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# FUZZY GREY RELATIONAL ANALYSIS FOR INFLUENCE FACTORS ON THE QUALITY OF CALCINED PETROLEUM COKE

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Real density and powder resistivity are key indicators for the quality of calcined petroleum coke (CPC). They are closely related to calcining parameters in shaft calciner. Fuzzy grey relational analysis was proposed to evaluate the effects of various factors (discharge rate per pot (DR), flue wall temperature of layers 2 ($T_2$) and 8 ($T_8$), volatiles (V), ash (A) and sulfur content (S) of green petroleum coke) on the quality of CPC. Results showed that by order of influence degree (most to least) to the real density, the factors were ranked as $T_2$, V, DR, $T_8$, A, and S, and for the powder resistivity, they were ranked as V, $T_2$, DR, $T_8$, and S.

*Keywords:* calcined petroleum coke; fuzzy grey relational analysis; shaft calciner; real density; powder resistivity

## INTRODUCTION

Petroleum coke is the key material for carbon anode in the aluminum electrolytic industry. Petroleum coke calcination is the primary process for carbon anode preparation. The quality of calcined petroleum coke (CPC) is directly related to the properties of carbon anode, which is the key factor affecting the stable operation of aluminum electrolysis production [1, 2]. Shaft calciner is an important calcined equipment for green petroleum coke [1]. Figure 1 shows a typical shaft calciner that comprises 24 vertical refractory pots; each pot is surrounded by eight layers of horizontally oriented heating flues. The entire calcination process is carried out in closed pots, and the combustion of volatiles (V) in the flues is the main source of heat in the shaft calciner. During the calcination process, there are various factors that affect the quality of CPC, such as the discharge rate, flue wall temperature, and V content of green petroleum coke. However, there are few reports about the influence of various calcining parameters on the quality of CPC.

Fuzzy grey relational analysis (FGRA) is an efficient evaluation method that is widely used in many fields, such as engineering [3] and buildings [4]. Zuo et al. [5] studied the influence of factors on the emission efficiency of micro-combustors using orthogonal design and FGRA and found the $H_2$/air equivalence ratio to be the most important factor. Cheng et al. [6] found that the deformation was the maximum influencing factor on stability of rock-bolt crane girder using FGRA.

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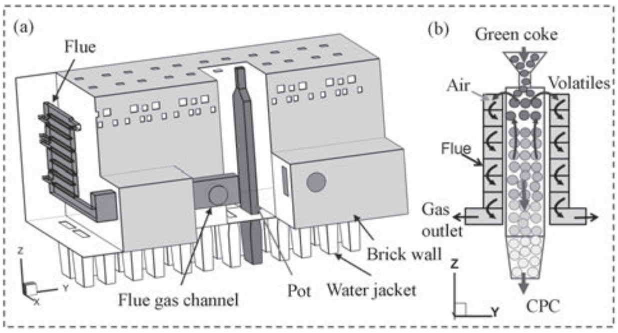


**Figure 1** Detailed structure of shaft calciner (a) three-dimensional diagram, (b) calcining process

In this work, FGRA was used to study the effects of various calcining parameters on the real density and powder resistivity of CPC to obtain the key affecting factors and provide reference for actual production.

## FGRA MODEL

This work aims to evaluate the influence degree of various factors on the CPC quality in shaft calciner using FGRA. The calculation steps for FGRA are as follows:

(1) Determine the reference matrix and comparison matrix

The reference matrix is a data sequence that reflects the behavior characteristics of the system. The variation of data in this matrix reflects the trend or change of the system, which is expressed as follows:

$$Y = \left[ y(1) \quad y(2) \quad \cdots \quad y(n) \right] \tag{1}$$

The data sequence that affects the behavior of the system is defined as the comparison matrix, which is expressed as follows:

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} x_1(1) & x_1(2) & \dots & x_1(n) \\ x_2(1) & x_2(2) & \dots & x_2(n) \\ \vdots & \vdots & \vdots & \vdots \\ x_m(1) & x_m(2) & \dots & x_m(n) \end{bmatrix} \quad (2)$$

where $Y$ is the reference matrix, $X$ is the comparison matrix, m is the number of factor vectors, and n is the number of operating conditions.

(2) Normalization of the original data series

Factors and variables typically have different physical meanings and data dimensions. For example, the flue wall temperature of layer 2 ($T_2$) can range from 1 250 °C to 1 350 °C, whereas the fluctuation range of ash (A) content of green petroleum coke is only 0,1 % – 0,3 %. To reduce the identification analysis error, the original data must be nondimensionalized before calculation. The standardized matrix can be obtained by using Equation (3).

$$x(k)' = \frac{x_i(k) - \min x_i(k)}{\max x_i(k) - \min x_i(k)} \quad (3)$$

(3) Calculation of grey correlation coefficient

$$\xi_i(k) = \frac{\Delta_{\min} + \rho\Delta_{\max}}{\Delta(k) + \rho\Delta_{\max}} \quad (4)$$

where $\rho$ is the resolution coefficient and $\Delta_{min}$ is the minimum absolute difference of the corresponding element between the reference matrix $Y$ and comparison matrix $X$, which can be calculated by using Equation (5). $\Delta_{max}$ is the maximum absolute difference, and it is calculated using Equation (6).

$$\Delta_{\min} = \min_{1\le j\le m} \min_{1\le k\le n} \left| y(k) - x_j(k) \right| \quad (5)$$

$$\Delta_{\max} = \max_{1\le j\le m} \max_{1\le k\le n} \left| y(k) - x_j(k) \right| \quad (6)$$

The essential meaning of the resolution coefficient $\rho$ is the weight of the maximum absolute difference. Its value should satisfy the anti-interference and the integrity of correlation degree, and it can be determined as follows:

$$\overline{\Delta} = \frac{1}{\mathrm{m} \cdot n} \sum_{j=1}^{m} \sum_{k=1}^{n} \Delta_i(k) \quad (7)$$

where $\overline{\Delta}$ is the mean value of absolute difference and $\Delta(k)$ is the absolute difference of point k between the reference matrix and the comparison matrix, which is defined using Equation (8).

$$\Delta_i(k) = \left| y(k) - x_i(k) \right| \quad (8)$$

The value interval of $\rho$ is described as $E_\Delta \le \rho \le 2E_\Delta$, where $E_\Delta = \overline{\Delta} / \Delta_{\max}$. The ratio of the absolute difference of mean value to the maximum absolute difference must meet the following conditions:

$$\begin{cases} \Delta_{\max} > 3\overline{\Delta}, E_\Delta \le \rho \le 1{,}5\,E_\Delta \\ \Delta_{\max} \le 3\overline{\Delta}, 1{,}5 E_\Delta \le \rho \le 2\,E_\Delta \end{cases} \quad (9)$$

(4) Calculation of fuzzy membership grade

The angle cosine method is used to judge the similarity of the two factors by calculating the cosine value of the angle between the two parameters, which is independent of the linear relation of the data.

$$r_1 = \frac{\sum_{k=1}^{n} y(k)x(k)}{\sqrt{\sum_{k=1}^{n} y(k)^2} \sqrt{\sum_{k=1}^{n} x(k)^2}} \quad (10)$$

(5) Calculation of Euclidean grey relational grade

$$r_2 = 1 - p\sqrt{\sum_{k=1}^{n} \left[ w(1 - \xi_i(k)) \right]^p} \quad (11)$$

where w is the weight and p is the distance parameter; when it is Hamming distance, p = 1; however, when it is Euclidean distance, p = 2.

(6) Calculation of fuzzy grey relational grade (FGRG)

$$r = \sqrt{\left(r_1^2 + r_2^2\right)/2} \quad (12)$$

## RESULTS AND DISCUSSION

The factors affecting the quality of CPC are determined as the discharge rate per pot (DR), $T_2$, $T_8$, V, A, and sulfur content (S) of green petroleum coke based on the practical productive experiences. Table 1 tabulates 20 samples of data obtained from production in a carbon factory.

Table 1 **Data of CPC quality and its influencing factors**

| DR / kg·h$^{-1}$ | $T_2$ / °C | $T_8$ / °C | A / % | V / % | S / % | Real density / g·cm$^{-3}$ | Powder resistivity / μΩ·m |
|---|---|---|---|---|---|---|---|
| 90,00 | 1 335 | 1 049 | 0,16 | 11,16 | 2,46 | 2,085 | 493,1 |
| 78,00 | 1 340 | 1 058 | 0,19 | 11,81 | 2,69 | 2,088 | 505,0 |
| 93,60 | 1 340 | 1 038 | 0,17 | 11,30 | 2,44 | 2,085 | 495,0 |
| 86,67 | 1 323 | 1 061 | 0,17 | 10,59 | 3,17 | 2,088 | 490,1 |
| 93,60 | 1 337 | 1 043 | 0,18 | 11,15 | 2,67 | 2,081 | 493,6 |
| 93,60 | 1 336 | 1 057 | 0,21 | 11,72 | 3,46 | 2,063 | 500,8 |
| 86,67 | 1 343 | 1 073 | 0,25 | 11,30 | 3,54 | 2,082 | 456,9 |
| 78,00 | 1 339 | 1 065 | 0,25 | 11,58 | 3,98 | 2,078 | 487,7 |
| 32,50 | 1 242 | 985 | 0,17 | 11,65 | 2,82 | 2,091 | 443,0 |
| 41,79 | 1 261 | 1 039 | 0,21 | 11,24 | 2,98 | 2,085 | 445,3 |
| 90,00 | 1 327 | 1 129 | 0,28 | 11,27 | 4,17 | 2,105 | 412,6 |
| 78,00 | 1 326 | 1 137 | 0,22 | 9,95 | 4,65 | 2,085 | 456,1 |
| 73,13 | 1 285 | 1 133 | 0,26 | 9,55 | 4,41 | 2,080 | 423,1 |
| 83,57 | 1 321 | 1 084 | 0,22 | 10,27 | 4,39 | 2,074 | 458,8 |
| 93,60 | 1 338 | 1 076 | 0,12 | 11,45 | 2,05 | 2,090 | 458,9 |
| 90,00 | 1 324 | 1 054 | 0,26 | 12,85 | 4,13 | 2,072 | 468,1 |
| 73,13 | 1 339 | 1 046 | 0,23 | 10,73 | 2,80 | 2,068 | 499,2 |
| 83,57 | 1 321 | 1 062 | 0,24 | 10,29 | 4,51 | 2,089 | 469,0 |
| 83,57 | 1 319 | 1 019 | 0,22 | 10,81 | 1,82 | 2,069 | 461,2 |
| 73,13 | 1 325 | 1 028 | 0,20 | 11,15 | 1,70 | 2,108 | 438,0 |

## FGRA for real density

The real density under various calcining parameters was taken as reference matrix $Y$. The six factors (DR,

$T_2$, $T_8$, V, A, and S) were considered as comparison matrix *X*.

$$\begin{bmatrix} Y \\ x_1 \\ \vdots \\ x_6 \end{bmatrix} = \begin{bmatrix} 2,085 & 2,088 & \cdots & 2,108 \\ 90,00 & 78,00 & \cdots & 73,13 \\ \vdots & \vdots & \vdots & \vdots \\ 2,46 & 2,69 & \cdots & 1,70 \end{bmatrix} \quad (13)$$

The dimensionless treatment of reference matrix and comparison matrix was carried out using Equation (3), and the results were as follows:

$$\begin{bmatrix} Y_0 \\ x_{10} \\ \vdots \\ x_{60} \end{bmatrix} = \begin{bmatrix} 0,489 & 0,556 & \cdots & 1,000 \\ 0,941 & 0,745 & \cdots & 0,665 \\ \vdots & \vdots & \vdots & \vdots \\ 0,258 & 0,336 & \cdots & 0,000 \end{bmatrix} \quad (14)$$

According to Equations (4)–(9), the grey relational coefficient matrix of each influencing factor can be obtained as follows:

$$\begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_6 \end{bmatrix} = \begin{bmatrix} 0,491 & 0,698 & \cdots & 0,565 \\ 0,502 & 0,512 & \cdots & 0,710 \\ \vdots & \vdots & \vdots & \vdots \\ 0,654 & 0,665 & \cdots & 0,303 \end{bmatrix} \quad (15)$$

In this work, all factors are considered to be mutually independent. Therefore, the weights are equal (w = 1/20). The above grey relational coefficient matrix was substituted into Equation (10) to calculate the fuzzy membership grades of DR, $T_2$, $T_8$, V, A, and S to real density; then according to Equation (11), the Euclidean grey relational grades of these six factors were acquired. Finally, the FGRGs of DR, $T_2$, $T_8$, V, A, and S to real density were calculated using Equation (12). All calculation results are shown in Table 2. Figure 2 shows the FGRGs of the six factors to the real density of CPC.

Table 2 **FGRA for real density**

| Name | DR | $T_2$ | $T_8$ | A | V | S |
|---|---|---|---|---|---|---|
| $r_1$ | 0,905 | 0,929 | 0,750 | 0,760 | 0,866 | 0,721 |
| $r_2$ | 0,812 | 0,812 | 0,867 | 0,841 | 0,865 | 0,838 |
| $r$ | 0,859 | 0,872 | 0,811 | 0,801 | 0,866 | 0,782 |

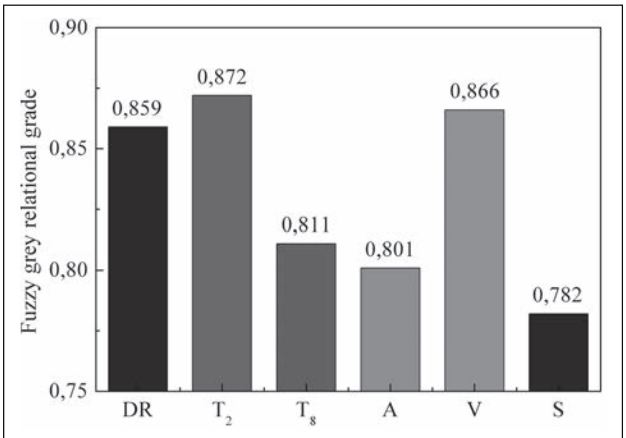


**Figure 2** FGRGs to real density

As show in figure 2, the FGRGs of DR, $T_2$, $T_8$, V, A, and S were 0,859, 0,872, 0,811, 0,801, 0,866, and 0,782, respectively, which means that $T_2$ was the most influential factor, followed by V, DR, $T_8$, and A. S was the least influential factor behind the real density of CPC.

## FGRA for powder resistivity

To analyze the influence degree of these six factors on the powder resistivity of CPC, the powder resistivity under various calcining parameters were defined as reference matrix *Y*, and the six factors (DR, $T_2$, $T_8$, V, A, and S) were taken to be comparison matrix *X*. They can be described as follows.

$$\begin{bmatrix} Y \\ x_1 \\ \vdots \\ x_6 \end{bmatrix} = \begin{bmatrix} 493,1 & 505,0 & \cdots & 438,0 \\ 90,00 & 78,00 & \cdots & 73,13 \\ \vdots & \vdots & \vdots & \vdots \\ 2,46 & 2,69 & \cdots & 1,70 \end{bmatrix} \quad (16)$$

The above matrixes can be dealt with by using Equation (3), and the dimensionless data were as follows:

$$\begin{bmatrix} Y_0 \\ x_{10} \\ \vdots \\ x_{60} \end{bmatrix} = \begin{bmatrix} 0,129 & 0,000 & \cdots & 0,725 \\ 0,059 & 0,255 & \cdots & 0,335 \\ \vdots & \vdots & \vdots & \vdots \\ 0,742 & 0,664 & \cdots & 1,000 \end{bmatrix} \quad (17)$$

Then, the grey relational coefficient matrix of the various influencing factors can be obtained according to Equations (4) – (9).

$$\begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_6 \end{bmatrix} = \begin{bmatrix} 0,856 & 0,618 & \cdots & 0,514 \\ 0,893 & 0,933 & \cdots & 0,430 \\ \vdots & \vdots & \vdots & \vdots \\ 0,402 & 0,383 & \cdots & 0,600 \end{bmatrix} \quad (18)$$

Similarly, the weights are considered to be equal (w = 1/20). According to Equation (10), the fuzzy membership grades of these six factors to the powder resistivity were acquired. Then, the Euclidean grey relational grades of these six factors to the powder resistivity were calculated by using Equation (11). Finally, according to Equation (12), the FGRGs of these six factors were calculated, and all results are listed in Table 3. Figure 3 shows the FGRGs of the six factors to powder resistivity.

As show in figure 3, there was a large difference in the FGRGs of these six factors. The FGRGs of DR, $T_2$, $T_8$, A, V, and S were 0,759, 0,799, 0,726, 0,706, 0,813, and 0,699, respectively. By order of influence degree (most to least) to the powder resistivity, the factors were ranked as V, $T_2$, DR, $T_8$, A, and S.

Table 3 **FGRA for powder resistivity**

| Name | DR | $T_2$ | $T_8$ | A | V | S |
|---|---|---|---|---|---|---|
| $r_1$ | 0,693 | 0,759 | 0,645 | 0,599 | 0,813 | 0,606 |
| $r_2$ | 0,820 | 0,836 | 0,798 | 0,798 | 0,813 | 0,782 |
| $r$ | 0,759 | 0,799 | 0,726 | 0,706 | 0,813 | 0,699 |

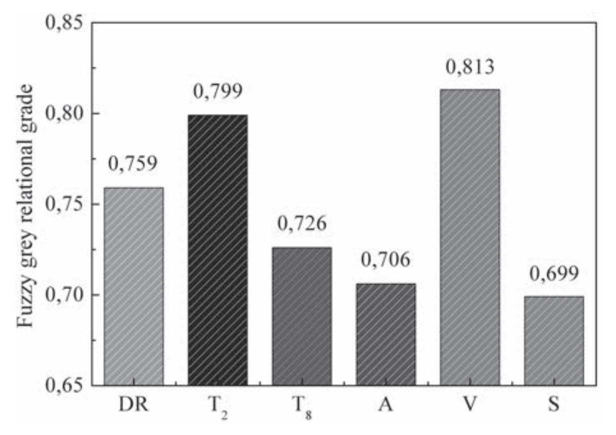


**Figure 3** FGRGs to powder resistivity

## CONCLUSIONS

In this work, FGRA was proposed to evaluate the effects of various factors (DR, $T_2$, $T_8$, V, A, and S of green petroleum coke) on the quality of CPC. Results showed that varying degrees of influence of the six factors on the real density and powder resistivity were obvious. The FGRGs of these six factors to real density were 0,859, 0,872, 0,811, 0,801, 0,866, and 0,782, respectively, which means that $T_2$ was the most influential factor, followed by V, DR, $T_8$, and A, and S was the least influential factor behind the real density of CPC. The FGRGs of these six factors to powder resistivity were 0,759, 0,799, 0,726, 0,706, 0,813, and 0,699, respectively. By order of the influence degree (most to least) to the powder resistivity, the factors were ranked as V, $T_2$, DR, $T_8$, A, and S. Therefore, V, $T_2$, and DR are the key factors affecting the quality of CPC and must be strictly controlled during production.

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**Note:** The professional translator for the English language is doctor J. D. Huang, Nanchang, China