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REGULAR PAPER

Non-overshooting PD and PID controllers design

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ABSTRACT

This paper involves the design of non-overshooting PD and PID controllers for some special plants. The PID controller parameters are determined to reach a stable closed-loop system with monotonically decreasing frequency response. Thus specific regions in the controller parameters space are obtained. Gain crossover frequency and phase isodamping property are employed to choose an appropriate solution among the obtained solutions. The performance of the proposed PD and PID controllers in position and velocity control of a laboratory DC servomechanism system is investigated through experimental tests.

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DC servomechanism system; non-overshooting step response; PID controllers; PD controllers

1. Introduction

Simple structure and easy tuning of PID controllers have made them as the most popular controllers for industrial applications [1]. A variety of approaches have been proposed to design PID controllers in the literature. PID controller coefficients could be adjusted to minimize a closed-loop system performance index by means of some optimization methods [2]. Internal Model Control (IMC) method has been employed for analytical design of PID controllers [3,4]. Tuning PID controllers based on some frequency specification such as gain margin, phase margin and gain crossover frequency has been considered, too [5]. In [6], a method to design PID controllers based on the phase margin and gain crossover frequency requirements has been proposed. In this work, the crossover frequency has been selected based on an integral performance index criteria. Moreover, adaptive control and auto-tuning methods have been utilized for PID controller design in the literature [7].

On the other hand, transient response control has been considered by the control engineers. Attaining a non-overshooting or minimum overshoot step response is required in some real plants. Thus several methods have been introduced to achieve this goal. In [8], necessary and sufficient conditions for state space models to achieve a non-overshooting step response was extracted. In [9], a min–max optimization approach to determine the optimum location of zeros to attain a minimum-overshoot transient response was employed. In [10], a compensator for special case of minimum phase systems to attain a non-overshooting closed-loop system step response was designed. A rational two-parameter controller to eliminate overshoot in closed-loop system step response was proposed in the literature [11]. Moreover, state feedback design to reach a non-overshooting step response has been reported [12]. Characteristic Ratio Assignment (CRA) method is one of the common approaches for transient response control. In this method, the characteristic ratios appropriately related to the denominator coefficients of a transfer function are assigned to reach a non-overshooting step response [13]. Manabe introduced Coefficient Diagram Method (CDM) to assign characteristic ratios [14]. A Butterworth filter pattern for characteristic ratios to achieve the desired transient response was proposed, too [15].

Designing PID controllers to reach a non-overshooting step response has been taken into consideration in the literature. In [16], the relation between the location of transfer function poles and zeros and step response overshoot has been extracted. This relation has been utilized to design a PID controller to avoid overshoot in the closed-loop system step response. In another work, a non-overshooting PI controller for variable-speed motor derives has been provided [17]. In [18], the Tabu search optimization method has been employed for adjustment of the PID controller parameters to attain a desired transient response. Widder's and Markov–Lucaks theorems have been employed to attain a non-negative error response [19]. In [20], a fuzzy PID controller to achieve a zero overshoot step response has been presented. The CRA and CDM approaches have been employed to design non-overshooting PID controllers for some real plants [21–23]. In [24], a modified version of the PID controller, called I+PD controller, has been designed using process step response and damping optimum criterion. In this approach, the

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characteristic ratios of the closed-loop transfer function are appropriately assigned to reach a satisfactory transient response. This method could not be applied to ordinary PD or PID controllers. Because, for these controllers, the obtained closed-loop transfer function is not all-pole and the characteristic ratios assignment method could not be utilized. Moreover, the magnitude optimum method has been employed to design PID controllers [25–28]. In this method, the PID controller parameters are appropriately calculated such that the magnitude of the closed-loop system frequency response becomes near to 1. This leads to the flatness of the frequency response in a wide range of frequencies. Consequently, a satisfactory transient response will be obtained. However, there is no guarantee that a non-overshooting step response could be obtained.

In this paper, designing PD and PID controllers to attain non-overshooting step responses for certain categories of systems is considered. According to an empirical principle, systems with monotonically decreasing frequency responses have low overshoot in their step responses [29]. Thus the PD and PID controller coefficients are selected to reach a closed-loop frequency response with monotonically decreasing property. Consequently, some inequalities in terms of PID controller parameters will be obtained. The numerical solution of these inequalities leads to some specific regions in the parametric space. This means that a variety of controllers could be designed to reach a non-overshooting closed-loop system step response. Among these controllers, those satisfying a predefined gain crossover frequency and phase isodamping property are selected. The isodamping property means that the phase of the loop gain frequency response is kept flat around the gain crossover frequency. This means that the closed-loop system is robust under gain uncertainties. Thus these uncertainties could not affect the minimum overshoot property of the step response. The simulation and experimental results on a DC velocity and position servo system demonstrate the capability of the so designed PD and PID controller.

The remainder of this paper is organized as follows. The proposed non-overshooting PD and PID controllers are given in Section 2. Section 3 investigates the performance of these controllers in position and velocity control of a laboratory DC motor. Finally, Section 4 concludes the paper.

2. The proposed non-overshooting PD and PID controllers

In this section, the proposed PD and PID controllers are introduced. These controllers are designed to meet the closed-loop system stability and achieve non-overshooting step responses. Moreover, gain crossover frequency and phase isodamping property are incorporated in the design procedure. Some special plants are chosen for these control purposes.

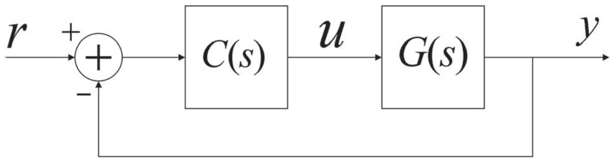

Figure 1. The unit negative feedback control structure.

Consider the unit negative feedback control structure in Figure 1. The controller $C(s)$ (may be a PD or PID controller) should be appropriately designed for a minimum phase plant $G(s)$ so that the closed-loop system does not exhibit overshoot in its step response. To attain this goal, the controller parameters could be adjusted such that the monotonically decreasing property for the magnitude of the closed-loop system frequency response will be fulfilled. Or

$$\left|H(j\omega_2)\right| < \left|H(j\omega_1)\right|, \quad \text{for}: \omega_2 > \omega_1, \tag{1}$$

where $H(j\omega)$ is the frequency response of the closed-loop system.

The following remarks could be expressed for using relation (1).

Remark 1: Condition (1) means that when the frequency ω increases, the numerator of $|H(j\omega)|$ increases smaller than its denominator.

Remark 2: Condition (1) is valid for minimum phase plants. This means that the closed-loop system should be minimum phase. Thus the plant should be minimum phase and the controller should be appropriately designed such that the closed-loop system is minimum phase, too.

Remark 3: If the condition (1) is satisfied, then a non-overshooting step response or a step response with very small overshoot will be obtained. For example, a second order system with complex conjugate poles with a damping ratio belonging to $(0.7, 1)$ satisfies the condition (1) but shows a small overshoot in its step response (below 5%). In control engineering applications, this small overshoot could be acceptable. Finally, for transfer functions with complex conjugate poles, condition (1) could result in a step response with a little overshoot.

2.1. Non-overshooting PID controller design for first-order systems

The plant transfer function is considered as

$$G(s) = \frac{K}{1 + Ts}, \tag{2}$$

where $K > 0$ and $T > 0$ are the steady-state gain and time constant parameters.

The PID controller with the following transfer function is considered:

$$C(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right), \tag{3}$$

where K_p, T_i and T_d are the proportional gain, integrator and derivative time constants, respectively.

The closed-loop transfer function is calculated as

$$H(s) = \frac{KK_p(T_i T_d s^2 + T_i s + 1)}{T_i(T + KK_p T_d)s^2 + T_i(1 + KK_p)s + KK_p}. \tag{4}$$

According to (4), the closed-loop system is stable if the following relations are satisfied

$$KK_p > 0, T_i \quad (1 + KK_p) > 0, \quad T_i(T + KK_p T_d) > 0. \tag{5}$$

Since $K > 0$, the following relations could be derived from (5)

$$K_p > 0, \quad T_i > 0, \quad T + KK_p T_d > 0. \tag{6}$$

This means that for the closed-loop system stability, K_p and T_i must be positive but T_d could be negative. On the other hand, to avoid undershoot in the closed-loop system step response the numerator coefficients of (4) should be positive. This means that

$$K_p T_i T_d > 0, \quad K_p T_i > 0, \quad K_p > 0. \tag{7}$$

Thus, to attain a step response without undershoot, T_d must be positive, too. Finally, we have

$$K_p > 0, \quad T_i > 0, \quad T_d > 0. \tag{8}$$

The magnitude square of the closed-loop system frequency response is calculated as

$$|H(j\omega)|^2 = K^2 K_p^2 \frac{T_d^2 T_i^2 \omega^4 + T_i(T_i - 2T_d)\omega^2 + 1}{\alpha(\omega)}, \tag{9}$$

where

$$\alpha(\omega) = T_i^2(T + KK_p T_d)^2\omega^4 + T_i\{T_i(1 + KK_p)^2 - 2KK_p(T + KK_p T_d)\}\omega^2 + K^2 K_p^2. \tag{10}$$

To realize monotonically decreasing condition (1), the following inequality should be fulfilled

$$\begin{aligned} &T_i^3\{T_i(T - T_d)(T + T_d + 2KK_p T_d) \\ &\quad - 2TT_d(T + KK_p T_d)\}\omega_1^2\omega_2^2(\omega_2^2 - \omega_1^2) \\ &+ T_i^2 T(T + 2KK_p T_d)(\omega_2^4 - \omega_1^4) \\ &+ T_i\{T_i(1 + 2KK_p) - 2KK_p T\}(\omega_2^2 - \omega_1^2) > 0. \end{aligned} \tag{11}$$

According to (1), $(\omega_2^2 - \omega_1^2)$ and $(\omega_2^4 - \omega_1^4)$ are positive. On the other hand, considering positive values for T_i, K_p and T_d, the term $T_i^2 T(T + 2KK_p T_d)$ is always positive. Thus the sufficient condition for realization of (11) is that the first and third terms in (11) are greater than or equal to zero. Thus we have

$$T_i(T - T_d)(T + T_d + 2KK_p T_d) \geq 2TT_d(T + KK_p T_d). \tag{12}$$

$$T_i(1 + 2KK_p) \geq 2KK_p T. \tag{13}$$

The realization of (12) requires the satisfaction of the following two relations:

$$T_d \leq T. \tag{14}$$

$$T_i \geq \frac{2TT_d(T + KK_p T_d)}{(T - T_d)(T + T_d + 2KK_p T_d)}. \tag{15}$$

To establish (13), the following condition should be met

$$T_i \geq \frac{2KK_p T}{1 + 2KK_p}. \tag{16}$$

Finally, attaining a stable closed-loop system with non-overshooting and non-undershooting step response requires the following conditions:

$$0 < T_d \leq T, K_p > 0, T_i > 0 \tag{17a}$$

$$T_i \geq \frac{2KK_p T}{1 + 2KK_p} \tag{17b}$$

$$T_i \geq \frac{2TT_d(T + KK_p T_d)}{(T - T_d)(T + T_d + 2KK_p T_d)}. \tag{17c}$$

Finally, the PID controller design for plant (2) is summarized in Algorithm 1.

Algorithm 1.

Step 1. Choose an arbitrary positive value for K_p and an arbitrary T_d, where $0 < T_d \leq T$.

Step 2. Now, find an appropriate value for T_i ensuring (17b) and (17c).

2.2. Non-overshooting PD controller design for integrating systems

In this section, the following type 1 second-order plant is considered

$$G(s) = \frac{K}{s(1 + Ts)}, \tag{18}$$

where K and T are arbitrary positive parameters. To track constant reference values, a PD controller with the following transfer function could be utilized for plant (18)

$$C(s) = K_p(1 + T_d s). \tag{19}$$

Considering plant (18) and controller (19), the following closed-loop system transfer function is obtained

$$H(s) = \frac{KK_p(T_d s + 1)}{Ts^2 + (1 + KK_p T_d)s + KK_p}. \tag{20}$$

According to (20), the necessary and sufficient conditions to reach a stable closed-loop system without undershoot in its step response are

$$K_p > 0, \quad T_d > 0. \tag{21}$$

The magnitude square of the closed-loop system frequency response is given by

$$|H(j\omega)|^2 = \frac{K^2K_p^2(T_d^2\omega^2+1)}{\psi(\omega)}, \tag{22}$$

where

$$\psi(\omega) = T^2\omega^4 + (K^2K_p^2T_d^2 + 2KK_p(T_d - T) + 1)\omega^2 + K^2K_p^2. \tag{23}$$

According to (1), the function (22) is monotonically decreasing if the following inequalities will be satisfied:

$$T^2(\omega_2^4 - \omega_1^4) + T_d^2T^2\omega_1^2\omega_2^2(\omega_2^2 - \omega_1^2) + (2KK_p(T_d - T) + 1)(\omega_2^2 - \omega_1^2) > 0. \tag{24}$$

Considering (21), the sufficient condition for establishment of (24) is

$$T_d \geq \frac{2KK_pT - 1}{2KK_p}. \tag{25}$$

Finally, if the following conditions are fulfilled, a closed-loop system step response with zero overshoot and undershoot could be obtained

$$\begin{aligned} &K_p > 0,\ T_d > 0 \\ &T_d \geq \frac{2KK_pT - 1}{2KK_p}. \end{aligned} \tag{26}$$

The PD controller design for plant (18) is illustrated in the following algorithm.

Algorithm 2.

Step 1. Select an arbitrary positive value for K_p.

Step 2. Now, choose an appropriate value for T_d satisfying($d2$).

2.3. Non-overshooting PID controller design for second-order systems

Consider the following stable second-order plant:

$$G(s) = \frac{K}{s^2 + As + B}, \tag{27}$$

where K, A and B are arbitrary positive constants. The closed-loop system transfer function for plant (27) and controller (3) becomes

$$H(s) = \frac{KK_p(T_iT_ds^2 + T_is + 1)}{T_is^3 + T_i(A + KK_pT_d)s^2 + T_i(B + KK_p)s + KK_p}. \tag{28}$$

It could be easily verified that the closed-loop system is a stable system without undershoot in its transient response, if

$$\begin{aligned} &K_p > 0, \quad T_i > 0, \quad T_d > 0, \\ &T_i > \frac{KK_p}{(A + KK_pT_d)(B + KK_p)}. \end{aligned} \tag{29}$$

Substituting $s = j\omega$ in (28) gives

$$|H(j\omega)|^2 = \frac{K^2K_p^2(T_i^2T_d^2\omega^4 + T_i(T_i - 2T_d)\omega^2 + 1)}{\beta(\omega)}, \tag{30}$$

where

$$\begin{aligned} \beta(\omega) = {} & T_i^2\omega^6 + T_i^2((A + KK_pT_d)^2 \\ & - 2(B + KK_p))\omega^4 + T_i(T_i(B + KK_p)^2 \\ & - 2KK_p(A + KK_pT_d))\omega^2 + K^2K_p^2. \end{aligned} \tag{31}$$

The monotonically decreasing condition (1) gives

$$\begin{aligned} & T_i^2(\omega_2^6 - \omega_1^6) + T_i^4T_d^2\omega_1^4\omega_2^4(\omega_2^4 - \omega_1^4) \\ & + T_i^3(T_i - 2T_d)\omega_1^2\omega_2^2(\omega_2^4 - \omega_1^4) \\ & + T_i^2(2AKK_pT_d + A^2 - 2B - 2KK_p)(\omega_2^4 - \omega_1^4) \\ & + T_i^3(-2T_d(A^2 + AKK_pT_d - 2B - 2KK_p) \\ & + T_i(A^2 + 2AKK_pT_d - 2B - 2KK_p \\ & - B^2T_d^2 - 2BKK_pT_d^2))\omega_1^2\omega_2^2(\omega_2^2 - \omega_1^2) \\ & + T_i(T_i(B^2 + 2BKK_p) - 2AKK_p)(\omega_2^2 - \omega_1^2) > 0. \end{aligned} \tag{32}$$

The sufficient conditions to realize (32) are

$$\begin{aligned} & T_i \geq 2T_d \\ & T_i \geq \frac{2AKK_p}{B^2 + 2BKK_p} \\ & T_d \geq \frac{2KK_p + 2B - A^2}{2AKK_p} \\ & A^2 + 2AKK_pT_d - 2B - 2KK_p \\ & \quad - B^2T_d^2 - 2BKK_pT_d^2 > 0 \\ & T_i \geq \frac{2T_d(A^2 + AKK_pT_d - 2B - 2KK_p)}{A^2} \\ & + 2KK_pT_d(A - BT_d) - 2B - 2KK_p - B^2T_d^2. \end{aligned} \tag{33}$$

Incorporating conditions (29) and (33) yields the following non-overshooting step response conditions for plant (27)

$$K_p > 0,\ T_i > 0,\ T_d > 0 \tag{34a}$$

$$T_i \geq 2T_d \tag{34b}$$

$$T_i > \frac{KK_p}{(A + KK_pT_d)(B + KK_p)} \tag{34c}$$

$$T_i \geq \frac{2AKK_p}{B^2 + 2BKK_p} \tag{34d}$$

$$T_d \geq \frac{2KK_p + 2B - A^2}{2AKK_p} \tag{34e}$$

$$A^2 + 2AKK_pT_d - 2B - 2KK_p - B^2T_d^2 - 2BKK_pT_d^2 > 0 \tag{34f}$$

$$T_i \geq \frac{2T_d(A^2 + AKK_pT_d - 2B - 2KK_p)}{A^2 + 2KK_pT_d(A - BT_d) - 2B - 2KK_p - B^2T_d^2}. \tag{34g}$$

Inequality (34f) could be rewritten as

$$(B^2 + 2BKK_p)T_d^2 - 2AKK_pT_d - A^2 + 2B + 2KK_p < 0. \tag{35}$$

If $A^2 \leq 2B$ is considered, then we have

$$\begin{aligned} &A^2K^2K_p^2 + (B^2 + 2BKK_p)(A^2 - 2B - 2KK_p) \\ &= (A^2 - 4B)K^2K_p^2 + 2B(A^2 - 3B)KK_p \\ &+ B^2(A^2 - 2B) < 0. \end{aligned} \tag{36}$$

On the other hand, $B^2 + 2BKK_p > 0$. Thus, according to (36), the left side of (35) should be positive. Therefore, the following limitation for the parameters of the plant (27) should be realized

$$A^2 > 2B. \tag{37}$$

Considering the constraint (37), relation (35) will be satisfied if the following relations are fulfilled:

$$K_p \leq \frac{A^2 - 2B}{2K}. \tag{38}$$

$$T_d < \frac{AKK_p}{B^2 + 2BKK_p} + \frac{\sqrt{A^2K^2K_p^2 + (B^2 + 2BKK_p)(A^2 - 2B - 2KK_p)}}{B^2 + 2BKK_p}. \tag{39}$$

Moreover, according to (38), the left side of (34e) is negative. This means that inequality (34e) will be automatically fulfilled. Thus relation (34) could be rewritten as

$$K_p > 0,\ T_i > 0,\ T_d > 0 \tag{40a}$$

$$K_p \leq \frac{A^2 - 2B}{2K} \tag{40b}$$

$$T_d < \frac{AKK_p}{B^2 + 2BKK_p} + \frac{\sqrt{A^2K^2K_p^2 + B(B + 2KK_p)(A^2 - 2B - 2KK_p)}}{B^2 + 2BKK_p} \tag{40c}$$

$$T_i > \frac{KK_p}{(A + KK_pT_d)(B + KK_p)} \tag{40d}$$

$$T_i \geq \frac{2AKK_p}{B^2 + 2BKK_p} \tag{40e}$$

$$T_i \geq 2T_d \tag{40f}$$

$$T_i \geq \frac{2T_d(A^2 + AKK_pT_d - 2B - 2KK_p)}{A^2 + 2KK_pT_d(A - BT_d) - 2B - 2KK_p - B^2T_d^2}. \tag{40g}$$

The following algorithm describes the details of the PID controller design for plant (27).

Algorithm 3.

Step 1. Choose a positive value for K_p ensuring (40b).

Step 2. Select a positive value for T_d satisfying (40b), (40c).

Step 3. Now, find an appropriate positive value for T_i satisfying (40d) –(40g).

2.4. Additional design conditions

Several controllers may satisfy conditions in (14), (23) or (31). Some criteria should be added to select appropriate controllers among them. Thus two conditions are added in the PID design procedure. The first condition is the gain crossover condition which guarantees unit magnitude for the loop gain frequency response in a predefined frequency ω_c. Or

$$\left|G(j\omega_c)C(j\omega_c)\right| = 1. \tag{41}$$

Condition (41) is required to reach a closed-loop system transient response with desired speed. For PD controller with only two design parameters, the additional condition (41) is enough. Moreover, for the PID controller with three design parameters, the following isodamping property should be fulfilled, too.

$$\frac{\mathrm{d}}{\mathrm{d}\omega}\angle[G(j\omega)C(j\omega)]\,\Big|_{\omega=\omega_c} = 0. \tag{42}$$

This means that the phase variations around the crossover frequency are negligible. This yields a closed-loop system robust to gain variations.

Applying conditions (41) and (42) on plant (2) yields the following relations between PID controller parameters

$$K_p = \frac{T_i\omega_c\sqrt{1+T^2\omega_c^2}}{K\sqrt{(1-T_iT_d\omega_c^2)^2+T_i^2\omega_c^2}}. \tag{43}$$

$$T_i = \frac{-a_1 \pm \sqrt{a_1^2+4a_2T}}{2a_2}, \tag{44}$$

where

$$a_1 = 1+T^2\omega_c^2+2TT_d\omega_c^2,$$
$$a_2 = \omega_c^2(T-T_d)(TT_d\omega_c^2-1). \tag{45}$$

Incorporating (17), (43) and (44) yields the PID controller parameters for plant (2). This means that the three-dimensional region obtained from conditions (17) is converted to a one-dimensional region in terms of parameter T_d. Several PID controller parameters may fulfil these relations. One of these controllers could be selected by the designer. The obtained solutions could vary by changing the crossover frequency parameter.

Remark 4: The gain crossover frequency ω_c should be appropriately selected such that inequalities (17) are fulfilled. This could be realized by trial and error. Moreover, this parameter determines the transient response speed of the closed-loop system. Increasing ω_c decreases the settling time of the closed-loop system step response. Moreover, increasing ω_c increases the control signal amplitude. Thus this parameter should be selected such that a satisfactory transient response with permissible control signal will be obtained.

For plant (18), applying (41) causes the following constraint on the PD controller parameters in (19)

$$K_p = \frac{\omega_c\sqrt{1+T^2\omega_c^2}}{K\sqrt{1+T_d^2\omega_c^2}}. \tag{46}$$

This converts the admissible two-dimensional parameter region obtained from (26) to a one-dimensional region in terms of parameter T_d.

Remark 5: For the PD controller design for plant (18), any arbitrary gain crossover frequency ω_c could be selected. Only the value of T_d should be selected such that inequality ($d2$) will be realized. However, this parameter determines the settling time of the closed-loop system step response and the maximum amplitude of the control signal.

Finally, for the second-order plant (27), the gain crossover frequency and isodamping property lead to the following relations between PID controller parameters:

$$K_p = \frac{T_i\omega_c\sqrt{(B-\omega_c^2)^2+A^2\omega_c^2}}{K\sqrt{(1-T_iT_d\omega_c^2)^2+T_i^2\omega_c^2}}. \tag{47}$$

$$T_i = \frac{-b_1 \pm \sqrt{b_1^2-4b_0b_2}}{2b_2}, \tag{48}$$

where

$$b_0 = \frac{A(B+\omega_c^2)}{(B-\omega_c^2)^2+A^2\omega_c^2},$$
$$b_1 = -(1+2b_0T_d\omega_c^2),\ b_2 = \omega_c^2(b_0(1+T_d\omega_c^2)-T_d). \tag{49}$$

Conditions (40) and constraints (47) and (48) yield a one-dimensional parameter region in terms of parameter T_d which could be considered as the solution region.

Remark 6: The gain crossover frequency ω_c should be chosen such that inequalities in (40) are satisfied. Substituting (47) and (48) in (40) will not lead to straightforward inequalities in terms of K_p, T_i, T_d and ω_c. Thus finding an appropriate value for ω_c for satisfaction of conditions (40) could be performed by software.

In the next section, the ability of the so-designed non-overshooting PD and PID controllers for the control of the mentioned plants is verified through some experimental and simulation tests.

3. Experimental and simulation results

In this section, the performance of the proposed PID and PD controllers is investigated. Three examples are given to show the effectiveness of non-overshooting PID controllers.

Example 1: Consider a modular DC servomotor system in Figure 1. As shown in Figure 2, a permanent magnet DC motor coupled with a tachometer to measure its angular velocity and a position potentiometer to measure its position is considered. An Advantech A/D and D/A interface is employed to implement the designed controllers in MATLAB real-time environment. To verify the effect of the load disturbance, a magnetic brake causing angular velocity decrement is provided.

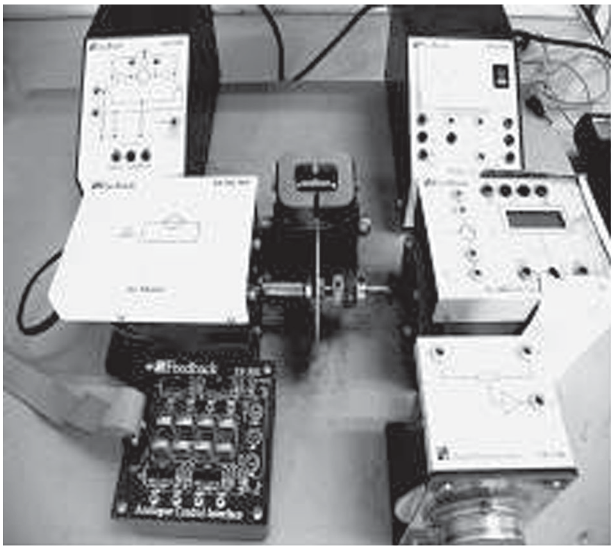

Figure 2. The DC servomotor plant in Example 1.

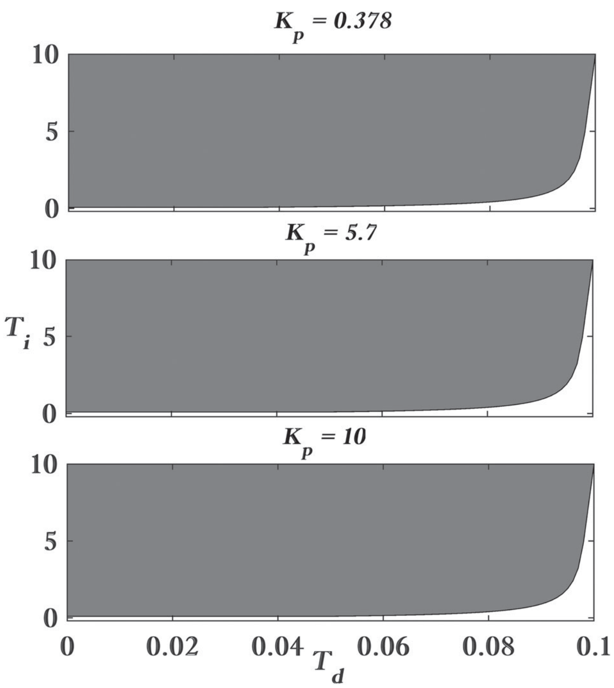

Figure 3. The $T_i - T_d$ region with different values of K_p in Example 1.

Employing a non-parametric identification approach leads to the following open loop transfer function for the speed servo system

$$G(s) = \frac{\omega(s)}{V(s)} = \frac{3.45}{0.1s + 1}, \tag{50}$$

where $\omega(t)$ is the motor angular velocity and $v(t)$ is the voltage applied to the servo amplifier block. The reference angular velocity is selected as 1000 rpm.

Figure 3 shows $(T_i - T_d)$ region with different values of K_p. According to (17), the parameter T_d should be in the range $(0, 0.1]$. But T_i is considered in the arbitrary range $(0, 10]$ due to drawing limitations. Among all possible controllers, a controller satisfying

Table 1. Controller parameters.

	K_P	T_i	T_d	ω_c
Example 1	0.378	0.122	0.002	12
Example 2	0.247	–	0.048	5
Example 3	0.992	4.629	0.98	2

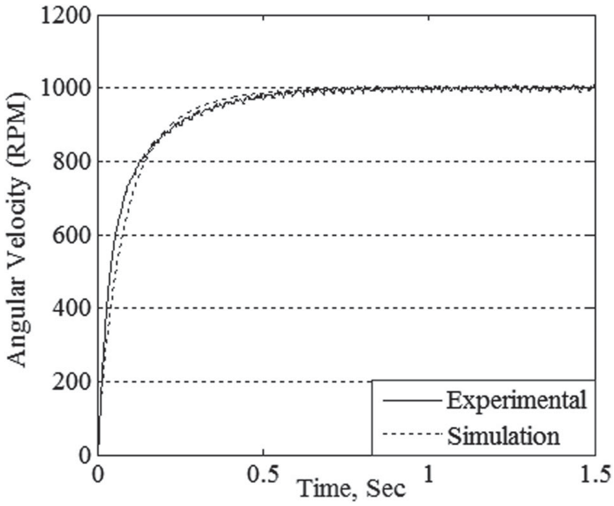

Figure 4. The experimental and simulation angular velocities for Example 1.

Table 2. Transient response specifications.

	Example 1	Example 2	Example 3
Rise time (s)	0.215	0.35	3.101
Settling time (s)	0.447	0.49	5.26

gain crossover frequency and isodamping property is selected. The corresponding gain crossover frequency and obtained PID controller parameters are presented in Table 1. Figure 4 compares the motor angular velocities obtained from simulation and experimental tests. The similarity of these responses to each other confirms the robust stability of the proposed controller. The obtained rising time and settling time for the experimental result are presented in Table 2. The voltage applied to the motor in the experimental test is given in Figure 5. Moreover, according to Figure 6, the effect of the load torque applied in $t = 1.5$ s is quickly eliminated.

Example 2: The position control of the DC motor in Example 1 is considered here. Considering the potentiometer gain, the following transfer function is obtained for the position DC servo plant

$$G(s) = \frac{\theta(s)}{V(s)} = \frac{21.113}{s(0.1s + 1)}, \tag{51}$$

where $\theta(t)$ is the motor shaft position and $v(t)$ is the voltage applied to the servo amplifier block.

The shaft position set value is chosen as 60°. The two-dimensional non-overshooting PD controller

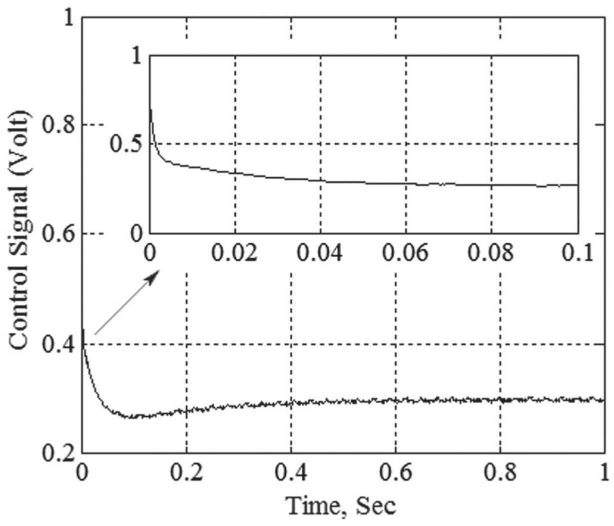

Figure 5. The control signal for Example 1.

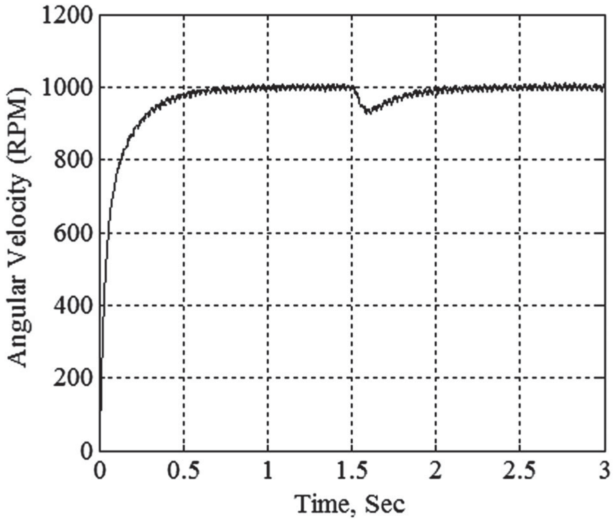

Figure 6. The load torque effect for Example 1.

parameter region is given in Figure 7. The parameters K_p and T_d are considered in the arbitrary ranges $(0, 10]$ and $(0, 0.1]$, respectively. An appropriate controller realizing the gain crossover frequency condition is selected in which its parameters are given in Table 1. As could be seen in Figure 8, the difference between the non-overshooting motor shaft position responses corresponding with simulation and experimental tests is negligible. The rising time and the settling time values for experimental test are shown in Table 2. Moreover, the control signal is shown in Figure 9.

Example 3: In this example, the ability of the proposed non-overshooting PID controller for controlling a second-order plant is verified. Consider a second-order plant with the following transfer function:

$$G(s) = \frac{5}{s^2 + 5s + 1}. \tag{52}$$

The $(T_i - T_d)$ region with different values of K_p ensuring (40b) is shown in Figure 10. The parameter T_i

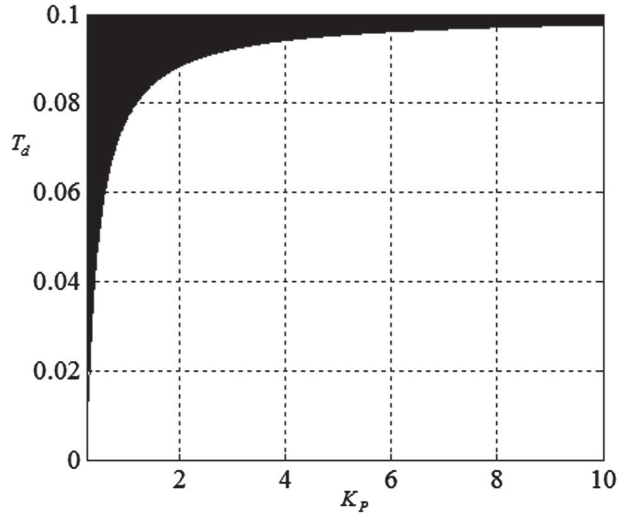

Figure 7. The two-dimensional region for PD controller parameters in Example 2.

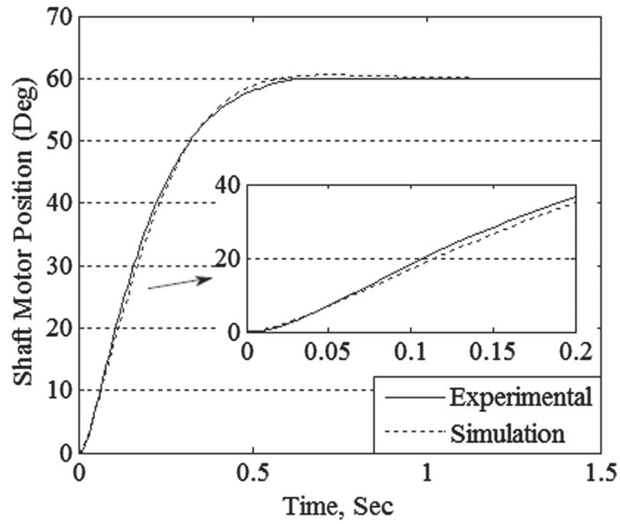

Figure 8. The experimental and simulation motor shaft positions for Example 2.

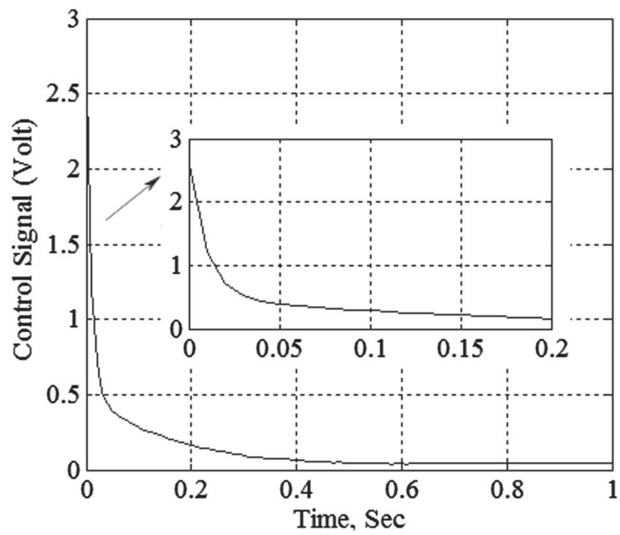

Figure 9. The control signal for Example 2.

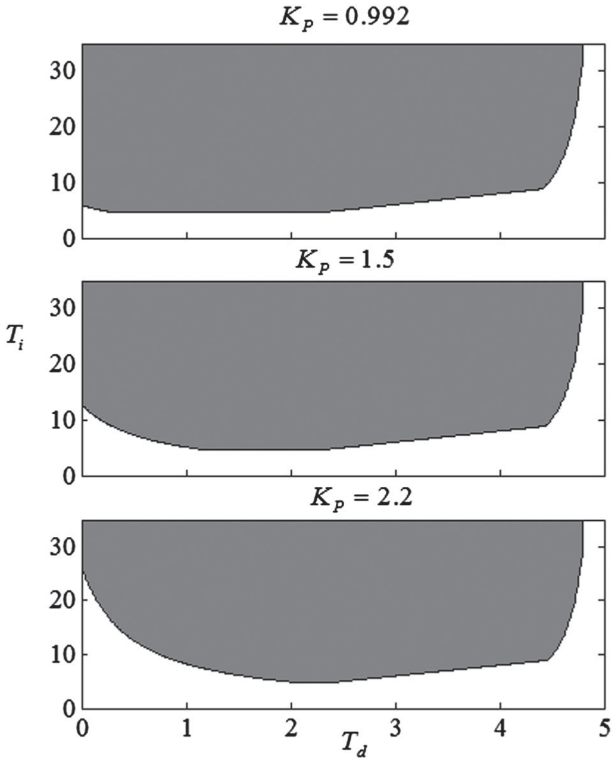

Figure 10. The $T_i - T_d$ region with different values of K_P in Example 3.

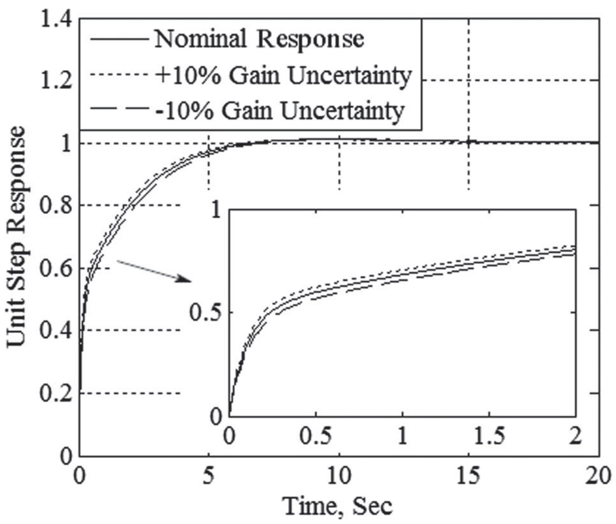

Figure 11. The effect of gain uncertainty in unit step response of Example 3.

is assumed in the arbitrary ranges (0, 35]. The range of T_d is obtained according to (40c). With an appropriate gain crossover frequency, the PID controller parameters are selected according to relations (40), (47), (48) and (49). The obtained controller coefficients are given in Table 1. Figure 11 compares the nominal unit step response with those obtained with $\pm10\%$ uncertainty in parameter K. This uncertainty does not have any effect on step response overshoot. The obtained rising time and settling time for nominal response are given in Table 2. Figure 12 shows the control signal.

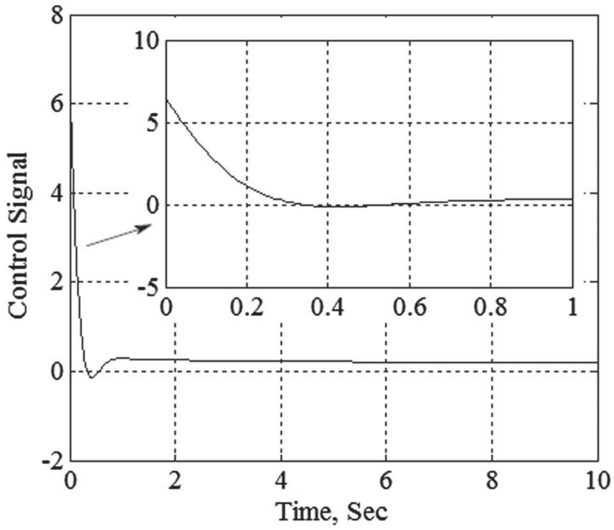

Figure 12. The control signal for Example 3.

4. Conclusion

This paper presents a novel analytical tuning method for the family of PID controllers. The design method is based on adjusting the PID controller parameters to avoid overshoot in the closed-loop system step response. This is accomplished by achieving a monotonically decreasing closed-loop system frequency response. Moreover, the loop gain phase is adjusted to be flat around the desired gain crossover frequency which makes the closed-loop system robust under gain uncertainties. The simulation results on a second-order plant and the experimental results on a laboratory DC position and speed servomechanism demonstrate the efficiency of the proposed non-overshooting PID controller, as well.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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