availability. Therefore, they examined a production system with regular process and rework. Produced items were categorized as perfect quality, rework-able or scrap items. The reworkable nonconforming items deteriorate over time, hence they affect the rework time and cost. A disposal strategy and two types of rework strategies are examined in their study; they derived according expressions for the average profit per time unit, which includes different relevant system costs. Pillai and Chandrasekharan [4] considered the flow of material through the production system as an absorbing Markov chain and characterised its uncertainty due to scrapping and reworking. They identified system parameters under scrapping and reworking, and estimated the quantity of raw materials needed for the manufacturing system. Studies related to production systems with the reworking of nonconforming products can also be found elsewhere [5-9].

In addition, due to limited production capacity in real-life manufacturing environments, sometimes, a partial outsourcing policy can be used to cope with occasional unsteady demands, or running short of in-house capacity, to allow the management to maintain a smooth operation and/or shorten the production cycle length. Chalos and Sung [10] proposed an agency model in which outsourcing strictly dominates in-house production and argued that firms outsource in order to improve managerial incentives. They established the conditions under which the firm is more productive with outsourcing. The optimal contract also shows whether or not the firm is publicly held. For a publicly held firm, the contract is constant. For a privately held supplier, the contract is likely to be of a cost-sharing type. Their findings offered preliminary incentive explanations for commonly observed outsourcing practices. Cachon and Harker [11] proposed a model of competition between two firms that face scale economies, specifically, each firm's cost per unit of demand decreases in demand. Two service providers with price and time-sensitive demand, and competition between two retailers with fixed-ordering costs and price-sensitive consumers, are considered in their study. For the two service providers case, they demonstrated an enviable situation where the lower cost firm may have a higher market share and a higher price. By allowing each firm to outsource their production process to a supplier, they showed that the firms strictly prefer to outsource. They concluded that scale economies provide a strong motivation for outsourcing that has not previously been identified in the literature. De Fontenay and Gans [12] examined the outsourcing choice of a downstream firm with its own upstream production resources. They considered the outsourced function involving resources is consistent with the resource-based view of the firm. From a bargaining perspective, they examined a downstream firm's decision whether to outsource to an independent or to an established upstream firm. Therefore, the downstream firm faces a trade-off between lower input costs afforded by independent competition, and higher resource value associated with those who can consolidate upstream capabilities. They showed that this trade-off is resolved in favour of outsourcing to an established firm. Gray et al. [13] studied how cost and quality priorities influence a manufacturer's tendency to outsource. They bridged the existing gap between research on manufacturing strategy and firm boundaries, and developed a theory-based model that links manufacturer's cost and quality priorities to its plans to outsource production. With the empirical analysis, based on survey data obtained from 867 manufacturing business units, they found that the competitive priority placed on cost played an integral role in sourcing decisions, but conformance quality priorities did not. The latter, they stated, may partially explain why there is an emergence of so many nonconforming products associated with outsourcing. In summary, their results provided theoretical insights for future research into how manufacturing managers can improve their decision making on outsourcing production. Studies related to various aspects of outsourcing policy can also be found elsewhere [14-19].

Also, unlike the conventional EMQ model that assumes a continuous inventory issuing policy, in vendor-buyer integrated supply chains, multi-delivery policy is often considered for distributing finished products to customers. Hill [20] studied a production-inventory model where a firm purchases a raw material, manufactures a product and ships a fixed quantity of the product to a single customer at fixed and regular intervals of time. The objective was to determine optimal purchasing and production schedule which minimizes the total cost of purchasing, manufacturing and stock-holding. Diponegoro and Sarker [21] derived an optimal ordering policy for raw materials and economic batch size for finished products that are delivered frequently at a fixed interval of time for a finite planning horizon. They also extended to the problem of compensating lost sales of finished products, and provided a lower bound on the optimal solution for the problem with lost sale. Grunder et al. [22] examined a supply chain which involves multiple supply links. Each link is considered as an integrated scheduling problem in which a set of identical jobs are first processed on a single machine, and then batch is delivered to a customer by a transporter. Each job has a customer demanded due date in each supply link, and the inventory cost is associated with any early completion job waiting to be delivered. The objective of their study was to find a joint schedule for each supply link that minimizes total cost of the supply chain. They showed that it is an NP-hard problem in the maximum capacity of the transporters, and proposed a dominance-related greedy algorithm and a genetic algorithm to the problem. Computational results were provided to illustrate the efficiency of the proposed heuristics. Studies related to various aspects of discontinuous multi-delivery policy in supply chains can also be found elsewhere [23-29]. Because little attention has been paid to the investigation of joint effects of rework, outsourcing and discontinuous distribution policy on the optimal lot size and product distribution decision, this paper is intended to bridge the gap.

2. DESCRIPTION AND MATHEMATICAL MODELLING

This section describes the proposed mathematical model for investigating the manufacturing lot size problem with rework, outsourcing and discontinuous inventory distribution policy. Suppose that a product has an annual demand rate of λ units and it can be produced at an annual rate of P units. All items produced are screened and the inspection cost is included in the unit production cost C. During the manufacturing process, an x portion of nonconforming items may be produced randomly, at a rate of d (hence $d = Px$). Under the ordinary assumption of EPQ model, P must be larger than the sum of λ and d in order to avoid stock-out (i.e., $P-d-\lambda > 0$). All nonconforming items are assumed to be repairable through a rework process that immediately follows the regular production in each cycle, at a rate of P_1 units per year.

In addition to in-house fabrication, in each replenishment cycle a π portion (where $0<\pi<1$) of the lot-size Q is outsourced. All outsourced items are received at the time when in-house rework process ($t_{2\pi}$) ends (see Figure 1) and these items are guaranteed to be perfect quality by the outsourcing contract.

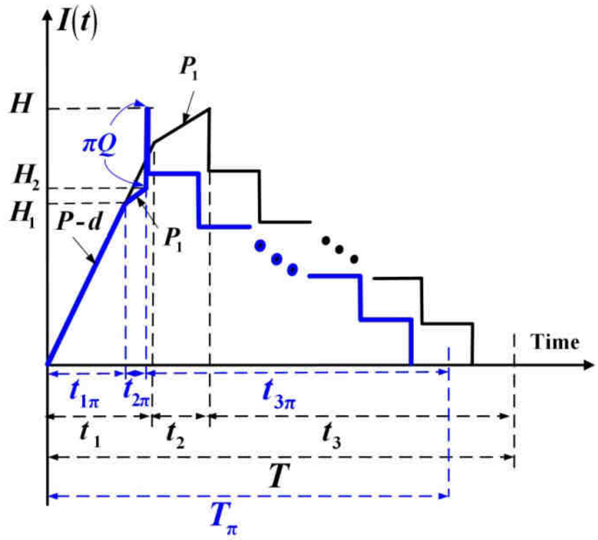

Fig. 1 *On-hand inventory level of perfect quality products in the proposed EMQ-based model with rework, outsourcing and discontinuous inventory distribution policy (in blue) compared to the EMQ-based model without considering outsourcing (in black)*

It is noted that when $\pi = 0$, the proposed system becomes a *100%* in-house 'make' system, and on the contrary, if $\pi = 1$, then the proposed system turns into a 'buy' system. Upon receipt of outsourcing items, fixed quantity of n instalments of the entire replenishment lot are distributed to the customer at a fixed interval of time during the production downtime $t_{3\pi}$.

Figure 2 illustrates the on-hand inventory level of nonconforming items during uptime and rework time of the proposed system.

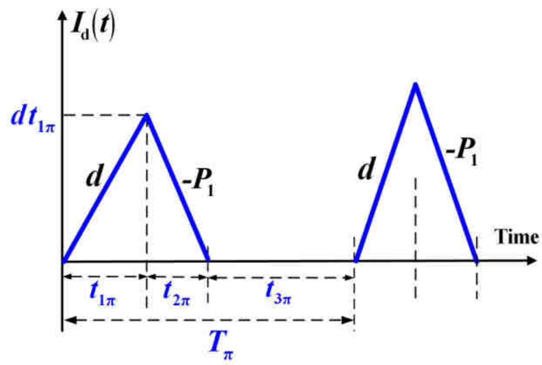

Fig. 2 *On-hand inventory level of nonconforming items during uptime and rework time of the proposed system*

Additional notation used in this study includes the following:

Q = the replenishment lot size of the proposed problem,

T_π = the replenishment cycle time,

C = unit manufacturing cost (including unit screening cost),

C_π = unit outsourcing cost,

K = in-house production setup cost per cycle,

K_π = fixed outsourcing (setup) cost per cycle,

h = holding cost per item per unit time,

C_R = rework cost per nonconforming item,

β_1 = the relating parameter between K_π and K (it is assumed that $K_\pi = (1 + \beta_1)K$, where $0 \le \beta_1 \le 1$),

β_2 = the relating parameter between C_π and C (it is assumed that $C_\pi = (1 + \beta_2)C$, where $\beta_2 \ge 0$),

H_1 = on-hand inventory of perfect quality items at the time in-house production ends,

H_2 = on-hand inventory of perfect quality items at the time rework process stops,

H = maximum level of on-hand inventory of perfect quality items after receiving the outsourced items,

$t_{1\pi}$ = production uptime for the proposed problem,

$t_{2\pi}$ = time required to rework all nonconforming items produced,

$t_{3\pi}$ = time required for distributing all items in a replenishment cycle,

t_1 = production uptime of the proposed system when $\pi = 0$,

t_2 = rework time of the proposed system when $\pi = 0$,

t_3 = delivery time of the proposed system when $\pi = 0$,

T = the cycle time of the proposed system when $\pi = 0$,

$I(t)$ = on-hand inventory level of perfect quality items at time t,

$I_d(t)$ = on-hand inventory level of nonconforming items at time t,

K_1 = fixed distributing cost per shipment,

C_T = unit distributing cost,

n = number of fixed quantity instalments of the finished lot to be delivered per cycle,

t_n = the fixed interval of time between each instalment delivered during $t_{3\pi}$,

h_2 = holding cost per item per unit time on the customer's side,

$I_c(t)$ = on-hand inventory level of stocks on the customer's side at time t,

$TC(Q, n)$ = total production-inventory-distribution cost per cycle for the proposed problem,

$E[TCU(Q, n)]$ = the long-run average costs per unit time for the proposed problem.

By observing Figures 1 and 2, we directly obtain the following equations:

$$t_{1\pi} = \frac{(1-\pi)Q}{P} = \frac{H_1}{P-d} \tag{1}$$

$$t_{2\pi} = \frac{x\left[(1-\pi)Q\right]}{P_1} \tag{2}$$

$$t_{3\pi} = T_\pi - t_{1\pi} - t_{2\pi} \tag{3}$$

$$T_\pi = t_{1\pi} + t_{2\pi} + t_{3\pi} = \frac{Q}{\lambda} \tag{4}$$

$$H_1 = (P-d)t_{1\pi} \tag{5}$$

$$H_2 = H_1 + P_1 t_{2\pi} \tag{6}$$

$$H = H_2 + \pi Q = Q \tag{7}$$

$$dt_{1\pi} = xPt_{1\pi} = x\left[(1-\pi)Q\right] \tag{8}$$

Total holding cost during delivery time $t_{3\pi}$ is [26] (see Figure 3):

$$h\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right)Ht_{3\pi} = h\left(\frac{1}{n^2}\right)\left(\frac{n(n-1)}{2}\right)Ht_{3\pi} = h\left(\frac{n-1}{2n}\right)Ht_{3\pi} \tag{9}$$

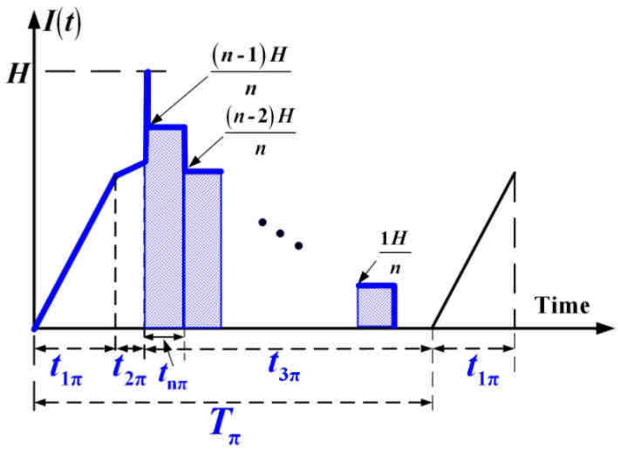

Fig. 3 *On-hand inventory of the finished items during $t_{3\pi}$ of the proposed system*

Delivery cost per shipment and total delivery costs per cycle are shown in Eqs. (10) and (11) as follow:

$$K_1 + C_T\left(\frac{H}{n}\right) \tag{10}$$

$$n\left[K_1 + C_T\left(\frac{H}{n}\right)\right] = nK_1 + C_T H \tag{11}$$

Total stock holding costs on the customer's side are [28]:

$$h_2 \frac{1}{2}\left[\frac{H}{n}t_{3\pi} + T_\pi\left(H - \lambda t_{3\pi}\right)\right] \tag{12}$$

Total production-inventory-distribution cost per cycle $TC(Q, n)$ consists of fixed and variable outsourcing costs, setup and variable in-house production costs, rework cost, fixed and variable product distribution costs, holding cost for reworked items in $t_{2\pi}$, holding cost for perfect quality and nonconforming items during $t_{1\pi}$, $t_{2\pi}$, and $t_{3\pi}$, and holding cost for stocks stored on the customer's side. Hence, $TC(Q, n)$ is:

$$TC(Q,\ n)=K_{\pi}+C_{\pi}(\pi Q)+K+C\left[(1-\pi)Q\right]+C_{R}x\left[(1-\pi)Q\right]+nK_{1}+C_{T}H+h_{1}\frac{dt_{1\pi}}{2}(t_{2\pi})+ +h\left[\frac{H_{1}+dt_{1\pi}}{2}(t_{1\pi})+\frac{H_{1}+H_{2}}{2}(t_{2\pi})+\left(\frac{n-1}{2n}\right)Ht_{3\pi}\right]+\frac{h_{2}}{2}\left[\frac{H}{n}t_{3\pi}+T_{\pi}(H-\lambda t_{3\pi})\right] \quad (13)$$

By substituting K_{π} and C_{π} in Eq. (13), following is obtained:

$$TC(Q,\ n)=(1+\beta_{1})K+(1+\beta_{2})C(\pi Q)+K+C\left[(1-\pi)Q\right]+C_{R}x\left[(1-\pi)Q\right]+nK_{1}+C_{T}H+ +h_{1}\frac{dt_{1\pi}}{2}(t_{2\pi})+h\left[\frac{H_{1}+dt_{1\pi}}{2}(t_{1\pi})+\frac{H_{1}+H_{2}}{2}(t_{2\pi})+\left(\frac{n-1}{2n}\right)Ht_{3\pi}\right]+\frac{h_{2}}{2}\left[\frac{H}{n}t_{3\pi}+T_{\pi}(H-\lambda t_{3\pi})\right] \quad (14)$$

In order to cope with the randomness of nonconforming rate x, the expected values of x are applied to cost analysis. By substituting all variables from Eqs. (1) to (8) into Eq. (14), the long-run average system costs per unit time for the proposed problem E[$TCU(Q, n)$] can be obtained as follows:

$$E\left[TCU(Q,\ n)\right]=\frac{E\left[TC(Q,\ n)\right]}{E\left[T\right]}=\frac{\lambda}{Q}\left[(1+\beta_{1})K+K+nK_{1}\right]+(1+\beta_{2})C\pi\lambda+\lambda(1-\pi)\left(C+C_{R}E\left[x\right]\right)+ +C_{T}\lambda+\frac{Q(h_{1}-h)}{2}\left(\frac{\lambda E\left[x\right]^{2}(1-\pi)^{2}}{P_{1}}\right)+\frac{hQ}{2}\left(1-\frac{\lambda(1-\pi)\pi}{P}+\frac{\lambda(1-\pi)E\left[x\right](1-2\pi)}{P_{1}}\right)+ +\frac{h_{2}Q\lambda(1-\pi)}{2}\left(\frac{1}{P}+\frac{E\left[x\right]}{P_{1}}\right)+\frac{(h_{2}-h)Q}{2n}\left[1-\lambda(1-\pi)\left(\frac{1}{P}+\frac{E\left[x\right]}{P_{1}}\right)\right] \quad (15)$$

2.1 DETERMINING THE OPTIMAL LOT SIZE AND SHIPMENTS

We first apply Hessian matrix equations [30] to Eq. (15) to prove that system cost function E[$TCU(Q, n)$] is convex, i.e., the following equation holds:

$$\left[Q \quad n\right]\cdot\begin{bmatrix}\frac{\partial^{2}E\left[TCU(Q,n)\right]}{\partial Q^{2}} & \frac{\partial^{2}E\left[TCU(Q,n)\right]}{\partial Q\partial n}\\ \frac{\partial^{2}E\left[TCU(Q,n)\right]}{\partial Q\partial n} & \frac{\partial^{2}E\left[TCU(Q,n)\right]}{\partial n^{2}}\end{bmatrix}\cdot\begin{bmatrix}Q\\ n\end{bmatrix}>0 \quad (16)$$

By using Eq. (15), the following partial derivatives can be obtained:

$$\frac{\partial E\left[TCU(Q,n)\right]}{\partial Q}=\frac{-(1+\beta_{1})K\lambda}{Q^{2}}-\frac{K\lambda}{Q^{2}}-\frac{nK_{1}\lambda}{Q^{2}}+\frac{(h_{1}-h)}{2}\left[\frac{\lambda E\left[x\right]^{2}(1-\pi)^{2}}{P_{1}}\right]+ +\frac{h}{2}\left[1-\frac{\lambda\pi(1-\pi)}{P}+\frac{\lambda E\left[x\right](1-\pi)(1-2\pi)}{P_{1}}\right]+\frac{h_{2}\lambda(1-\pi)}{2}\left(\frac{1}{P}+\frac{E\left[x\right]}{P_{1}}\right)+ +\frac{(h_{2}-h)}{2n}\left[1-\lambda(1-\pi)\left(\frac{1}{P}+\frac{E\left[x\right]}{P_{1}}\right)\right] \quad (17)$$

$$\frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q^2}=\frac{2\left[(2+\beta_1)K+nK_1\right]\lambda}{Q^3} \tag{18}$$

$$\frac{\partial E\left[TCU(Q,n)\right]}{\partial n}=\frac{\lambda K_1}{Q}-\frac{(h_2-h)Q}{2n^2}\left[1-\lambda(1-\pi)\left(\frac{1}{P}+\frac{E[x]}{P_1}\right)\right] \tag{19}$$

$$\frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial n^2}=\frac{(h_2-h)Q}{n^3}\left[1-\lambda(1-\pi)\left(\frac{1}{P}+\frac{E[x]}{P_1}\right)\right] \tag{20}$$

$$\frac{\partial E\left[TCU(Q,n)\right]}{\partial Q\partial n}=-\frac{\lambda K_1}{Q^2}-\frac{(h_2-h)}{2n^2}\left[1-\lambda(1-\pi)\left(\frac{1}{P}+\frac{E[x]}{P_1}\right)\right] \tag{21}$$

By substituting Eqs. (18), (20) and (21) in Eq. (16), the following is obtained:

$$[Q \quad n]\cdot\begin{bmatrix}\dfrac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q^2} & \dfrac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q\partial n}\\ \dfrac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q\partial n} & \dfrac{\partial^2 E\left[TCU(Q,n)\right]}{\partial n^2}\end{bmatrix}\cdot\begin{bmatrix}Q\\ n\end{bmatrix}=\left[\frac{2\lambda}{Q}(2+\beta_1)K\right]>0 \tag{22}$$

Because λ, Q, $(2+\beta_1)$, and K are all positive, Eq. (22) is positive, too. Therefore, $E[TCU(Q, n)]$ is a strictly convex function for all Q and n different from zero. It follows that, in order to derive the optimal replenishment lot size and optimal number of shipments, one can set first derivatives of $E[TCU(Q, n)]$ with respect to Q and with respect to n equal to zero and solve the linear system of Eqs. (17) and (19). With further derivations, the following is obtained:

$$Q^* = \sqrt{\frac{2\left[(2+\beta_1)K+nK_1\right]\lambda}{\dfrac{(h_1-h)\lambda(1-\pi)^2E[x]^2}{P_1}+h\left[1-\dfrac{\lambda(1-\pi)\pi}{P}+\dfrac{\lambda(1-\pi)E[x](1-2\pi)}{P_1}\right]+h_2\lambda(1-\pi)\left(\dfrac{1}{P}+\dfrac{E[x]}{P_1}\right)+\dfrac{(h_2-h)}{n}\left[1-\lambda(1-\pi)\left(\dfrac{1}{P}+\dfrac{E[x]}{P_1}\right)\right]}} \tag{23}$$

and:

$$n^* = \sqrt{\frac{(2+\beta_1)K(h_2-h)\left(1-\dfrac{\lambda(1-\pi)}{P}-\dfrac{\lambda E[x](1-\pi)}{P_1}\right)}{K_1\left(\dfrac{(h_1-h)\lambda E[x]^2(1-\pi)^2}{P_1}+h_2\lambda(1-\pi)\left(\dfrac{1}{P}+\dfrac{E[x]}{P_1}\right)+h\left(1-\dfrac{\lambda\pi(1-\pi)}{P}+\dfrac{\lambda E[x](1-\pi)(1-2\pi)}{P_1}\right)\right)}} \tag{24}$$

3. NUMERICAL EXAMPLE AND DISCUSSION

This section uses a numerical example to demonstrate applicability of our research results. Consider that a product can be manufactured at an annual rate P = *20,000* units to meet its demand rate of λ = *4,000* units per year. In-house production setup cost K is *$5,000* per cycle and unit manufacturing cost C = *$100* (which includes the inspection cost). In each replenishment lot Q per cycle, a portion $\pi = 0.4$ is outsourced with a fixed outsourcing cost $K_\pi = \$1,500$ (i.e., $\beta_1 = -0.7$) and outsourcing cost $C_\pi = \$120$ per unit (i.e., $\beta_2 = 0.2$).

In-house production process may produce an x portion of nonconforming items which follows a uniform distribution over the interval $[0, 0.2]$. All nonconforming items are reworked and repaired right after the completion of regular production process (i.e., in the end $t_{1\pi}$), at a rate of $P_1 = 5,000$ units per year and with an additional cost of $C_R = \$60$ per reworked item.

All outsourcing items are received at the end of the rework process (i.e., in the end $t_{2\pi}$) and upon the receipt of outsourced products, fixed quantity n instalments of the entire replenishment lot are distributed to the customer, at a fixed interval of time during the delivery time $t_{3\pi}$. Additional values of system variables include the following:

K_1 = $\$800$, fixed product distribution cost per shipment,

C_T = $\$0.5$, unit distribution cost,

h = $\$30$, holding cost per unit per year,

h_1 = $\$40$, holding cost per reworked item per year,

h_2 = $\$80$, holding cost per item per year on the customer's side.

By applying Eqs. (23), (24) and (15), the optimal replenishment lot size per cycle, optimal number of shipments, and the long-run average costs per unit time for the proposed problem can be obtained, respectively as follows: $Q^* = 1,126$, $n^* = 3$, and $E[TCU(Q^*, n^*)] = \$511,648$. The function $E[TCU(Q, n)]$ with respect to Q and n is illustrated in Figure 4.

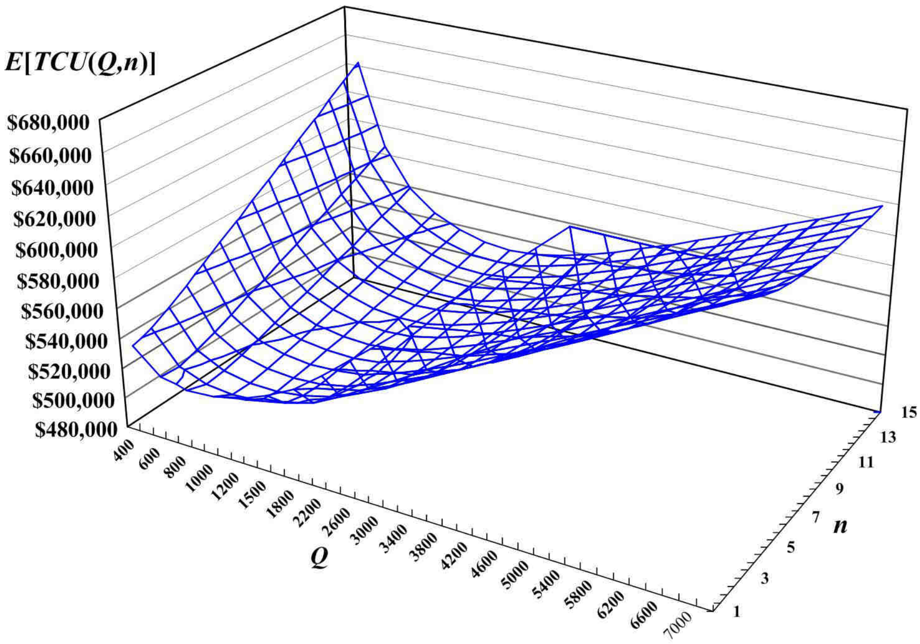

Fig. 4 *The expected system cost function E[TCU(Q, n)] with respect to Q and n*

The joint effects of the variations of outsourcing proportion π and C_π/C on the expected system cost function $E[TCU(Q, n)]$ are depicted in Figure 5. It is noted that there is a breakeven point on C_π/C (i.e., $1+\beta_2$), once C_π/C exceeds the breakeven rate, as the outsourcing proportion π increases, the expected system cost $E[TCU(Q, n)]$ increases; and as C_π/C ratio raises, $E[TCU(Q, n)]$ significantly increases.

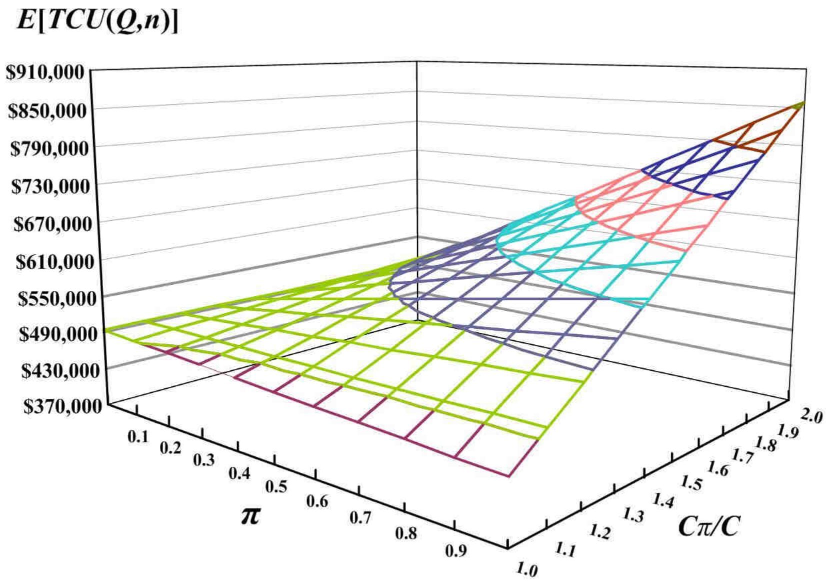

Fig. 5 *Joint effects of variations of outsourcing proportion π and C_π/C on the expected system cost function E[TCU(Q, n)]*

Further analysis indicates that the breakeven β_2 ratio is *0.1201* or $C_\pi = (1 + \beta_2)C = \$112.01$ (see Figure 6). It can provide insight information for 'make-or-buy' decision making. For instance, if the system parameters for any given manufacturing firm are known, then our proposed model can analyze and offer the breakeven outsourcing cost per item to the management regarding their make-or-buy decision. This means that if β_2 falls below this critical ratio, then the manufacturing firm is at an advantage to outsource the entire replenishment lot.

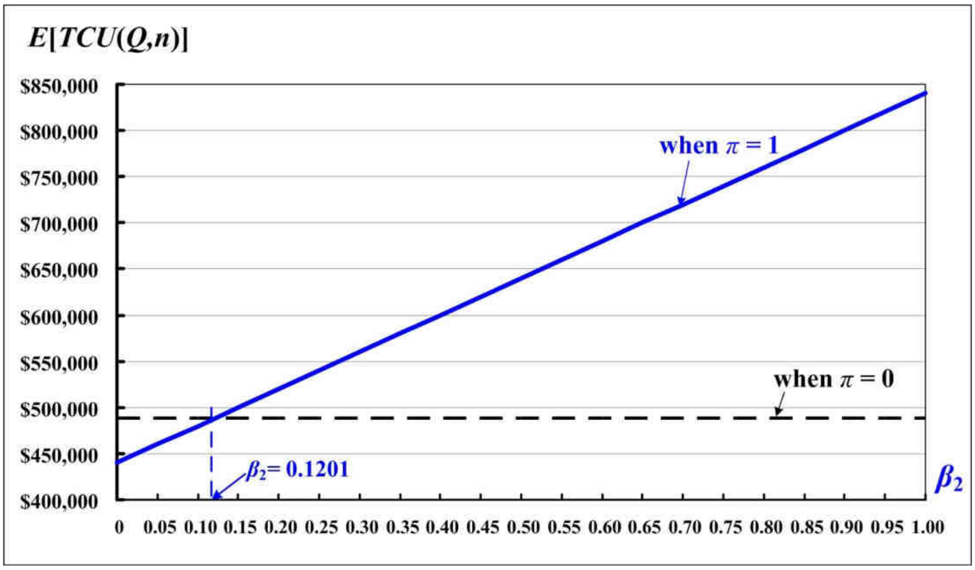

Fig. 6 *Analysis of the breakeven point of C_π/C (or β_2) in the proposed system*

In general, the outsourcing policy is used to resolve the short supply of in-house capacity at an increased cost on $E[TCU(Q, n)]$. By applying the results of the proposed model, effect of variation of outsourcing proportion π on the production uptime $t_{1\pi}$ can be specifically analysed, as exhibited in Figure 7. It is noted that as the outsourcing proportion π increases, the in-house production uptime $t_{1\pi}$ declines significantly. For instance, at $\pi = 0.4$ the production uptime $t_{1\pi}$ decreases from *0.0487* years (at $\pi = 0$) to *0.0327* years, or an uptime reduces by *32.85%*, at an increased cost of *$23,615* on $E[TCU(Q, n)]$ (i.e., *$511,648* - *$488,033*), or *4.84%* of total system costs.

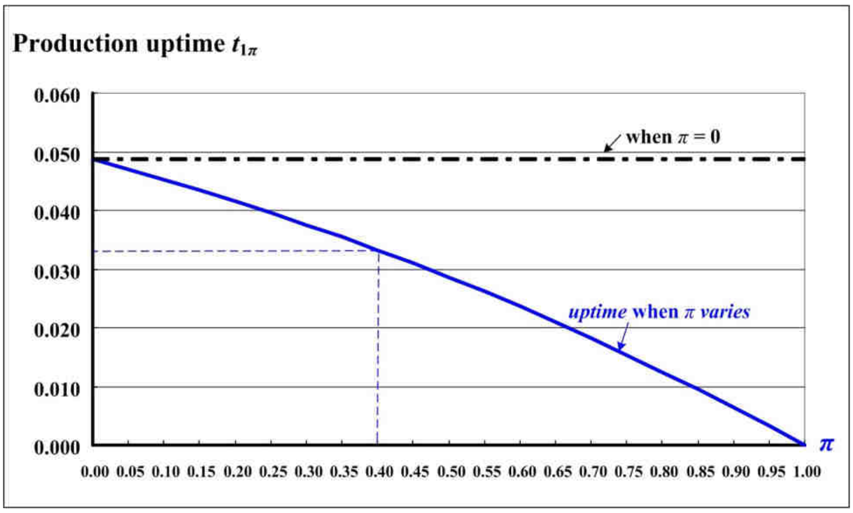

Fig. 7 *Effect of variation of outsourcing proportion π on the production uptime* $t_{1\pi}$

The proposed model can also provide information regarding the breakeven C_R/C ratio. For example, for a known unit outsourcing cost (e.g., C_π = *$110*, or β_2 = *0.1*), the analytical result from our proposed system indicates that breakeven C_R/C ratio is *0.3983* (see Figure 8). In other words, if the in-house unit reworking cost is over *$39.83* (since C = *$100* in this example), then the management of the firm is at an advantage (in terms of cost savings) to outsource the entire replenishment lot.

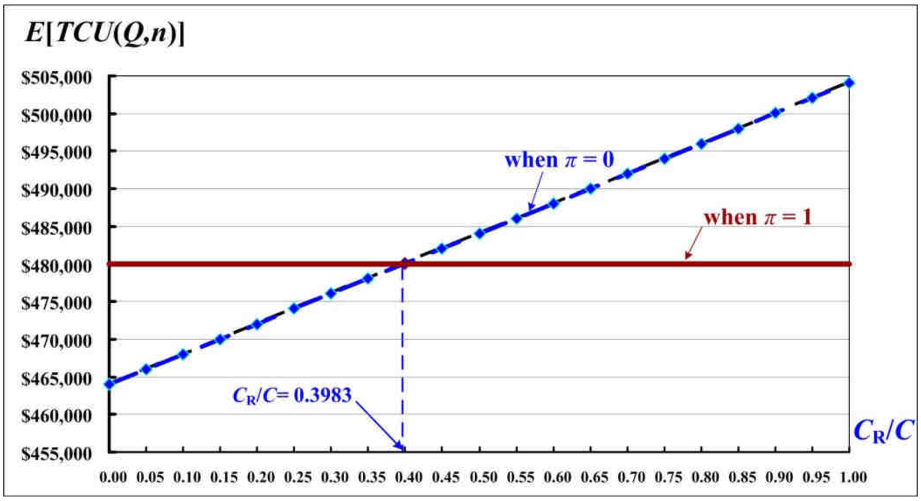

Fig. 8 *Analysis of the breakeven point of* C_R/C *in the proposed system*

4. CONCLUSIONS

A precise analytical model is developed in this study to explicitly interpret a manufacturing lot size and product distribution problem with rework, outsourcing and discontinuous inventory distribution policy. With a help of the optimization technique, the closed-form optimal lot size and shipment solutions are derived. With thorough exploration and robust analysis of such a realistic problem, various important insight information can be revealed (refer to Section 3: Figures 4-8) for supporting different managerial decision-making processes. For future study, one may consider the effect of stochastic demand rate on the optimal operating policy of this specific problem.

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