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Wei Yue & Liyuan Wang

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REGULAR PAPER

Event-triggered autonomous platoon control against probabilistic sensor and actuator failures

Wei Yue[a] and Liyuan Wang[b*]

[a]School of Information Sciences and Technology, Dalian Maritime University, Dalian Shi, China; [b]School of Control Science and Engineering, Dalian University of Technology, Dalian Shi, China

ABSTRACT
This paper is concerned with event-triggered autonomous platoon control with probabilistic sensor and actuator failures. A new platoon model is established, in which the effect of event-triggered scheme and probabilistic failures are involved. Based on the model, the criteria for the exponential stability and co-designing both the trigger parameters and the output feedback are derived by using the Lyapunov method. The theoretical results show that the proposed controller would be able to safely maintain a smaller inter-vehicle spacing and the platoon would be string stable. The effectiveness and advantage of the presented methodology are demonstrated by numerical simulations.

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1. Introduction

Autonomous platoon control system (APCS) is a vehicle-following control system which automatically accelerates and decelerates so as to keep a small inter-vehicle distance [1,2]. There are so many advantages of moving vehicle based on the notion of platoons, such as driving safety and comfort, reducing fuel consumption and air pollution, and improving the throughput in the urban traffic [3,4]. As a result, a lot of research works on platoon control have been extensively studied in [5–7].

Since vehicles in a platoon are coupled, disturbances acting on one vehicle may inevitably affect the others, rendering spacing errors to amplify along the platoon which is called string instability [8,9]. Therefore, an important aspect of vehicle platoon control, beyond stabilizing each of the individual vehicles involved, is the problem of ensuring string stability, or stability of the platoon of vehicles as a whole.

To guarantee string stability and maintain the desired space, much research has been proposed in [3–17]. As stated in [6], there have been two control strategies, i.e. the bidirectional following and predecessor and leader following. First, the bidirectional following strategy is a platoon control scheme by which the information of its following and preceding vehicle should be employed. This scheme is decentralized, since the control information can be obtained by on-board sensors alone. Still, the nearest neighbour following control suffers from the high sensitivity to the length of the vehicular platoon and lower performance compared with the predecessor and leader following strategy. Such as in [8], the authors investigate optimal control strategies for a nearest neighbour following with an increasing number of vehicles and show that some related linear quadratic regulator (LQR) problems are ill-posed. A mistuning control method is designed in [9] to improve the stability margin of the platoon system. In order to enhance the coherence of the nearest neighbour following control scheme, an optimal controller was designed in [10].

Due to these weaknesses of the bidirectional structure, most of the platoon-control research work has been based on the predecessor and leader following platoon control structure. This control scheme is advantageous because, apart from its simplicity in achieving string stability, it utilizes the wireless communication technology to increase the performance of the platoon. However, the use of the wireless communication immediately causes some questions on the effect of communication constraints. Under this framework, these works present in [11] studied the effects of communication delays on string stability; longitudinal platoon control and state estimation via communication channels with packed-dropout are addressed in [12]; a decentralized communication and control strategy is presented in [13] for automated driving assistance to a platoon of vehicles in heavy traffic and scarce visibility.

In contrast to the aforementioned literature, the main focus in this paper is on how to deal with the following three aspects. First, ignoring the frequent operation on the actuator brings an uncomfortable experience to the passengers and increases the fuel

CONTACT Wei Yue 148098475@qq.com; weiy@dlmu.edu.cn
* Present address: College of Mechanical and Electrical Engineering, Dalian Minzu University, Dalian, China.

consumption. In [14], the authors discuss how the frequent operation affects the fuel consumption. In [15], a model predictive control method was discussed, which can minimize the frequent operation on the throttle. However, these control methods suggested are not applicable to the APCS. Second, without considering the contingent failures might happen to the on-board sensors in practical cars for reasons such as poor visibility due to rain or sandstorm and interference of radar signals [16,17]. The combined actuator fault is the third aspect that may add to the limitations since the actuator failure will cause a wrong operation on the actuator speed growing or reducing. Previous works on actuator failure detection and control related to vehicle control have been carried out in [18,19]. However, the detection technologies suggested are not applicable to the fully APCS, which is still an open and challenging problem.

The aim of this paper is to design an autonomous platoon control method within an event-triggered framework. We first model the platoon system that takes full consideration of the probabilistic failures. Sufficient conditions for the existence of output feedback controllers are derived in the context of an event-triggered scheme, which ensure the exponential stability of the platoon system. With these conditions, the individual vehicle stability and string stability can be guaranteed with a desired exponential decay rate. As will be shown later in numerical simulations, the presented method can serve as an effective algorithm for practical use.

The remainder of this paper is organized as follows. Section 2 introduces the problem formulation of platoon control with sensor and actuator failures taken into consideration. Section 3 presents an event-triggered controller for dealing with probabilistic failures. Section 4 obtains the sufficient conditions for the controller to achieve string stability. Numerical simulations are shown in Section 5. Finally, Section 6 presents the conclusion.

2. Problem formulations

Consider a platoon system consisting of n vehicles (see Figure 1) running in a horizontal environment. Denote by z_i, v_i and a_i the ith (i = 0, ..., $n-1$) vehicle's position, velocity and acceleration, with i = 0 standing for the lead vehicle and the others being followers. Each follower vehicle periodically broadcasts its position, velocity and acceleration to the following vehicle in the platoon one by one. The lead vehicle periodically broadcasts its position, velocity and acceleration to all the follower vehicles in the platoon. All followers are equipped with GPS devices and to measure the distance and relative velocity between it and its preceding vehicle.

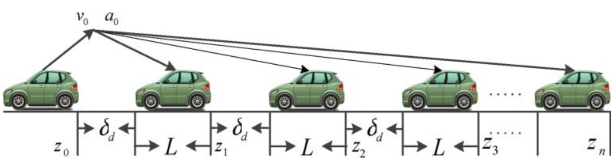


Figure 1. Autonomous platoon systems.

2.1. Platoon system dynamics modelling

Consider a platoon of n vehicles, as shown in Figure 1, where i = 0 stands for the leading vehicle. The spacing error for the ith following vehicle can be defined as

$$\delta_i = z_{i-1} - z_i - \delta_d - L \ (z_0 = 0 \text{ in } \delta_1), \tag{1}$$

where δ_d is the desired vehicle spacing, z_{i-1} and z_i denote the position of two consecutive vehicles, and L is the length of the vehicle. Then, the dynamics of the ith following vehicle can be modelled by the following nonlinear differential equations:

$$\begin{aligned} \dot{\delta}_i &= v_{i-1} - v_i, \\ \dot{v}_i &= a_i, \\ \dot{a}_i &= f_i(v_i, a_i) + g_i(v_i)c_i, \end{aligned} \tag{2}$$

where $v_i$and a_i are the velocity and acceleration, respectively, c_i is the control input of the ith vehicle's engine, with $c_i > 0$ and $c_i < 0$ representing the throttle input and the brake input, respectively. $f_i(v_i, a_i)$ and $g_i(v_i)$ are given by

$$f_i(v_i, a_i) = -\frac{1}{\varsigma_i}\left(\dot{v}_i + \frac{\sigma A_i c_{\text{di}}}{2m_i} v_i^2 + \frac{d_{\text{mi}}}{m_i}\right) - \frac{\sigma A_i c_{\text{di}} v_i a_i}{m_i},$$

$$g_i(v_i) = \frac{1}{\varsigma_i m_i},$$

with σ being the specific mass of the air. For the ith vehicle, m_i is the vehicle mass, A_i is the cross-sectional area, $\sigma A_i c_{\text{di}}/2m_i$ is the air resistance, c_{di} is the drag coefficient, d_{mi} is the mechanical drag and ς_i is the engine time constant.

For (2), we adopt the following feedback linearization control law:

$$c_i = u_i m_i + \sigma A_i c_{\text{di}} v_i^2/2 + d_{\text{mi}} + \varsigma_i \sigma A_i c_{\text{di}} v_i a_i, \tag{3}$$

where u_i is the additional input signal to be designed so that the closed-loop system can satisfy certain performance criteria. After introducing (3), the third equation in (2) becomes

$$\dot{a}_i(t) = -\frac{1}{\varsigma_i} a_i(t) + \frac{1}{\varsigma_i} u_i(t). \tag{4}$$

Define $x(t) = \text{Col}[x_i(t)]_{i=1}^{n-1}$, $u(t) = \text{Col}[u_i(t)]_{i=1}^{n-1}$ and $y(t) = \text{Col}[y_i(t)]_{i=1}^{n-1}$, respectively, as the state, the control and the measurement output vectors, where "Col" represents the column vector, $x_i(t) = [\delta_i \quad v_i \quad a_i]^T$ and $y_i(t) = [\delta_i \quad v_{i-1} - v_i \quad a_{i-1} - a_i \quad v_0 - v_i a_0 - a_i]^T$, $i = 1, \ldots, n-1$. Based on (2) and (4), the state-space equation of the platoon system can be written as

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{5}$$

where

$$A = \begin{bmatrix} A_v & 0 & \cdots & 0 \\ A_d & A_v & \cdots & 0 \\ \cdots & \ddots & \ddots & \cdots \\ 0 & \cdots & A_d & A_v \end{bmatrix}, \; B = \begin{bmatrix} B_v & 0 & \cdots & 0 \\ 0 & B_v & \cdots & 0 \\ \cdots & \ddots & \ddots & \cdots \\ 0 & \cdots & 0 & B_v \end{bmatrix},$$

$$B_v = [0 \quad 0 \quad 1/\varsigma_i]^T,$$

$$A_v = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1/\varsigma_i \end{bmatrix}, \; A_d = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

Similarly, the output equation is written as

$$y(t) = Cx(t), \tag{6}$$

For each following vehicle, the kernel controller to be designed is in the following output feedback form:

$$u_i(t) = K_i y_i(t), \tag{7}$$

where $K_i = [k_p \quad k_v \quad k_a \quad k_{vl} \quad k_{al}]$ is the controller gain to be determined.

2.2. Construction of event-triggered framework

As is well known, the periodic sampling mechanism has been widely used in APCS. However, it may often lead to sending many unnecessary signals to the controller, which in turn will increase the fuel consumption. Therefore, for the control of the platoon systems shown in Figure 2, in order to achieve fuel economy, it is significant to introduce an event-triggered mechanism which decides whether the newly sampled information should be sent to the controller. As shown in Figure 2, an event generator is constructed between the sensor and the controller, which decides when to transmit the measurement output to the controller by a specified trigger condition; the state is sampled regularly by the sampler of the sensor with period h and is fed into the event generator, which will be given in the sequel. The following function of the event-triggered platoon system architecture in Figure 2 is expected:

(1) The state of the vehicles i is sampled at time kh by sampler with a given period h. The next state is at time $(k + 1)h$.

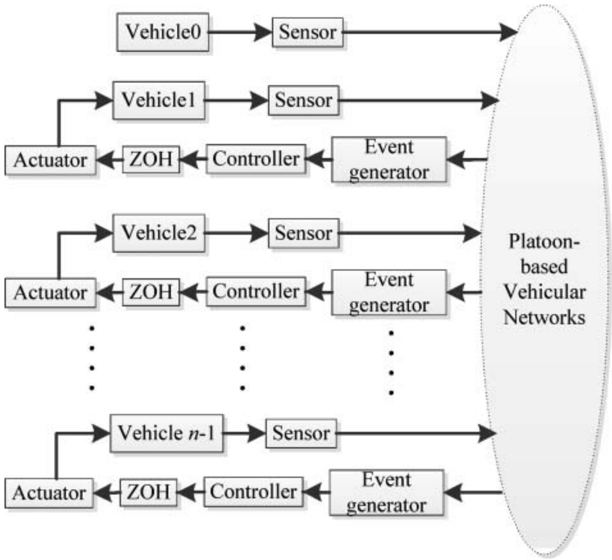


Figure 2. Architecture of the event-triggered platoon system.

(2) As shown in Figure 2, the event generator is constructed between the sensor and the controller, which uses the sampled state to determine whether the newly sampled state will be sent out to the controller. Considering the probabilistic failures, we adopt the following judgement algorithm:

$$[\mathrm{E}\{\rho x(k+j)h)\} - \mathrm{E}\{\rho x(kh)\}]^{\mathrm{T}}\Omega[\mathrm{E}\{\rho x(k+j)h)\} \\ -\mathrm{E}\{\rho x(kh)\}] \le \mu \mathrm{E}\{\rho x(k+j)h)\}]^{\mathrm{T}}\Omega[\mathrm{E}\{\rho x(k+j)h)\}], \tag{8}$$

where Ω is a positive weighting matrix, $j \in \{0, 1, 2, \ldots\}$, $\mu \in [0, 1]$ and $\rho = \mathrm{diag}\{\rho_1, \rho_2, \ldots, \rho_n\}$ with $\rho_i = \mathrm{diag}\{\rho_p, \rho_v, \rho_a, \rho_{vl}, \rho_{al}\}$ is the failures' status matrix of the ith vehicle, and ρ_p, ρ_v, ρ_a, ρ_{vl} and ρ_{al} are five unrelated random variables.

(3) Under the event-triggered scheme (8), the release times are assumed to be t_0h, t_1h, t_2h, ..., where t_0 is the initial time. $s_jh = t_{j+1}h - t_jh$ denotes the release period of event generator in (8). Considering the effect of the transmission delay on the wireless communication network, we suppose the time-varying delay from the lead vehicle is τ_k and $\tau_k \in [0, \tilde{\tau}]$, where $\tilde{\tau}$ is a positive integer. Therefore, the measurement output $x_i(t_0h)$, $x_i(t_1h)$, $x_i(t_2h)$, ... will arrive at the following vehicle at the instants $t_0h + s_0$, $t_1h + s_1$, $t_2h + s_2$, ..., respectively.

Considering the effect of the communication delay, under the event generator with (8), the controller in (7) can be rewritten as

$$u(t) = KCx(t_kh), \; t \in [t_kh + \tau_k, t_{k+1}h + \tau_{k+1}], \tag{9}$$

where $K = \mathrm{diag}\{K_i\}_1^{n-1}$.

Remark 2.1: Under the event-triggered scheme (8), the set of release instants $\{t_0h, t_1h, t_2h, \ldots\}$ is a subset of the sampled instants $\{0, 1, 2, \ldots\}$. Moreover, the amount of $\{t_0h, t_1h, t_2h, \ldots\}$ depends on not only the parameter μ_i, but also on the variation of the preceding vehicle's state, that is to say, if the change of the lead vehicle and the preceding vehicle's states is not serious, there will be no further actions on the following vehicles.

2.3. Effect of sensor and actuator failures

Now, we are in a position to show the sensors and actuators used in the platoon system and the problems caused by the failures as indicated in the Table 1.

In this research, the sensor-failure model in [20] was utilized to describe the failure phenomenon in GPS, wheel speed sensor and accelerometer, namely $\delta_i^f(t) = \rho_i^\delta \delta_i(t)$, $v_i^f(t) = \rho_i^v v_i(t)$, $a_i^f(t) = \rho_i^a a_i(t)$, where $0 \le \rho_i^\delta \le \overline{\rho}_i^\delta$, $0 \le \rho_i^v \le \overline{\rho}_i^v$ and $0 \le \rho_i^a \le \overline{\rho}_i^a$, with $\overline{\rho}_i^\delta$, $\overline{\rho}_i^v$ and $\overline{\rho}_i^a$. Then, the controller (9) can be written in the following form:

$$u(t) = KC\rho_s x(t_kh),\ t \in [t_kh + \tau_k, t_{k+1}h + \tau_{k+1}], \quad (10)$$

where $\rho_s = \text{diag}\{\rho_{s1}, \rho_{s2}, \ldots, \rho_{s(n-1)}\}$ with $\rho_{si} = \text{diag}\left\{\rho_{si}^\delta, \rho_{si}^v, \rho_{si}^a\right\}$ and the mathematical expectation and variance of $\rho_{si}^m (m = \delta, v, a)$ are β_{si}^m and λ_{si}^m, respectively.

Based on (10), we consider the actuator failures as in [21], then (10) can be described as follows:

$$u(t) = \rho_a KC\rho_s x(t_kh),\ t \in [t_kh + \tau_k, t_{k+1}h + \tau_{k+1}], \quad (11)$$

where $\rho_{ac} = \text{diag}\{\rho_{ac1}, \rho_{ac2}, \ldots, \rho_{ac(n-1)}\}$ represents the actuator failure state and $0 \le \rho_{aci} \le \overline{\rho}_{aci}$ with $\overline{\rho}_{aci} \ge 1$. The mathematical expectation and variance of ρ_{aci} are β_{aci} and λ_{aci}, respectively.

Under the controller (10), the closed-loop platoon system for $t \in [t_kh + \tau_k, t_{k+1}h + \tau_{k+1}]$, $k = 0, 1, 2, \ldots$ can be written in the following form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B\rho_{ac}KC\rho_s x(t_kh), \\ &= Ax(t) + B\overline{\rho}_{ac}KC\overline{\rho}_s x(t_kh) + \nu x(t_kh), \end{aligned} \quad (12)$$

where $\nu = B\overline{\rho}_{ac}K(\rho_s - \overline{\rho}_s) + B(\rho_{ac} - \overline{\rho}_{ac})K\overline{\rho}_s + B(\rho_{ac} - \overline{\rho}_{ac})K(\rho_s - \overline{\rho}_s)$,

$$\overline{\rho}_{ac} = \text{diag}\left\{\beta_{ac1}, \beta_{ac2}, \ldots, \beta_{ac(n-1)}\right\} = \sum_{i=1}^{n-1} \beta_{aci} L_{ac}^i,$$

$$\overline{\rho}_s = \text{diag}\left\{\beta_{s1}^m, \beta_{s2}^m, \ldots, \beta_{s(n-1)}^m\right\} = \sum_{i=1}^{n-1} \beta_{si}^m L_s^i,$$

$$\text{E}\left\{(\rho_{ac} - \overline{\rho}_{ac})^2\right\} = \text{diag}\left\{\lambda_{ac1}^2, \ldots, \lambda_{ac(n-1)}^2\right\},$$

$$\text{E}\left\{(\rho_s - \overline{\rho}_s)^2\right\} = \text{diag}\left\{(\beta_{s1}^m)^2, (\beta_{s2}^m)^2, \ldots, (\beta_{s(n-1)}^m)^2\right\},$$

$$L_a^i = \text{diag}(\underbrace{0, \ldots, 0}_{i-1}, 1, \underbrace{0, \ldots, 0}_{n-1-i}),$$

$$L_s^i = \text{diag}(\underbrace{0, \ldots, 0}_{i-1}, 1, \underbrace{0, \ldots, 0}_{n-1-i}).$$

For the convenience of forthcoming discussion, consider the following two cases:

Case 1: If $t_kh + h + \tilde{\tau} \ge t_{k+1}h + \tau_{k+1}$, where $\tilde{\tau} = \max\tau_k$, define a function $\tau(t)$ as

$$\tau(t) = t - t_kh,\ t \in [t_kh + \tau_k, t_{k+1}h + \tau_{k+1}], \quad (13)$$

Clearly, following from (13) that

$$\tau_k \le \tau(t) \le (t_{k+1} - t_k)h + \tau_{k+1} \le h + \tilde{\tau}. \quad (14)$$

Case 2: If $t_kh + h + \tilde{\tau} < t_{k+1}h + \tau_{k+1}$, consider the following intervals:

$$[t_kh + \tau_k, t_kh + h + \overline{\tau}] \text{ and } [t_kh + jh + \tau_k, t_kh + jh + h + \overline{\tau}].$$

Since $\tau_k \le \overline{\tau}$, it can be easily shown that there exists d_{iM} such that

$$t_kh + d_Mh + \overline{\tau} \le t_{k+1}h + \tau_{k+1} \le t_kh + d_Mh + h + \overline{\tau}.$$

Moreover, $x(t_kh)$ and $t_kh + jh$ with j= 0, 1, ..., d_M satisfy (8):

$$\text{Let} \begin{cases} I_0 = [t_kh + \tau_k, t_kh + h + \overline{\tau}], \\ I_j = [t_kh + jh + \tau_k, t_kh + jh + h + \overline{\tau}], \\ I_{d_M} = [t_kh + d_Mh + \overline{\tau}_i, t_{k+1}h + \tau_{k+1}], \end{cases} \quad (15)$$

where j= 0, 1, ...,$d_M - 1$, one can get

$$[t_kh + \tau_k, t_{k+1}h + \tau_{k+1}] = \bigcup_{j=0}^{j=d_M} I_j.$$

$$\text{Define } \tau(t) = \begin{cases} t - t_kh, & t \in I_{d_{iM}} \\ t - t_kh - jh, & t \in I_j \\ t - t_kh - d_Mh, & t \in I_{d_M} \end{cases}, \quad (16)$$

Then, we have

$$\begin{cases} t_k \le \tau(t) < h + \overline{\tau}, & t \in I_0 \\ t_k \le \overline{\tau} \le \tau(t) \le h + \overline{\tau}, & t \in I_j \\ t_k \le \overline{\tau} \le \tau(t) \le h + \overline{\tau}, & t \in I_{d_M} \end{cases}, \quad (17)$$

Table 1. Complete set of sensor and actuator.

On-board sensor and actuator	Failure phenomenon
Wheel speed sensor	Lost velocity information
GPS	Lost position information
Accelerometer	Lost acceleration information
Throttle actuator	Platoon break-up
Brake actuator	Rear-end collision

where the third row in (10) holds because $t_{k+1}h + \tau_{k+1} \le t_k h + (d_M + 1)h + \overline{\tau}$. Obviously,

$$0 \le \tau_i \le \tau_i(t) \le h + \overline{\tau}_i \triangleq \tau_{im},\ t \in [t_k h + \tau_k, t_{k+1}h + \tau_{k+1}]. \tag{18}$$

In Case 1, for $t \in [t_k h + \tau_k, t_{k+1}h + \tau_{k+1})$, define $e_k(t) = 0$; in Case 2, define

$$\overline{\rho}_s e_k(t) = \begin{cases} 0, & t \in I_0 \\ \overline{\rho}_s x(t_k h) - \overline{\rho}_s x(t_k h + jh), & t \in I_j \\ \overline{\rho}_s x(t_k h) - \overline{\rho}_s x(t_k h + d_M h), & t \in I_{d_M} \end{cases}. \tag{19}$$

From (19) and the triggering algorithm (8), it can be easily seen that, for $t \in [t_k h + \tau_k, t_{k+1}h + \tau_{k+1}]$,

$$e_k^T(t)\overline{\rho}_s^T \Omega \overline{\rho}_s e_k(t) \le \mu x^T(t - \tau(t))\overline{\rho}_s^T \Omega \overline{\rho}_s \tilde{x}(t - \tau(t)). \tag{20}$$

According to (19), we can deduce that

$$e_k(t) = \begin{cases} 0, & t \in I_0 \\ x(t_k h) - x(t_k h + jh), & t \in I_j \\ x(t_k h) - x(t_k h + d_M h), & t \in I_{d_M} \end{cases}. \tag{21}$$

Utilizing $\tau(t)$ and $e_k(t)$, the closed-loop platoon system (12) can be rewritten as

$$\begin{aligned} \dot{x}(t) = Ax(t) + B\overline{\rho}_{ac}K\overline{\rho}_s[x(t - \tau(t)) + e_k(t)] \\ + v[x(t - \tau(t)) + e_k(t)], \end{aligned} \tag{22}$$

where $t \in [t_k h + \tau_k, t_{k+1}h + \tau_{k+1}]$.

2.4. The objective

Our objective of this paper is to design an event-triggered-based controller for the platoon system to meet the following criteria:

(i) Individual vehicle stability: the entire closed-loop platoon system is exponentially mean square stable (EMSS).

(ii) Steady-state performance: the relative velocity errors $\Delta v_i(t)$ approach to zero for all vehicles.

(iii) String stability: the oscillations are not amplifying with vehicle index due to any manoeuver of the lead vehicle, namely $\|G(jw)\| \le 1$ for any w, where $G(s) = \delta_i(s)/\delta_{i-1}(s)$ with $\delta_i(s)$ and $\delta_{i-1}(s)$denotes the Laplace transforms of the spacing error $\delta_i(t)$ and $\delta_{i-1}(t)$, respectively.

Before giving the main results on the controller design, we first give two definitions and two lemmas.

Definition 2.1 [22]**:** For a given function $V{:}C^b_{F_0}([-\tau_m, 0], R^n) \times S$, its infinitesimal operator Γ is defined as $\Phi(V\eta(t)) = \lim_{\Delta \to 0^+}[\mathrm{E}(V(\eta_t + \Delta) \mid \eta_t) - V(\eta_t)]$.

Definition 2.2 [22]**:** System (18) and (19) is said to be EMSS if there exist constants $\alpha > 0$ and $\beta > 0$ such that, for $t \ge 0$,

$$\mathrm{E}(\|x(t)\|^2) \le \alpha e^{-\beta t}\mathrm{E}\left\{\sup_{-\tau_m \le s \le 0} \|\varphi(s)\|^2\right\}.$$

Lemma 2.1 [23]**:** For any vectors $x, y \in R^n$, and positive definite matrix $Q \in R^{n \times n}$, the following inequality holds:

$$2x^T y \le x^T Q x + y^T Q^{-1} y.$$

Lemma 2.2 [23]: E_1, E_2 and Ω are matrices with appropriate dimensions, $\tau(t)$ is a function of t and $0 \le \tau(t) \le \tau_M$, then $\tau(t)\mathrm{E}_1 + (\tau_M - \tau(t))\mathrm{E}_2 + \Omega < 0$, if and only if $\tau_M \mathrm{E}_1 + \Omega < 0$, $\tau_M \mathrm{E}_2 + \Omega < 0$.

3. Event-triggered controller design

One event-triggered controller for the platoon system with sensor and actuator failures is derived in this section. This controller is designed based on Lyapunov's second method.

We first give the EMSS condition for the platoon system (22) in the following theorem:

Theorem 3.1: *For the given scalars τ_M, β^m_{si}, β_{aci}, λ^m_{si}, λ_{aci}, $\mu \in [0, 1]$ and feedback gain* K*, the closed-loop platoon system in (22) is EMSS, if there exist matrices $P > 0$, $Q > 0$, $R > 0$, $\Omega > 0$, N and M such that for $g = 1, 2$ and the following inequalities hold:*

$$\Sigma(g) \triangleq \begin{bmatrix} \Sigma_{11} + \Gamma + \Gamma^T & \Sigma^g_{12} & \sqrt{\tau_M}\overline{A}^T & \Sigma_{14} & \Sigma_{15} \\ * & -R & 0 & 0 & 0 \\ * & * & -R^{-1} & 0 & 0 \\ * & * & * & \Sigma_{44} & 0 \\ * & * & * & * & \Sigma_{55} \end{bmatrix} < 0, \tag{23}$$

where
$\Sigma^1_{12} = \sqrt{\tau_M}N,\ \Sigma^2_{12} = \sqrt{\tau_M}M,$
$\Gamma = [N \quad M - N \quad -M \quad 0],$
$\overline{A} = [A \quad B\overline{\rho}_{ac}K\overline{\rho}_s \quad 0 \quad B\overline{\rho}_{ac}K\overline{\rho}_s],$
$W_{ij} = 2\tau_m(\beta^2_{aci}(\gamma^m_{sj})^2 + \lambda^2_{aci}(\beta^m_{sj})^2 + \lambda^2_{aci}(\gamma^m_{sj})^2),$
$\Sigma_{14} = [\Lambda_1 \quad \Lambda_2 \quad \cdots \quad \Lambda_n],$
$\Lambda_i = \left[\Lambda^T_{i1} \quad \Lambda^T_{i2} \quad \cdots \quad \Lambda^T_{i(n-1)}\right],$
$\Lambda_{ij} = \sqrt{W_{ij}}\left[0 \quad BL^i_{ac}KL^i_s \quad 0 \quad 0\right],$
$\Sigma_{15} = \left[\Pi_1 \quad \Pi_2 \quad \cdots \quad \Pi_{(n-1)}\right],$

$$\Pi_i = \begin{bmatrix} \Pi_{i1}^T & \Pi_{i2}^T & \cdots & \Pi_{i(n-1)}^T \end{bmatrix},$$
$$\Pi_{ij} = \sqrt{W_{ij}}\begin{bmatrix} 0 & 0 & 0 & BL_{ac}^i K L_s^i \end{bmatrix},$$
$$\Sigma_{44} = \Sigma_{55} = \mathrm{diag}\{-R^{-1}, -R^{-1}, \ldots, -R^{-1}\},$$

$$\Sigma_{11} = \begin{bmatrix} PA + A^TP + Q & * & * & * \\ \overline{\rho}_s K^T \overline{\rho}_{ac} B^T P & \sum_i^{n-1} \mu(\beta_{si}^m)^2 (L_s^i)^T \Omega L_s^i & * & * \\ 0 & 0 & -Q & * \\ 0 & 0 & 0 & -\sum_i^{n-1} (\beta_{si}^m)^2 (L_s^i)^T \Omega L_s^i \end{bmatrix}.$$

Proof: See Appendix 1.

Remark 3.1: Theorem 3.1 supplies a sufficient condition for the platoon system to be EMSS, implying that the control objective (i) and (ii) can be achieved. We now proceed to give a reliable controller design method through selecting a constant α to minimize $\sum_{i=1}^{n-1}(\alpha_i - \alpha)$in the following theorem:

Theorem 3.2: *For the given scalars τ_M, β_{si}^m, β_{aci}, λ_{si}^m, λ_{aci}, $\varepsilon > 0$ and $\mu \in [0, 1]$, the closed-loop platoon system in (22) with controller gain $K = YX^{-1}$ is EMSS if there exist matrices $X > 0$, $\tilde{Q} > 0$, $\tilde{R} > 0$, $\Omega > 0$, $\tilde{N}$ $\tilde{M}$ and Y such that for $g = 1, 2$, and the following inequalities hold:*

$$\begin{bmatrix} \tilde{\Sigma}_{11} + \tilde{\Gamma} + \tilde{\Gamma}^T & \tilde{\Sigma}_{12}^g & \sqrt{\tau_{iM}}\tilde{A}^T & 0 & \tilde{\Sigma}_{15} & 0 & 0 & \tilde{\Sigma}_{18} & \tilde{\Sigma}_{19} \\ * & -\tilde{R} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -2\varepsilon X + \varepsilon^2 \tilde{R} & 0 & \tilde{\Sigma}_{35} & 0 & 0 & 0 & 0 \\ * & * & * & \tilde{\Sigma}_{44} & 0 & \tilde{\Sigma}_{46} & 0 & 0 & 0 \\ * & * & * & * & \tilde{\Sigma}_{55} & 0 & \tilde{\Sigma}_{57} & 0 & 0 \\ * & * & * & * & * & \tilde{\Sigma}_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \tilde{\Sigma}_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \tilde{\Sigma}_{88} & 0 \\ * & * & * & * & * & * & * & * & \tilde{\Sigma}_{99} \end{bmatrix} < 0, \quad (24)$$

where

$$\tilde{\Sigma}_{11} = \begin{bmatrix} AX + XA^T + \tilde{Q} & \alpha B\overline{\delta}_{ac}Y & 0 & \alpha B\overline{\delta}_{ac}Y \\ * & \mu\tilde{\Omega} & 0 & 0 \\ * & * & -\tilde{Q} & 0 \\ * & * & * & -\tilde{\Omega} \end{bmatrix},$$
$$\tilde{\Sigma}_{12}^1 = \sqrt{\tau_M}\tilde{N},\ \tilde{\Sigma}_{12}^2 = \sqrt{\tau_M}\tilde{M},$$
$$\tilde{\Gamma} = \begin{bmatrix} \tilde{N} & \tilde{M} - \tilde{N} & -\tilde{M} & 0 \end{bmatrix},$$
$$\tilde{A} = [AX \quad \alpha B\overline{\rho}_{ac}Y \quad 0 \quad \alpha B\overline{\rho}_{ac}Y],$$
$$\tilde{\Sigma}_{15} = \begin{bmatrix} B\overline{\delta}_{ac}Y & B\overline{\delta}_{ac}Y & 0 & 0 \\ 0 & 0 & \overline{\delta}_s - \alpha I & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{\delta}_s - \alpha I \end{bmatrix},$$

$$\tilde{\Sigma}_{18} = \begin{bmatrix} 0 & \cdots & 0 \\ L_s^1 & \cdots & L_s^{n-1} \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix},\ \tilde{\Sigma}_{19} = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ L_s^1 & \cdots & L_s^{n-1} \end{bmatrix},$$
$$\tilde{\Sigma}_{35} = [\sqrt{\tau_M}B\overline{\rho}_{ac}Y \quad 0 \quad 0 \quad 0],$$
$$\tilde{\Sigma}_{44} = \mathrm{diag}\{\underbrace{-2\varepsilon X + \varepsilon^2\tilde{R}, \ldots, -2\varepsilon X + \varepsilon^2\tilde{R}}_{2(n-1)^2}\},$$
$$\tilde{\Sigma}_{55} = \mathrm{diag}\{-X, -X, -X, -X\},$$
$$\tilde{\Sigma}_{46} = \begin{bmatrix} \tilde{\Lambda}_1 & \tilde{\Lambda}_2 & \cdots & \tilde{\Lambda}_{n-1} \end{bmatrix},$$
$$\tilde{\Lambda}_i = \begin{bmatrix} 0_{1\times(i-1)(n-1)} & \tilde{C}_{ji}^T & 0_{(n-1-i)(n-1)\times 1} \end{bmatrix}^T,$$
$$\tilde{\Sigma}_{57} = \begin{bmatrix} \tilde{\Delta}_1 & \tilde{\Delta}_2 & \cdots & \tilde{\Delta}_{n-1} \end{bmatrix},$$
$$\tilde{\Delta}_i = \tilde{\Lambda}_i,\ \tilde{C}_{ji}^T = \sqrt{W_{ji}}\begin{bmatrix} Y^TL_{ac}^1B^T & \cdots & Y^TL_{ac}^1B^T \end{bmatrix},$$
$$\tilde{\Sigma}_{66} = \Theta_{77} = \Theta_{88} = \Theta_{99} = \mathrm{diag}\{\underbrace{-X, \ldots, -X}_{(n-1)^2}\}$$

Furthermore, a candidate controller gain can be given by $K = YX^{-1}$.

Proof: See Appendix 2.

Remark 3.2: The proposed Theorem 3.2 gives upper bounds τ^* for the delay which guarantee the stability of the individual vehicle. This means that the theorem can help us to evaluate the wireless network used in the platoon systems.

4. String stability

In the above section, considerations have been focused primarily on the EMSS of all the individual vehicles in the platoon system. This section is concerned with the issue of string stability, which is associated with objectives (iii) given in Section 2. Here, we first give a result on string stability, and then derive an additional set of constraints to guarantee zero steady-state velocity error. The analysis and results are based on the event-triggered controller (7) obtained above and assume all vehicles have the same fault parameters, namely $\rho_{si}^{\delta} = \rho_{si}^{v} = \rho_{si}^{a} = \rho_s^*$ and $\rho_{aci} = \rho_{ac}^*$.

Consider the ith following vehicle under the control of the presented event-triggered controller. By using Equation (1), we can have the following equation about its spacing error:

$$\dddot{\delta}_i(t) = \dot{a}_{i-1}(t) - \dot{a}_i(t). \quad (25)$$

Substituting (11) into (4), we have

$$\dot{a}_i(t) = -\frac{1}{\varsigma}a_i(t) + \frac{1}{\varsigma}k_i y_i.$$

Combining with (25) and considering the faults in (11), the equation about spacing error can be written as

$$\begin{aligned}\varsigma\dddot{\delta}_i(t) = &-\ddot{\delta}_i(t) - \rho_s^*[k_p\delta_i(t-\tau) - k_v\dot{\delta}_i(t-\tau)\\ &-k_a\ddot{\delta}_i(t-\tau) - k_{vl}\dot{\delta}_i(t-\tau) - k_{al}\ddot{\delta}_i(t-\tau)\\ &+k_p\delta_{i-1}(t-\tau) + k_v\dot{\delta}_{i-1}(t-\tau)\\ &+k_a\ddot{\delta}_{i-1}(t-\tau)]\rho_{ac}^*. \end{aligned} \quad (26)$$

Taking the Laplace transform of Equation (26), we can get

$$G(s) = \frac{\delta_i(s)}{\delta_{i-1}(s)} \times \frac{\rho_s^*(k_p + k_v s + k_a s^2)\rho_{ac}^* e^{-\tau s}}{\varsigma s^3 + s^2 + \rho_s^*[k_p + (k_v + k_{vl})s + (k_a + k_{al})s^2]\rho_{ac}^* e^{-\tau s}}. \quad (27)$$

Based on this transfer function, we have the following result on string stability:

Theorem 4.1: for the platoon-spacing error system (26), $|\delta_i(jw)/\delta_{i-1}(jw)| \leq 1$ holds for any$w > 0$, if the following conditions are satisfied:

$$\begin{cases} \text{(a) } \varsigma k_p/\rho_s^*\rho_{ac}^* - k_v - k_{vl} \leq 0,\\ \text{(b) } k_{al} + k_a = \varsigma(k_v + k_{vl})/\rho_s^*\rho_{ac}^*,\\ \text{(c) } k_{vl}^2 + 2(k_v k_{vl} - k_p k_{al}) - 2k_p \geq 0,\\ \text{(d) } 1 - k_a^2 + 2\tau(\varsigma k_p/\rho_s^*\rho_{ac}^* - k_v - k_{vl}) \geq 0. \end{cases} \quad (28)$$

Proof: First, we write $|\delta_i(jw)/\delta_{i-1}(jw)|$ as

$$G(jw) = |\frac{\delta_i(jw)}{\delta_{i-1}(jw)}| = \sqrt{\frac{a}{a+b}},$$

where

$$\begin{aligned} a = &(k_p - k_a w^2)^2 + k_v^2 w^2,\\ b = &[k_{vl}^2 + 2(k_v k_{vl} - k_p k_{al}) - 2k_p\cos(\tau w)]w^2\\ &+2(\varsigma k_p/\rho_s^*\rho_{ac}^* - k_v - k_{vl})\sin(\tau w)w^3\\ &+[1 + k_{al}^2 + 2k_a k_{al} + 2[(k_{al} + k_a)\\ &-\varsigma(k_v + k_{vl})/\rho_s^*\rho_{ac}^*]\cos(\tau w)]w^4\\ &-2\varsigma(k_{al} + k_a)/\rho_s^*\rho_{ac}^*\sin(\tau w)w^5 + (\varsigma/\rho_s^*\rho_{ac}^*)^2 w^6 \end{aligned}$$

Since $a > 0$, $|\delta_i(jw)/\delta_{i-1}(jw)| \leq 1$ holds true, i.e. the platoon is string stable, if $b \geq 0$. From (28(a)) and the fact that $\sin(\eta w) \leq \tau w$, we have for $w > 0$ that

$$\begin{aligned} &2(\varsigma k_p/\rho_s^*\rho_{ac}^* - k_v - k_{vl})\sin(\tau w)w^3\\ &\leq 2\tau(\varsigma k_p/\rho_s^*\rho_{ac}^* - k_v - k_{vl})w^4. \end{aligned} \quad (29)$$

Using the condition (28(b)), we have

$$\begin{aligned} b \geq &[k_{vl}^2 + 2(k_v k_{vl} - k_p k_{al}) - 2k_p\cos(\tau w)]w^2\\ &+[1 + k_{al}^2 + 2k_a k_{al} + 2\tau(\varsigma k_p/\rho_s^*\rho_{ac}^* - k_v - k_{vl})]w^4\\ &-2\varsigma(k_{al} + k_a)/\rho_s^*\rho_{ac}^*\sin(\tau w)w^5 + (\varsigma/\rho_s^*\rho_{ac}^*)^2 w^6. \end{aligned}$$

Since ς, k_p, k_a and k_{al} are all positive, and the fact that $\cos(\tau w)$, $\sin(\tau w) \leq 1$, one can get

$$\begin{aligned} b \geq &[k_{vl}^2 + 2(k_v k_{vl} - k_p k_{al}) - 2k_p]w^2 + [1 - k_a^2\\ &+ 2\tau(\varsigma k_p/\rho_s^*\rho_{ac}^* - k_v - k_{vl})]w^4. \end{aligned} \quad (30)$$

Thus, if the conditions (28(c,d)) hold, then $b \geq 0$. This completes the proof.

Remark 4.1: It should be noted that the conditions for achieving platoon control require combining Theorems 3.2 and 4.1. This produces an upper bound for the time delay, that is,

$$\tau \leq \left\{\frac{1 - k_a^2}{2(k_v + k_{vl} - \varsigma k_p/\rho_s^*\rho_{ac}^*)}, \tau^*\right\}. \quad (31)$$

5. Simulations

For the numerical simulations, we consider the platoon-control system in Figure 1, each of which consists of 10 vehicles, which run in a virtual environment established using a system-build software package in MATLAB. Comparisons are made between the proposed controller and the method in [15]. The following parameters are used in the simulations: minimum vehicle distance $d_0 = 3$m, length of the vehicle $L_i = 4$ m and the engineer time constant $\varsigma_i = 0.25$. The other parameters used in the simulations are the same as [17], namely specific mass of the air $\sigma = 1.2$kg/m^3, cross-sectional area of vehicle $A_i = 2.2$m^2, drag coefficient $c_{di} = 0.35$, vehicle mass $m_i = 1464$kg and mechanical drag $d_{mi} = 5$N.

In the simulation, we suppose that the sensor's failure $\overline{\rho}_{s1} = \overline{\rho}_{s2} = \cdots = \overline{\rho}_{s9} = \mathrm{diag}\{0.8, 0.8, 0.8\}$, with $\lambda_{s1}^{\delta} = \lambda_{s2}^{\delta} = \cdots = \lambda_{s9}^{\delta} = 0.15$, $\lambda_{s1}^{v} = \lambda_{s2}^{v} = \cdots = \lambda_{s9}^{v} = 0.3$, $\lambda_{s1}^{a} = \lambda_{s2}^{a} = \cdots = \lambda_{s9}^{a} = 0.15$ and the actuator's failure $\overline{\rho}_{ac} = \mathrm{diag}\{0.65, 0.65, 0.65, 0.65, 0.65, 0.65, 0.65, 0.65, 0.65\}$ with $\lambda_{ac1} = \lambda_{ac2} = \cdots = \lambda_{ac9} = 0.15$, $\alpha = 18.06$, $\varepsilon = 1$. The sampling period is $h = 0.2$ms,the corresponding parameter μ= 0.03. The probabilistic sensor and actuator failures for the first vehicle are shown in Figure 3(a,b), respectively. Then, from Theorems 3.2 and 4.1, we can get the upper bound of the time delay $\tau_M = 0.9486$. By setting the delay $\tau = 0.9$, the controller

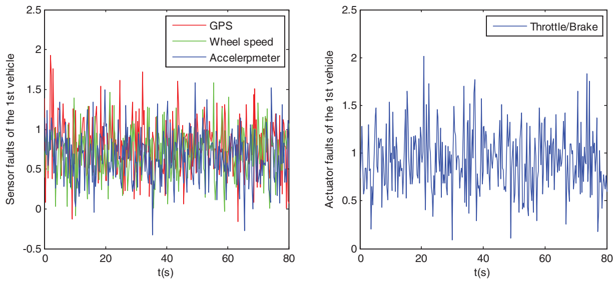


Figure 3. (a) Sensor failures; (b) actuator failures.

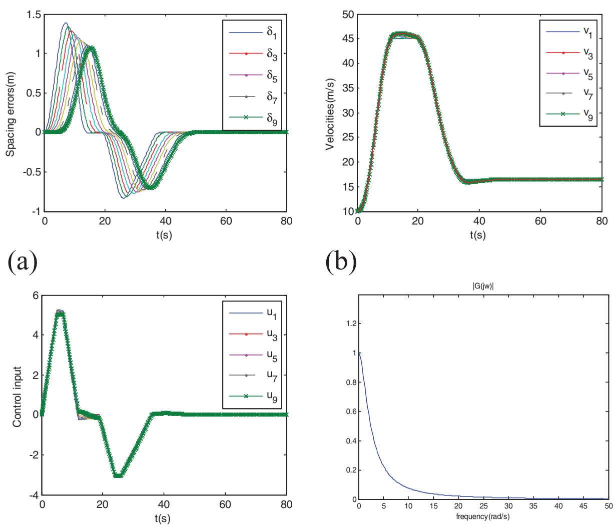
gains are obtained as

$$K = \text{diag}\{K_i\}_1^9, \\ = [9.001 \quad 0.2110 \quad 3.000 \quad 14.214 \quad 0.6068]$$

5.1. Lead vehicle's rapid acceleration and deceleration scenario

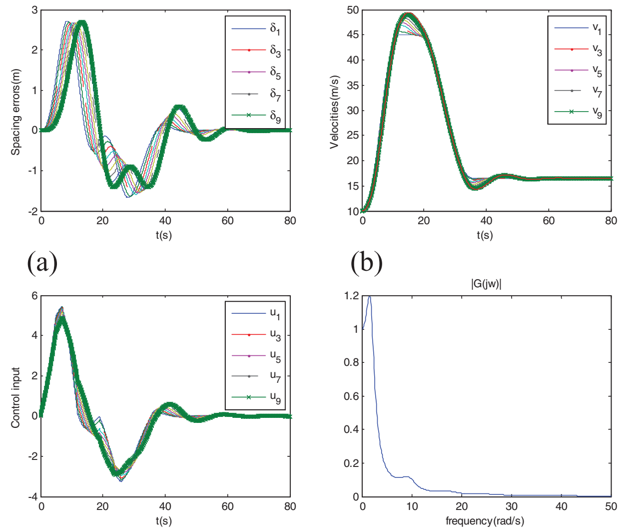
In this scenario, it is assumed that all vehicles in the platoon are running at the same initial speed of 10 m/s with desired spacing as 1 m. At 5 s, the lead vehicle accelerates at 2m/s^2 from 10 to 45 m/s and at 19 s, the preceding car decelerates at the acceleration of -3m/s^2 from 45 to 17 m/s. All the following vehicles are controlled to follow it by using the proposed controller and the algorithms in [15]. The results are shown in Figures 4 and 5, respectively.

During the acceleration stage, the maximum spacing errors and velocities for all the following vehicles in the platoon system under the proposed controller are 1.48 m and 45.3m/s, respectively. As shown in Figure 4, the whole platoon can achieve tracking control accuracies with a smooth control input. In this same case, when the method suggested in [15] is used, the system is string unstable (see Figure 5). The maximum spacing error and velocity are 2.8 m and 49.1m/s, respectively, which are much higher than in our case as shown in Figure 5(a,b). During the deceleration stage, it is found that the whole platoon can hold string stability, and the maximum spacing error and velocity are 0.74 m and 16.4m/s, respectively, as shown in Figure 4. In contrast, as shown in Figure 5(a), the maximum spacing error is −1.75 m, which means a rear-end collision is happening, and the platoon is string instability.



(a) (b) (c) (d)

Figure 4. Ten-vehicle platoon system under proposed controller: responses and spacing propagation characteristics: (a) spacing errors; (b) velocities; (c) control input; (d) frequency response $\exists\, w\, |G(jw)| \leq 1$.



(a) (b) (c) (d)

Figure 5. Ten-vehicle platoon system under controller in [15]: responses and spacing propagation characteristics: (a) spacing errors; (b) velocities; (c) control input; (d) frequency response $\exists\, w\, |G(jw)| > 1$.

5.2. The platoon suffered unknown disturbances

Various types of disturbances typical for real operation conditions have been considered. Figure 6(a) represents an illustration of the capabilities of the proposed methodology to cope with noisy and measurement errors. We assume zero initial conditions, using noisy measurements of δi, vi and ai, where the noise is assumed to be white and zero mean with standard deviations 0.02 m, 0.05 m/s and 0.02 m/s^2, respectively. In the stochastic case, the efficiency of the controller described in Section 3 is obvious and the results show that the maximum absolute spacing error does not exceed 0.8 m. Figure 6(b) illustrates the situation when the second and third vehicles lose the information of the lead vehicle's velocity and acceleration; vehicles 2 and 3 are obviously string unstable, but the rest of the vehicles still meet the platoon-control objective (3) in Section 2.

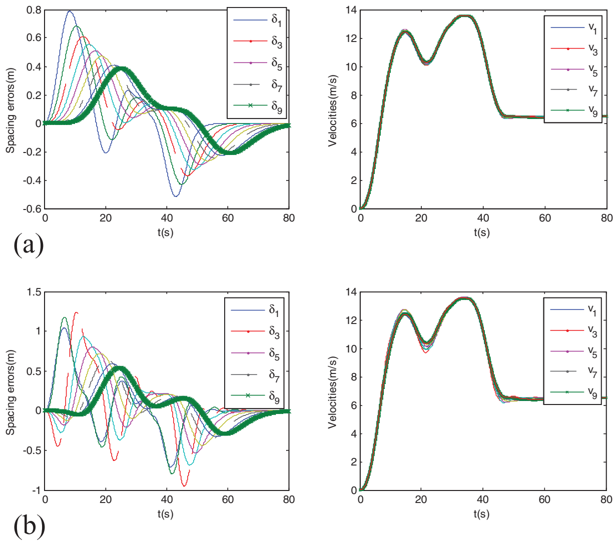


Figure 6. Ten-vehicle platoon system under proposed controller: (a) effect of noisy and measurement errors; (b) loss of lead vehicle's information.

6. Conclusions

In this paper, we have established an event-triggered control scheme for the autonomous platoon control of vehicles with sensor's and actuator's failure. To reduce the negative effect of the failures, an event-triggered control method based on the Lyapunov method was proposed. The simulation results show the presented method is in general superior to the existing result, and a safer and smoother transient performance can be achieved by properly choosing the design parameters.

An interesting future topic that merits this research is how to improve the closed-platoon system performance by using more than two vehicles' information. In this scheme, the wireless communication constraints should be considered. These issues raise various open problems that are worth investigating. One possibility is to apply the method presented by Li and Dong [24,25] to autonomous vehicular platoon control.

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References

[1] Vahidi A, Eskandarian A. Research advances in intelligent collision avoidance and adaptive cruise control. IEEE Trans Intell Transp Syst. 2003;4(3):143–153.
[2] Li S, Li K, Rajamani R, et al. Model predictive multiobjective vehicular adaptive cruise control. IEEE Trans Control Syst Technol. 2011;19(3):556–566.
[3] Jovanovic MR, Bamieh B. On the ill-posedness of certain vehicular platoon control problems. IEEE Trans Automat Control. 2005;50(9):1307–1321.
[4] Seiler P, Pant A, Hedrick JK. Disturbance propagation in vehicle strings. IEEE Trans Automat Control. 2004;49(10):1835–1841.
[5] Huang S, Ren W. Design of vehicle following control systems with actuator delays. Int J Syst Sci. 1997;28 (2):145–151.
[6] Middleton R, Braslavsky J. String stability in classes of linear time invariant formation control with limited communication range. IEEE Trans Automat Control. 2010;55(7):1519–1530.
[7] Bamieh B, Jovanovic MR, Mitra P, et al. Coherence in large-scale networks: dimension dependent limitations of local feedback. IEEE Trans Automat Control. 2012;57(9):2235–2249.
[8] Hao H, Barooah P. Stability and robustness of large platoons of vehicles with double-integrator models and nearest neighbor interaction. Int J Robust Nonlinear Control. 2013;23(18):2097–2122.
[9] Barooah P, Mehta PG, Hespanha JP. Mistuning-based control design to improve closed-loop stability margin of vehicular platoons. IEEE Trans Automat Control. 2009;54(9):2100–2113.
[10] Liu F, Fardad M, Jovanovic MR. Optimal control of vehicular formations with nearest neighbor interactions. IEEE Trans Automat Control. 2012;57(9):2203–2218.
[11] Liu XH, GoldSmith A. Effects of communication delay on string stability in vehicle platoons. Paper presented at: The IEEE Intelligent Transportation Systems Conference; Oakland, CA: IEEE. 2001. p. 625–630.
[12] Guo G, Yue W. Hierarchical platoon control with heterogeneous information feedback. IET Control Theory Appl. 2011;5(15):1766–1781.
[13] Guo G, Yue W. Sampled-data cooperative adaptive cruise control of vehicles with sensor failures. IEEE Trans Intell Transp Syst. 2014;6(15):2404–2418.
[14] Andrea C, Vincenzo M, Sergio MS. Vehicle's energy estimation using low frequency speed signal. In: PA Ioannou, editor. IEEE Conference on Intelligent Transportation Systems; Anchorage, AK: IEEE; 2012. p. 16–19.
[15] Dominik M, Harald W, Harald K, et al. Cooperative adaptive cruise control applying stochastic linear model predictive control strategies. In: T. Parisini, editor. Proceedings of the European Control Conference; Linz, Austria: IEEE; 2015. p. 15–17.
[16] Rasshofer RH, Spies M, Spies H. Influences of weather phenomena on automotive laser radar systems. Adv Radio Sci. 2011;9:49–60.
[17] Hassen AA. Indicators for the signal degradation and optimization of automotive radar sensors under adverse weather conditions [dissertation]. Darmstadt: Technische Universität Darmstadt; 2006.
[18] Douglas RK, Speyer DL. Fault detection and identification with application to advanced vehicle control systems. Proceedings of the California PATH Research; 1995 Dec 5; Los Angeles, CA. UCB-ITS-PRR-95-26

[19] Patwardhan S, Tomizuka M. Robust failure detection in lateral control for IVHS. Proceedings of the American Control Conference; Jun 1992. p. 1768–1772.
[20] Garg V, Hedrick JK. Fault detection filters for a class of nonlinear systems. Proceedings of the American Control Conference; 1995 Jun 21–23; San Diego, CA. p. 1647–1651.
[21] Yue W, Wang LY, Guo G. Event-triggered platoon control of vehicles with time-varying delay and probabilistic faults. Mech Syst Signal Process. 2017;87(2):96–114.
[22] Khasminskii R. Stochastic stability of differential equations. Vol. 66. Heidelberg: Springer; 2011.
[23] Tian E, Yue D, Zhang Y. Delay-dependent robust H∞ control for T–S fuzzy system with interval time-varying delay. Fuzzy Sets Syst. 2009;160(12):1708–1719.
[24] Zhang Y, Dong H, Zhang X, et al. Rational solutions and lump solutions to the generalized(3+1)-dimensional shallow water-like equation, computers and mathematics with Applications. Comput Math Appl. 2017;73:246–252.
[25] He DH, Yong GB, Shu YB. Generalized fractional supertrace identity for Hamiltonian structure of NLS–MKdV hierarchy with self-consistent sources. Anal Math Phys. 2016;6(2):199–209.

Appendix

Here, we present the proofs of Theorems 3.1 and 3.2.

A.1. Proof of Theorem 3.1

Define a Lyapunov function as

$$V(x_t) = x^T(t)Px(t) + \int_{t-\tau_M}^{t} x^T(s)Qx(s)ds + \int_{t-\tau_M}^{t}\int_{s}^{t} \dot{x}^T(\theta)R\dot{x}(\theta)d\theta ds, \tag{A1.1}$$

where P, Q and R are positive-definite matrices with appropriate dimensions.

According to Definition 2.1 for $V(x_t)$ and taking expectation on it, one can derive

$$\begin{aligned}\mathrm{E}\{\Psi V(x_t)\} &= 2x^T(t)P\{Ax(t) + B\overline{\rho}_{ac}p\overline{\rho}_s[x(t-\tau(t)) \\ &\quad + e_k(t)] \quad + x^T(t)Qx(t) - x^T(t-\tau_M)Qx(t-\tau_M) \\ &\quad + \mathrm{E}\{\tau_M\dot{x}^T(t)R\dot{x}(t)\} - \int_{t-\tau_M}^{t} \dot{x}^T(s)Q\dot{x}(s)ds \\ &\quad + \Gamma_1 + \Gamma_2,\end{aligned} \tag{A1.2}$$

where

$$\Gamma_1 = 2\xi^T(t)N[x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{x}(s)ds] = 0, \tag{A1.3}$$

$$\Gamma_2 = 2\xi^T(t)M[x(t-\tau(t)) - x(t-\tau_M) - \int_{t-\tau_M}^{t-\tau(t)} \dot{x}(s)ds] = 0, \tag{A1.4}$$

where N_{i1} and N_{i2} are matrices with appropriate dimensions, and

$$\xi^T(t) = \begin{bmatrix} x^T(t) & x^T(t-\tau(t)) & x^T(t-\tau_M) & w^T(t-\tau(t)) \end{bmatrix}.$$

By Lemma 2.1, we have

$$\begin{aligned}-2\xi^T(t)N\int_{t-\tau(t)}^{t} \dot{x}(s)ds &\le \tau(t)\xi^T(t)NR^{-1}N^T\xi(t) \\ &\int_{t-\tau(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds,\end{aligned} \tag{A1.5}$$

$$\begin{aligned}-2\xi^T(t)M\int_{t-\tau_M}^{t-\tau(t)} \dot{x}(s)ds &\le (\tau_M - \tau(t))\xi^T(t)MR^{-1}M^T\xi(t) \\ &\int_{t-\tau_M}^{t-\tau(t)} \dot{x}^T(s)R\dot{x}(s)ds.\end{aligned} \tag{A1.6}$$

Notice that

$$\begin{aligned}\mathrm{E}\{\tau_M\dot{x}^T(t)R\dot{x}(t)\} &= \mathrm{E}\tau_M[Ax(t) + B\overline{\rho}_{ac}K\overline{\rho}_s(x(t-\tau(t)) \\ &\quad + e_k(t)) + \nu(x(t-\tau(t))]^T R[Ax(t) \\ &\quad + B\overline{\rho}_{ac}K\overline{\rho}_s(x(t-\tau(t)) + e_k(t)) \\ &\quad + \nu(x(t-\tau(t))] \\ &= \tau_M[Ax(t) + B\overline{\rho}_{ac}K\overline{\rho}_s[x(t-\tau(t)) + e_k(t)]]^T R[Ax(t) \\ &\quad + B\overline{\rho}_{ac}K\overline{\rho}_s[x(t-\tau(t)) + e_k(t)]] \\ &\quad + B\overline{\rho}_{ac}K\overline{\rho}_s e_k(t) \\ &\quad + \mathrm{E}\{\tau_M x^T(t-\tau(t))\nu^T R\nu x(t-\tau(t))\} \\ &\quad + \mathrm{E}\{2\tau_M x^T(t-\tau(t))\nu^T R\nu e_k(t)\} \\ &\quad + \mathrm{E}\{\tau_M e_k^T(t))\nu^T R\nu e_k(t)\}.\end{aligned} \tag{A1.7}$$

According to Lemma 2.2, the term $2\tau_M x^T(t-\tau(t))$ $\nu^T R\nu e_k(t)$ in (A1.7) satisfies

$$\begin{aligned}2\tau_M x^T(t-\tau(t))\nu^T R\nu e_k(t) &\le \tau_M x^T(t-\tau(t))\nu^T R\nu x(t-\tau(t)) \\ &\quad + \tau_M e_k^T(t)\nu^T R\nu e_k(t).\end{aligned} \tag{A1.8}$$

Then, we get

$$\begin{aligned}\mathrm{E}\{\tau_M\dot{x}^T(t)R\dot{x}(t)\} &\le \tau_M[Ax(t) + B\overline{\rho}_{ac}K\overline{\rho}_s[x(t-\tau(t)) + e_k(t)]]^T R \\ &\quad [Ax(t) + B\overline{\rho}_{ac}K\overline{\rho}_s[x(t-\tau(t)) + e_k(t)]] \\ &\quad + \mathrm{E}\{2\tau_M x^T(t-\tau(t))\nu^T R\nu x(t-\tau(t))\} \\ &\quad + \mathrm{E}\{2\tau_M e_k^T(t)\nu^T R\nu e_k(t)\}.\end{aligned} \tag{A1.9}$$

Recalling (12), we obtain

$$\begin{aligned}&\mathrm{E}2\tau_M \mathrm{x}^\mathrm{T}(\mathrm{t}-\tau(\mathrm{t}))\nu^\mathrm{T}\mathrm{R}\nu\mathrm{x}(\mathrm{t}-\tau(\mathrm{t}))\} \\ &\quad = 2\tau_M\mathrm{E}\{x^T(t-\tau(t)), \\ &[B\overline{\rho}_{ac}K(\rho_s - \overline{\rho}_s) + B(\rho_{ac} - \overline{\rho}_{ac})K\overline{\rho}_s \\ &\quad + B(\rho_{ac} - \overline{\rho}_{ac})K(\rho_s - \overline{\rho}_s)]^T R\,[B\overline{\rho}_{ac}K(\rho_s - \overline{\rho}_s)\end{aligned}$$

$$+B(\rho_{ac}-\overline{\rho}_{ac})K\overline{\rho}_s + B(\rho_{ac}-\overline{\rho}_{ac})K(\rho_s-\overline{\rho}_s)]\,x(t-\tau(t))$$
$$= 2\tau_M x^T(t-\tau(t))\Big[(B\overline{\rho}_{ac}K(\rho_s-\overline{\rho}_s))^T RB\overline{\rho}_{ac}K(\rho_s-\overline{\rho}_s) + (B(\rho_{ac}-\overline{\rho}_{ac})K\overline{\rho}_s)^T RB(\rho_{ac}-\overline{\rho}_{ac})K\overline{\rho}_s + (B(\rho_{ac}-\overline{\rho}_{ac})K(\rho_s-\overline{\rho}_s))^T RB(\rho_{ac}-\overline{\rho}_{ac})K(\rho_s-\overline{\rho}_s)]\, x(t-\tau(t)). \quad (A1.10)$$

Noting that

$$\mathrm{E}\{2\tau_M(B\overline{\rho}_{ac}K(\rho_s-\overline{\rho}_s))^T RB\overline{\rho}_{ac}K(\rho_s-\overline{\rho}_s)\} = \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} 2\tau_M\beta_{aci}^2(\gamma_{sj}^m)^2(BL_{ac}^iKL_s^j)^T RBL_{ac}^iKL_s^j, \quad (A1.11)$$

$$\mathrm{E}\{2\tau_M(B(\rho_{ac}-\overline{\rho}_{ac})K\overline{\rho}_s)^T RB(\rho_{ac}-\overline{\rho}_{ac})K\overline{\rho}_s\} = \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} 2\tau_M\lambda_{aci}^2(\beta_{sj}^m)^2(BL_{ac}^iKL_s^j)^T RBL_{ac}^iKL_s^j, \quad (A1.12)$$

$$\mathrm{E}\{2\tau_M(2B(\rho_{ac}-\overline{\rho}_{ac})K(\rho_s-\overline{\rho}_s))^T RB(\rho_{ac}-\overline{\rho}_{ac})K(\rho_s-\overline{\rho}_s))\} = \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} 2\tau_M\lambda_{aci}^2(\gamma_{sj}^m)^2(BL_{ac}^iKL_s^j)^T RBL_{ac}^iKL_s^j. \quad (A1.13)$$

Combining (A1.10, A1.13), we get

$$\mathrm{E}2\tau_M x^T(t-\tau(t))\nu^T R\nu x(t-\tau(t))\} = \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} W_{ij}x^T(t-\tau(t))(BL_{ac}^iKL_s^j)^T RBL_{ac}^iKL_s^j x(t-\tau(t)), \quad (A1.14)$$

where $W_{ij} = 2\tau_M(\beta_{aci}^2(\gamma_{sj}^m)^2 + \lambda_{aci}^2(\beta_{sj}^m)^2 + \lambda_{aci}^2(\gamma_{sj}^m)^2)$.

Using the same method as (A1.14), we have

$$\mathrm{E}2\tau_M e_k^T(t)\nu^T R\nu e_k(t)\} = \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} W_{ij}e_k^T(t)(BL_{ac}^iKL_s^j)^T RBL_{ac}^iKL_s^j e_k(t). \quad (A1.15)$$

Substituting (A1.3, A1.7, A1.14, A1.15) into (A1.2) and combining (20), we can get

$$\mathrm{E}\{\Psi V(x_t)\} \le 2x^T(t)P\{Ax(t) + B\overline{\rho}_{ac}p\overline{\rho}_s[x(t-\tau(t)) + e_k(t)] + x^T(t)Qx(t) - x^T(t-\tau_M)Qx(t-\tau_M)$$
$$+ \tau_M[Ax(t) + B\overline{\rho}_{ac}K\overline{\rho}_s[x(t-\tau(t)) + e_k(t)]]^T R\,[Ax(t) + B\overline{\rho}_{ac}K\overline{\rho}_s[x(t-\tau(t)) + e_k(t)]]$$
$$+\sum_{i=1}^{n-1}\sum_{j=1}^{n-1} W_{ij}x^T(t-\tau(t))(BL_{ac}^iKL_s^j)^T RBL_{ac}^iKL_s^j x(t-\tau(t))$$
$$+\sum_{i=1}^{n-1}\sum_{j=1}^{n-1} W_{ij}e_k^T(t)(BL_{ac}^iKL_s^j)^T RBL_{ac}^iKL_s^j e_k(t)$$
$$+ 2\xi^T(t)N[x(t) - x(t-\tau(t))] + 2\xi^T(t)M[x(t-\tau(t)) - x(t-\tau_M)]$$
$$+ \tau(t)\xi^T(t)NR^{-1}N^T\xi(t) + (\tau_M - \tau(t))\xi^T(t)MR^{-1}M^T\xi(t)$$
$$+ \mu\sum_{i=1}^{n-1}(\beta_{sj}^m)^2x^T(t-\tau(t))(L_s^j)^T\Omega L_s^j x(t-\tau(t)) + \sum_{i=1}^{n-1}(\beta_{sj}^m)^2 e_k^T(t)(L_s^j)^T\Omega L_s^j e_k(t)$$
$$= \xi^T(t)(\Theta + \tau(t))NR^{-1}N^T + (\tau_M - \tau(t))MR^{-1}M^T\xi(t), \quad (A1.16)$$

where $\Theta = \Psi_{11} + \Gamma + \Gamma^T + \overline{A}^T R\overline{A}$ with

$$\Psi_{11} = \begin{bmatrix} PA + A^TP + Q & * & * & * \\ \rho_s K^T\overline{\rho}_{ac}B^TP & \Delta + \sum_{i=1}^{n-1}\mu(\beta_{si}^m)^2(L_s^j)^T\Omega L_s^j & * & * \\ 0 & 0 & -Q & * \\ \rho_s K^T\overline{\rho}_{ac}B^TP & 0 & 0 & \Delta - \sum_{i=1}^{n-1}\mu(\beta_{si}^m)^2(L_s^j)^T\Omega L_s^j \end{bmatrix}$$

$$\Delta = \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} W_{ij}(BL_{ac}^iKL_s^j)^T RBL_{ac}^iKL_s^j,$$

$$\Gamma = [N \quad M-N \quad -M \quad 0].$$

By using Lemma 2.2, we can conclude from (A1.1) that there exists a constant ϑ such that

$$\mathrm{E}\{\Psi V(x_t)\} \le -\vartheta\mathrm{E}(\|x(t)\|^2), \quad (A1.17)$$

where $\vartheta = \min\{\vartheta_{\min}\Sigma(g)\}$. Define a new function as

$$W(x_t) = e^{\varepsilon t}\nu(x_t). \quad (A1.18)$$

Its infinitesimal operator Ψ is given by

$$\Psi W(x_t) = \varepsilon e^{\varepsilon t}\nu(x_t) + e^{\varepsilon t}\Psi\nu(x_t).$$

Then, we can get

$$\mathrm{E}W(x_t) - \mathrm{E}W(x_0) = \int_0^t \varepsilon e^{\varepsilon t}\mathrm{E}\{V(x_t)\}ds + \int_0^t e^{\varepsilon t}\mathrm{E}\{\Psi V(x_t)\}ds. \quad (A1.19)$$

By using the method in [23], we know that there exists a positive number αsuch that for $t \ge 0$

$$\mathrm{E}\{V(x_t)\} \le \alpha \underbrace{\sup}_{-\tau_M \le s \le 0} \mathrm{e}^{-\varepsilon s}\mathrm{E}\{\|\psi(s)\|^2\}. \quad \text{(A1.20)}$$

Since $V(x_t) \ge \lambda_{\min}(P)x^T(t)x(t)$, then for $t \ge 0$, from (A1.20), we can obtain

$$\mathrm{E}\{x^T(t)x(t)\} \le \overline{\alpha} \underbrace{\sup}_{-\tau_M \le s \le 0} \mathrm{e}^{-\varepsilon s}\mathrm{E}\{\|\psi(s)\|^2\}, \quad \text{(A1.21)}$$

where $\overline{\alpha} = \alpha/\lambda_{\min}(P)$. The proof is completed.

A2. Proof of Theorem 3.2

Proof: Defining $X = P^{-1}$ and separating $\overline{\rho}_{ac}$ with $\overline{\rho}_{ac} - \alpha I$ and αI, then from (A1.2), we can get

$$\overline{\Sigma}(g) + \begin{bmatrix} PB\overline{\delta}_{ac}K \\ 0_{4\times1} \\ \sqrt{\tau_M}B\overline{\rho}_{ac}K \\ 0_{2(n-1)^2\times1} \end{bmatrix} \begin{bmatrix} 0 & \overline{\delta}_s - \alpha I & 0_{1\times[2(n-1)^2+2]} \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ \overline{\delta}_s - \alpha I \\ 0_{[2(n-1)^2+4]\times1} \end{bmatrix} + \begin{bmatrix} K^T\overline{\rho}_{ac}B^TP & 0_{1\times4} & \sqrt{\tau_M}K^T\overline{\rho}_{ac}B^T & 0_{1\times2(n-1)^2} \end{bmatrix}$$

$$\begin{bmatrix} PB\overline{\delta}_{ac}K \\ 0_{[2(n-1)^2+5]\times1} \end{bmatrix} \begin{bmatrix} 0_{1\times3} & \overline{\delta}_s - \alpha I & 0_{1.\times2(n-1)^2} \end{bmatrix} + \begin{bmatrix} 0_{3\times1} \\ \overline{\delta}_s - \alpha I \\ 0_{2[(n-1)^2+1]\times1} \end{bmatrix}$$

$$\begin{bmatrix} K^T\overline{\rho}_{ac}B^T & 0_{1\times[2(n-1)^2+5]} \end{bmatrix} \le \begin{bmatrix} PB\overline{\rho}_{ac}K \\ 0_{4\times1} \\ \sqrt{\tau_M}P\overline{\rho}_{ac}K \\ 0_{[2(n-1)^2+2]\times1} \end{bmatrix} X \begin{bmatrix} PB\overline{\rho}_{ac}K \\ 0_{4\times1} \\ \sqrt{\tau_M}P\overline{\rho}_{ac}K \\ 0_{[2(n-1)^2+2]\times1} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ \overline{\delta}_s - \alpha I \\ 0_{[2(n-1)^2+4]\times1} \end{bmatrix} X^{-1} \begin{bmatrix} 0 \\ \overline{\delta}_s - \alpha I \\ 0_{[2(n-1)^2+4]\times1} \end{bmatrix}^T$$

$$+ \begin{bmatrix} PB\overline{\delta}_{ac}K \\ 0_{[2(n-1)^2+5]\times1} \end{bmatrix} X \begin{bmatrix} PB\overline{\delta}_{ac}K \\ 0_{[2(n-1)^2+5]\times1} \end{bmatrix}^T$$

$$+ \begin{bmatrix} 0_{3\times1} \\ \overline{\delta}_s - \alpha I \\ 0_{2[(n-1)^2+1]\times1} \end{bmatrix} X^{-1} \begin{bmatrix} 0_{3\times1} \\ \overline{\delta}_s - \alpha I \\ 0_{2[(n-1)^2+1]\times1} \end{bmatrix}^T, \quad \text{(A2.1)}$$

where $\overline{\Sigma}(g)$ is derived from $\Sigma(g)(g = 1, 2)$ by replacing $PB\overline{\rho}_{ac}K\overline{\rho}_s$, $\overline{\rho}_sK^T\overline{\rho}_{ac}B^TP$, $\sqrt{\tau_M}B\overline{\rho}_{ac}K\overline{\rho}_s$and $\sqrt{\tau_M}\overline{\rho}_sK^T\overline{\rho}_{ac}B^T$ by $\alpha PB\overline{\rho}_{ac}K$, $\alpha K^T\overline{\rho}_{ac}B^TP$, $\alpha\sqrt{\tau_M}B\overline{\rho}_{ac}K$and $\alpha\sqrt{\tau_M}K^T\overline{\rho}_{ac}B^T$, respectively.

$$\overline{\overline{\Sigma}}(g) + \sum_{i=1}^{n-1}\left\{ \begin{bmatrix} 0_{[6+(n-1)^2+(i-1)(n-1)]\times1]} \\ C_{ji} \\ 0_{(n-1-i)(n-1)\times1} \end{bmatrix} \begin{bmatrix} 0_{1\times3} & L_s^2 & 0_{1\times2[(n-1)^2+1]} \end{bmatrix}\right.$$

$$\left.\begin{bmatrix} 0_{1\times3} \\ L_s^2 \\ 0_{1\times2[(n-1)^2+1]} \end{bmatrix} \begin{bmatrix} 0_{1\times[6+(n-1)^2+(i-1)(n-1)]} & C_{ji}^T & 0_{1\times(n-1-i)(n-1)} \end{bmatrix}\right\}$$

$$+\sum_{i=1}^{n-1}\left\{ \begin{bmatrix} 0_{[6+(i-1)(n-1)\times1]} \\ C_{ji} \\ 0_{(n-1)(2n-2-i)\times1} \end{bmatrix} \begin{bmatrix} 0 & L_s^2 & 0_{1\times[4+2(n-1)^2]} \end{bmatrix}\right.$$

$$\left. + \begin{bmatrix} 0 \\ L_s^2 \\ 0_{[4+2(n-1)^2]\times1} \end{bmatrix} \begin{bmatrix} 0_{1\times[6+(i-1)(n-1)]} & C_{ji}^T & 0_{1\times[2(n-1)^2-i(n-1)]} \end{bmatrix}\right\}$$

$$\le \overline{\overline{\Sigma}}(g) + \sum_{i=1}^{n-1}\left\{ \begin{bmatrix} 0_{[6+(n-1)^2+(i-1)(n-1)]\times1]} \\ C_{ji} \\ 0_{(n-1-i)(n-1)\times1} \end{bmatrix}\right.$$

$$X \begin{bmatrix} 0_{[6+(n-1)^2+(i-1)(n-1)]\times1]} \\ C_{ji} \\ 0_{(n-1-i)(n-1)\times1} \end{bmatrix}^T$$

$$\left.\begin{bmatrix} 0_{1\times3} \\ L_s^2 \\ 0_{1\times2[(n-1)^2+1]} \end{bmatrix} X^{-1} \begin{bmatrix} 0_{1\times3} \\ L_s^2 \\ 0_{1\times2[(n-1)^2+1]} \end{bmatrix}^T \right\}$$

$$+\sum_{i=1}^{n-1}\left\{ \begin{bmatrix} 0_{[6+(i-1)(n-1)\times1]} \\ C_{ji} \\ 0_{(n-1)(2n-2-i)\times1} \end{bmatrix} X \begin{bmatrix} 0_{[6+(i-1)(n-1)\times1]} \\ C_{ji} \\ 0_{(n-1)(2n-2-i)\times1} \end{bmatrix}^T\right.$$

$$\left. + \begin{bmatrix} 0 \\ L_s^2 \\ 0_{[4+2(n-1)^2]\times1} \end{bmatrix} X^{-1} \begin{bmatrix} 0 \\ L_s^2 \\ 0_{[4+2(n-1)^2]\times1} \end{bmatrix}^T \right\}, \quad \text{(A2.2)}$$

where $\overline{\overline{\Sigma}}(g)$ is derived from $\overline{\Sigma}(g)(g = 1, 2)$ by deleting $B L_{ac}^iKL_s^j$ and its transposes from the last $(n-1)^2$ columns and rows, and

$$C_{ji} = \sqrt{W_{ji}} \begin{bmatrix} BL_{ac}^1K \\ \vdots \\ BL_{ac}^{n-1}K \end{bmatrix}.$$

Combining (A1.2, A2.1, A2.2) and applying Schur complement, we can get

$$\begin{bmatrix} \Theta_{11} + \Gamma + \Gamma^T & \Sigma_{12}^g & \sqrt{\tau_{iM}}\hat{A}^T & 0 & \Theta_{15} & 0 & 0 & \Theta_{18} & \Theta_{19} \\ * & -R & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -PR^{-1}P & 0 & \Theta_{35} & 0 & 0 & 0 & 0 \\ * & * & * & \Theta_{44} & 0 & \Theta_{46} & 0 & 0 & 0 \\ * & * & * & * & \Theta_{55} & 0 & \Theta_{57} & 0 & 0 \\ * & * & * & * & * & \Theta_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Theta_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \Theta_{88} & 0 \\ * & * & * & * & * & * & * & * & \Theta_{99} \end{bmatrix} < 0, \quad \text{(A2.3)}$$

where $\Theta_{11} =$

$$\begin{bmatrix} PA + A^TP + Q & \alpha PB\overline{\delta}_{ac}K & 0 & \alpha PB\overline{\delta}_{ac}K \\ * & \sum_i^{n-1}\mu(\beta_{si}^m)^2(L_s^i)^T\Omega L_s^i & 0 & 0 \\ * & * & -\tilde{Q} & 0 \\ * & * & * & \sum_i^{n-1}(\beta_{si}^m)^2(L_s^i)^T\Omega L_s^i \end{bmatrix},$$

$$\hat{A} = [\,PA \quad \alpha\sqrt{\tau_M}PB\overline{\rho}_{ac}K \quad 0 \quad \alpha\sqrt{\tau_M}PB\overline{\rho}_{ac}K\,],$$

$$\Theta_{15} = \begin{bmatrix} PB\overline{\delta}_{ac}KX & PB\overline{\delta}_{ac}KX & 0 & 0 \\ 0 & 0 & \overline{\delta}_s - \alpha I & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{\delta}_s - \alpha I \end{bmatrix},$$

$$\Theta_{18} = \begin{bmatrix} 0 & \cdots & 0 \\ L_s^1 & \cdots & L_s^{n-1} \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}, \Theta_{19} = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ L_s^1 & \cdots & L_s^{n-1} \end{bmatrix},$$

$$\Theta_{35} = [\,\sqrt{\tau_M}B\overline{\rho}_{ac}KX \quad 0 \quad 0 \quad 0\,],$$

$$\Theta_{44} = \mathrm{diag}\{\underbrace{-PR^{-1}PR, \ldots, -PR^{-1}P}_{2(n-1)^2}\},$$

$$\Theta_{55} = \mathrm{diag}\{-X, -X, -X, -X\},$$

$$\Theta_{46} = \left[\,\hat{\Lambda}_1\,\hat{\Lambda}_2 \cdots \hat{\Lambda}_{n-1}\right],$$

$$\tilde{\Lambda}_i = \left[\,0_{1\times(i-1)(n-1)} \quad X\tilde{C}_{ji}^T \quad 0_{(n-1-i)(n-1)\times 1}\,\right]^T,$$

$$\Theta_{57} = \left[\,\hat{\Delta}_1\,\hat{\Delta}_2 \cdots \hat{\Delta}_{n-1}\right], \hat{\Delta}_i = \hat{\Lambda}_i,$$

$$\Theta_{66} = \Theta_{77} = \Theta_{88} = \Theta_{99} = \mathrm{diag}\{\underbrace{-X, \ldots, -X}_{(n-1)^2}\}.$$

Since $(R - \varepsilon^{-1}P)R^{-1}(R - \varepsilon^{-1}P) \geq 0$, we have

$$-PR^{-1}P \leq -2\varepsilon P + \varepsilon^2 R. \tag{A2.4}$$

Substituting $-PR^{-1}P$ and $-2\varepsilon P + \varepsilon^2 R$ into (A2.3), we get

$$\begin{bmatrix} \Theta_{11} + \Gamma + \Gamma^T & \Sigma_{12}^g & \sqrt{\tau_{iM}}\hat{A}^T & 0 & \Theta_{15} & 0 & 0 & \Theta_{18} & \Theta_{19} \\ * & -R & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -2\varepsilon P + \varepsilon^2 R & 0 & \Theta_{35} & 0 & 0 & 0 & 0 \\ * & * & * & \hat{\Theta}_{44} & 0 & \Theta_{46} & 0 & 0 & 0 \\ * & * & * & * & \Theta_{55} & 0 & \Theta_{57} & 0 & 0 \\ * & * & * & * & * & \Theta_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Theta_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \Theta_{88} & 0 \\ * & * & * & * & * & * & * & * & \Theta_{99} \end{bmatrix} < 0, \tag{A2.5}$$

where $\hat{\Theta}_{44} = \mathrm{diag}\{\underbrace{-2\varepsilon P + \varepsilon^2 R, \ldots, -2\varepsilon P + \varepsilon^2 R}_{2(n-1)^2}\}$.
Multiplying both sides of (A2.5) by $\mathrm{diag}\{\underbrace{X, \ldots, X}_{2(n-1)^2+6}, I, \ldots, I\}$, and denoting $\tilde{\Omega} = X[\sum_i^{n-1} (\beta_{si}^m)^2 (L_s^i)^T \Omega L_s^i]X$, $\tilde{Q} = XQX$, $\tilde{R} = XRX$ $\tilde{N} = XNX$, $\tilde{M} = XMX$ and $Y = KX$. Then Equation (24) can be obtained, which completes the proof.