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Complementarity between energy and physical capital in a simple model of economic growth

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ABSTRACT

Energy is one of the most important factors of growth, needed to power physical capital. This implies complementarity between these two factors, but in most theoretical papers, substitutability is assumed. In this paper, the main question is if complementarity between energy and physical capital hampers balanced growth or not. In order to answer this question, we consider the simple economic growth model with two sectors – output production and energy production – where energy is complementary to physical capital. Two forms of capital – physical and human – are distributed in this model between two sectors to maximise output. We consider the equilibrium state of this model, analyse comparative statics and derive necessary conditions for balanced growth. We also analyse some of the effects of technological progress. The analysis brings us to the conclusion that, in general, it leads to an increase in output, but that changes in technology of production have an ambiguous influence on output. We also derive explicit paths of growth for all macroeconomic variables. We find that, in general, if the obtained conditions are fulfilled then complementarity is not an obstacle to economic growth, subject to our assumptions which are quite standard in this kind of research.

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1. Introduction

Economic growth models primarily focus on the description of relationships between the accumulation of physical and human capital (or other factors of production), technological progress and economic growth. Sometimes, they are concentrated around the description of technological progress itself. Occasionally, their objects of interest are optimal depletion of natural resources, emission of pollution or the production and distribution of energy which powers the existing physical capital. The most common reason for considering growth models is an analysis as to whether a balanced and stable growth is possible when stocks of production factors are changing over time, whether increasing or decreasing, or in other theoretical circumstances.

The related question of distribution of resources, especially physical capital, is one of the main topics in economic growth theory. Therefore, another interesting issue is how

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much of existing physical capital should be devoted to different sectors, in particular to energy production and to output production, considering that non-consumed output is later invested, increasing stock of physical capital to maintain balanced economic growth, while also raising the demand for the energy. It is also interesting to determine how much of the output should be invested and how much physical capital should be accumulated to maintain a balanced growth of physical capital. We use the terminology of balanced growth of physical capital to describe growth that occurs when shares of capital devoted to different sectors in the economy are constant in time and, additionally, the production of the energy grows at the same rate as physical capital to continuously power all the existing and newly accumulated physical capital. Otherwise, there exists a stock of capital that is unused due to lack of energy, or there is over-production of the energy, which cannot be stored and is an obvious waste of resources.

Theoretical and empirical studies show that energy, mostly used to power the physical capital needed to produce output, is one of the most important factors of growth. More physical capital requires more energy, as shown in many empirical studies. With higher stocks of physical capital, more energy is used. This leads to the conclusion that energy has to be complementary to the physical capital, which is in contrast to many theoretical studies, where substitutability between these two factors is assumed. The direct consequence of this assumption is the erroneous result that the entire growing energy demand of the economy is satisfied with a use of only a single unit of the energy or the energy resource used in the energy production. In that situation, obviously, sustainable and balanced growth is possible. However, if complementarity rather than substitutability between energy and growing stock of physical capital is the case in a real-life production process, then the crucial question seems to be if balanced growth is still possible in the long run. Also, the specific kind of 'golden rule of accumulation' should also be considered to calculate optimal investment rate in a case of complementarity between accumulated capital and energy.

In this paper the main question is if complementarity between energy and physical capital hampers balanced growth or not. To answer this question, we consider a simple two sector model of economic growth without an *explicit* technological progress, but with mentioned complementarity. The model includes the problem of optimal production factors distribution. Existing stocks of capital, both physical and human, are distributed in this model between two sectors: the output production sector and energy production sector. Energy is needed to power the existing stock of physical capital. The economy also invests in both forms of capital, which increases stocks of physical and human capital. More physical capital requires more energy. To produce more energy, more inputs of physical and human capital are needed in the second sector. This implies that the accumulation of both forms of capital needs to be balanced to maximise the output of the economy. If it is not, then there is over- or under-production of the energy with respect to the existing (and growing) stock of physical capital. If accumulation of physical capital is too fast, then there exists a stock of physical capital that cannot be powered. If accumulation is too slow, then economic growth is not sustainable. In the paper we analyse and derive some necessary conditions that need to be fulfilled to achieve balanced growth. We consider the problem of optimal investments rate in the economy and analyse it in detail. We also analyse some of the effects of technological progress. Existing models rarely assume complementarity between energy and physical capital, and to our best knowledge, this paper is one of the first papers where this feature is presented and considered.¹ We find that, in general, if

the obtained conditions are fulfilled, then complementarity is not an obstacle to economic growth, conditional on our other assumptions as is quite standard in this kind of research.

The structure of this paper is as follows: in the next section, we conduct literature review. In Section 3, we describe the proposed model. In Section 4, we define, consider and analyse the equilibrium state of the model. In Section 5, we analyse comparative statics of the size of production due to changes in key parameters in the equilibrium. In Section 6, we derive paths for key variables in the model: size of output, stock of physical and human capital and consumption. In Section 7, we obtain the golden rule of accumulation of human capital, and in Section 8, we conclude with some important remarks.

2. Literature review

Energy is one of the most important factors of growth (Janda & Torkhani, 2016; Pablo-Romero & Sánchez-Braza, 2015; Stern, 2011; Stern & Cleveland, 2004; Suri & Chapman, 1998). Some empirical studies, including Warr and Ayres (2010), show cointegration between time series of energy use and G.D.P., which implies a long-run relationship between these two variables. Research conducted by Costantini and Martini (2010), Gross (2012) and Lee, Chang, and Chen (2008) show Granger causality from energy consumption to economic growth, which confirms energy as a factor of growth. Stern and Cleveland (2004) state that ‘the theoretical and empirical evidence indicates that energy use and output are tightly coupled with energy availability playing a key role in enabling growth’. This approach can also be observed in papers such as Arbex and Perobelli (2010), where a growth model is integrated with an input–output model. Despite that, there are a plenty of papers with different and controversial results, such as a lack of cointegration between energy use and output (for example, Yu & Jin, 1992 or Soytaş & Sari, 2003), cointegration only in a few countries, no such cointegration in others (among many others, Masih & Masih, 1996) and different directions of Granger causality (Chontanawat, Hunt, & Pierse, 2008; Kapusuzoglu & Karan, 2013; Masih & Masih, 1996; Pablo-Romero & Sánchez-Braza, 2015). A literature review concerning many studies investigating Granger causality in different countries can be found in Altunbas & Kapusuzoglu, 2011). Coers and Sanders (2013) and Stern and Cleveland (2004) discuss those and many other papers with surprising results. Kalimeris, Richardson, and Bithas (2014) conduct a meta-analysis on causality between energy and G.D.P. research results and had no clear conclusions. The question is also if different kinds of energy have similar empirical qualities, but Bruns and Gross (2013) shows that the time series of different kinds of energies are highly correlated with total energy.

Energy is mostly used to power physical capital which is necessary to produce output (Acemoglu, 2009; Frondel & Schmidt, 2002; Pablo-Romero & Sánchez-Braza, 2015; Stern, 2011; Stern & Cleveland, 2004; Van der Zwaan, Gerlagh, & Schratzenholzer, 2002; Van Zon & Yetkiner, 2003). More physical capital requires more energy, meaning that energy has to be complementary, or at least gross complementary, to the physical capital factor of production. The elasticities of substitution between energy and physical capital has been a subject of estimation in many empirical papers, such as Christopoulos and Tsionas (2002), Fiorito and Van den Bergh (2011), Medina and Vega-Cervera (2001) and Sorrell (2014). Estimations based on cross-sectional international data usually show substitutability between these two factors, while estimations based on time series show complementarity.

Different elasticity of substitution can be obtained even when a different type of energy is used. For example, [Kim and Heo \(2013\)](#) show different elasticities of substitution between fossil fuels and physical capital than between electricity and physical capital. Even though [Solow \(1987\)](#) explains these controversies in empirical research, there are a lot of studies with different and surprising results (a summary of these discrepancies may be found in [Koetse, De Groot, & Florax, 2008](#)). However, in general, in economic growth models, energy is introduced as a substitutable factor in the Cobb–Douglas production function (see, among many others, [Fröling, 2011](#); [Hassler, Krusell, & Olovsson, 2016](#); [Hung Nguyen & Nguyen Van, 2008](#)). Only rarely is a different approach used, usually in the form of the constant elasticity of substitution (C.E.S.) production function (see [Hassler, Krusell, & Olovsson, 2012, 2016](#); [Macías & Matilla-García, 2015](#); [Mizanski, 2013](#); [Smulders & De Nooij, 2003](#); [Sorrell, 2014](#); [Van der Zwaan et al., 2002](#); [Van Zon & Yetkiner, 2003](#)). An approach where a source of energy (natural resources) is non-substitutable to physical capital is also present in the literature (see for example [Di Maria & Valente, 2008](#); [Stuermer & Schwerhoff, 2013](#); [Voosholz, 2014](#)).

Technological progress seems to play an important role in this question and is present in many theoretical studies (for example [Grimaud & Rougé, 2003, 2005](#); [Pérez-Barahona, 2011](#)). It affects the relationship between physical capital and the required amount of energy in one of three ways ([Bretschger, 2005](#); [Foster, 2014](#); [Hassler et al., 2016](#); [Jaffe, Newell, & Stavins, 2005](#); [Ketteni, Mamuneas, & Pashardes, 2013](#); [Pittel & Rübhelke, 2010](#); [Popp, Newell, & Jaffe, 2010](#); [Sorrell, 2010, 2014](#); [Stern, 2011](#); [Vollebergh & Kemfert, 2005](#)). First of all, technological progress helps to create new generations of physical capital that require less energy. Second, it allows for the formation of new technologies of energy production, which helps to produce more energy with identical inputs of energy production factors and to use energy sources, such as fossil fuels, more effectively. Finally, technological progress allows for the discovery of new sources of energy, such as new kinds of fuels or renewable sources such as solar, wind and water. Nevertheless, according to the International Energy Agency, almost 80% of energy demand in the world is still satisfied only with the use of non-renewable fossil fuels.

The economy produces not only the goods with which to satisfy particular demands of the economic entities, but also the energy to power the existing stock of physical capital. Demand for this energy comes, in general, from two sources – households and firms. According to the data from the US Energy Information Administration (2012), in the United States, domestic energy consumption constitutes only 10% of total energy consumption, transportation requires 28% and the rest, 62% of total energy consumption, is consumed by industries and electric power companies.² Therefore, most of the energy produced in the economy satisfies the demand of firms, and therefore primarily powers physical capital which is used in the production of the output.

3. Model

In this section, we describe the economic growth model that is used in this paper to consider sustainable growth when energy and physical capital are complementary. We use the simple standard growth theory setup, which is in the spirit of [Mankiw et al. \(1992\)](#) and [Solow \(1956\)](#), [Swan \(1956\)](#). We consider a two-sector economy, endowed with the stocks of physical and human capital. Shares of both stocks are allocated between two different

activities: in the production sector to produce an output and in the energy production sector to produce energy to power the existing stock of physical capital. Similarly to [Antony \(2007\)](#), [Pittel and Bretschger \(2010\)](#) and [Stamford da Silva \(2008\)](#), in the production sector the economy engages a share ν of total stock of physical capital and a share u of total stock of human capital, where $\nu, u \in [0; 1]$. The economy produces a single good, and the production function has a constant return to scale and the standard, Cobb–Douglas form³:

$$Y = A(\nu K)^\alpha (uH)^{1-\alpha} \quad (1)$$

where Y stands for the level of output, A denotes the existing level of technology in production, K stands for physical capital, H for human capital and $\alpha \in (0; 1)$ is an elasticity of output with respect to physical capital.⁴ $K(0) = K_0$ and $H(0) = H_0$ denote the starting capital endowment of the economy.

Existing and used stock of physical capital requires certain amounts of energy ([Dalgaard & Strulik, 2007](#)). Therefore, unlike other studies,⁵ we assume complementarity between the flow of produced energy and physical capital, so the flow of energy needs to be at least proportional to the stock of K :

$$E \geq dK \quad (2)$$

where E is an output of the energy production sector. The interpretation of this equation is as follows: one unit of physical capital requires at least d units of energy. For simplicity, d in this model is constant and exogenous, but in general, it depends on the existing technology of energy production and technological advancement and quality of physical capital⁶ ([Hung Nguyen & Nguyen Van, 2008](#)).

In the energy production sector, share $1 - \nu$ of physical capital and share $1 - u$ of human capital are used to produce the energy. The energy production function, for mathematical simplicity, is also the Cobb–Douglas form:

$$E = B((1 - \nu)K)^\beta ((1 - u)H)^{1-\beta} \quad (3)$$

where B is a level of technology used in this sector and $\beta \in (0; 1)$ is an elasticity of energy production with respect to physical capital. $1 - \nu$ and $1 - u$ are also in the $[0; 1]$ interval.

The assumption of constant return to scale in energy production seems to be controversial. There is no empirical evidence that the energy production function should have Cobb–Douglas form. Nevertheless, the basic argument is as follows: a given amount of energy is produced with a certain input of stocks of physical and human capital. If we double these stocks, production of energy seems also to increase twofold. All in all, two identical energy factories with the same input of physical and human capital should each produce the same output of energy. This suggests constant returns to scale.

The economy spends a part, s_H , of output on investment in human capital. Therefore, the evolution of human capital is in a manner similar to the [Mankiw, Romer, and Weil \(1992\)](#) model⁷:

$$\dot{H} = s_H \cdot Y - \lambda_H H \quad (4)$$

where λ_H is the depreciation rate of human capital. The economy also spends a constant share, s_K , of its output on investment in physical capital. Finally, the evolution of physical

capital is in a standard fashion:

$$\dot{K} = s_K \cdot Y - \lambda_K K \quad (5)$$

where λ_K denotes the depreciation rate of physical capital. The output is, therefore, distributed between consumption and investments in two different forms of capital:

$$Y = C + s_K \cdot Y + s_H \cdot Y \quad (6)$$

which entails the consumption equation:

$$C = (1 - s_K - s_H)Y \quad (7)$$

The economy has to distribute the existing stocks of human and physical capital between two activities: output production and energy production. The latter is necessary to power physical capital and is used in both sectors. The economy also decides the distribution of the final output between three purposes: investments in physical capital (share s_K), investments in human capital (share s_H) and consumption (share $(1 - s_K - s_H)$, see Equation (7)).

The level of investments in physical capital have to be high enough to establish a balanced growth of stock of K . This is needed to ensure that the economy has the possibility to power the entire stock of physical capital. Any excessive investment is redundant because without energy, other units of K cannot be used in any of the sectors. Of course, the economy can move more physical and human capital to energy production, but this will lower output and, as a consequence, consumption; therefore, this solution cannot be optimal from the point of view of the entire economy. We derive the relationships between all macroeconomic variables in the equilibrium state required to preserve a balanced growth path.

In the presented model, there are some parameters that are independent of the choice of the economy. Most of them depend on the level of technology and therefore have to be treated as exogenous. Those parameters are $A, B, \lambda_K, \lambda_H, \alpha, \beta$ and d . Technological progress is explicitly absent in this model. This choice is made only because of mathematical simplicity and clarification of the results, but obviously, inclusion of technological progress in this model would not change the main conclusions. Nevertheless, we consider some effects of technological progress by obtaining signs of derivatives of basic macroeconomic variables with respect to certain technological parameters. We expect that technological progress increases the value of parameters A and B and reduces the value of parameters d, α and β . Analysis of these derivatives gives some information about the effects of technological progress.

Parameters such as s_K, s_H, u and v (all contained in $[0, 1]$ interval) depend on the choice of agents in the economy. Therefore, we need some kind of criteria to choose their proper value. We define the equilibrium state and consider this problem in the next section. We also consider some conditions that need to be fulfilled to reach a balanced growth path. Also, we conduct comparative statics of the equilibrium state and derive a formula for the share of investment in physical capital that depends on the share of investments in human capital.

4. Equilibrium analysis

In our analysis, we define equilibrium as the state when shares of physical and human capital devoted to both sectors are constant and also when constant shares of output are devoted to investments in both forms of capital. In this state, we also expect that the economy is able to produce enough energy to power all existing physical capital, so there is no stock of K that stays unused in any of sectors.

We denote by $g_X = \frac{\dot{X}}{X}$ the growth rate of variable X . We start the analysis considering Equation (2). In equilibrium, the amount of energy produced has to be equal to dK and proportional to the stock of physical capital that exists in the economy and is used in any sector (output production or energy production). Indeed, if E is lower than dK , there is not enough energy produced to power the entire stock of physical capital. This means that both stocks of capital are not distributed optimally, or there is a share of K that is not used in any sectors. In that case, investments in physical capital are not necessary, which means that gross investments in K should be equal to zero until the production of energy will satisfy demand. On the other hand, if E is strictly greater than dK , which means that more energy is produced than is needed to power the existing stock of K , then production of the energy sector is too high. Excessive shares of physical and human capital may be moved into the output production sector to increase the efficiency of the economy.⁸

To sum up, to reach optimal distribution of both forms of capital, $E = dK$, which leads to following equation:

$$dK = B((1 - \nu)K)^\beta ((1 - u)H)^{1-\beta}. \tag{8}$$

Equation (8) implies the following relationship between the stock of physical and human capital in the equilibrium state. When the economy is able to power all existing physical capital and use all units of it in one of the two sectors:

$$K = \left(\frac{B(1 - \nu)^\beta}{d} \right)^{\frac{1}{1-\beta}} (1 - u)H. \tag{9}$$

Proportionality (9) implies the obvious conclusion that $g_K = g_H$ in the equilibrium state. The equality of these two growth rates and constant return to scale in production function (Equation (1)) implies that $g_Y = g_K = g_H$ and that the output is equal to:

$$Y = Av^\alpha \left(\frac{B(1 - \nu)^\beta}{d} \right)^{\frac{\alpha}{1-\beta}} (1 - u)^\alpha u^{1-\alpha} H. \tag{10}$$

Therefore, the production function is of the AK form (see Barro & Sala-i-Martin, 2003), implied by the complementarity between energy and physical capital. The equilibrium linear relationship between the final output and stock of human capital depends on the shares of physical and human capital devoted to both sectors (ν and u), technology used in production and in energy production (α , β , A and B), and energy efficiency d .

The economy distributes stocks of capital between two sectors to maximise output. The following two equations are first order conditions for this maximisation:

$$\frac{\partial Y}{\partial \nu} = Av^\alpha \left(\frac{B(1 - \nu)^\beta}{d} \right)^{\frac{\alpha}{1-\beta}} (1 - u)^\alpha u^{1-\alpha} H (\alpha \nu^{-1} - \frac{\beta \alpha}{1 - \beta} (1 - \nu)^{-1}) = 0, \tag{11}$$

$$\frac{\partial Y}{\partial u} = Av^\alpha \left(\frac{B(1-v)^\beta}{d} \right)^{\frac{\alpha}{1-\beta}} (1-u)^\alpha u^{1-\alpha} H(-\alpha(1-u)^{-1} + (1-\alpha)u^{-1}) = 0. \quad (12)$$

This leads to the conclusion that shares of physical and human capital devoted to the production sector are given by the following formulae:

$$v = 1 - \beta, \quad (13)$$

$$u = 1 - \alpha. \quad (14)$$

Therefore, optimal shares of physical and human capital devoted to the production sector to maximise output are equal to the elasticities of (respectively) production and energy production with respect to human capital. This result confirms the important role of human capital and technology of production in both sectors in the modelled economy.

Equations (4) and (5) lead to following formulae for growth rates:

$$g_H = \frac{\dot{H}}{H} = s_H \cdot \frac{Y}{H} - \lambda_H, \quad (15)$$

$$g_K = \frac{\dot{K}}{K} = s_K \cdot \frac{Y}{K} - \lambda_K. \quad (16)$$

We proved in Equation (9) that in equilibrium $g_H = g_K$. Therefore,=,

$$s_K \cdot \frac{Y}{K} - \lambda_K = s_H \cdot \frac{Y}{H} - \lambda_H \quad (17)$$

which implies the following relationship between two shares of investments necessary to fulfil in equilibrium to guarantee balanced growth of both forms of capital:

$$\begin{aligned} s_K &= s_H \cdot \frac{K}{H} + (\lambda_K - \lambda_H) \frac{K}{Y} \\ &= s_H \left(\frac{B(1-v)^\beta}{d} \right)^{\frac{1}{1-\beta}} (1-u) + (\lambda_K - \lambda_H) \frac{\left(\frac{B(1-v)^\beta}{d} \right)^{\frac{1-\alpha}{1-\beta}} (1-u)^{1-\alpha}}{Av^\alpha u^{1-\alpha}}. \end{aligned} \quad (18)$$

With a given share of investments in human capital, one can obtain an equilibrium share of the investments in physical capital. The reverse is also true: with a given s_K , it is possible to obtain s_H . Relationship (18) determines the value of two shares of investments necessary for balanced growth (growth on equal rates of growth) of both forms of capital.

The size of the overall consumption of the economy with Equations (7), (10) and (18) is as follows:

$$C = \left(1 - s_H \left(\left(\frac{B(1-v)^\beta}{d} \right)^{\frac{1}{1-\beta}} (1-u) + 1 \right) - (\lambda_K - \lambda_H) \frac{\left(\frac{B(1-v)^\beta}{d} \right)^{\frac{1-\alpha}{1-\beta}} (1-u)^{1-\alpha}}{A\nu^\alpha u^{1-\alpha}} \right) \cdot A\nu^\alpha \left(\frac{B(1-v)^\beta}{d} \right)^{\frac{\alpha}{1-\beta}} (1-u)^\alpha u^{1-\alpha} H. \tag{19}$$

Therefore, consumption and stock of human capital are proportional in the economy, which entails $g_C = g_H$.

In the next section, we consider the obtained path of output. We also analyse comparative statics and discuss the obtained results.

5. Comparative statics of optimal size of production

In the previous section, we proved that to maximise output $v = 1 - \beta$ and $u = 1 - \alpha$. Optimal shares of both forms of capital engaged in the production of output entails the following formula for the production function:

$$Y = A(1 - \beta)^\alpha \left(\frac{B\beta^\beta}{d} \right)^{\frac{\alpha}{1-\beta}} \alpha^\alpha (1 - \alpha)^{1-\alpha} H. \tag{20}$$

The equilibrium’s final output depends on a few technological parameters and is proportional to the stock of human capital. With an increase in H , production also increases at the same growth rate, but this increase depends on the values of A , B , d , α and β . We now consider the changes that occur with a small increase in the value of one of the parameters, with other things equal:

$$\frac{\partial Y}{\partial A} = (1 - \beta)^\alpha \left(\frac{B\beta^\beta}{d} \right)^{\frac{\alpha}{1-\beta}} \alpha^\alpha (1 - \alpha)^{1-\alpha} H > 0, \tag{21}$$

$$\frac{\partial Y}{\partial B} = A(1 - \beta)^\alpha \frac{\alpha}{1 - \beta} \frac{1}{B} \left(\frac{B\beta^\beta}{d} \right)^{\frac{\alpha}{1-\beta}} \alpha^\alpha (1 - \alpha)^{1-\alpha} H > 0, \tag{22}$$

$$\frac{\partial Y}{\partial d} = -A(1 - \beta)^\alpha \frac{\alpha}{1 - \beta} \frac{1}{d} \left(\frac{B\beta^\beta}{d} \right)^{\frac{\alpha}{1-\beta}} \alpha^\alpha (1 - \alpha)^{1-\alpha} H < 0, \tag{23}$$

$$\begin{aligned} \frac{\partial Y}{\partial \alpha} &= AH(1 - \beta)^\alpha \alpha^\alpha (1 - \alpha)^{1-\alpha} \left(\frac{B\beta^\beta}{d} \right)^{\frac{\alpha}{1-\beta}} \\ &\cdot \left(\ln(1 - \beta) + \ln \left(\frac{B\beta^\beta}{d} \right) \cdot \frac{1}{1 - \beta} + \ln \left(\frac{\alpha}{1 - \alpha} \right) \right), \end{aligned} \tag{24}$$

$$\frac{\partial Y}{\partial \beta} = AH(1 - \beta)^\alpha \alpha^\alpha (1 - \alpha)^{1-\alpha} \left(\frac{B\beta^\beta}{d} \right)^{\frac{\alpha}{1-\beta}} \frac{\alpha}{(1 - \beta)^2} \cdot \left(\ln(\beta) - \ln \left(\frac{B}{d} \right) \right). \tag{25}$$

With an increase in the value of technological parameters A and B , final production also increases (see Equations (21) and (22)). An increase in A affects Y directly: A is the total

factor productivity (T.F.P.) in the production function, so higher values of this parameter allow the economy to reach higher levels of production without changes in the structure of engaged production factors. A small change in B affects Y indirectly: an increase in B allows for the production of more energy and, as a consequence, more physical capital with the same inputs of K and H can also be powered. Also, the same amount of energy can be produced with the use of a smaller share of both forms of capital engaged in the energy production sector, which gives the economy an opportunity to vary the distribution of K and H between the two sectors (for example, to allocate more capital into the output production sector) and produce more.

Parameter d governs the required amount of energy that is needed to power the existing stock of physical capital. One unit of physical capital requires d units of energy to be fully powered. Technological progress affects this parameter in the sense that another generation of K , one more technologically advanced, requires less energy. Therefore, we expect d to decrease over time. A decrease in d increases the size of production because less energy is needed to power the same size of K . As a consequence, this means that less of K and H are engaged in energy production, indicating that more of both forms of capital can be used in the production process. That in turn implies a higher Y .

The effects of change in α and β are not clear. Both parameters are connected with the technology of production and are elasticities of the production (respectively) of output and energy regarding physical capital. Whenever the production of output or production of energy becomes less ‘capital-intensive’ and more ‘skill-intensive’ (decrease in value of the parameters α and β), the reaction of final output is inconclusive and depends on the value of all the other parameters in the model. It is more likely to predict a sign of a derivative in equilibrium in Equation (25): $\beta \in (0, 1)$, therefore, $\ln(\beta) < 0$ and, because of technological progress, B is expected to rise and d is expected to decline. Therefore B/d is expected to eventually be greater than one, which implies that $\ln(\frac{B}{d}) > 0$. Therefore, starting from a certain moment in time, $\frac{\partial Y}{\partial \beta} < 0$. Interpretation of this fact may be as follows: with a decline in capital intensity in the production of energy, less of K is devoted to the production of the output and more of it is sent to energy production to power the same amount of physical capital stock. This affects the size of Y in negative way.

A similar effect is expected with any change in α , but the situation is more complex here. An increase in α implies a rise in the capital intensity in the production of the output and, due to the optimal value of v and u , a decline in the share of human capital devoted to the production of output occurs (and, at the same time, a rise in a share of H devoted to production of energy). This redistribution of human capital between the two sectors obviously affects the size of the output in both sectors. In the short-run, energy production rises and output declines. In the long-run, this implies changes in the shares of investments. With a greater production of energy, it is possible to accumulate greater stock of K because there are opportunities to do so.

In the next section, we obtain paths of growth for key variables in this model.

6. Paths of growth

In this section, we derive explicit solutions for all the differential equations and paths of growth for the key variables in this model. Equation (4), together with Equation (20), leads to the following linear first degree differential equation, which describes the evolution of

human capital:

$$\dot{H} = (s_H \cdot D_1 - \lambda_H)H \tag{26}$$

where $D_1 = A(1 - \beta)^\alpha \left(\frac{B\beta^\beta}{d}\right)^{\frac{\alpha}{1-\beta}} \alpha^\alpha (1 - \alpha)^{1-\alpha} > 0$. The solution to the differential Equation (26) is as follows:

$$H = H_0 e^{(s_H \cdot D_1 - \lambda_H)t} \tag{27}$$

Therefore, $g_H = s_H \cdot D_1 - \lambda_H$. This path and proportionality (Equation (9)) implies

$$K = D_2 \cdot H = D_2 H_0 e^{(s_H \cdot D_1 - \lambda_H)t} \tag{28}$$

where $D_2 = \left(\frac{B\beta^\beta}{d}\right)^{\frac{1}{1-\beta}} \alpha > 0$. In Equation (28), $K(0) = D_2 H_0$ which may be different to K_0 , the starting physical capital endowment of the economy. If $K_0 > D_2 H_0$, then a part of the physical capital $K_0 - D_2 H_0$ is not used in any sectors for a certain period of time until the stock of human capital reaches the required level to power K . On the other hand, if $K_0 < D_2 H_0$, then, for a while, the accumulation of physical capital has to be more intensive to increase the stock of K .

With a given path of growth for H and K and using Equation (20), it is possible to derive a path of growth for Y :

$$Y = D_1 H_0 e^{(s_H \cdot D_1 - \lambda_H)t} \tag{29}$$

Finally, thanks to Equation (19), we obtain a path of growth for the overall consumption:

$$C = (1 - s_H (D_2 + 1) - (\lambda_K - \lambda_H) \frac{D_2}{D_1}) \cdot D_1 H_0 e^{(s_H \cdot D_1 - \lambda_H)t} \tag{30}$$

All the variables in the economy grow at the same growth rate, $g_H = s_H \cdot D_1 - \lambda_H$. Therefore, the share of investments in human capital governs the growth of the entire economy. Increasing s_H increase the rate of economic growth. This outcome is consistent with the basic results in the economic growth theory; in the long run the growth rate is equal to the growth rate of technological progress (see [Acemoglu, 2009](#); [Barro & Sala-i-Martin, 2003](#)).

In the next section, we focus on the consumption path and obtain the golden rule of accumulation of human capital - the value of s_H that maximises the consumption of the economy.

7. Golden rule of accumulation

Because of Equation (30), it is possible to obtain the golden rule of accumulation of human capital. We consider Equation (30) to find the value of s_H that maximises consumption. The first order condition of this maximisation is as follows:

$$\frac{\partial C}{\partial s_H} = D_1 H_0 e^{(s_H \cdot D_1 - \lambda_H)t} \left(-(D_2 + 1) + \left(1 - s_H (D_2 + 1) - (\lambda_K - \lambda_H) \frac{D_2}{D_1} \right) \cdot D_1 \right) = 0 \tag{31}$$

which implies

$$s_H^* = \frac{D_1 - D_2 - 1 - (\lambda_K - \lambda_H) D_2}{D_1 (D_2 + 1)} \tag{32}$$

Equation (32) is a formula for the share of investments in human capital that maximises consumption. The result can be obtained by substituting the D_1 and D_2 derivative with a respect to given parameter of the model.

8. Conclusions

In this paper, we consider the economic growth model with complementarity between energy and physical capital. We propose the model of two-sector economy that has two forms of capital: physical and human, which are distributed between the two activities: output production and energy production. We analyse the equilibrium state in the model, where physical capital grows in such a rate that the distribution of it between sectors and the accumulation of human capital ensures that the produced amount of energy is always proportional to the stock of physical capital. Therefore, it is necessary to obtain shares of two forms of capital that are engaged in both sectors and the rate of physical and human accumulation.

We find that to maximise the output, shares of physical and human capital devoted to output production are equal to the elasticities of (respectively) output and energy production with respect to human capital. If the elasticity of the output production with respect to human capital is higher then more physical capital should be devoted to this sector. Accordingly, if the elasticity of energy production with respect to human capital is higher, then more human capital should be devoted to the production sector. This distribution is optimal in the sense that it maximises output. Additionally, energy production is high enough that the entire stock of physical capital in the economy is powered and available to use.

To ensure that both forms of capital grow at the same rate, we obtain shares of investments in physical and human capital. It turns out that with a given share of investments in, for example, human capital, the other one depends on it and the other parameters in the model. This affects consumption, which in turn also grows at the same rate as human capital. Therefore, choice of share of investments in human capital has an influence on the entire economy (Ayres, Van den Bergh, Lindenberger, & Warr, 2013) and determines the paths of growth of all macroeconomic variables, as in the fundamental works by Aghion and Howitt (1998), Barro and Sala-i-Martin (2003), Lucas (1988) and Romer (1990, 1994). This result is consistent with the standard growth theory that indicates that the entire path of growth is determined by the rate of technological progress (see, for example, Acemoglu, 2009; Barro & Sala-i-Martin, 2003). In the proposed model, technological progress is explicitly absent, but the accumulation of human capital reflects the growing productive skills and knowledge of the labour force, which can be interpreted in terms of advancements of technology.⁹

We also analyse how changes in the parameters of the model affect the size of output. The conducted analysis brings us to the conclusion that technological progress in general leads to an increase in the output, which is consistent with the results obtained by, for example, Grimaud and Rougé (2003, 2005), Pérez-Barahona (2011) and Van der Zwaan et al. (2002), but changes in the technology of production (which affect the elasticities of both productions with respect to the given factors) have an ambiguous effect on the output.

As in Hassler et al. (2016) and Pérez-Barahona (2011), we derive explicit paths of growth of all macroeconomic variables, which may be useful when attempting to forecast. The growth of human capital determines the growth of all other variables. Therefore, their paths of growth are strongly associated with the path of growth of human capital, which also is a standard result (see, for example, Aghion & Howitt, 1998; Barro & Sala-i-Martin, 2003; Lucas, 1988; Romer, 1990, 1994). We also obtain an optimal rule of accumulation: the

share of investments in human capital that maximises consumption. The accumulation of human capital directly affects production (in the form of the *AK* relationship), but on the other hand, a higher share of investment reduces consumption. These two effects allow for the establishment of an optimal size of investments in human capital. This accumulation not only maximises consumption, but also ensures the balanced growth of all other variables, to produce enough energy to power all available physical capital as is necessary to maximise the output (Stern, 2011; Stern & Cleveland, 2004).

The analysis of the presented model clearly shows that it is possible to achieve a balanced, sustainable path of growth for each basic macroeconomic variable, even when complementarity between some of the production factors is assumed. Empirical studies show that energy is more likely to be a complement to physical capital; therefore, unlike many other studies, we do not assume substitutability between all the factors of production, only between physical capital and human capital in both sectors of production. The presented model may be treated as a benchmark for more complex studies.

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Notes

1. Only in a few papers, mostly considering natural resources as a source of energy, is substitutability between produced energy and physical capital not present. These papers include Amigues et al. (2004), Costanza and Daly (1992), Daly (1997), Daly (1999), Georgescu-Roegen (1979), Malaczewski (2016) and Van Zon and Yetkiner (2003).
2. For comparison, in Europe, according to the data for 2010, the share of domestic energy consumption in total energy consumption was c.a. 27%.
3. This assumption is quite standard and widely used in these kinds of economic growth models, see Acemoglu (2009).
4. We do not consider labour or population growth explicitly; therefore, all variables may be considered as in per capita units.
5. Among many others, Fröling (2011) and Hung Nguyen and Nguyen Van (2008).
6. We note that there are some arguments that an increase in energy efficiency creates a rise in the rate of economic growth and, as a consequence, energy demand, see Howarth (1997) and Madlener and Alcott (2009).
7. We denote by \dot{X} the first derivative with respect to time, therefore $\frac{dX}{dt} = \dot{X}$.
8. We assume that the economy decides to not have any stock of idle capital. Therefore, enough energy to power all existing physical capital is produced.
9. This approach to human capital is also consistent with the standard growth theory; see Acemoglu (2009), Barro and Sala-i-Martin (2003), Lucas (1988) and Romer (1990).

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