A GROWTH THEORY BASED ON WALRASIAN GENERAL EQUILIBRIUM, SOLOW-UZAWA GROWTH, AND HECKSCHER-OHLIN TRADE THEORIES

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ABSTRACT

The purpose of this study is to analyse the role of preferences and technological differences between countries in determining dynamics of capital accumulation, wealth and income distribution within countries and between countries, and patterns of trade in a dynamic general equilibrium framework. The model is built by integrating Walrasian general equilibrium, neoclassical growth, and H-O international trade theories. The model is built for any number of countries and each country is composed of three production sectors and heterogeneous households. The national growth mechanism is the same as that in neoclassical growth theory. Labour and capital distributions among sectors and among countries are determined under perfect competition and free trade. The model synthesizes the well-known H-O and the Oniki-Uzawa trade models, Solow-Uzawa neoclassical growth theory, and Walrasian general equilibrium theory with Zhang's utility function. We simulated the model with three national economies and with two groups of households for each country. We identified the existence of equilibrium points and plot motion of the dynamic system. We also conducted a comparative dynamic analysis.

KEY WORDS

CLASSIFICATION

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INTRODUCTION

It is a challenging question in trade theory to study the role of preferences and technological differences between countries in determining dynamics of capital accumulation, wealth and income distribution within countries and between countries, and patterns of trade in a dynamic general equilibrium framework. Traditional economic theories do not offer an integrated framework to address these issues on the basis of microeconomic foundation. This study makes a contribution to the literature of trade theory by integrating Walrasian general equilibrium, neoclassical growth, and Heckscher-Ohlin (H-O) international trade theories with Zhang's approach to household behaviour.

The H-O model is a key model in formal international theory [1, 2]. The standard H-O model is for a two-countries global economy. The two economies have the same technology for producing two goods using labour and capital inputs under perfect competition and constant returns to scale. Input factors are mobile between sectors in each economy, but immobile between countries. There is no international borrowing and lending. Moreover, as Chen [3; pp.923-924] observes, "Yet explaining trade in terms of differences in preferences is no longer in the spirit of the Heckscher-Ohlin model in which trade arises because of differences in relative factor proportions." There are other models which attempt to generalize or extend the H-O model in different directions [4]. But there is no model for any number of economies with endogenous wealth in the H-O framework. This study generalizes the H-O theory in the sense that it is Ricardian as it allows cross-country differences in technology and labour productivity. Not only generalizing the H-O model and the Ricardian static approach, we also introduce endogenous wealth as a main engine of global economic growth. We model wealth accumulation on the basis of the Oniki-Uzawa model (for instance [5]). The Oniki-Uzawa model describes an economy with two countries and two goods with fixing saving rates. This study is also influenced by the Walrasian general equilibrium theory (see e.g. [6-9]). The theory is mainly concerned with market equilibrium with economic mechanisms of production, consumption, and exchanges with heterogeneous industries and households. This study is Walrasian in the sense that for given levels of wealth there are competitive market equilibriums with heterogeneous industries and households. Moreover, we deviate from traditional approaches in modelling behaviour of households. The model in this study is an integration of the growth models by Zhang [10, 11]. This article is organized as follows. Section 2 develops a global growth model of multi-national economies and heterogeneous households. Section 3 studies dynamic properties of the model and calibrates the model. Section 4 carries out comparative dynamic analysis. Section 5 concludes the study. The results of Section 3 are proved in the Appendix.

THE MODEL

The model in this study is influenced by some typical dynamic H-O models and the neoclassical trade growth theory. It is an integration of the two models by Zhang [10, 11]. We consider a world economy composed of any number of national economies, indexed by $j = 1, \ldots, J$. Different from the standard H-O model which studies a world economy composed of only two countries and two goods [4], this study assumes that each national economy produces three goods. Each country produces a homogeneous capital consumer good which can be used as capital and consumption. This sector is called industrial sector, which is similar to the homogenous sector in the traditional neoclassical trade growth model [12]. Capital good is freely mobile between national economies and sectors. No tariff is charged on any good. Services are supplied domestically and are not internationally tradable. Domestic households consume services. Each country is also specified at producing a good called global commodity. A global commodity is possibly consumed by all economies but can be

supplied only by the single economy. Global commodities are pure consumption goods. Production sectors are neoclassical, which also implies that all earnings of firms are distributed in the form of payments to factors of production. Following the traditional H-O model, immobile labour between economies is assumed. Wealth is owned by households and saving is undertaken only by households.

The population of country j is grouped into J_n groups, indexed with subscripts n, $n = 1, \ldots, J_n$. Group (j, n)'s population is constant and denoted by $\bar{N}_{jn}$. Let N_j stands for country j's flow of labour services for production. We have

$$N_j = \sum_{n=1}^{J_n} h_{jn}\, \bar{N}_{jn}\,, \tag{1}$$

where h_{jn} is the fixed level of human capital of group (j, n). The national labour force is fully employed by the three sectors. We use subscripts i, s and m to denote industrial sector, service sector, and country j's global commodity sector, respectively. We use (j, q) to index sector $q = i, s, m$ in country j Variable $p_{jq}(t)$ denotes the price of product (j, q) Wages differ between countries and equal within each economy. Perfect competition implies that labour and capital earn their marginal products, and firms earn zero profits. Let $w_j(t)$ and $r(t)$ denote the wage rate in the j-th country, and rate of interest, respectively. Capital depreciation rates vary between countries. The capital employed by country j depreciates at a constant rate δ_{jk}. We use $N_{jq}(t)$ and $K_{jq}(t)$ to stand for the labour force and capital stocks employed by sector q in country j Variable $F_{jq}(t)$ represents the output level of sector q in country j Variables $K_j(t)$ and $\bar{K}_j(t)$ represent respectively the capital stocks employed and the wealth owned by country j.

NEOCLASSICAL PRODUCTION FUNCTIONS

The production function of section q in country j is taken on the following form

$$F_{jq}(t) = A_{jq}\, K_{jq}^{\alpha_{jq}}(t) N_{jq}^{\beta_{jq}}(t), \quad A_{jq}, \ \alpha_{jq}, \ \beta_{jq} > 0, \quad \alpha_{jq} + \beta_{jq} = 1, \tag{2}$$

where A_{jq}, α_{jq} and β_{jq} are positive parameters. Traditional H-O theory assumes variations in capital and labour endowments with identical technology between countries.

The marginal conditions

The rate of interest, wage rate, and prices are determined by markets. The marginal conditions imply

$$r(t) + \delta_{jk} = \frac{\alpha_{jq}\, p_{jq}(t) F_{jq}(t)}{K_{jq}(t)}, \quad w_j(t) = \frac{\beta_{jq}\, p_{jq}(t) F_{jq}(t)}{N_{jq}(t)}. \tag{3}$$

The current income and disposable income

This study uses Zhang's utility function to describe behaviour of households [4]. Let $\bar{k}_{jn}(t)$ stand for the wealth of household (j, n). Per household's current income from the interest payment $r(t)\, \bar{k}_{jn}(t)$ and the wage payment $w_{jn}(t) = h_{jn}(t)\, w_j(t)$ is

$$y_{jn}(t) = r(t)\bar{k}_{jn}(t) + h_{jn}\, w_j(t).$$

The per capita disposable income which is used for saving and consumption is

$$\hat{y}_{jn}(t) = y_{jn}(t) + \bar{k}_{jn}(t) = (1 + r(t))\bar{k}_{jn}(t) + h_{jn}\, w_j(t). \tag{4}$$

The budgets and utility functions

Let $c_{jnq}(t)$ stands for consumption level of consumer good q by household (j, n). We use $s_{jn}(t)$ to stand for the saving made at the current time by household (j, n). Household (j, n) is faced with the following budget constraint

$$c_{jni}(t) + p_{js}(t)c_{jns}(t) + s_{jn}(t) + \sum_{q=1}^{J} p_{qm}(t)c_{jnq}(t) = \hat{y}_{jn}(t).$$ (5)

We assume that consumers' utility function is a function of the consumption levels of goods, services and the saving as follows

$$U_{jn}(t) = c_{jni}^{\xi_{ji0}}(t) c_{jns}^{\chi_{jn0}}(t) s_{jn}^{\lambda_{jn0}}(t) \prod_{q=1}^{J} c_{jnq}^{\gamma_{0jnq}}(t), \quad \xi_{ji0}, \chi_{jn0}, \gamma_{0jnq}, \lambda_{jn0} > 0,$$ (6)

where ξ_{ji0} is called household (j, n)'s propensity to consume industrial goods, χ_{jn0} propensity to consume domestic service, γ_{jnq0} propensity to consume global commodity j and λ_{jn0} propensity to own wealth. Maximizing (6) subject to (5) yields

$$c_{jni}(t) = \xi_{jn}\hat{y}_{jn}(t), \, p_{js}(t)c_{jns}(t) = \chi_{jn}\hat{y}_{jn}(t), \, s_{jn}(t) = \lambda_{jn}\hat{y}_{jn}(t), \, p_{qm}(t)c_{jnq}(t) = \gamma_{jnq}\hat{y}_{jn}(t),$$ (7)

where

$$\xi_{jn} \equiv \rho_{jn}\xi_{0jn}, \quad \chi_{jn} \equiv \rho_{jn}\chi_{0jn}, \quad \lambda_{jn} \equiv \rho_{jn}\lambda_{0jn}, \quad \gamma_{jnq} \equiv \rho_{jn}\gamma_{jnq0},$$

$$\rho_{jn} \equiv \frac{1}{\xi_{0jn} + \gamma_{0jn} + \lambda_{0jn} + \sum_{v}^{J} \gamma_{0jnv}}.$$

Wealth accumulation

According to the definitions of $s_{jn}(t)$ and $\bar{k}_{jn}(t)$ wealth change of household (j, n) is saving minus dissaving. That is

$$\dot{\bar{k}}_{jn}(t) = s_{jn}(t) - \bar{k}_{jn}(t).$$ (8)

Factors being fully employed

We use $K_j(t)$ to stand for the capital stocks employed by country j. The capital stock employed by a country is fully employed by the three sectors. That is

$$K_{ji}(t) + K_{jm}(t) + K_{js}(t) = K_j(t).$$ (9)

The labour force is fully employed by the three sectors

$$N_{ji}(t) + N_{jm}(t) + N_{js}(t) = N_j.$$ (10)

Market clearing for services

The demand and supply of services balance in each national market

$$\sum_{n=1}^{J_n} c_{jns}(t)\bar{N}_{jn} = F_{js}(t).$$ (11)

Market clearing in global commodity markets

The demand and supply of tradable goods balance in global markets

$$\sum_{j=1}^{J} \sum_{n=1}^{J_n} c_{jnq}(t)\bar{N}_{jn} = F_{qm}(t).$$ (12)

Market clearing in capital markets

The global capital production is equal to the global net savings. That is

$$\sum_{j=1}^{J} \sum_{n=1}^{J_n} [s_{jn}(t) + c_{jn}(t) - \bar{k}_{jn}(t)]\bar{N}_{jn} + \sum_{j=1}^{J} \delta_{kj} K_j(t) = \sum_{q=1}^{J} F_{qi}(t).$$ (13)

Wealth balance

The wealth owned by the global population is equal to the total global wealth

$$\sum_{j=1}^{J} \sum_{n=1}^{J_n} \bar{k}_{jn}(t) \bar{N}_{jn} = \sum_{j=1}^{J} K_j(t) = K(t). \tag{14}$$

We built a generalized dynamic H-O model with endogenous wealth accumulation. Through the construction process we see that it is structurally a unification of the Walrasian general equilibrium, neoclassical growth theory, and the H-O theory with Zhang's approach to the household behaviour.

THE DYNAMICS AND EQUILIBRIUM

The previous section developed a global growth model with any number of national economies. As the system is nonlinear and is of high dimension, it is difficult to obtain analytical properties of the system. Before calibrating the model, we show that dynamics of J national economies with $\sum_{n=1}^{J} J_n$ groups of people can be expressed by $\sum_{n=1}^{J} J_n$ differential equations. We first introduce variables

$$z_1 \equiv \frac{r + \delta_{1k}}{w_1} \ , \quad \left\{ \bar{k}_{jn} \right\} \equiv \left(\bar{k}_{12}, \dots, \bar{k}_{JJ_n} \right).$$

The following lemma shows how to follow the dynamics of global economic growth with initial conditions.

Lemma

The motion of the global economy with J national economies and $\sum_{n=1}^{J} J_n$ groups of people is described by the following $\sum_{n=1}^{J} J_n$ differential equations with $\sum_{n=1}^{J} J_n$ variables

$$\dot{z}_1(t) = \widetilde{\Lambda}_1(z_1(t), \{\bar{k}_{jn}(t)\}), \ \dot{\bar{k}}_{jn}(t) = \widetilde{\Lambda}_{jn}(z_1(t), \{\bar{k}_{jn}(t)\}), j = 1, 2, \dots, J, \ n = 1, \dots, J_n, (j, n) \neq (1, 1), \tag{15}$$

where $\widetilde{\Lambda}_1(t)$ and $\widetilde{\Lambda}_j(t)$ are functions of $z_1(t)$ and $\{\bar{k}_j(t)\}$ defined in the appendix. The values of the other variables are given as functions of $z_1(t)$ and $\{\bar{k}_j(t)\}$ at any point in time by the following procedure: $r(t)$ by (A2) $\rightarrow w_j(t)$ by (A2) $\rightarrow w_{jn}(t) = h_{jn}(t) \cdot w_j(t)$ by (A2) $\rightarrow z_j(t)$ by (A3) $\rightarrow p_{jq}(t)$ by (A4) $\rightarrow \bar{k}_{11}(t)$ by (A17) $\rightarrow \hat{y}_{jq}(t)$ by (A5) $\rightarrow K(t)$ by (A15) $\rightarrow K_j(t)$ by (A14) $\rightarrow N_{jm}(t)$ by (A12) $\rightarrow N_{ji}(t)$ by (A9) $\rightarrow N_{js}(t)$ by (A8) $\rightarrow K_{jq}(t)$ by (A1) $\rightarrow F_{jq}(t)$ by (2) $\rightarrow c_{jni}(t), \ c_{jns}(t), \ c_{jnm}(t)$ and $s_{jn}(t)$ by (7).

We now follow the motion of the global economy with the computational procedure given in the lemma. We consider a world economy with 3 national economies and each national economy with 2 groups. For simulation, we specify values of the parameters as follows:

$$\delta_{1k} = 0,05, \ \ \delta_{2k} = 0,055, \ \ \delta_{3k} = 0,06,$$

$$\begin{pmatrix} \bar{N}_{11} \\ \bar{N}_{12} \\ \bar{N}_{21} \\ \bar{N}_{22} \\ \bar{N}_{31} \\ \bar{N}_{32} \end{pmatrix} = \begin{pmatrix} 30 \\ 50 \\ 20 \\ 80 \\ 20 \\ 80 \end{pmatrix}, \begin{pmatrix} h_{11} \\ h_{12} \\ h_{21} \\ h_{22} \\ h_{31} \\ h_{32} \end{pmatrix} = \begin{pmatrix} 10 \\ 8 \\ 8 \\ 6 \\ 6 \\ 4 \end{pmatrix}, \begin{pmatrix} A_{1i} \\ A_{2i} \\ A_{3i} \\ \alpha_{1i} \\ \alpha_{2i} \\ \alpha_{3i} \end{pmatrix} = \begin{pmatrix} 1,4 \\ 1,3 \\ 1,2 \\ 0,32 \\ 0,31 \\ 0,3 \end{pmatrix}, \begin{pmatrix} A_{1s} \\ A_{2s} \\ A_{3s} \\ \alpha_{1s} \\ \alpha_{2s} \\ \alpha_{3s} \end{pmatrix} = \begin{pmatrix} 1 \\ 0,9 \\ 0,8 \\ 0,3 \\ 0,32 \\ 0,32 \end{pmatrix}, \begin{pmatrix} A_{1m} \\ A_{2m} \\ A_{3m} \\ \alpha_{1m} \\ \alpha_{2m} \\ \alpha_{3m} \end{pmatrix} = \begin{pmatrix} 1,3 \\ 1,2 \\ 1,1 \\ 0,34 \\ 0,3 \\ 0,31 \end{pmatrix},$$

$$
\begin{pmatrix} \xi_{110} \\ \chi_{110} \\ \lambda_{110} \\ \gamma_{1101} \\ \gamma_{1102} \\ \gamma_{1103} \end{pmatrix} = \begin{pmatrix} 0,05 \\ 0,07 \\ 0,7 \\ 0,02 \\ 0,01 \\ 0,01 \end{pmatrix}, \quad
\begin{pmatrix} \xi_{120} \\ \chi_{120} \\ \lambda_{120} \\ \gamma_{1201} \\ \gamma_{1202} \\ \gamma_{1203} \end{pmatrix} = \begin{pmatrix} 0,05 \\ 0,08 \\ 0,65 \\ 0,02 \\ 0,01 \\ 0,01 \end{pmatrix}, \quad
\begin{pmatrix} \xi_{210} \\ \chi_{210} \\ \lambda_{210} \\ \gamma_{2101} \\ \gamma_{2102} \\ \gamma_{2103} \end{pmatrix} = \begin{pmatrix} 0,05 \\ 0,08 \\ 0,65 \\ 0,01 \\ 0,02 \\ 0,01 \end{pmatrix}, \quad
\begin{pmatrix} \xi_{220} \\ \chi_{220} \\ \lambda_{220} \\ \gamma_{2201} \\ \gamma_{2202} \\ \gamma_{2203} \end{pmatrix} = \begin{pmatrix} 0,05 \\ 0,085 \\ 0,6 \\ 0,01 \\ 0,02 \\ 0,01 \end{pmatrix},
$$

$$
\begin{pmatrix} \xi_{310} \\ \chi_{310} \\ \lambda_{310} \\ \gamma_{3101} \\ \gamma_{3102} \\ \gamma_{3103} \end{pmatrix} = \begin{pmatrix} 0,05 \\ 0,09 \\ 0,6 \\ 0,01 \\ 0,01 \\ 0,02 \end{pmatrix}, \quad
\begin{pmatrix} \xi_{320} \\ \chi_{320} \\ \lambda_{320} \\ \gamma_{3201} \\ \gamma_{3202} \\ \gamma_{3203} \end{pmatrix} = \begin{pmatrix} 0,05 \\ 0,095 \\ 0,55 \\ 0,01 \\ 0,01 \\ 0,02 \end{pmatrix}.
$$

Country 1 and 2's populations are, respectively, 10 and 20. We consider equal depreciation rates of physical capital within an economy and differ between economies. The total factor productivities are different between economies. Countries are also different in preferences. We specify the initial conditions as follows:

$$
z_1(0) = 0,042, \; \bar{k}_{12}(0) = 68, \; \bar{k}_{21}(0) = 65, \; \bar{k}_{22}(0) = 36, \; \bar{k}_{31}(0) = 30, \; \bar{k}_{32}(0) = 24.
$$

The motion of the system is given in Figure 1. In the figure the national incomes and the global income are defined as follows:

$$
Y_j(t) \equiv F_{ji}(t) + p_{js}(t)F_{js}(t) + p_j(t)F_{jj}(t), \quad Y(t) \equiv Y_1(t) + Y_2(t) + Y_3(t), \quad E_j(t) \equiv \bar{K}_j(t) - K_j(t).
$$

From Figure 1 we observe that the system becomes stationary in the long term. Following the procedure in the lemma, we calculate the equilibrium values of the variables as follows
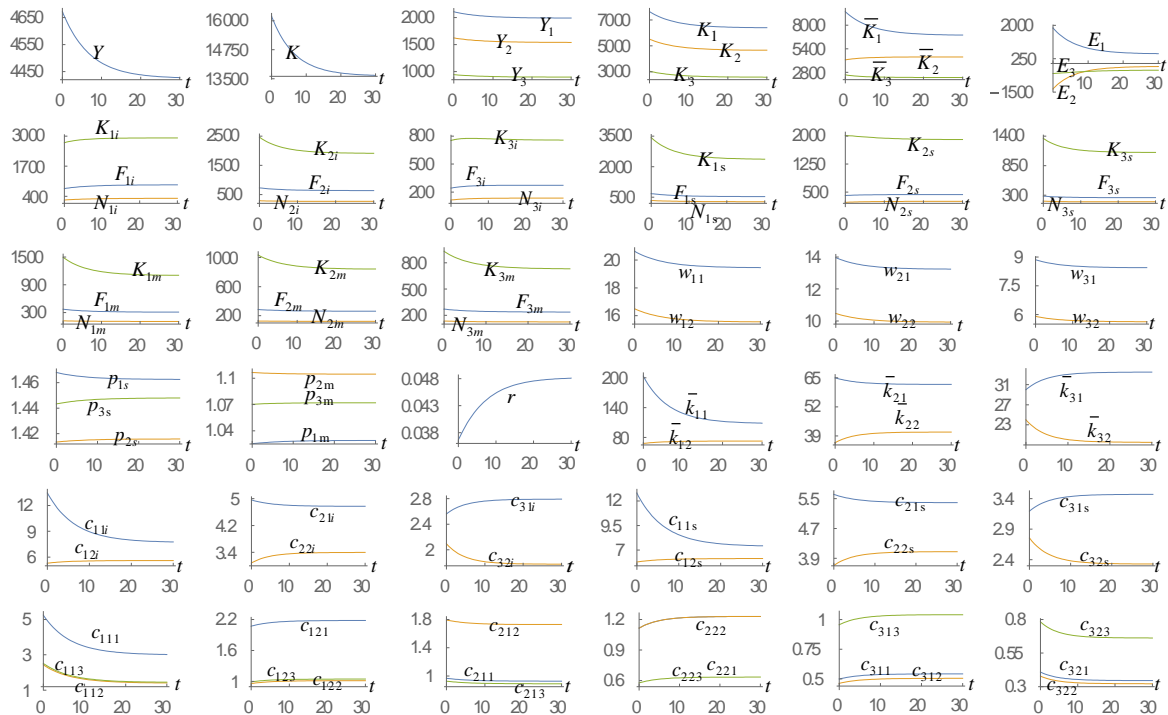

Figure 1. The motion of the global economy.

$$
\begin{pmatrix} Y \\ K \\ r \\ Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} 4425 \\ 13614 \\ 0.0048 \\ 1987 \\ 1539 \\ 898 \end{pmatrix},\ \begin{pmatrix} K_1 \\ K_2 \\ K_3 \\ \overline{K}_1 \\ \overline{K}_2 \\ \overline{K}_3 \end{pmatrix} = \begin{pmatrix} 6482 \\ 4652 \\ 2580 \\ 6881 \\ 4502 \\ 2232 \end{pmatrix},\ \begin{pmatrix} E_1 \\ E_3 \\ E_3 \\ F_1 \\ F_2 \\ F_3 \end{pmatrix} = \begin{pmatrix} 498 \\ -150 \\ -349 \\ 898 \\ 635 \\ 273 \end{pmatrix},\ \begin{pmatrix} F_{1s} \\ F_{2s} \\ F_{3s} \\ F_{1m} \\ F_{2m} \\ F_{3m} \end{pmatrix} = \begin{pmatrix} 528 \\ 434 \\ 256 \\ 309 \\ 261 \\ 237 \end{pmatrix},\ \begin{pmatrix} N_{1i} \\ N_{2i} \\ N_{3i} \\ N_{1s} \\ N_{2s} \\ N_{3s} \end{pmatrix} = \begin{pmatrix} 314 \\ 265 \\ 136 \\ 278 \\ 253 \\ 179 \end{pmatrix},
$$

$$
\begin{pmatrix} N_{1m} \\ N_{2m} \\ N_{3m} \\ K_{1i} \\ K_{2i} \\ K_{3i} \end{pmatrix} = \begin{pmatrix} 108 \\ 122 \\ 125 \\ 2925 \\ 1907 \\ 756 \end{pmatrix},\ \begin{pmatrix} K_{1s} \\ K_{2s} \\ K_{3s} \\ K_{1m} \\ K_{2m} \\ K_{3m} \end{pmatrix} = \begin{pmatrix} 2358 \\ 1905 \\ 1094 \\ 1099 \\ 839 \\ 730 \end{pmatrix},\ \begin{pmatrix} w_{11} \\ w_{12} \\ w_{21} \\ w_{22} \\ w_{31} \\ w_{31} \end{pmatrix} = \begin{pmatrix} 19,4 \\ 15,6 \\ 13,2 \\ 9,9 \\ 8,4 \\ 5,6 \end{pmatrix},\ \begin{pmatrix} p_{1s} \\ p_{2s} \\ p_{3s} \\ p_{1m} \\ p_{2m} \\ p_{3m} \end{pmatrix} = \begin{pmatrix} 1,46 \\ 1,42 \\ 1,45 \\ 1,03 \\ 1,11 \\ 1,07 \end{pmatrix},\ \begin{pmatrix} c_{11i} \\ c_{12i} \\ c_{21i} \\ c_{22i} \\ c_{31i} \\ c_{32i} \end{pmatrix} = \begin{pmatrix} 7,7 \\ 5,61 \\ 4,77 \\ 3,4 \\ 2,79 \\ 1,77 \end{pmatrix}.
$$

$$
\begin{pmatrix} c_{11s} \\ c_{12s} \\ c_{21s} \\ c_{22s} \\ c_{31s} \\ c_{32s} \end{pmatrix} = \begin{pmatrix} 7,37 \\ 6,14 \\ 5,39 \\ 4,08 \\ 3,47 \\ 2,33 \end{pmatrix},\ \begin{pmatrix} c_{111} \\ c_{121} \\ c_{211} \\ c_{221} \\ c_{311} \\ c_{321} \end{pmatrix} = \begin{pmatrix} 2,99 \\ 2,18 \\ 0,93 \\ 0,66 \\ 0,54 \\ 0,35 \end{pmatrix},\ \begin{pmatrix} c_{112} \\ c_{122} \\ c_{212} \\ c_{222} \\ c_{312} \\ c_{322} \end{pmatrix} = \begin{pmatrix} 1,39 \\ 1,02 \\ 1,73 \\ 1,23 \\ 0,51 \\ 0,32 \end{pmatrix},\ \begin{pmatrix} c_{113} \\ c_{123} \\ c_{213} \\ c_{223} \\ c_{313} \\ c_{323} \end{pmatrix} = \begin{pmatrix} 1,44 \\ 1,05 \\ 0,89 \\ 0,63 \\ 1,04 \\ 0,66 \end{pmatrix},\ \begin{pmatrix} \overline{k}_{11} \\ \overline{k}_{12} \\ \overline{k}_{21} \\ \overline{k}_{22} \\ \overline{k}_{31} \\ \overline{k}_{32} \end{pmatrix} = \begin{pmatrix} 108 \\ 73 \\ 62 \\ 41 \\ 33,5 \\ 19,5 \end{pmatrix}.
$$

It is straightforward to calculate the six eigenvalues as follows

$$\{-0,215,\ -0,194,\ -0,188,\ -0,1691,\ -0,169,\ -0,149\}.$$

This implies that the world economy is stable. We can effectively conduct comparative dynamic analysis.

COMPARATIVE DYNAMIC ANALYSIS

We simulated the motion of the dynamic system. This section carries out comparative dynamic analysis. As we can follow the motion of the global economy, it is straightforward to provide transitory and long-term effects of changes in any parameter on the global economy. It is important to ask questions such as how a change in one country's conditions affects the national economy and global economies. First, we introduce a variable $\overline{\Delta}x(t)$ to stand for the change rate of the variable $x(t)$ in percentage due to changes in the parameter value.

THE TOTAL FACTOR PRODUCTIVITY OF COUNTRY 1'S INDUSTRIAL SECTOR RISES

Variation in incomes across countries is much determined by technological differences. We examine effects of an improvement of productivity in country 1's industrial sector as follows: A_{1i}: $1,4 \rightarrow 1,45$. The simulation results are plotted in Figure 2. Figure 1 shows how the system moves. Figure 2 shows how the variables deviate from the variables in Figure as the parameter is shifted. As the system variables interact in nonlinearly, it is tedious to interpret why variables vary over time in a clear manner, even though it is not difficult to see by observing the motions in the plots. The global income rises. The global physical capital falls initially and rises in the long term. Country 1's national output and capital employed rise and the other two economies' national incomes and capital stocks employed fall initially and

slightly change in the long term. Country 1's national wealth falls initially and rises in the long term. The other two countries' national wealth slightly changes. Countries 2 and 3's trade surpluses are improved initially and are slightly changed in the long term. Country 1's surplus is deteriorated initially and is changed slightly. Countries 1's and 2's industrial sectors expand and country 3's industrial sector shrinks. Country 1's service sector shrinks initially and expands in the long term. The other two countries' service sectors change slightly in the long term. The wage rates in country rise and the wage rates in the other two economies fall initially and change slightly in the long term. The rate of interest rises initially and changes slightly in the long term. The prices of country 1's service and specified good rise. The corresponding prices change slightly. The wealth per household of country 1 rise in the long term and the wealth levels per household in the other two economies change slightly in the long term. The representative households of country consume more industrial goods and services in the long term. The consumption levels of the other two economies' representative households are slightly affected. Country 1's households increase their consumption levels of all country-specified goods. The other two economies' households reduce their consumption levels of country 1' specified good and change slightly consumption levels of the two economies' specified goods.
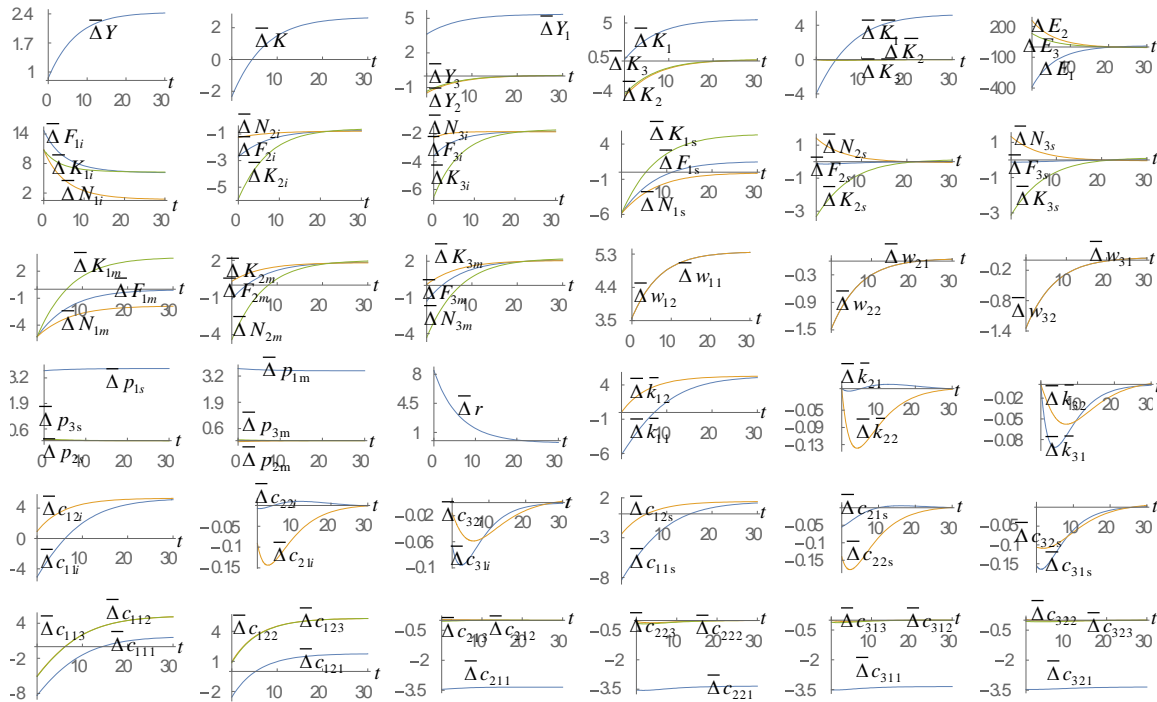
Figure 2. The total factor productivity of country 1's industrial sector rises.

GROUP (1, 1)'S PROPENSITY TO CONSUME COUNTRY 1'S SERVICE RISES

We now study what happen to the global economy if country 1's group 1 increases the propensity to consume country 1's service as follows: $\chi_{110}: 0{,}07 \to 0{,}075$. The simulation results are plotted in Figure 3. As group (1, 1) would like to spend more out of the disposable income, the global income and physical capital fall. All economies' national incomes and levels of physical capital employed fall. Country 1's wealth falls and the other two economies' levels of wealth change slightly. All wage rates fall and rate of interest rises. There are economic structural changes as illustrated in Figure 3. Group 1's wealth falls and other groups' levels of wealth change slightly.
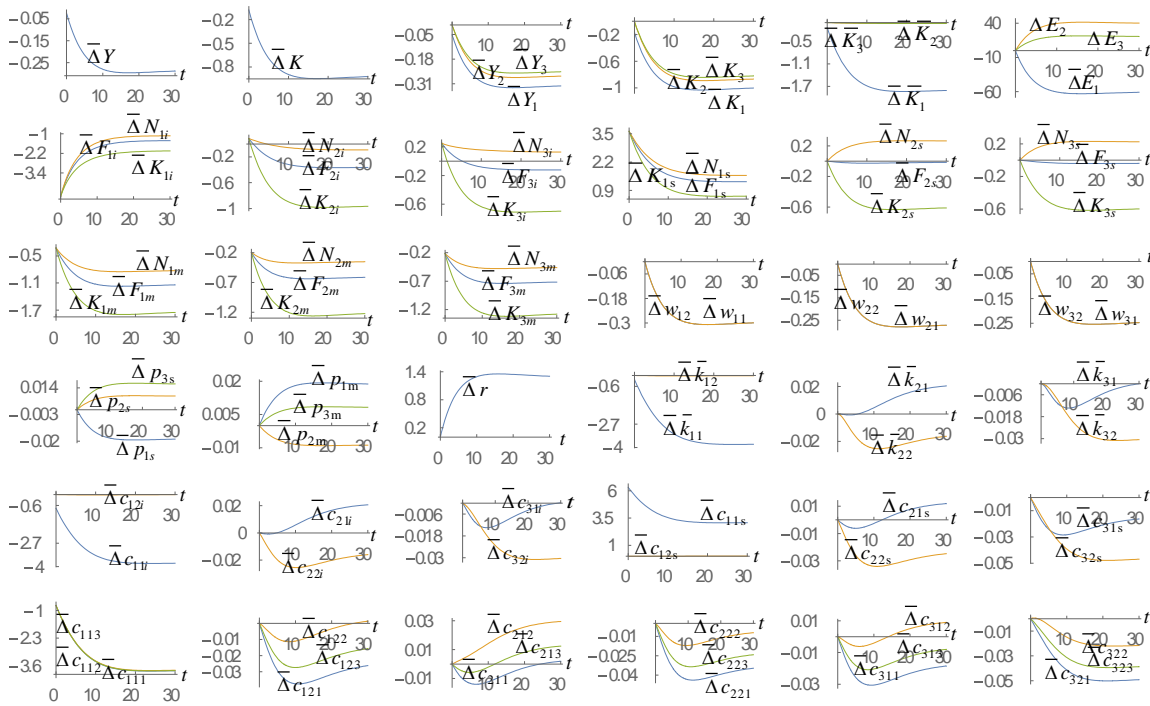
Figure 3. Group (1, 1)'s propensity to consume country 1's service rises.

GROUP (3, 2)'S POPULATION RISES

Chen [3] observed: "There have been few attempts in the literature to explain long-run comparative advantage in terms of differences in initial factor endowment ratios among countries." The task is difficult as one needs a dynamic general equilibrium theory to properly deal with the issue. The theory proposed in this study can properly deal with issues because it is a dynamic general equilibrium theory. We now examine what happen in the global economy if group (3, 2)'s population rises as follows: $N_{32}: 80 \rightarrow 90$. The simulation results are plotted in Figure 4. The global income and physical capital rise. Country 3's national income, capital stock employed and wealth are all increased. The other two economies' macro variables change slightly. All micro economic variables are effected slightly. Hence, the rise in the population changes mainly global and national macro variables and has weak impact on micro variables.

GROUP (1, 1)'S PROPENSITY TO CONSUME COUNTRY 2'S SPECIFIED GOOD RISES

We now examine what happen in the global if group (1, 1)'s propensity to consume country 2's specified good rises as follows: $\gamma_{0112}: 0,01 \rightarrow 0,015$. The simulation results are plotted in Figure 5. The global income, global physical capital, and national incomes fall. The capital stock employed by all the economies fall. All the wage rates fall and rate of interest rises. Country 1's wealth falls and the other two economies' levels of national wealth change slight. There are also economic structural changes.

CONCLUDING REMARKS

This article examined the role of preferences and technological differences between countries in determining dynamics of capital accumulation, wealth and income distribution within countries and between countries, and patterns of trade in a dynamic general equilibrium framework by integrating Walrasian general equilibrium, neoclassical growth, and H-O
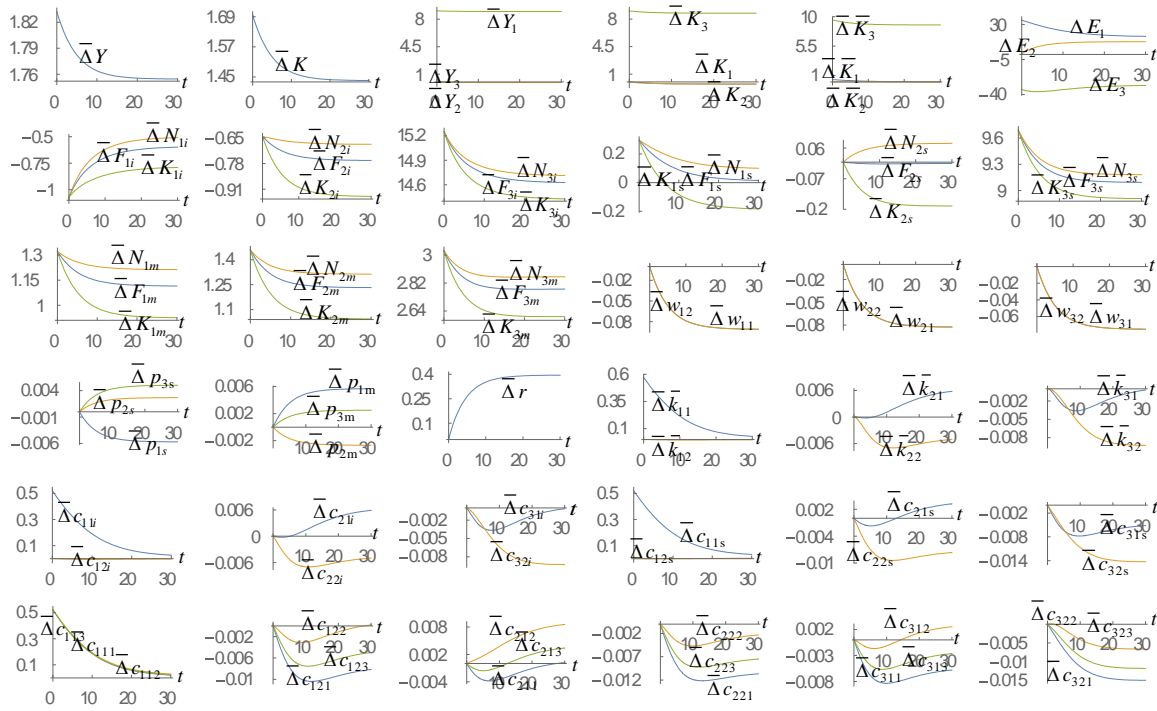
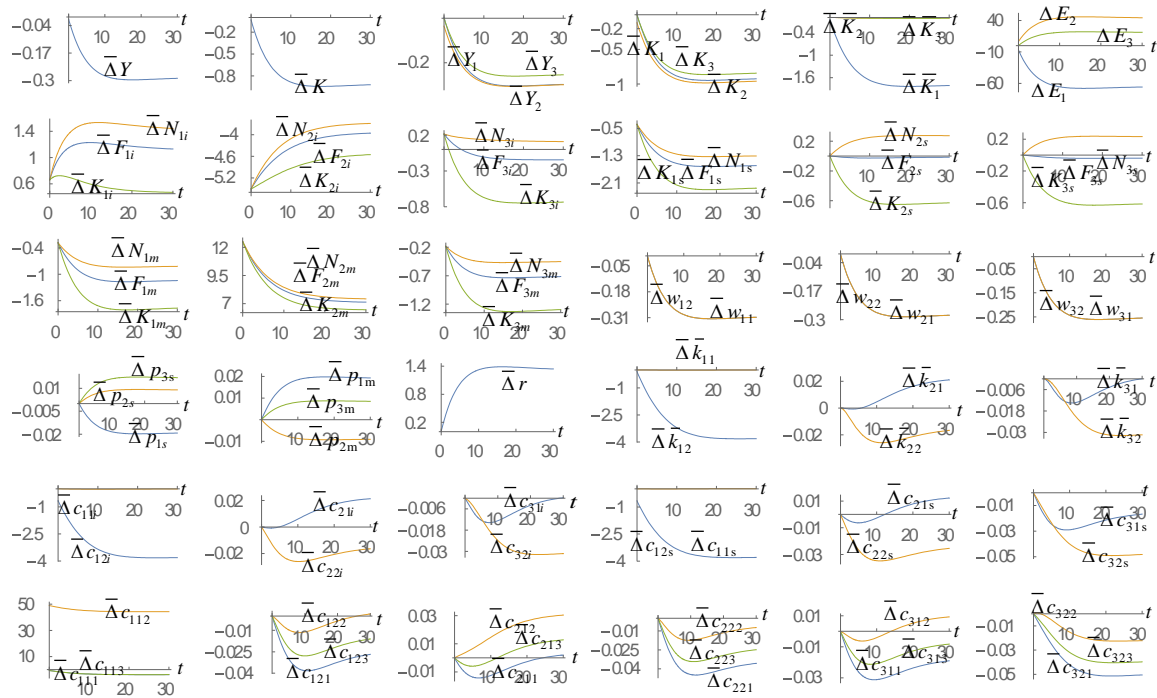**Figure 4.** Group (3, 2)'s population rises.

Figure 5. Group (1, 1)'s propensity to consume country 2's specified good rises.

international trade theories. It is built for any number of countries. Each country is composed of heterogeneous households. The national growth mechanism is the same as that in neoclassical growth theory in the sense that endogenous wealth accumulation is the engine of growth. Labour and capital distribution between sectors and between countries are determined under perfect competition and free trade. The model synthesized the well-known H-O and the Oniki-Uzawa trade models, Solow-Uzawa neoclassical growth theory, and Walrasian general equilibrium theory with Zhang's utility function. We simulated the model

with three national economies and each country with two groups of households. We identified existence of equilibrium points and plotted motion of the dynamic system. We also conducted comparative dynamic analysis. It should be noted that although the model is structurally general, it can be refined and generalized in different directions. For instance, we might get more insights from further simulation. Our comparative dynamic analysis is limited to a few cases. Our model is based on some of most well-known models in the literature of economic theory. We can generalize the model on the basis of the literature. There are many trade models which explicitly emphasize technological change and human capital accumulation as sources of global growth (see e.g. [4, 13]).

APPENDIX: PROVING THE LEMMA

We now derive dynamic equations for global economic growth. From equations (3), we have

$$z_j \equiv \frac{r + \delta_{jk}}{w_j} = \frac{\overline{\alpha}_{ji} N_{ji}}{K_{ji}} = \frac{\overline{\alpha}_{jm} N_{jm}}{K_{jm}} = \frac{\overline{\alpha}_{js} N_{js}}{K_{js}}, \tag{A1}$$

where $\overline{\alpha}_{jq} \equiv \alpha_{jq} / \beta_{jq}$. From (3) and (A1), we have

$$r(z_j) = \frac{\alpha_{ji} A_{ji} z_j^{\beta_{ji}}}{\overline{\alpha}_{ji}^{\beta_{ji}}} - \delta_{jk}, \quad w_j(z_j) = \frac{r + \delta_{jk}}{z_j}. \tag{A2}$$

From (A2) we have

$$z_j = \left(\frac{r(z_1) + \delta_{jk}}{\alpha_{ji} A_{ji}} \right)^{1/\beta_{2i}} \overline{\alpha}_{ji}. \tag{A3}$$

From (3) and (A1)

$$p_{jq} = \frac{w_j z_j^{\alpha_{jq}}}{\beta_{jq} A_{jq} \overline{\alpha}_{jq}^{\alpha_{jq}}}. \tag{A4}$$

From the definition of $\hat{y}_j$, we have

$$\hat{y}_{jn} = (1 + r)\overline{k}_{jn} + h_{jn} w_j. \tag{A5}$$

From (9) and (A1) we have

$$\overline{\alpha}_{ji} N_{ji} + \overline{\alpha}_{jm} N_{jm} + \overline{\alpha}_{js} N_{js} = z_j K_j. \tag{A6}$$

From (7) and (11) we have

$$\sum_{n=1}^{J_n} \chi_{jn} \hat{y}_{jn} \overline{N}_{jn} = \frac{w_j N_{js}}{\beta_{js}}. \tag{A7}$$

Insert (A5) in (A7)

$$N_{js} = \omega_j + \sum_{n=1}^{J_n} \omega_{jn} \overline{k}_{jn}, \tag{A8}$$

where

$$\omega_j \equiv \beta_{js} \sum_{n=1}^{J_n} h_{jn} \chi_{jn} \overline{N}_{jn}, \quad \omega_{jn} \equiv \beta_{js} \left(\frac{1+r}{w_j} \right) \chi_{jn} \overline{N}_{jn}.$$

Insert (A8) in (10)

$$N_{ji} = N_j - N_{jm} - \omega_j - \sum_{n=1}^{J_n} \omega_{jn} \overline{k}_{jn}. \tag{A9}$$

Insert (7) in (12)

$$\sum_{j=1}^{J}\sum_{n=1}^{J_n}\gamma_{jnq}\,\hat{y}_{jn}\,\overline{N}_{jn} = p_{qm}\,F_{qm}. \tag{A10}$$

Insert (3) in (A10)

$$\sum_{j=1}^{J}\sum_{n=1}^{J_n}\gamma_{jnq}\,\hat{y}_{jn}\,\overline{N}_{jn} = \frac{w_q\,N_{qm}}{\beta_{qm}}. \tag{A11}$$

Insert (A5) in (A11)

$$N_{qm} = R_{qm}\sum_{j=1}^{J}\sum_{n=1}^{J_n}\overline{\gamma}_{jnq}\,\overline{k}_{jn} + \frac{W_q\,\beta_{qm}}{w_q}, \tag{A12}$$

where

$$R_{qm} \equiv \frac{(1+r)\beta_{qm}}{w_q}, \quad \overline{\gamma}_{jnq} \equiv \gamma_{jnq}\,\overline{N}_{jn}, \quad n_{jq} \equiv \sum_{n=1}^{J_n}h_{jn}\overline{\gamma}_{jnq}, \quad W_q \equiv \sum_{j=1}^{J}n_{jq}\,w_j.$$

Insert (10) in (A6)

$$\overline{\alpha}_{ji}\,N_j + \tilde{\alpha}_{jm}\,N_{jm} + \tilde{\alpha}_{js}\,N_{js} = z_j\,K_j, \tag{A13}$$

where

$$\tilde{\alpha}_{jm} \equiv \overline{\alpha}_{jm} - \overline{\alpha}_{ji}, \quad \tilde{\alpha}_{js} \equiv \overline{\alpha}_{js} - \overline{\alpha}_{ji}.$$

Substituting (A8) and (A12) into (A13)

$$K_j = u_j + \sum_{f=1}^{J}\sum_{n=1}^{J_n}\tilde{\gamma}_{fnj}\,\overline{k}_{fn} + \sum_{n=1}^{J_n}\overline{\omega}_{jn}\,\overline{k}_{jn}, \tag{A14}$$

where

$$u_j \equiv \left(\overline{\alpha}_{ji}\,N_j + \tilde{\alpha}_{js}\,\omega_j + \frac{\tilde{\alpha}_{jm}\,W_j\,\beta_{jm}}{w_j}\right)\frac{1}{z_j}, \quad \tilde{\gamma}_{fnj} \equiv \frac{\tilde{\alpha}_{jm}\,R_{jm}\,\overline{\gamma}_{fnj}}{z_j}, \quad \overline{\omega}_{jn} \equiv \frac{\omega_{jn}\,\tilde{\alpha}_{js}}{z_j}.$$

From (14) and (A14) we have

$$K = u + \sum_{j=1}^{J}\sum_{n=1}^{J_n}\tilde{\gamma}_{jn}\,\overline{k}_{jn}, \tag{A15}$$

where

$$u \equiv \sum_{j=1}^{J}u_j, \quad \tilde{\gamma}_{jn} \equiv \overline{\omega}_{jn} + \sum_{f=1}^{J}\tilde{\gamma}_{jnf}.$$

From (14) and (A15) we have

$$\sum_{j=1}^{J}\sum_{n=1}^{J_n}\overline{k}_{jn}\,\overline{N}_{jn} = u + \sum_{j=1}^{J}\sum_{n=1}^{J_n}\tilde{\gamma}_{jn}\,\overline{k}_{jn}, \tag{A16}$$

Solve (A16) with $\overline{k}_{11}$ as the variable

$$\overline{k}_{11} \equiv \varphi\left(z_1, \{\overline{k}_{jq}\}\right) = \left(u + \sum_{j=1}^{J}\sum_{n=1,(j,n)\neq(1,1)}^{J_n}\left(\tilde{\gamma}_{jn} - \overline{N}_{jn}\right)\overline{k}_{jn}\right)\frac{1}{\overline{N}_{11} - \tilde{\gamma}_{11}}, \tag{A17}$$

where $\{\overline{k}_{jn}\} = \{\overline{k}_{12}, …, \overline{k}_{JJ_n}\}$. It is straightforward to confirm that all the variables can be expressed as functions of z_1 and $\{\overline{k}_{jq}\}$ by the following procedure: r by (A2) $\rightarrow w_j$ by (A2) $\rightarrow$ $w_{jn} = h_{jn}w_j$ by (A2) $\rightarrow z_j$ by (A3) $\rightarrow p_{jq}$ by (A4) $\rightarrow \overline{k}_{11}$ by (A17) $\rightarrow \hat{y}_{jq}$ by (A5) $\rightarrow K$ by (A15) $\rightarrow K_j$ by (A14) $\rightarrow N_{jm}$ by (A12) $\rightarrow N_{ji}$ by (A9) $\rightarrow N_{js}$ by (A8) $\rightarrow K_{jq}$ by (A1) $\rightarrow F_{jq}$ by (2) $\rightarrow$ $c_{jni}, c_{jns}, c_{jnm}$ and s_{jn} by (7). From this procedure and (8), we have

$$\dot{\bar{k}}_{11} = \widetilde{\Lambda}_0\left(z_1, \left\{\bar{k}_{jn}\right\}\right) \equiv s_{11} - \varphi, \tag{A18}$$

$$\dot{\bar{k}}_{jn} = \widetilde{\Lambda}_{jn}\left(z_1, \left\{\bar{k}_{jn}\right\}\right) \equiv s_{jn} - \bar{k}_{jn}. \tag{A19}$$

Here, we do not provide explicit expressions of the functions as they are tedious. Taking derivatives of equation (A17) with respect to *t* yields

$$\dot{\bar{k}}_{11} = \frac{\partial \phi}{\partial z_1} \dot{z}_1 + \sum_{j=1}^{J} \sum_{n=1, (j,n) \neq (1,1)}^{J_n} \widetilde{\Lambda}_{jn} \frac{\partial \phi}{\partial \bar{k}_{jn}}, \tag{A20}$$

where we use (A19). Equal (A20) and (A18)

$$\dot{z}_{11} = \left(\widetilde{\Lambda}_0 - \sum_{j=1}^{J} \sum_{n=1, (j,n) \neq (1,1)}^{J_n} \widetilde{\Lambda}_{jn} \frac{\partial \phi}{\partial \bar{k}_{jn}} \right) \left(\frac{\partial \phi}{\partial z_{11}} \right)^{-1}. \tag{A21}$$

In summary, we proved the lemma.

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