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# 'Guaranteed lowest prices: do they facilitate collusion?': Revisited 

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#### Abstract

We examine the effect of guaranteed lowest price clauses (G.L.P.). First, we correct the proof of Logan and Lutter's main result that it is the unique equilibrium outcome for firms adopting G.L.P. to charge collusive prices in a simultaneous pricing game, if one uses the trembling-hand perfect equilibrium as the solution concept. Second, we extend their argument to a sequential pricing game in which one firm chooses its price before the other, given that both firms adopt G.L.P. We show that collusive prices is the unique equilibrium outcome in this game even without resorting to any stringent refinement like the trembling-hand perfect equilibrium.


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## 1. Introduction

In 1963, G.E. (General Electric) issued a price-book containing information about prices of its turbine generators. Shortly thereafter, Westinghouse followed G.E. by publishing a similar price-book. In the period, there was no evidence that they kept any contact with each other, but they maintained high prices in following years. This case triggered the Department of Justice's new antitrust doctrine arguing that an agreement between firms does not require direct explicit communication between them but that the agreement can also be established through a public communication. At that time, collusion-facilitating practices such as 'guaranteed lowest price' clauses (hereafter G.L.P.) began to receive attention in antitrust cases.

In fact, many firms offer consumers lowest-price guarantees in which they promise to match any lower price charged by rivals. Also, many authors study the effect of G.L.P., especially the possibility that it can facilitate collusion. The intuition is quite clear. If a firm cuts its price when both firms adopt G.L.P., it has the effect of lowering the rival firm's price by the same amount. Therefore, firms would have no incentive to lower prices under G.L.P.

Early theoretical studies on G.L.P. include Holt and Scheffman (1987), Belton (1987) and Logan and Lutter (1989). Holt and Scheffman (1987) considered several facilitating practices jointly, namely G.L.P., Most-Favoured-Customer Clauses (M.F.C.), and public

[^0]advance notification. Logan and Lutter (1989) considered G.L.P. only. Schnitzer (1994) considered G.L.P. and M.F.C. separately. More recently, authors expanded their interests beyond 'meet-the-competition' clauses (M.C.C.), equivalent to G.L.P., to 'beat-the-competition' clauses (B.C.C.).

In this paper, we will examine the robustness of the collusion-facilitating effect of G.L.P. by using Logan and Lutter (1989) as a benchmark model. Logan and Lutter (1989), in their interesting paper, argued that the unique equilibrium in the symmetric two stage G.L.P. game is for both firms to adopt G.L.P. in the first stage and collusive prices in the second stage (Theorem 1). In the proof, they used two arguments. One is that when both firms select G.L.P., the set of Nash equilibria in the subgame is $E_{N} \equiv\left\{(p, p) \mid p^{N} \leq p \leq p^{C}\right\}$ where $p^{N}$ is the symmetric Nash price and $p^{C}$ is the symmetric collusive price. The second is that the unique (trembling-hand) perfection-like refinement (hereafter, simply trembling-hand perfect equilibrium) among them is $\left(p^{C}, p^{C}\right)$; Theorem 2 . In this paper, we show that their arguments have flaws, although the second argument itself is correct. We correct the proof of their main result that it is the unique equiibrium outcome for firms adopting G.L.P. to charge collusive prices in a simultaneous pricing game. More importantly, we extend their argument to a sequential pricing game in which one firm chooses its price before the other, given that both firms adopt G.L.P. We show that collusive prices $\left(p^{C}, p^{C}\right)$ are the unique equilibrium outcome in this game even without resorting to any stringent refinement like the trembling-hand perfect equilibrium. This implies that the collusion-facilitating nature of G.L.P. is robust to the pricing sequence and even strengthened in the sequential pricing game, which is more plausible in reality.

Despite the extensive theoretical literature on G.L.P., empirical literature is more limited, mainly due to the difficulty in collecting price data before and after introduction of G.L.P. To the best of our knowledge, Hess and Gerstner (1991) seems to be the first empirical work on this topic. They used price data at supermarkets and showed that G.L.P. relieves competition, i.e., concluded that no evidence of cut-throat price competition is found. Arbatskaya, Hviid, and Shaffer (2004) showed by using advertised tyre prices that a tyre retailer's own price-matching or price-beating guarantee has no significant effect on the retailer's advertised tyre price, but that the price tends to rise as the percentage of firms announcing low-price guarantees increases. Also, Arbatskaya et al. (2004) obtained the result that the majority of low-price guarantees are not consistent with their use as a facilitating device. Mañez (2006), using price data on U.K. supermarkets, showed that B.C.C. was not a collusive device leading to higher prices but an advertising tool to signal the low prices of the supermarket. Empirical studies on G.L.P. show its other roles besides facilitating price collusion. Moorthy and Zhang (2006) empirically showed that adopting G.L.P. signals a low-price positioning, while not adopting G.L.P. signals high-service positioning.

Due to the difficulties in data collection, experimental methods have been also considered as an alternative for testing the effect of G.L.P. on prices. Deck and Wilson (2003) is the first experimental research on G.L.P. They showed that the G.L.P. pricing algorithm leads to higher prices than the undercutting algorithm. Subsequent experimental results of Fatas and Mañez (2007), Mago and Pate (2009), and Fatas, Georgantzís, Mañez, and Sabater (2013) also strongly support the collusion-enhancing effects of G.L.P.


Figure 1. The effect of a deviation from Nash prices. Source: Authors.

## 2. Simultaneous pricing

We closely follow the model of Logan and Lutter (1989) in which two firms produce differentiated goods with identical marginal costs normalised to zero. They consider a twostage game between the two firms, where they decide whether or not to adopt G.L.P. in the first stage and they select prices in the second stage. In this note, we focus on the case where both firms adopt G.L.P. This is the only case in which Logan and Lutter obtained the collusive outcome.

Figure 1 provides clear intuition for why prices $(p, p)$ such that $p \leq p^{N}$ as well as $(p, p)$ such that $p^{N} \leq p \leq p^{C}$ can be equilibrium outcomes. Consider any non-negative symmetric prices ( $p, p$ ) such that $p^{N} \leq p \leq p^{C}$. If a firm, say Firm 1, slightly lowers its price, it moves the new pair of effective prices not to point $A$ but to point $A^{\prime}$ on the 45 degree line, because it accompanies a fall in the effective price of Firm 2 by the same amount. This makes Firm 1 worse off, implying that it has no incentive to cut its price. The intuition for a lower profit is that a cut in its price does not increase its demand at all, since the price cut is immediately and automatically matched by the rival firm. Meanwhile, it has no incentive to raise its price, either, because a price increase does not imply moving to point $B$ but bouncing back to the original point. A price increase cannot be maintained effectively because it must be matched with the lower price by the rival firm. Combining the two arguments together, we can see that any such symmetric vector is a Nash equilibrium outcome. Now, consider any non-negative symmetric prices $(p, p)$ such that $0 \leq p<p^{N}$. The same logic can be applied. Any price cut by a firm is effectively accompanied by a matching price cut by the rival firm, so it cannot be profitable. Hence, there will be no incentive to cut its price. Also, any unilateral price increase would have no real effect. So, any such price vector is also a Nash equilibrium outcome.


Figure 2. Best-response correspondences. Source: Authors.

Formally, let $\tilde{p}=\min \left\{p_{1}, p_{2}\right\}$ be the effective price for both firms. That is, $\tilde{p}=p_{1}$ if $p_{1}<p_{2}$, $\tilde{p}=p_{2}$ if $p_{1}>p_{2}$ and $\tilde{p}=p_{1}=p_{2}$ if $p_{1}=p_{2}$. Note that a firm's own price is not important to its demand as long as it is above the rival's price. All that matters is the price of the other firm. Taking this into account, we can figure out the best response (B.R.) correspondence of each firm as in Figure 2.
Proposition 1 The set of Nash equilibria is $E$.
The proof is immediate from the best-response correspondences in Figure 2. This proposition is important because this result seems to suggest that prices even below Nash prices could be supported under G.L.P. If this is the case, G.L.P. can induce not only collusive prices higher than Nash prices, but also non-collusive prices lower than Nash prices. That is, the anti-competitive implication of G.L.P. becomes ambiguous. However, due to the following proposition, it is not possible for G.L.P. to induce lower prices than Nash prices if one uses a stronger solution concept.
Proposition 2 (Logan and Lutter) ( $p^{c}, p^{c}$ ) is the unique trembling-hand perfect equilibrium of this simultaneous pricing game.

In the appendix, we provide a corrected proof of Logan and Lutter by allowing perturbations to prices above $p^{C}$ (up to a certain upper bound $\bar{p}$ ). ${ }^{1}$ The result of Logan and Lutter remains unaffected even if we allow more perturbations. Intuitively, this is because $p^{C}$ is still the best response to the rival's higher price (charged with some small probability).

## 3. Sequential pricing

In this section, we extend the analysis of Logan and Lutter to the sequential model in which Firm 1 behaves as a Stackelberg leader by choosing its price earlier than Firm 2. ${ }^{2}$ We are


Figure 3. Iso-profit curves of Firm 1. Source: Authors.
interested in two issues. First, does G.L.P. still facilitate collusion in this sequential game? Second, is there any strategic advantage to either firm? That is, is there any first-mover advantage or second-mover advantage? ${ }^{3}$

The standard solution concept for (complete information) sequential games is the subgame perfect equilibrium which employs backward induction. For backward induction, consider the best-response correspondence of firm 2 given G.L.P. which is described in Figure 3. Firm 1 will choose the price on this B.R. correspondence that yields its maximum profit. The next proposition is our second main result.
Proposition $3 \quad\left(p^{c}, p^{c}\right)$ is the unique subgame perfect Nash equilibrium outcome of the sequential price game.

The proof is immediate from B.R. correspondence of Firm 2 and $L$-shaped iso-profit curves of Firm 1 drawn in Figure 3. This proposition has the implication that we do not need to resort to a stringent refinement such as trembling-hand perfect equilibrium to obtain the unique outcome if firms set prices even slightly in a sequential manner. The intuitive reason is that the first mover can initiate coordination on the most preferred outcome among a continuum of Nash prices. Although this game has the unique equilibrium outcome, it does not mean that this game has the unique equilibrium, since there are continuum of equilibria yielding the same outcome. That is, any price vector of $E_{1} \cup E_{2}$ can be a subgame perfect equilibrium, although all of them yield the same effective prices ( $p^{C}, p^{C}$ ). Also, note that there is no first mover advantage nor second mover advantage. As long as both firms adopt G.L.P., $\left(p^{C}, p^{C}\right)$ is a focal point in the sense that $p^{C}$ is the dominant strategy for each firm.

## 4. Empirical application

To find the effects of G.L.P. clauses on prices empirically, we use the data-set of electronics prices collected by Lundberg (2008). In the Swedish electronics market, there are a number

Table 1. Prices in selected models in El-Giganten, ONOFF, Siba and expert.

| Model | El-Giganten | ONOFF | Siba | Expert |
| :--- | :---: | :---: | :---: | :---: |
| Sony Ericsson W890i | 3149 | 3390 | 3490 | 3590 |
| LG KU990 Viewty | 3579 | 3835 | 3890 | 3789 |
| Nokia 5310 Xpress Music | 1779 | 2107 | 2190 | 1999 |
| Panasonic TH-42PX80E | 9690 | 9788 | 9736 | 9699 |
| Sony Blu-ray spelare BDP-S300 B | 3990 | 2890 | 3490 | 2790 |
| Apple Ipod Classic 80 GB Silver | 1979 | 2169 | 2290 | 2159 |
| Canon Digital IXUS 80 IS | 2190 | 2390 | 2289 | 2289 |
| Canon EOS 450D 18-55 IS | 7490 | 7795 | 7490 | 7749 |
| Canon EOS 400D + 18-55/3.5-5.6 | 5490 | 5765 | 5765 | 5929 |
| Olympus Digitalkamera MY840 | 1829 | 1990 | 1838 | 1819 |
| Sony DAVIS10 | 5490 | 6290 | 5990 | 6279 |
| Philips HTS6600 | 4990 | 4969 | 4990 | 5259 |
| CREATIVE Webcam Live Cam Optia | 379 | 495 | 380 | 529 |

Source: Lundberg (2008).

Table 2. Result of paired $t$-test.

|  | El-Giganten and ONOFF | Siba and Expert |
| :--- | :---: | :---: |
| Mean | 4072.9615 | 4142.5769 |
| Variance | $6,848,171.9359$ | $6,894,342.8269$ |
| Observations | 13 | 13 |
| Pearson Correlation | 0.9982 |  |
| Hypothesised Mean Difference | 0 |  |
| df | 12 |  |
| $t$ Stat | -1.5870 |  |
| $\mathrm{P}(T<=t)$ one-tail | 0.0692 |  |
| t Critical one-tail | 1.7823 |  |
| $\mathrm{P}(T<=t)$ two-tail | 0.1385 |  |
| $t$ Critical two-tail | 2.1788 |  |

Source: Authors.
of firms; El-Gigantel, ONOFF, Siba, Expert, Expert Stormarknad, ELON, Media Market and Euronics. Among them, the first two firms (El-Giganten and ONOFF) adopt G.L.P., while the others do not. Table 1 describes prices for 13 different products at four firms (El-Giganten, ONOFF, Siba and Expert).

Now, our question is whether the average prices of firms adopting G.L.P. (EL-Giganten and ONOFF) are higher than the average prices of firms not adopting G.L.P. (Siba and Expert). For the comparison, we perform a paired $t$-test by using the data-set in Table 1. Table 2 shows our result of the paired $t$-test.

From Table 2, we can tell that the average price of El-Giganten and ONOFF adopting G.L.P. (4072.9615) is lower than that of Siba and Expert not adopting G.L.P. (4142.5769). However, Table 2 also shows that the t -statistic is -1.5870 , which belongs to the $95 \%$ confidence interval [-2.1788, 2.1788], where 2.1788 is the $t$ critical two-tailed value. This implies that the price difference is not statistically significant at the $95 \%$ confidence level. This leads us to conclude that our theoretical result that G.L.P. does facilitate collusion for higher prices is not empirically supported by this data-set, although the general methodology provided in this section could be applied to a much larger data-set to tell whether the anti-competitive effect of G.L.P. is empirically supported or not.

## 5. Conclusion

In this note, we corrected the proof of Logan and Lutter (1989)'s result, although their result still remains valid. We also strengthened their main finding that G.L.P. can facilitate collusion by showing that the collusive prices are the unique equilibrium of the sequential price game without invoking a stringent refinement like trembling-hand perfection.

Our result has an important policy implication that under G.L.P. practice, equilibrium prices cannot be lower than Nash prices, rather they must be higher than Nash prices. Therefore, G.L.P. practice is unambiguously facilitating collusion.

## Notes

1. A high price that makes each firm's demand fall to zero can be an upper bound.
2. This sequential model is just to demonstrate the robustness of our result. It is often used in models of price leadership. See, for example, Rotemberg and Saloner (1990) for price leadership.
3. It is well known from Gal-Or (1985) that there is the second-mover advantage in price competition games, whereas there is the first-mover advantage in output competition games.

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## Disclosure statement

No potential conflict of interest was reported by the authors.

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## Appendix

Proof of Proposition 2: Since trembling hand perfection is generally defined for games with finite strategy sets, we approximate the game by replacing the uncountable strategy sets with discretised finite sets. Let $\pi_{i}\left(p_{i}, p_{j}\right)$ be the profit function of firm $i$. We assume that there exists $\bar{p}$ such that for any $p_{i} \geq \bar{p}, \pi\left(p_{i}, p_{j}\right) \leq 0$ for any $p_{j}$. Let $\Delta_{1} \equiv p^{C}$ and $\Delta_{2} \equiv \bar{p}-p^{C}$. Then, we can divide the interval $\left[0, p^{C}\right]$ into $n \Delta_{1}$ subintervals and the interval $\left[p^{C}, \bar{p}\right]$ into $n \Delta_{2}$ subintervals with equal length for a sufficiently large value of $n$ such that both $n \Delta_{1}$ and $n \Delta_{2}$ are integers. Now, there are $n \Delta$ subintervals each of which have length of $\frac{1}{n}$ where $\Delta=\Delta_{1}+\Delta_{2}$. Take any $p(k)=\frac{k}{n}$ for $k=0,1,2, \cdots, n \Delta$. (Note that $p(k)=p^{C}$ if $k=n \Delta_{1}$ and $p(k)=\bar{p}$ if $k=n \Delta$.)
(i) First, we will show that $p^{C}$ is a symmetric (trembling-hand) perfect equilibrium price. Consider a totally mixed strategy $\sigma=\left(\sigma_{0}, \sigma_{1}, \cdots, \sigma_{n \Delta_{1}}, \cdots, \sigma_{n \Delta}\right)=\left(\varepsilon_{0}, \varepsilon_{1}, \cdots, 1-\sum_{k \neq n \Delta_{1}} \varepsilon_{k}, \cdots, \varepsilon_{n \Delta}\right)$. It is easy to see that as $\varepsilon_{k} \downarrow 0 \forall k$, the sequence of totally mixed strategies converges to $p^{C}$. We have

$$
E\left(\pi_{i}\left(p^{C}, \sigma\right)\right)=\sum_{k \neq n \Delta_{1}-1} \varepsilon_{k} \pi_{i}(p(k), p(k))+\left(1-\sum_{k \neq n \Delta_{1}} \varepsilon_{k}\right) \pi_{i}\left(p^{C}, p^{C}\right)+\sum_{k \geq n \Delta_{1}+1} \varepsilon_{k} \pi_{i}\left(p^{C}, p^{C}\right)
$$

since $\tilde{p}=p(k)$ for all $k<n \Delta_{1}$ and $\tilde{p}=p^{C}$ for all $k>n \Delta_{1}$. For $p(k)<p^{C}$, say, $p\left(k_{0}\right)$ where $k_{0}<n \Delta_{1}$, we have

$$
\begin{aligned}
E\left(\pi_{i}\left(p\left(k_{0}\right), \sigma\right)\right)= & {\left[\sum_{k \leq k_{0}-1} \varepsilon_{k} \pi_{i}(p(k), p(k))+\sum_{k_{0} \leq k \leq n \Delta_{1}-1} \varepsilon_{k} \pi_{i}\left(p\left(k_{0}\right), p\left(k_{0}\right)\right)\right] } \\
& +\left(1-\sum_{k \neq n \Delta_{1}} \varepsilon_{k}\right) \pi_{i}\left(p\left(k_{0}\right), p\left(k_{0}\right)\right)+\sum_{k \geq n \Delta_{1}+1} \varepsilon_{k} \pi_{i}\left(p\left(k_{0}\right), p\left(k_{0}\right)\right)
\end{aligned}
$$

since $\tilde{p}=p(k)$ if $k<k_{0}$ and $\tilde{p}=p\left(k_{0}\right)$ if $k \geq k_{0}$. We will compare $E\left(\pi_{i}\left(p^{C}, \sigma\right)\right)$ and $E\left(\pi_{i}\left(p\left(k_{0}\right), \sigma\right)\right)$ term by term. Since $p(k)$ is increasing in $k$ and $\pi_{i}(p(k), p(k))$ is increasing in $p_{k}$ until $p(k)=p^{C}$ (for the firstterm comparison) and $\pi_{i}\left(p^{C}, p^{C}\right) \geq \pi_{i}(p, p)$ for all $p$ (for the second- and third-term comparison), we can see that $E\left(\pi_{i}\left(p^{C}, \sigma\right)\right)>E\left(\pi_{i}\left(p\left(k_{0}\right), \sigma\right)\right)$, implying that $E\left(\pi_{i}\left(p^{C}, \sigma\right)\right)>E\left(\pi_{i}(p(k), \sigma)\right)$ for all $p(k)<p^{C}$. For $p(k)>p^{C}$, say, $p\left(k_{1}\right)$ where $k_{1}>n \Delta_{1}$, we have

$$
\begin{aligned}
E\left(\pi_{i}\left(p\left(k_{1}\right), \sigma\right)\right)= & \sum_{k \leq n \Delta_{1}-1} \varepsilon_{k} \pi_{i}(p(k), p(k))+\left(1-\sum_{k \neq n \Delta_{1}} \varepsilon_{k}\right) \pi_{i}\left(p^{C}, p^{C}\right) \\
& +\left[\sum_{n \Delta_{1}+1 \leq k \leq k_{1}-1} \varepsilon_{k} \pi_{i}\left(p^{C}, p^{C}\right)+\sum_{k \geq k_{1}} \varepsilon_{k} \pi_{i}\left(p\left(k_{1}\right), p\left(k_{1}\right)\right)\right]
\end{aligned}
$$

since $\tilde{p}=p(k)$ if $k<k_{1}$ and $\tilde{p}=p\left(k_{1}\right)$ if $k \geq k_{1}$. We will compare $E\left(\pi_{i}\left(p^{C}, \sigma\right)\right)$ and $E\left(\pi_{i}\left(p\left(k_{1}\right), \sigma\right)\right)$ term by term. Since $p(k)$ is increasing in $k$ and $\pi_{i}(p(k), p(k))$ is increasing in $p_{k}$ until $p(k)=p^{C}$ (for the firstterm comparison) and $\pi_{i}\left(p^{C}, p^{C}\right) \geq \pi_{i}(p, p)$ for all $p$ (for the second- and third-term comparison), we can see that $E\left(\pi_{i}\left(p^{C}, \sigma\right)\right)>E\left(\pi_{i}\left(p\left(k_{0}\right), \sigma\right)\right)$, implying that $E\left(\pi_{i}\left(p^{C}, \sigma\right)\right)>E\left(\pi_{i}(p(k), \sigma)\right)$ for all $p(k)<p^{C}$. Since $p^{C}$ is the best response to a sequence of $\sigma$ that converges to $p^{C}$, it is the perfect equilibrium.
(ii) Next, we will show that any $p(k)<p^{C}$ cannot be a symmetric perfect equilibrium price. Consider a totally mixed strategy $\sigma^{\prime}=\left(\varepsilon_{0}, \varepsilon_{1}, \cdots, 1-\sum_{k \neq k_{2}} \varepsilon_{k}, \cdots, \varepsilon_{n \Delta}\right)$ for any arbitrary $k_{2}<n \Delta_{1}$. Clearly, $\sigma^{\prime}$ converges to $p\left(k_{2}\right)$ as $\varepsilon_{k} \downarrow 0$. We have

$$
\begin{aligned}
& E\left(\pi_{i}\left(p\left(k_{2}\right), \sigma^{\prime}\right)\right)=\sum_{k \neq k_{2}} \varepsilon_{k} \pi_{i}(p(k), p(k))+\left(1-\sum_{k \neq k_{2}} \varepsilon_{k}\right) \pi_{i}\left(p\left(k_{2}\right), p\left(k_{2}\right)\right)+\sum_{k>k_{2}} \varepsilon_{k} \pi_{i}\left(p\left(k_{2}\right), p\left(k_{2}\right)\right), \\
& E\left(\pi_{i}\left(p^{C}, \sigma^{\prime}\right)\right)=\sum_{k \neq k_{2}} \varepsilon_{k} \pi_{i}(p(k), p(k))+\left(1-\sum_{k \neq k_{2}} \varepsilon_{k}\right) \pi_{i}\left(p\left(k_{2}\right), p\left(k_{2}\right)\right) \\
& +\sum_{k_{2}<k<n \Delta_{1}} \varepsilon_{k} \pi_{i}\left(p\left(k_{2}\right), p\left(k_{2}\right)\right)+\sum_{k \geq n \Delta_{1}} \varepsilon_{k} \pi_{i}\left(p^{C}, p^{C}\right) .
\end{aligned}
$$

Since $p^{C}=\arg \max _{p} \pi_{i}(p, p)$, it is clear that $E\left(\pi_{i}\left(p^{C}, \sigma^{\prime}\right)\right)>E\left(\pi_{i}\left(p\left(k_{2}\right), \sigma^{\prime}\right)\right)$, implying that $p\left(k_{2}\right)$ is not a best response to $\sigma^{\prime}$. Therefore, $p\left(k_{2}\right)$ cannot be a symmetric perfect equilibrium price.
(iii) Finally, it remains to show that any asymmetric price vector ( $p^{C}, p$ ) for $p>p^{C}$ cannot be a perfect equilibrium. It is trivial to see this, because we already showed in (i) that $p^{C}$ (not any $p>p^{C}$ ) is the best response to a sequence of $\sigma$ defined in (i), implying that ( $p^{C}, p$ ) for $p \neq p^{C}$ cannot be a perfect equilibrium.


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