

Simple analytical models for the calculation of the electric field radiated by the base station antenna

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SUMMARY

This paper outlines four different models for the assessment of the electric field radiated by the base station antenna system, thus featuring the Free Space Model (FS), Perfect Ground Model (PG), Fresnel Reflection Coefficient Model (RC) and Modified Image Theory Model (MIT). The results obtained with these approaches are compared to the results computed via Numerical Electromagnetics Code (NEC). Several variables have been varied to examine the accuracy of each calculation model. The calculations have been undertaken for the far field only. As many European countries perform the field assessment using the FS model, the final goal of this work is to come up with the most convenient model for the field calculation.

KEY WORDS: *electric field strength estimation; free space; perfect ground; modified image theory method; Fresnel equation; far field area.*

1. INTRODUCTION

The presence of high frequency (HF) electromagnetic fields in the environment due to base station antenna systems has caused a continuous public concern regarding possible radiation hazard. Namely, at HF exposures the human body may absorb a significant amount of the radiated energy, as the dimensions of organs are comparable to the wavelength of the incident field and the dominant biological effect is the tissue heating [1–2]. The problem is by itself twofold including: first, the assessment of the external field (incident field dosimetry), and second, the rate of power deposition in tissue due to the exposure to HF radiation and the related temperature distribution within the body has to be determined.

This paper is the sequel to two previous articles of the authors [3–4] and deals with the simple analytical relations for the incident field dosimetry based on the different schemes of ray tracing algorithm (arising from the geometrical optics approach [5, 6] implementation. Four

different models for the assessment of the field radiated by the base station antenna are presented; the antenna insulated in Free Space (FS), the antenna above Perfect Ground (PG) and the antenna above a lossy half space. The half-space model uses the reflection coefficient stemming from the Snell's law and Fresnel's equation [7-8] but also from the MIT approximation [9-10]. The paper is organized as follows: Mathematical Model is presented in Section II, and it is followed by a number of illustrative computational results. Finally, some concluding remarks are given. The results obtained by three different models are compared to the results computed via Numerical Electromagnetics Code (NEC) [11]. Several variables have been varied to examine the accuracy of a given approximation. All calculations have been carried out for the far field zone.

2. MATHEMATICAL MODELS FOR INCIDENT DOSIMETRY OF THE BASE STATION ANTENNAS

2.1 FREE SPACE APPROXIMATION

The Free Space approximation neglects effects due to reflections and scattering, thus taking into account only the incident field, Figure 1.

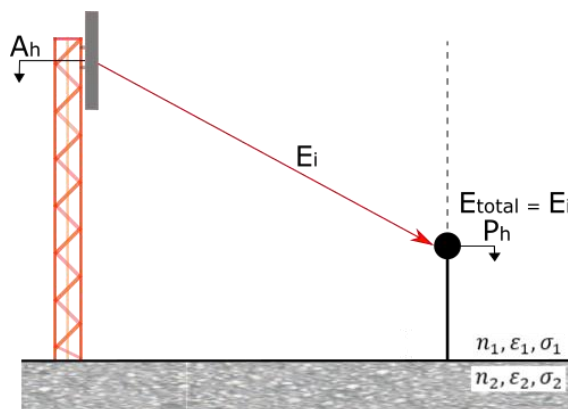


Fig. 1 Free Space Approximation

As documented in [12] the electric field at some point in free space in the far-field zone is given by:

$$E_{total} = E_i = \frac{\sqrt{30 * N * P * 10^{10} \frac{G_i}{d_i}}}{d_i} \quad (V/m) \quad (1)$$

where N is number of channels, P is radiated power (W), d_i is the distance between the antenna and the calculation point (m) and G_i is antenna gain of incident field (dB).

Formula (1) gives the absolute value of the total electric field according to the assumptions given at the beginning of the section.

2.2 PERFECT GROUND APPROXIMATION

A simple way to take into consideration the effect of the reflected wave on total electric field is to assume the ground to be perfectly conducting, Figure 2.

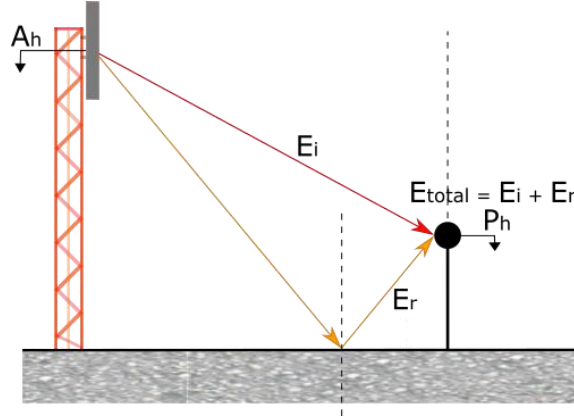


Fig. 2 Perfect Ground Approximation

Therefore, the total electric field is:

$$\vec{E}_{total} = \vec{E}_i + \vec{E}_{r_PG} \quad (V/m) \quad (2)$$

where:

$$E_{r_PG} = \frac{\sqrt{30 * N * P * 10^{10} \frac{Gr}{dr}}}{dr} \quad (V/m) \quad (3)$$

with d_r being the distance that wave travels from the antenna to the calculation point and G_r is the antenna gain.

2.3 HALF-SPACE MODEL FEATURING FRESNEL'S APPROACH

The total field composed from both, incident and reflected field, can also be calculated using RC or MIT method, respectively, Figure 3.

Reflection coefficient arising from RC approach is given by:

$$\Gamma_{Fresnel} = \frac{Z_1 * \cos \alpha - Z_2 * \cos \beta}{Z_1 * \cos \alpha + Z_2 * \cos \beta} \quad (4)$$

where Z_1 is the free space impedance (377Ω) and Z_2 is given by:

$$Z_2 = \sqrt{\frac{j\mu_0 \cdot \omega}{\sigma_2 + j\varepsilon_0\varepsilon_2 \cdot \omega}} \quad (5)$$

The angle β stems from Snell's law, as follows:

$$n_1 * \sin \alpha = n_2 * \sin \beta \quad (6)$$

where n_2 is given by:

$$n_2 = \sqrt{\varepsilon_2} \quad (7)$$

Thus, the reflected electric field is:

$$\vec{E}_{r_Fresnel} = \Gamma_{Fresnel} * \vec{E}_{r_PG} \quad (V/m) \quad (8)$$

and the total electric field strength is:

$$\vec{E}_{total} = \vec{E}_i + \vec{E}_{r_Fresnel} \quad (V/m) \quad (9)$$

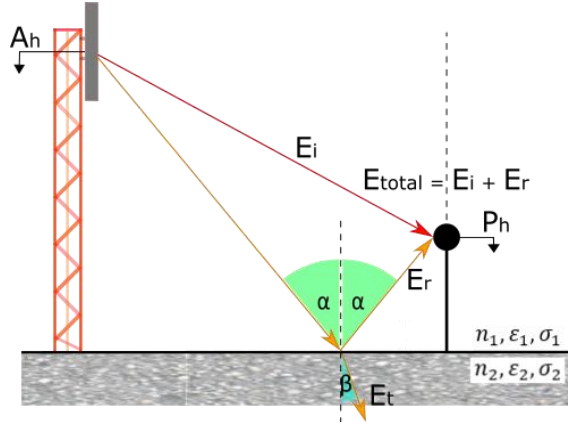


Fig. 3 Antenna above a lossy half space

2.4 HALF-SPACE MODEL FEATURING MIT APPROACH

Reflection coefficient arising from MIT approach is given by:

$$\Gamma_{MIT} = \frac{\epsilon_{eff} - \epsilon_0}{\epsilon_{eff} + \epsilon_0} \quad (10)$$

where:

$$\epsilon_{eff} = \epsilon_r \epsilon_0 + \frac{\sigma}{j\omega} \quad (F/m) \quad (11)$$

Thus, the reflected electric field is:

$$\vec{E}_{r_MIT} = \Gamma_{MIT} * \vec{E}_{r_PG} \quad (V/m) \quad (12)$$

Therefore, the total electric field is:

$$\vec{E}_{total} = \vec{E}_i + \vec{E}_{r_MIT} \quad (V/m) \quad (13)$$

3. THE RESULTS

3.1 ANTENNA CONFIGURATION

Antenna configuration of interest is 2 m long and consists of 8 dipoles and metal grid with the total radiated power of 100 W. The operating frequency is 936.8 MHz. Simulations are carried out by varying specific ground conductivity σ_2 , relative permittivity ϵ_2 and the antenna height A_h . Values of these variables used in simulations are shown in Table 1. The theoretical antenna pattern is shown in Figure 4.

Table 1 Values of variables

variable	1	2	3	4	5	6
σ_2 (S/m)	10^{-3}	0.01	0.1	10	10^3	10^6
ϵ_2	10	40	80	110	150	-
A_h (m)	10	20	30	40	50	-

Other variables used in simulations are set to be constant, i.e.: radiated power ($P = 100$ W), channel number ($N = 1$), calculation point height ($P_h = 2$ m).

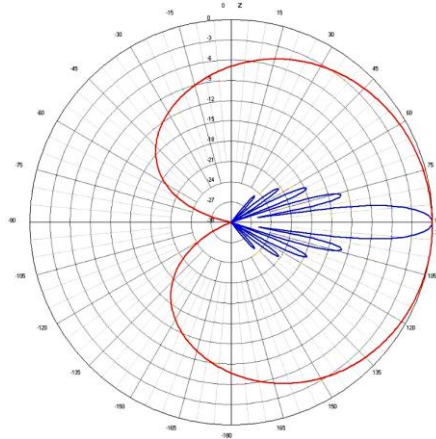


Fig. 4 Perfect Ground Approximation

As already stated, all calculations have been carried out for the far field zone [13]:

$$d \geq 2 * D^2 / \lambda \quad (V/m) \tag{14}$$

where d is horizontal distance from the antenna, D is the largest antenna dimension and λ is wavelength. Calculating with $D = 2$ m and $\lambda = 0.32$ m, far field area is related to distances from 25 m. Therefore, the electric field is calculated for distances between 25 m to 400 m away from antenna with the step of 1 m (total 376 data per calculation).

3.2 VARIATIONS OF THE GROUND CONDUCTIVITY

The differences in obtained results among the proposed methods with respect to the conductivity variations are presented in Tables 2 and 3, and Figures 5 and 6.

Table 2 The highest absolute differences between each method and NEC simulation in mV/m

σ_2 (S/m)	FS-NEC	PG-NEC	RC-NEC	MIT-NEC
0.001	70,87	198,11	51,32	174,27
	36,99%	220,19%	53,48%	193,69%
0.01	70,99	198,07	51,42	174,25
	37,02%	220,06%	53,60%	193,21%
0.1	72,00	197,59	52,91	174,51
	37,36%	217,61%	55,16%	192,43%
10	94,74	110,89	128,75	110,42
	42,37%	66,50%	89,87%	66,22%
1000	130,55	124,54	126,98	124,54
	49,42%	77,61%	79,13%	77,61%
10^6	140,06	126,50	126,58	126,50
	47,12%	77,88%	77,93%	77,88%

Table 3 Percentage of results for the given method within distortion up to 25% of NEC results

σ_2 (S/m)	FS-NEC	PG-NEC	RC-NEC	MIT-NEC
0.001	63%	29%	93%	32%
0.01	62%	29%	93%	32%
0.1	62%	30%	93%	32%
10	44%	48%	93%	48%
1000	39%	93%	93%	93%
10 ⁶	37%	93%	93%	93%

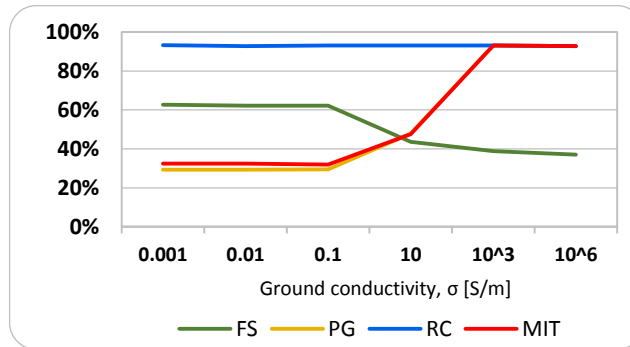


Fig. 5 Percentage of results for the given method within distortion up to 25% of NEC results for different σ_2

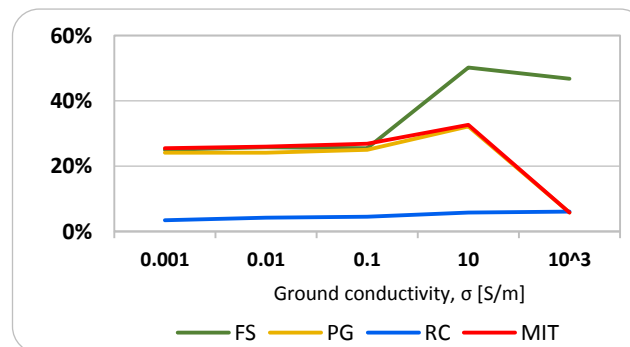


Fig. 6 Percentage of results for the given method with distortion between 25% and 50% of NEC results for different σ_2

The increase of ground conductivity has negligible influence on total electric field strength. The RC approximation appears to be optimal while FS method, on the other hand, provides more accurate results than PG or MIT method for conductivity values less than 10 S/m. Increasing the ground conductivity to 1000 S/m or even 10⁶ S/m, PG, MIT and RC models provides similar results, while FS method fails to ensure valid results.

3.3 VARIATIONS OF THE RELATIVE PERMITTIVITY

The differences in simulated results among the proposed methods for the permittivity variations are presented in Tables 4 and 5 and Figures 7 and 8.

Table 4 The highest absolute differences between each method and NEC simulation in mV/m

ϵ_2	FS-NEC	PG-NEC	RC-NEC	MIT-NEC
10	70,99	198,07	51,42	174,25
	37,02%	220,06%	53,60%	193,21%
40	52,73	163,15	100,26	156,76
	32,13%	130,85%	84,07%	125,72%
80	66,64	141,63	115,95	138,39
	63,57%	97,27%	89,70%	95,05%
110	72,46	131,26	120,00	128,90
	65,49%	84,16%	90,04%	82,65%
150	77,66	121,05	122,41	119,32
	67,04%	73,01%	89,46%	71,96%

Table 5 Percentage of results for the given method within distortion up to 25% of NEC results

ϵ_2	FS-NEC	PG-NEC	RC-NEC	MIT-NEC
10	62%	29%	93%	32%
40	86%	30%	94%	32%
80	88%	32%	93%	31%
110	88%	32%	93%	34%
150	86%	35%	93%	36%

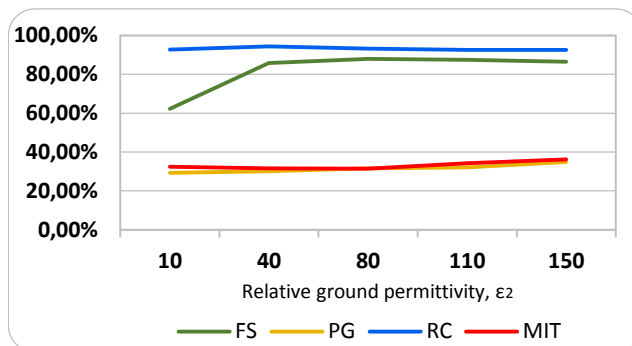


Fig. 7 Percentage of results for the given method within distortion up to 25% of NEC results for different ϵ_2

RC method is considered to be the most accurate approach with approximately 93% of data with distortion up to 25% of NEC results. FS method provides better results as relative permittivity values increase above 10. PG and MIT methods show similar results but also suffer from the low accuracy with maximum 36% of data with distortion up to 25% of NEC results.

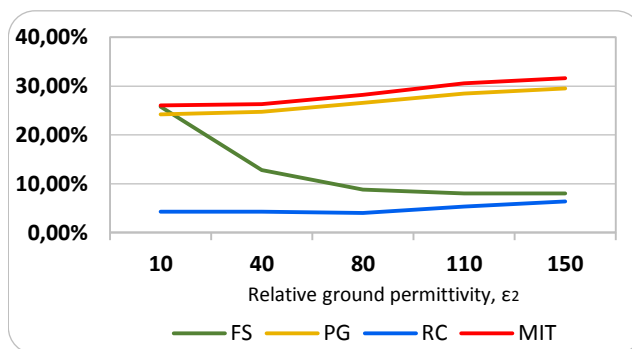


Fig. 8 Percentage of results for the given method with distortion between 25% and 50% of NEC results for different ϵ_2

3.4 VARIATIONS OF THE ANTENNA HEIGHT

The differences in simulated results among the proposed methods for the antenna height variations are presented in Tables 6 and 7, as well in Figures 9 and 10, respectively.

The higher the antenna height above the ground, the lower are differences in the results obtained by NEC and FS simulations. PG and MIT methods again show the worst approximation with the maximum of 36% of data with distortion up to 25% of NEC results.

Although differences between the results obtained via RC model and NEC increase with antenna height, RC method still remains the most accurate approximation.

Table 6 The highest absolute differences between each method and NEC simulation in mV/m

A_h (m)	FS-NEC	PG-NEC	RC-NEC	MIT-NEC
10	151,52	346,48	90,17	306,73
	41,12%	153,31%	29,59%	85,04%
20	70,99	198,07	51,42	174,25
	37,02%	220,06%	53,60%	193,21%
30	47,28	137,63	34,98	121,21
	86,52%	251,55%	37,85%	221,54%
40	23,82	74,19	29,28	63,13
	26,47%	107,64%	40,74%	72,19%
50	16,40	60,20	24,85	49,80
	26,04%	108,46%	43,42%	89,73%

Table 7 Percentage of results for the given method within distortion up to 25% of NEC results

A_h (m)	FS-NEC	PG-NEC	RC-NEC	MIT-NEC
10	31%	14%	98%	16%
20	62%	29%	93%	32%
30	73%	27%	89%	29%
40	81%	34%	88%	39%
50	86%	27%	85%	38%

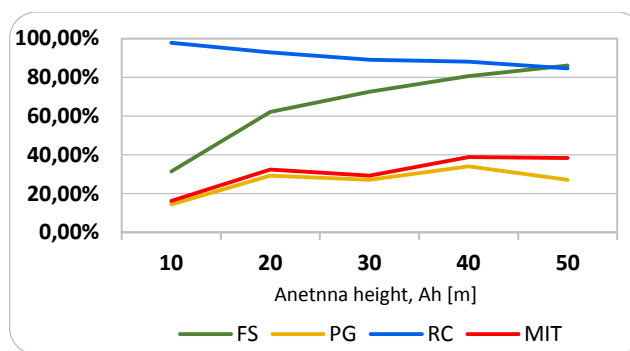


Fig. 9 Percentage of results for the given method within distortion up to 25% of NEC results for different A_h

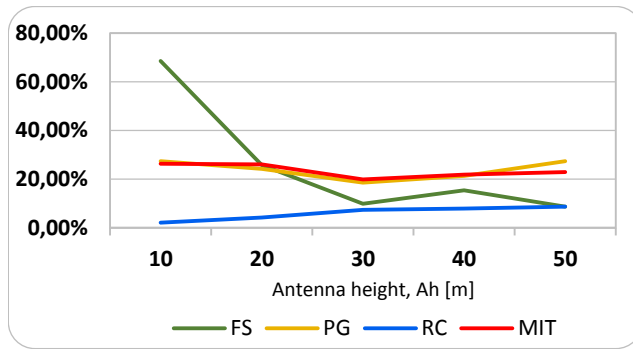


Fig. 10 Percentage of results for the given method with distortion between 25% and 50% of NEC results for different A_h

3.5 RADIATED FIELD VS. DISTANCE

According to the presented results, it can be seen that RC approach shows the smallest distortions compared to NEC simulation (Figures 11 and 12). The PG and MIT methods provide similar results with a satisfactory approximation only at high ground conductance values (Figure 13). For all the other values and all the other calculations these two methods show the worst approximation of all. FS method provides acceptable approximation for higher relative permittivity values and for higher A_h values (above 30 m). For those specific cases FS method is similar to RC model and NEC, respectively (Figure 14).

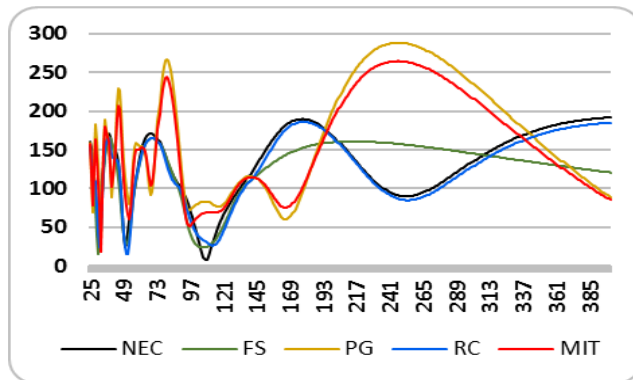


Fig. 11 Electric field computed via different approaches compared to NEC simulation in mV/m at specific distance from antenna ($\sigma_2 = 1000 \text{ S/m}$, $\epsilon_2 = 110$ and $A_h = 40 \text{ m}$)

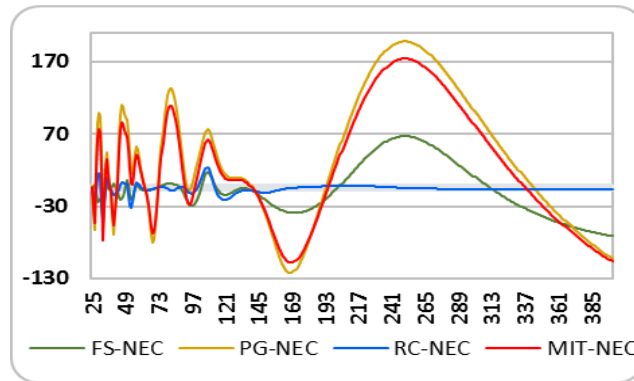


Fig. 12 Differences in electric field strength between the given method and NEC simulation in mV/m at specific distance from antenna ($\sigma_2 = 0.01$ S/m, $\epsilon_2 = 10$ and $A_h = 20$ m)

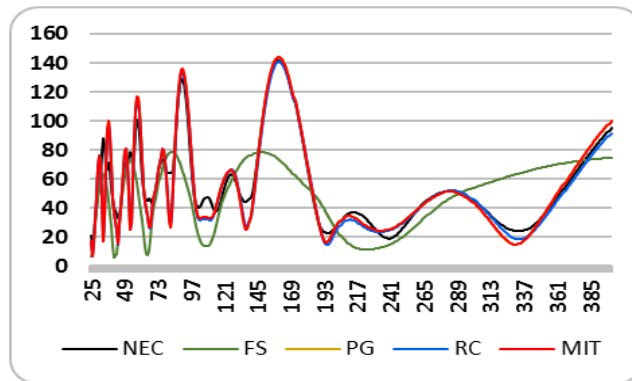


Fig. 13 Electric field computed via different approaches compared to NEC simulation in mV/m at specific distance from antenna ($\sigma = 1000$ S/m, $\epsilon_2 = 110$ and $A_h = 40$ m)

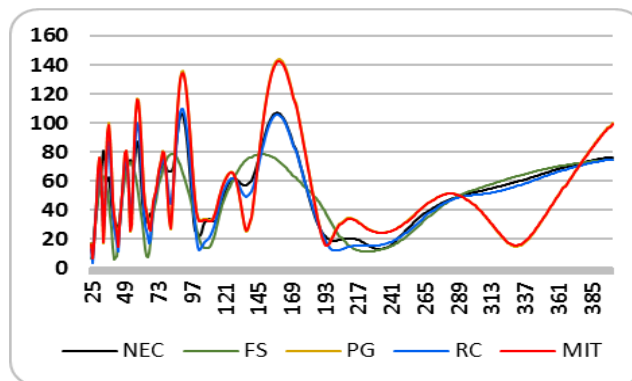


Fig. 14 Electric field computed via different approaches compared to NEC simulation in mV/m at specific distance from antenna ($\sigma = 0.01$ S/m, $\epsilon_2 = 110$ and $A_h = 40$ m)

4. CONCLUSION

The paper deals with four different models for the assessment of the electric field radiated by the base station antenna system: the antenna insulated in Free Space (FS), the antenna above Perfect Ground (PG) and the antenna above a lossy half space. The half-space is taken into account via Fresnel Reflection Coefficient (RC) and Modified Image Theory Method (MIT), respectively. The results obtained by these models are compared to the results obtained by NEC as benchmark.

The PG and MIT methods are considered to be satisfactory approximations only for high ground conductivity values and provide similar results. The FS method provides a good approximation for higher relative permittivity values (above 10) and for higher Ah values as well. Finally, RC approach provides rather satisfactory results in all cases.

5. APPENDIX A

The power flux P (radiated power of an antenna) is given by integral:

$$\oint \vec{E} \times \vec{H} \, d\vec{S} = \oint S d\vec{S} \quad (\text{A1})$$

where product $\vec{E} \times \vec{H}$ presents the power density, i.e. Poynting vector S .

Poynting vector in the free space for root-mean-square values can be written, as follows:

$$S = \frac{E^2}{Z_0} = \frac{E^2}{120\pi} \quad (\text{A2})$$

Combining (A.1) and (A.2) for the case of an isotropic radiator yields:

$$S = \frac{E^2}{120\pi} = \frac{P}{4\pi * r^2} \quad (\text{A3})$$

and the electric field strength is given by:

$$E = \frac{\sqrt{30 * P}}{r} \quad (\text{A4})$$

Taking into account the number of antenna channels N , as well as horizontal and vertical antenna gain G follows:

$$E = \frac{\sqrt{30 * N * P * 10^{\frac{G}{10}}}}{r} \quad (\text{A5})$$

Note that expression (A.5) is related to directional radiators.

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