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Automatika
Journal for Control, Measurement, Electronics, Computing and Communications

ISSN: 0005-1144 (Print) 1848-3380 (Online) Journal homepage: http://www.tandfonline.com/loi/taut20

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To cite this article: Ali Bekir Yildiz (2017) Determination of unified time constants of switching circuits in terms of averaged-nodal equations, Automatika, 58:1, 11-17, DOI: 10.1080/00051144.2017.1305600

To link to this article: https://doi.org/10.1080/00051144.2017.1305600

Published online: 03 Apr 2017.

AUTOMATIKA, 2017
VOL. 58, NO. 1, 11–17
https://doi.org/10.1080/00051144.2017.1305600

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Determination of unified time constants of switching circuits in terms of averaged-nodal equations

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ABSTRACT
Switching circuits have variable structural topologies. In every topology, they have different dynamics and time constants. Resistances of switching elements vary between a very small value and a very large value during conduction mode and disconnection mode. Therefore, equivalent circuit modelling of switching circuits requires averaging the on-state and off-state resistances of switching elements over one switching period. Determining averaged switch models allows to determine unified time constants of variable structural switching circuits. In this paper, analytically, it is shown how to calculate the equivalent average resistance and to derive unified time constants. The formulation method is based on nodal equations, very suitable for the analysis of switching circuits.

ARTICLE HISTORY
Received 13 November 2016
Accepted 2 February 2017

KEYWORDS
Unified time constant; nodal equation; switching circuit; averaged switch model

1. Introduction

Power electronic circuits have variable topologies due to the switching action of devices. Since these circuits operate in a switched mode, after switching, the switching circuit behaves as a linear circuit because the remaining elements of circuit consist of passive elements such as resistors, capacitors and inductors. In other words, every switched state is reduced into a linear circuit structure. So, they can be analysed with an adequate method for linear circuits. Time constants relating to every linear topology in a switched circuit can be obtained separately. But, these time constants contain only information about the related topology. In order to model the behaviour of switching circuits during the whole switching period and to determine their transient-state duration, it is necessary to determine unified time constants which cover the whole switching period and topologies.

The time constants are important parameters in system analysis. They refer to the transient response of circuits and determine the dynamics of circuits. The time constants are obtained from the eigenvalues in the time domain. They are also related to the poles of the circuit's transfer functions. Determination of time constants provides valuable information for circuit designers. Time constants, indirectly eigenvalues, are particularly useful in the design of feedback systems in which relative stability, dynamics and other complete response characteristics are important functions.

The time constants relate to the eigenvalues in time domain and the poles of transfer functions in s-domain. In [1], a general method based on the Laplace expansion for determining the transfer function of a wide variety of linear electronic circuits is given. A method for estimation poles and zeroes of linear active circuit transfer functions is described in [2]. Hauksdottir et al. [3] gave closed-form expressions, for real and/or complex eigenvalues, of transfer function responses. Hagiwara [4] used the eigenvalue approach to calculate the zeros of the system. In [5], the transfer functions of circuits are expressed in terms of time and transfer constants calculated under different combinations of shorted and opened energy-storing elements. All these studies are related to non-switched linear circuits.

The difficulty of determining the time constants arises from obtaining the system equations. Especially, this situation is more challenging for variable structural switching circuits. In general, time constants of circuits are obtained from the state-space formulation, a form of differential equations [6–9]. But, this method has some restrictions in obtaining the system equations, as will be explained in Section 2. Therefore, in the works using the state-space formulation, time constants of switching circuits are determined under restricted conditions.

The switching process leads to different actions and different time constants for every switched state. It needs to determine the unified time constants valid for the whole switching period in studying the dynamic of systems. The main contribution of this paper is that it gives a systematic and generalized method to compute unified time constants of switching circuits. Time constants are determined according to averaged-nodal equations in Laplace domain, in which the system

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equations are easily obtained and have no restrictions. The rest of the paper is organized as follows. In Section 2, the formulation method related to deriving system equations is given. In Section 3, modelling techniques of switches are summarized. In Section 4, averaging circuit approach is explained and the proposed switch model is introduced. Section 5 gives an illustrative example of the approach. Section 6 concludes the paper.

2. Formulation method

In general, the system equations used for determining time constants are expressed according to the state-space formulation. Although the state-space method, based on the graph theoretical approach, has the smallest number of variables, it involves intensive mathematical process and has major limitations in the formulation of circuit equations. Some of these limitations arise because the state variables are capacitor voltages and inductor currents. Every circuit element cannot be easily included into the state equations. Especially, there are some restrictions in the analysis of switching circuits. Writing and solving of the state-space equations for each topology of switched networks (2^n possible topological states for n switches) is very difficult. This is especially true for switched networks since the number of the state variables may change and state-variable discontinuities may be present, from one topology to the next. This causes the limitation in the number of switches.

Modified nodal analysis (MNA), which is the other popular technique in circuit analysis, is more elegant and suitable for the analysis of switching circuits. The classical nodal method, the predecessor of the modified nodal method, has some restrictions. As an extension to the classical nodal method, the MNA was first introduced by Ho et al. [10] to overcome its shortcomings and has been developed more by including many circuit elements (transformers, semiconductor devices, short circuits, etc.) into system equations. Stamps are developed that express the contributions of circuit elements to system equations. Because of their systematic nature, the system equations can also be obtained by inspection.

The modified nodal equations in the time domain and Laplace domain can be written in the following form:

$$Gx(t) + C\frac{dx}{dt} = Bu(t) \tag{1}$$

$$GX(s) + sCX(s) = BU(s) \tag{2}$$

G, C and B are coefficient matrices. All conductances and frequency-independent values (connections of elements, short-circuit element, etc.) arising in the formulation are stored in matrix G, whereas the values of capacitors and inductors are stored in matrix C because they are associated with the frequency. $u(t)$ represents the source vector containing the independent current and voltage sources. $x(t)$ is the vector of unknowns. In this method, there are both voltage variables (nodal voltages) and current variables (currents of inductors, independent and controlled voltage sources, short-circuit elements, etc.). Let us rearrange Equation (2) as follows:

$$GX(s) + sCX(s) = BU(s) \rightarrow (G + sC)X(s) = BU(s)$$

$$A(s)X(s) = BU(s) \tag{3}$$

where $A(s) = (G + sC)$. Matrix $A(s)$ is also called the characteristic matrix. The determinant of the characteristic matrix has fractional and polynomial form as follows:

$$\det(A(s)) = \frac{Q(s)}{R(s)} \tag{4}$$

The numerator of the determinant of the characteristic matrix ($Q(s)$) polynomial is called the characteristic equation. The eigenvalues, indirectly poles, of any circuit are derived from the roots of the characteristic polynomial, $Q(s) = 0$. Time constants are determined by taking the inverse of the eigenvalues.

3. Modelling of switches

In switching circuits, switches appear physically in the form of diodes, thyristors, transistors, CMOS pairs, etc., and can be characterized at various levels of abstraction. Among which, full semiconductor models, binary valued element model and ideal switch model are most often used. General-purpose circuit simulations, like SPICE, have been tried for the analysis of cyclically switching circuits by using built-in switch models. But these models are highly nonlinear. Such detailed micro-modelling of semiconductor devices as used in general-purpose circuit simulation programme is unnecessary for studying switching circuits, which always operate in the switched mode.

In many papers, the switches are considered to be ideal [11–14]. An ideal switch has zero resistance in on-state, zero admittance in off-state, and switching from one state to another is instantaneous. The ideal switch introduces some problems not commonly encountered when analysing non-switched analogue networks. The Dirac impulses of voltage and current are possible at the switching instants, when inductor currents and capacitor voltages are discontinuous. In ideal switch approach, the Dirac impulses must be considered for the analysis, although they are only present as an intermediate step. Neglect of the impulses in the simulation of switching circuits with ideal switches leads to erroneous solutions. In [14], a complete analysis which takes into consideration such a case is made. Moreover, writing and solving of system equations for each topology of switched networks (2^n possible topological states, n: ideal switches) are necessary. These problems can be avoided if non-ideal switch models are used.

The most often used non-ideal switch model is a binary-valued element approach. To that end, a binary-valued resistance or an inductance is used. In binary-valued resistance approach, a switch is modelled by a small resistance R_{on} when on-state, a large resistance R_{off} when off-state. Similarly, switches can be modelled as binary-valued inductors (a small-valued inductance L_{on}, when on-state, a large-valued inductance L_{off} when off-state). In [15], a non-ideal switch model comprised a switched inductor in parallel with a resistor-capacitor series network is used. In [16], devices are modelled as binary-valued inductances. The main advantage of using non-ideal switch models allows to study switching circuits with only one topology. Accordingly, it requires the use of a constant system equation. Of course, during the simulation, values of switching elements in system equations must be consistently updated.

In this paper, for the switch model, the binary-valued resistance approach is used because of the following reasons:

1. To obtain a constant circuit topology and to use only one system equation: so, it is unnecessary to deal with 2^n possible topological states as in ideal switch approach.
2. To avoid the use of Dirac impulses like in ideal switch approach.
3. To apply the averaged-nodal equation concept, proposed in this paper and explained in Section 4: the proposed concept can only be used with binary-valued switch approach. In this paper, the binary-valued resistance is used. But, the binary-valued inductance can be also preferred.

The proposed switch model associated with the formulation method is given in Section 4.

4. Averaging circuit approaches

The essential principles for averaged circuit models are proposed by Middlebrook-Cuk [17]. For DC-DC converters containing controlled switches, they developed the state-space averaging method. These converters contain absolutely one active switch (Q) and one diode (D) (for example, Buck converter circuit in Figure 3). In [17], the converter circuit is defined by two different topologies and equations with a switching period (T_s) and duty-cycle (d). They used the ideal switch concept and the state-space formulation. State equations related to every topology are obtained separately. For example, for Buck converter circuit, the state-space equations for state "1" and state "2" are given as follows:

$$\begin{aligned} \dot{x} &= A_1x + B_1u \\ y &= C_1x + D_1u \end{aligned}, \quad \begin{aligned} \dot{x} &= A_2x + B_2u \\ y &= C_2x + D_2u \end{aligned} \tag{5}$$

Equation (5) corresponds to the exact topological solution over one switching period (T_s), since the circuit topology changes periodically according to the closing and opening of switches. In order to obtain a unified form which covers the whole switching period (T_s), the average of the circuit equations (Equation 5) is taken. In this manner, the state-space averaging equations are obtained as follows:

$$\begin{aligned} \dot{x} &= [A_1d + A_2(1-d)]x + [B_1d + B_2(1-d)]u \\ y &= [C_1d + C_2(1-d)]x + [D_1d + D_2(1-d)]u \end{aligned} \tag{6}$$

There are two main disadvantages of this approach. As explained in Section 3, the ideal switch introduces some problems not commonly encountered when analysing non-switched analogue networks. Therefore, the ideal switch concept is not a good approximation. Second, state equations relating to every topology must be obtained separately. There are 2^n possible topological states. Besides, according to [17], if the switching period (T_s) is much smaller than the time constants of the system, the averaging solution for system equations can be obtained. In other words, their solution is only valid for continuous-conduction mode. The state-space averaging method is also used for the analysis of switching circuits in [18–21].

4.1. Averaged-nodal-equation-based approach

In this paper, an averaged-equation system concerning generalized nodal equations is proposed. To obtain only one topology and to avoid the disadvantages of ideal switch concept, switching elements are modelled as a binary-valued resistance. During the simulation, element values related to switches in system equations are only updated.

Because of the switch model, the circuit structure is unique and system equations are valid for all switching topologies. Accordingly, the unknown vector, $X(s)$, remains unchanged. Since a binary-valued resistance approach for switches is used, the matrix G, to which switching elements contribute, in Equation (2), will only be updated during the simulation. If a binary-valued inductance approach is used for switching elements, then the matrix C will be updated. Let us consider s-domain equations in Equation (2). The system equations for the first state (dT) with "1" and for the second state ($(1-d)T$) with "2" according to two positions of switches are given as follows:

$$G_1X(s) + sCX(s) = BU(s),\ G_2X(s) + sCX(s) = BU(s) \quad (7)$$

Equation (7) corresponds to the exact topological solution over one switching period (T_s). As in [17], let us apply the averaging approach to system equations. In order to obtain a unified form which covers the whole switching period (T_s), the average of the circuit equations (Equation 7) is taken. So, the averaged-nodal equations are obtained as follows:

$$G_1dX(s) + sCdX(s) + G_2(1-d)X(s) + sC(1-d)X(s) = BdU(s) + B(1-d)U(s) \quad (8)$$

$$\underbrace{[G_1d + G_2(1-d)]}_{G_{av}} X(s) + sCX(s) = BU(s) \quad (9)$$

$$G_{av}X(s) + sCX(s) = BU(s)$$

$$\underbrace{[G_{av} + sC]}_{A(s)} X(s) = BU(s) \quad (10.a)$$

$$G_{av}x(t) + C\frac{dx}{dt} = Bu(t) \quad (10.b)$$

The resulting statements in Equation (10) are arranged in s-domain and t-domain. Equation (10) is in the form of the averaged-nodal equation. Equation (10.a) has the same form as Equation (3). Accordingly, the unified time constants are obtained from the numerator of the determinant of the characteristic matrix, $A(s)$, as explained in Section 2. After determining the unified time constants, the averaged exact solutions can be obtained in time domain from Equation (10.b).

As seen from Equations (10.a) and (10.b), matrices B, C and X remain unchanged. Only matrix G is averaged, (G_{av}). The reason is that the resistance values relating to switching elements, in the matrix G, change, whereas, values of other linear elements remain unchanged for the whole switching period because of the constant circuit structure. Essentially, the average of resistance value in the switch model is used in matrix G. This situation is expressed in Figure (1). The illustrated appearance in Figure 1 is related to two positions of switches. Here, R_1 expresses "On-state resistance" or "Off-state resistance" of any switch during the first state (dT). Similarly, R_2 expresses "On-state resistance" or "Off-state resistance" of any switch during the second state ($(1-d)T$).

Figure 1(a) represents a switch which is On-state during the first state (dT) and Off-state during the second state ($(1-dT)$). Figure 1(b) represents a switch which is Off-state during the first state (dT) and On-state during the second state ((1 − dT)). For both

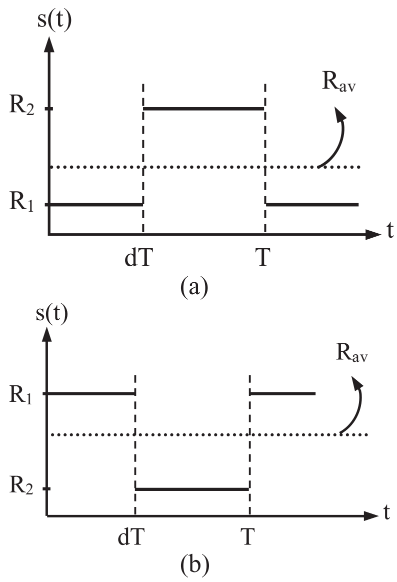

Figure 1. Changes in switch on-resistance and off-resistance with time.

representations, the averaged resistance value of a switch over one switching period is analytically calculated as follows:

$$R_{av} = \frac{dTR_1 + (1-d)TR_2}{T} = dR_1 + (1-d)R_2 \quad (11)$$

The illustration in Figure (1) provides a basis of the proposed approach for the analysis of switching circuits. Regarding this, the proposed switch model is illustrated in Figure 2. This model is valid for both an opening switch and a closing switch. Switching element is modelled by an averaged resistor. By this model, a switching circuit will have only one topology in spite of switching process. The averaged solutions related to all variables of any switching circuit are obtained. Accordingly, unified time constants valid for the whole period and transient-state analysis are determined by a constant topology. Various averaged-element approaches are also applied to switching circuits in [22–24].

5. Illustrative example

Consider the switching circuit in Figure 3. E = 20 V, R_L = 5 Ω, L = 5 mH, C = 50 µF, F_s = 1 kHz (switching frequency), d = 0.5 (duty cycle). In the circuit, there are two possible switching states: (i) Q is on and D is off, (ii) Q is off and D is on. The time constants and

Figure 2. The average-based switch model.

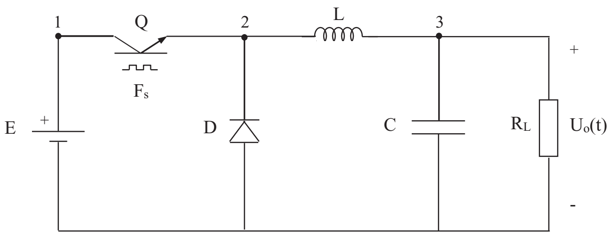

Figure 3. Buck converter circuit.

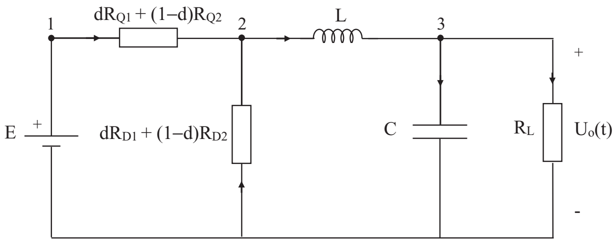

Figure 4. Averaged Buck converter circuit.

the averaged exact solutions will be obtained for two different resistance values of switches.

a. First values for switches
   i. Q is on and D is off: $R_{Q1} = 10^{-6}\ \Omega$ (transistor on-resistance), $R_{D1} = 10^{6}\ \Omega$ (diode off-resistance).
   ii. Q is off and D is on: $R_{Q2} = 10^{6}\ \Omega$ (transistor off-resistance), $R_{D2} = 10^{-6}\ \Omega$ (diode on-resistance),
b. Second values for switches
   i. Q is on and D is off: $R_{Q1} = 10^{-2}\ \Omega$ (transistor on-resistance), $R_{D1} = 10^{2}\ \Omega$ (diode off-resistance).
   ii. Q is off and D is on: $R_{Q2} = 10^{2}\ \Omega$ (transistor off-resistance), $R_{D2} = 10^{-2}\ \Omega$ (diode on-resistance)

As explained in Section 4.1, the switches are replaced with their averaged models. After the active switch (Q) and the diode (D) are replaced by averaged resistors, the unified circuit model is valid for the whole period. The averaged circuit model is given in Figure 4. The averaged circuit has the same topology as the converter. According to this circuit structure, the switching circuit can be expressed by only one equation system.

In system equations, all time-dependent variables of circuit are changed by averaged values. The nodal-equation system of averaged converter circuit is obtained as follows:

$$G_{av}X(s) + sCX(s) = BU(s) \rightarrow A(s)X(s) = BU(s)$$

Using the system equations, let us obtain the characteristic equation and unified time constants. After substituting the element values into the system equations, the determinants of the coefficient matrix (A), related to Equation (4), are obtained for two different resistance values of switches, as follows:

a. For $R_{Q1} = 10^{-6}\ \Omega$, $R_{Q2} = 10^{6}\ \Omega$, $R_{D1} = 10^{-6}\ \Omega$, $R_{D2} = 10^{6}\ \Omega$

$$\det(A(s)) = \frac{Q(s)}{R(s)} = \frac{s^2 + 4000s + 4 \times 10^6}{4} \qquad (13)$$

b. For $R_{Q1} = 10^{-2}\ \Omega$, $R_{Q2} = 10^{2}\ \Omega$, $R_{D1} = 10^{-2}\ \Omega$, $R_{D2} = 10^{2}\ \Omega$

$$\det(A(s)) = \frac{Q(s)}{R(s)} = \frac{s^2 + 4000s + 4 \times 10^6}{40000} \qquad (14)$$

As seen from Equations (13) and (14), for both states, the characteristic equations, $Q(s)$, are the same. Consequently, time constants remain unchanged and they do not depend on switch resistance values.

The characteristic equation: $Q(s) = s^2 + 4000s + 4 \times 10^6$. The roots of characteristic equation ($Q(s)$) give the eigenvalues of the second-order circuit: $\alpha_1 \cong -1.99 \times 10^3$, $\alpha_2 = -2 \times 10^3$. Unified time constants are computed by taking inverse of the eigenvalues:

$$\tau_1 = \frac{1}{|-1.99 \times 10^3|} \cong 0.5025\ \text{ms}\ (= \tau_{max}),$$
$$\tau_2 = \frac{1}{|-2|} = 0.5\ \text{ms}$$

The duration of the transient-state depends on the maximum time constant: $5\tau_{max} \cong 2.512\ \text{ms}$

These unified time constants are valid for the whole switching period, and they describe all system dynamics. After determining the unified time constants, the averaged exact solutions in time domain can be obtained. To that end, the system equations in Equation (12) are converted into time-domain form as in Equation (10.b). The characteristics related to averaged model of converter system can be obtained by using any integration method. Moreover, the averaged circuit model given in Figure 4 can be also analysed

$$\begin{bmatrix} dG_{Q1} + (1-d)G_{Q2} & -dG_{Q1} - (1-d)G_{Q2} & 0 & 0 & 1 \\ -dG_{Q1} - (1-d)G_{Q2} & dG_{Q1} + (1-d)G_{Q2} + dG_{D1} + (1-d)G_{D2} & 0 & 1 & 0 \\ 0 & 0 & G_L + sC & -1 & 0 \\ 0 & 1 & -1 & -sL & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ I_L \\ I_E \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} U(s) \qquad (12)$$

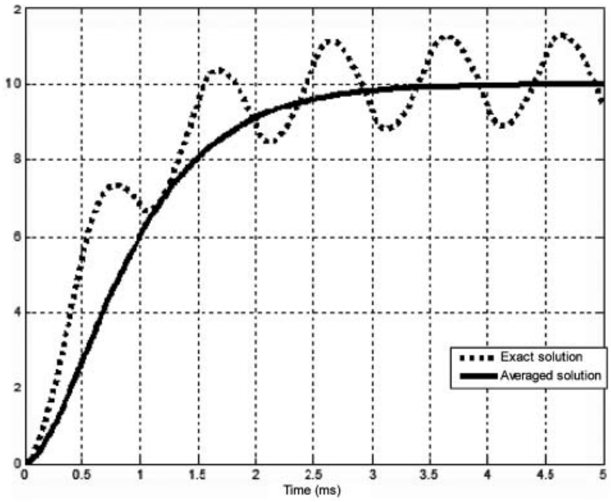

Figure 5. Exact and averaged solution of capacitor voltage.

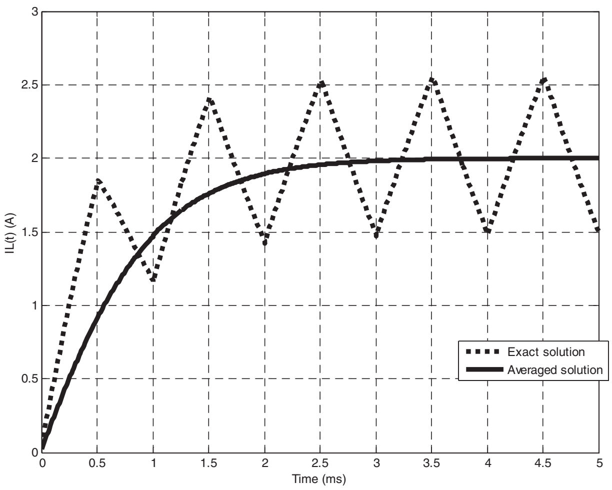

Figure 6. Exact and averaged solution of inductor current.

with any computer simulation program (PSPICE, etc.). Both the exact time-domain topological solutions (using different topologies respectively relating to every state of active switch) and the results of averaged circuit (using the time-domain form of Equation 12) are shown together in Figures 5 and 6. Here, the waveforms of the current ($i_L(t)$) flowing through the inductor and the voltage across the capacitor ($u_C(t)$) are given. The characteristics are the same for two different resistance values of switches.

It is seen that the averaging solutions pass through the middle of the exact solutions in Figures 5 and 6. And also, the averaged solutions have also the same duration of the transient-state, which is determined according to unified time constants.

6. Conclusions

Since switching circuits have variable structural topologies, obtaining system equations for every topology makes the analysis of switching circuits difficult. Unlike other switch models, in order to obtain a unified circuit structure, binary-valued element approach is used. Averaging the on-state and off-state resistances of switching elements over one switching period allows to obtain the equivalent circuit model of switching circuits. This modelling of switches is a good approach to model switching circuits with a unified equation. The desired analysis and designs are easily realized by the averaged circuit and unified-nodal equations. By using the proposed approach, the unified time constants valid for the whole period are derived. At first, the averaged system equations are obtained in s-domain. After finding unified time constants, for obtaining averaged time-domain solutions, the same system equations are converted into time domain.

The proposed unified model and analysis method is applied for Buck converter. The proposed technique can be also applied for other switching converters (Boost, Buck-Boost, Cuk). The averaged circuit model has the same topology as switching converter. The only difference is that the active switch and the diode are replaced with averaged resistors. So, it is very easy to obtain the large-signal model for any switching converter. The essential properties of the model are simple, general and unified.

Disclosure statement

No potential conflict of interest was reported by the author.

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