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Some Additional Light Scattering Functions for Polystyrene Latexes*

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For the evaluation of the experimental results on polystyrene latexes some additional light scattering functions were computed. The data of i_1 and i_2 as well as the real and imaginary parts of the complex amplitude functions $R(S_1)$, $I(S_1)$, $R(S_2)$ and $I(S_2)$ for $m = 1.15$ and 1.25 and $\gamma = 45^0$, 90^0 and 135^0 are tabulated. From these results theoretical specific intensities at 90^0, dissymmetries and polarization ratios were calculated. Functions for $m = 1.199$, 1.210 and 1.214 were obtained by graphical interpolation. The problem of the location of maxima and minima of intensity of angularly scattered light is discussed.

INTRODUCTION

For light scattering investigations of polystyrene latexes the Mie functions for relative refractive index m in the range 1.15 — 1.25 are necessary[1]. A part of these functions is now available[2-8], but to cover the whole range necessary in the evaluation of the experimental results a set of additional functions had to be calculated**.

The light scattering functions from the Mie theory for spherical particles in the m range mentioned are worked out by Gumprecht and Sliepcevich[9] and Pangonis, Heller, and Jacobson[10]. The latter tables are especially useful for our purposes owing to the fairly narrow intervals in relative diameter α and relative refractive indices m close enough to 1.20 (this is the m value[11] of a polystyrene latex in water at the wavelength of light in vacuum $\lambda_0 = 546.1$ mμ).

RESULTS AND DISCUSSION

For the experimental work, carried out in these laboratories[12], we needed functions with relative refractive indices 1.199, 1.200, 1.210 and 1.214, corresponding to the wavelengths of the mercury spectrum lines in vacuum 578.0, 546.1, 435.8 and 404.7 mμ, respectively. It is possible to obtain these functions from functions for $m = 1.15$, 1.20 and 1.25 by interpolations. For these values of m only values of the specific turbidity are completely available[2]. The angular intensity functions i_1 and i_2 exist only for $m = 1.20$, $\alpha = 0.2(0.2)7.0$, and for the angles $\gamma = 45^0$, 90^0 and 135^0 [8]. Here i_1 and i_2 are

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** After completing this work Pangonis and Heller published *Angular Scattering Functions for Spherical Particles* (Wayne State University Press, Detroit 1960) containing i/α^3 functions for $m = 1.05(0.05)1.30$. The author of this paper hopes that the presented tabulations, obtained independently and consisting of yet unpublished data, can be useful.

defined as usually[8,9], and γ is the angle between the direction of propagation of the scattered light and the reversed direction of propagation of the incident beam.

For this reason i_1 and i_2 for $m = 1.15$ and 1.25 for $\gamma = 45^0$, 90^0 and 135^0 were computed. The additional functions for $m = 1.20$, $\gamma = 90^0$, $\alpha = 7.2(0.2)9.2$ were computed, too. The computations were performed with the aid of the light scattering functions $R(A_n)$, $I(A_n)$, $R(B_n)$ and $I(B_n)$ tabulated by Pangonis, Heller, and Jacobson* and the functions of partial derivatives of Legendre polynomials π_n and $[x\,\pi_n - (1-x^2)\,\pi_n']$ tabulated for $\gamma = 90^0$ by Gumprecht and Sliepcevich[13] and for $\gamma = 45^0$ and 135^0 by Kerker and Matijević[8]. The numerical results of the computations are given in Tables I — VII. Together with these functions the real and imaginary parts of the complex amplitude functions[14] $R(S_1)$, $I(S_1)$, $R(S_2)$ and $I(S_2)$, necessary for the definition of phase relations and calculation of Stokes parameters, are presented.

From these results all necessary functions for the practical use can be derived.

For measurements at 90^0 it is convenient to make use of the specific intensity[3,15,16] $(I_{90}/I_0c)_0$. For unpolarized incident light the following expression holds

$$(I_{90}/I_0c)_0 = \frac{3\,\rho_1}{2\lambda\alpha^3\,\rho_2}\,i_{90},$$

where $i_{90} = (i_1+i_2)_{90}/2$, I_0 and I_{90} are the intensities, per unit solid angle, of the incident and scattered beam, respectively, c the concentration in grams of dispersed spheres per gram of solution, λ the wavelength of light in the scattering medium, α the relative diameter ($\alpha = \pi D/\lambda$, where D is the diameter of the sphere), and ρ_1 and ρ_2 the densities of the solution and of the spheres, respectively. For the practical reasons specific intensities for $\lambda = 409.4$ mμ, corresponding to $\lambda_0 = 546.1$ mμ (the wavelength of the green mercury line in vacuum) and $\varrho_1 = \varrho_2$ were calculated. As shown earlier[2,16], the values obtained by measurements at other wavelengths can be easily *standardized* to the green mercury line. Fig. 1 shows the course of theoretical values of $(I_{90}/I_0c)_0$ as a function of α. Values for $m = 1.20$ are calculated from data of Kerker and Matijević, those for $m = 1.15$ and 1.25 arised from present calculations.

The theoretical dissymmetries $(1/z)_0 = (i_1+i_2)_{45}/(i_1+i_2)_{135} = i_{45}/i_{135}$ are represented in Fig. 2. Here index zero denotes the values extrapolated to zero concentration. Because of the wide range of z values covered by calculations, a logarithmic plot is used.

The polarization ratio** $(\rho_{90,u})_0 = (\rho_{90})_0 = (i_2/i_1)_{90}$ plotted *vs.* α is shown in Fig. 3 (for spheres it is immaterial whether the incident beam is unpolarized or the linear polarized components are used). Here again a logarithmic presentation is taken and values for $m = 1.20$ are from Kerker and Matijević.

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* Here it should be noted that several erratical signs exist both in the tables of Gumprecht and Sliepcevich and those of Pangonis, Heller, and Jacobson. So at Gumprecht and Sliepcevich $R(B_n)$ [they denote it as $R(P_n)$] for $m = 1.20$, $\alpha = 1$ and $n = 1$ must have a negative sign. In tables of Pangonis, Heller, and Jacobson: $R(B_n)$ for $m = 1.15$, $\alpha = 2.60$, $n = 2$ must be positive; $R(B_n)$ for $m = 1.15$, $\alpha = 3.80$, $n = 1$ must be negative *etc.*, because of their periodical change.

** In our preceding paper[16] ϱ is named *depolarization*, a term which is frequently in use. The term *polarization ratio* is better.

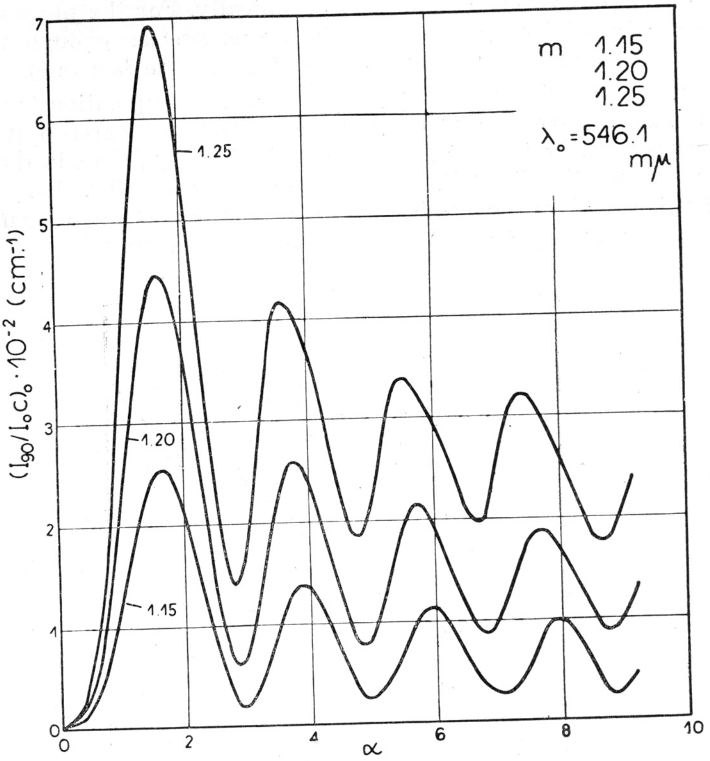

Fig. 1. **Specific** intensity at 90° *versus* relative diameter obtained from Mie theory for $\varrho_1 = \varrho_2$, $\lambda_0 = 546.1$ mμ, $m = 1.15$, 1.20 and 1.25.

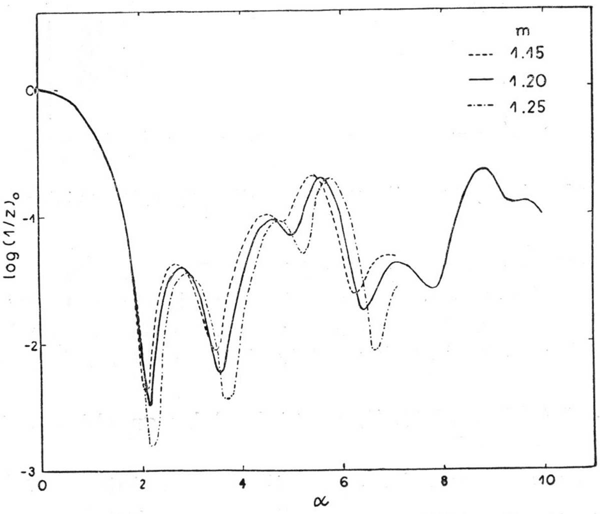

Fig. 2. Logarithm of the function of dissymmetry *versus* relative diameter obtained from Mie theory for $m = 1.15$, 1.20 and 1.25.

The interpolations were carried out graphically. For the purpose of application to experimental data this procedure was precise enough to evaluate the data to three figures (with a variation of ±1 in the last one).

In the case of specific turbidity the method of interpolation is worked out extensively[17]. It was shown that by plotting $\log(\tau/c)_0$ *vs.* $\log(m\text{-}1)$ at constant α straight lines result, if α is not too great. For the interpolations in the neighborhood of $m = 1.200$ the linearity is maintained for $\alpha < 3.0$. This method of interpolation is based on the application of the Rayleigh-Gans formula[18], where for m not far from unity $(\tau/c)_0$ is proportional to $(m - 1)^2$.

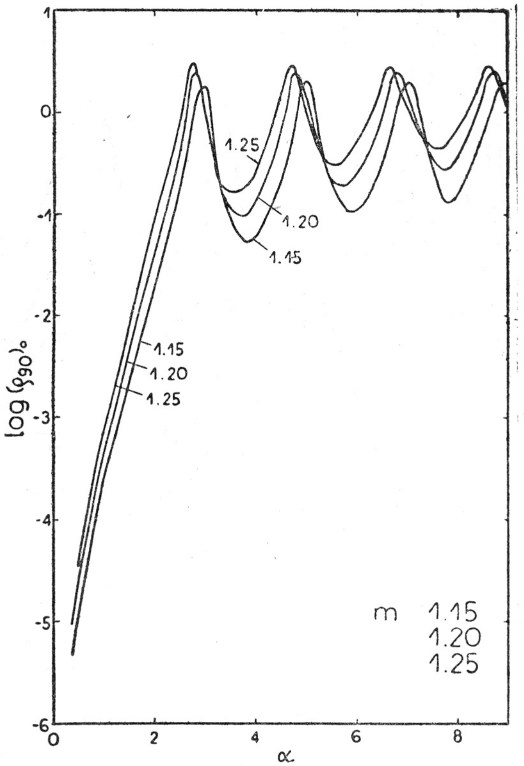

Fig. 3. Logarithm of polarization ratio *versus* relative diameter obtained from Mie theory for $m = 1.15$, 1.20 and 1.25.

For interpolations of specific intensities $(I_{90}/I_0c)_0$ the same method was used. Since from the Rayleigh-Gans theory[18] $(I_{90}/I_0c)_0$ is proportional to $(m - 1)^2$, a linear relationship in a plot of $\log(I_{90}/I_0c)_0$ *vs.* $\log(m - 1)$ can be expected for smaller values of α. This linearity exists till $\alpha = 2.2$ for interpolations about $m = 1.200$.

The values of $(\tau/c)_0$ and $(I_{90}/I_0c)_0$ obtained by graphical interpolation are given in Tables VIII and IX.

Dissymmetries and polarization ratios were interpolated by plotting $\log(1/z)_0$ and $\log(\rho_{90})_0$, respectively, *versus* m. The results are presented in Tables X and XI.

The problem of locations of maxima and minima of intensity of angularly scattered light still remains to be elucidated. Nakagaki and Heller[6] reported recently the results of the calculation from the Mie theory for $m = 1.20$ and unpolarized incident light. Here a few remarks should be done. Nakagaki and Heller supposed that the positions of maxima in plots of i_γ *vs.* α correspond necessarily to the locations of maxima and minima in the angular scattering diagram. This is only partially the case. There still exists a minimum in the Rayleigh region, for $0 < \alpha < 1.40$ and $m = 1.20$, and on the other hand the first order maximum of Nakagaki and Heller does not appear in the angular scattering diagram. By drawing i_γ *vs.* γ plots from values calculated by Ashley and Cobb*[7] for $m = 1.20$ and values calculated for some smaller values of α in this Laboratory, generally something smaller γ-values for the locations of minima and greater γ-values for those of maxima could be observed.

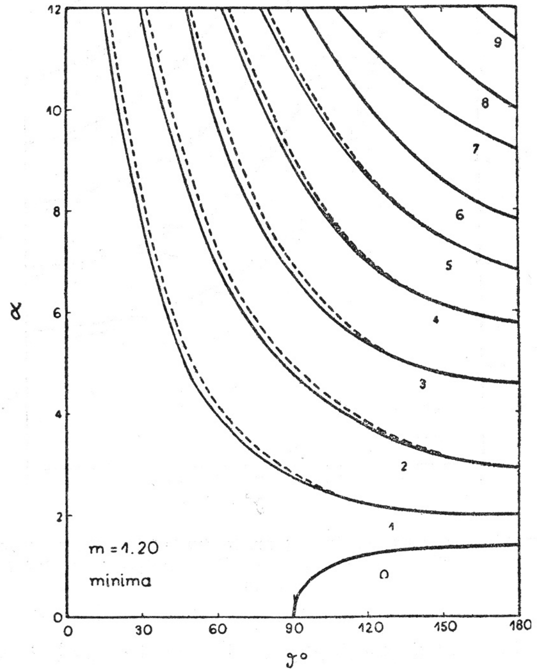

Fig. 4. Locations of minima *versus* scattering angle ($m = 1.20$). Full curves: data of Nakagaki and Heller[8]; dotted curves: estimations from i_γ *vs.* γ plots; numbers refer to the order of curves.

The minimum, mentioned above, is due to $(1 + \cos^2\gamma)$-factor in the Rayleigh-Gans equation. For extremely small values of α this minimum lies at 90^0. Owing to the assymmetry of the scattering envelope, the minimum shifts to smaller values of γ, at first slowly and later disappears very quickly at $\alpha < 1.40$ for $m = 1.20$. This minimum will be denoted as a *zero order minimum.*

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* Two errors could be perceived in their tabulations. The value of $i = (i_1 + i_2)/2$ for $\gamma = 0^0$ and $\alpha = 2.0$ should be 0.003415, and the value $i = 2.419$ for $\gamma = 60^0$ and $\alpha = 5.0$ is very probably incorrect.

In Figs. 4 i 5 a graphical presentation of the locations of extrema depending on the scattering angle is given. Here full curves represent the data of Nakagaki and Heller. For the sake of convenience the scattering angle is taken as $\vartheta = 180^0 - \gamma$. Dotted curves are estimated from the i_γ *vs.* γ plots. The numbers refer to the order of a curve.

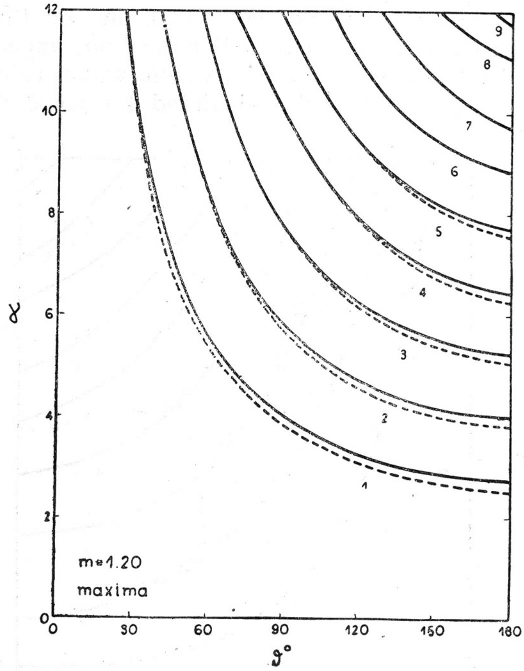

Fig. 5. Locations of maxima *versus* scattering angle ($m = 1.20$). Full curves: data of Nakagaki and Heller[8]; dotted curves: estimations from i_γ *vs.* γ plots; numbers refer to the order of curves.

It seems that the real dependence of the locations of maxima and minima of an angular scattering diagram on particle size deviates for a small amount from the locations of extreme values of the i_γ *vs.* α diagram for fixed angles. This conclusion agrees very well with our experimental results obtained on monodisperse polystyrene latexes[12]. The best way for obtaining a precise chart of locations of extrema lies in the rigorous and time consuming computation of angular scattering intensities. This difficulty increases with the increasing value of α. For $m = 1.20$, $\alpha = 10$, 8 minima and 7 maxima exist, and for more accurate locations of extrema smaller intervals in γ must be taken.

TABLE I

Intensity Functions from Mie Theory

$m = 1.15$ $\gamma = 45^0$

α	$R(S_1)$	$I(S_1)$	i_1	$R(S_2)$	$I(S_2)$	i_2
0.2	+0.000769	—0.000000	0.0_6591	+0.000544	—0.000000	0.0_6296
0.4	+0.005943	—0.000025	0.0_43532	+0.004196	—0.000018	0.4_41761
0.6	+0.018920	—0.000266	0.0_33580	+0.013334	—0.000188	0.0_31778
0.8	+0.041071	—0.001369	0.001689	+0.028846	—0.000966	0.0_38330
1.0	+0.070753	—0.004591	0.005027	+0.049442	—0.003229	0.002455
1.2	+0.10265	—0.011451	0.010668	+0.071128	—0.007998	0.005123
1.4	+0.12793	—0.022624	0.016878	+0.087365	—0.015578	0.007875
1.6	+0.13591	—0.036445	0.019799	+0.090280	—0.024341	0.008743
1.8	+0.11724	—0.047987	0.016047	+0.073251	—0.029904	0.006260
2.0	+0.067809	—0.049629	0.007061	+0.034230	—0.025564	0.001825
2.2	—0.007946	—0.033542	0.001188	—0.021332	—0.004774	0.0_34778
2.4	—0.096665	+0.004227	0.009362	—0.079358	+0.034255	0.007471
2.6	—0.17860	+0.059983	0.035494	—0.12249	+0.084251	0.022102
2.8	—0.23430	+0.12201	0.069781	—0.13685	+0.12904	0.035380
3.0	—0.25023	+0.17470	0.093137	—0.11857	+0.14937	0.036372
3.2	—0.22000	+0.20315	0.089669	—0.075427	+0.13086	0.022813
3.4	—0.14456	+0.19589	0.059271	—0.023654	+0.070625	0.005547
3.6	—0.032461	+0.14584	0.022322	+0.018867	—0.019433	0.0_37336
3.8	+0.099528	+0.051233	0.012530	+0.040975	—0.11557	0.015036
4.0	+0.22635	—0.082436	0.058032	+0.043081	—0.19120	0.038414
4.2	+0.31697	—0.23989	0.15802	+0.035295	—0.22666	0.052620
4.4	+0.33990	—0.39293	0.26993	+0.031147	—0.21580	0.047540
4.6	+0.27446	—0.50004	0.32536	+0.039092	—0.16659	0.029282
4.8	+0.12419	—0.51437	0.28000	+0.056871	—0.094571	0.012178
5.0	—0.075900	—0.40072	0.16633	+0.073728	—0.013644	0.005622
5.2	—0.26399	—0.15496	0.093704	+0.078633	+0.067994	0.010806
5.4	—0.35663	+0.18189	0.16027	+0.086615	+0.13482	0.025678
5.6	—0.35298	+0.52655	0.40184	+0.046060	+0.21118	0.046720
5.8	—0.20806	+0.77966	0.65116	+0.019510	+0.25630	0.066071
6.0	+0.013778	+0.86212	0.74344	—0.002977	+0.26668	0.071127
6.2	+0.22952	+0.74210	0.60340	—0.020309	+0.22898	0.052844
6.4	+0.38164	+0.44606	0.34461	—0.040731	+0.14012	0.021294
6.6	+0.36466	+0.050574	0.13554	—0.073381	+0.015299	0.005619
6.8	+0.29662	—0.35033	0.21072	—0.12481	—0.11438	0.028661
7.0	+0.10483	—0.67546	0.46723	—0.17374	—0.21479	0.076324
8.0	—0.60566	—0.14413	0.38760	+0.14714	—0.10633	0.032957
9.0	+1.23995	+0.85207	2.2635	+0.099382	+0.44362	0.20667

TABLE II

Intensity Functions from Mie Theory

$m = 1.15$ $\gamma = 90^0$

α	$R(S_1)$	$I(S_1)$	i_1	$R(S_2)$	$I(S_2)$	i_2
0.2	+0.000773	—0.000000	0.0_6598	—0.000000	+0.000000	0.000000
0.4	+0.006087	—0.000025	0.0_43705	—0.000012	+0.000000	0.0_914
0.6	+0.019974	—0.000266	0.0_33990	—0.000096	+0.000000	0.0_892
0.8	+0.045289	—0.001376	0.002053	—0.000407	+0.000004	0.0_6166
1.0	+0.082886	—0.004655	0.006892	—0.001262	+0.000036	0.0_51594
1.2	+0.13072	—0.011814	0.017226	—0.003195	+0.000197	0.0_41024
1.4	+0.18351	—0.024137	0.034258	—0.007008	+0.000846	0.0_44983
1.6	+0.23345	—0.041487	0.056218	—0.013666	+0.002898	0.0_31952
1.8	+0.27177	—0.061940	0.077696	—0.023806	+0.008235	0.0_36345
2.0	+0.29042	—0.082457	0.091145	—0.036629	+0.019756	0.001732
2.2	+0.28274	—0.099904	0.089924	—0.048479	+0.040205	0.003967
2.4	+0.24363	—0.11138	0.071763	—0.052735	+0.069361	0.007592
2.6	+0.16942	—0.11344	0.041572	—0.042765	+0.10141	0.012113
2.8	+0.059796	—0.10076	0.013728	—0.017134	+0.12688	0.016392
3.0	—0.079757	—0.065818	0.010693	+0.019657	+0.13675	0.019087
3.2	—0.23408	—0.001747	0.054796	+0.059609	+0.12749	0.019808
3.4	—0.37876	+0.091058	0.15175	+0.095871	+0.10031	0.019252
3.6	—0.48778	+0.19948	0.27772	+0.12445	+0.058342	0.018891
3.8	—0.54488	+0.30115	0.38759	+0.14329	+0.004342	0.020550
4.0	—0.54740	+0.37605	0.44107	+0.15014	—0.059973	0.026140
4.2	—0.49955	+0.41486	0.42165	+0.14057	—0.13230	0.037262
4.4	—0.40360	+0.41665	0.33648	+0.10838	—0.20662	0.054438
4.6	—0.25855	+0.38093	0.21196	+0.049241	—0.27118	0.075963
4.8	—0.065934	+0.30189	0.095485	—0.034415	—0.31042	0.097543
5.0	+0.16114	+0.17524	0.056674	—0.13139	—0.31074	0.11382
5.2	+0.39224	—0.11355	0.16675	—0.22449	—0.26607	0.12119
5.4	+0.56914	—0.26191	0.39252	—0.29658	—0.18059	0.12057
5.6	+0.69825	—0.51148	0.74917	—0.33356	—0.066032	0.11562
5.8	+0.72030	—0.72344	1.04221	—0.33373	+0.064131	0.11549
6.0	+0.65999	—0.85984	1.17491	—0.29586	+0.19862	0.12699
6.2	+0.53721	—0.90462	1.10693	—0.21826	+0.32672	0.15439
6.4	+0.36775	—0.86019	0.87516	—0.10165	+0.43398	0.19867
6.6	+0.12748	—0.73374	0.55462	+0.048541	+0.50152	0.25388
6.8	+0.090512	—0.52594	0.28481	+0.21761	+0.51154	0.30903
7.0	—0.35324	—0.22815	0.17683	+0.37602	+0.45411	0.34761
8.0	—0.51910	+1.45354	2.3822	+0.39913	—0.47108	0.38123
9.0	+0.64114	+0.14850	0.43311	—0.76928	—0.47692	0.81924

TABLE III

Intensity Functions from Mie Theory

$m = 1.15$ $\gamma = 135^0$

α	$R(S_1)$	$I(S_1)$	i_1	$R(S_2)$	$I(S_2)$	i_2
0.2	+0.000777	—0.000000	0.0_6604	—0.000550	+0.000000	0.0_6302
0.4	+0.006231	—0.000025	0.0_43883	—0.004412	+0.000018	0.0_41947
0.6	+0.021058	—0.000266	0.0_34435	—0.014942	+0.000188	0.0_32233
0.8	+0.049801	—0.001383	0.002482	—0.035426	+0.000980	0.001256
1.0	+0.096354	—0.004719	0.009306	—0.068804	+0.003355	0.004745
1.2	+0.16338	—0.012177	0.026840	—0.11728	+0.008709	0.013830
1.4	+0.25194	—0.025670	0.064132	—0.18212	+0.018577	0.033511
1.6	+0.36190	—0.046642	0.13315	—0.26391	+0.034449	0.070837
1.8	+0.49251	—0.076417	0.24841	—0.36287	+0.058242	0.13507
2.0	+0.64216	—0.11730	0.42613	—0.47813	+0.093172	0.23729
2.2	+0.80670	—0.17289	0.68065	—0.60586	+0.14354	0.38767
2.4	+0.97762	—0.24611	1.01631	—0.73860	+0.21224	0.59057
2.6	+1.14387	—0.33613	1.42142	—0.86658	+0.29775	0.83962
2.8	+1.29503	—0.43754	1.86854	—0.98296	+0.39491	1.12216
3.0	+1.42482	—0.54393	2.3260	—1.08484	+0.49971	1.42660
3.2	+1.52930	—0.65138	2.7630	—1.16900	+0.61226	1.74143
3.4	+1.60324	—0.75935	3.1470	—1.22747	+0.73363	2.0449
3.6	+1.63744	—0.86783	3.4343	—1.24915	+0.86033	2.3005
3.8	+1.61941	—0.97267	3.5686	—1.22559	+0.98334	2.4690
4.0	+1.53867	—1.06266	3.4967	—1.15392	+1.09288	2.5259
4.2	+1.39345	—1.12262	3.2020	—1.03440	+1.18199	2.4671
4.4	+1.19206	—1.14230	2.7258	—0.86695	+1.24523	2.3022
4.6	+0.94449	—1.12247	2.1520	—0.65229	+1.27599	2.0536
4.8	+0.65222	—1.06904	1.56823	—0.39490	+1.26743	1.76231
5.0	+0.31010	—0.98014	1.05684	—0.10206	+1.21607	1.48925
5.2	—0.079505	—0.84326	0.71741	+0.22066	+1.12113	1.30563
5.4	—0.48442	—0.64810	0.65470	+0.54961	+0.97959	1.26166
5.6	—0.92302	—0.39933	1.01143	+0.93056	+0.78512	1.48235
5.8	—1.33982	—0.11097	1.80744	+1.29230	+0.53389	1.95508
6.0	—1.74227	+0.20959	3.0794	+1.63189	+0.23096	2.7164
6.2	—2.1192	+0.56290	4.8078	+1.93300	—0.11012	3.7486
6.4	—2.4471	+0.94268	6.8769	+2.1881	—0.47466	5.0130
6.6	—2.7379	+1.32903	9.2622	+2.3916	—0.85375	6.4484
6.8	—2.8960	+1.70072	11.2793	+2.5431	—1.24486	8.0170
7.0	—3.0149	+2.0522	13.3009	+2.6223	—1.64398	9.5790
8.0	—2.1290	+3.1001	14.1432	+1.62558	—3.0777	12.1149
9.0	+0.66690	+1.93422	4.1860	—1.07299	—2.4759	7.2813

TABLE IV

Intensity Functions from Mie Theory

$m = 1.25$ $\gamma = 45^0$

α	$R(S_1)$	$I(S_1)$	i_1	$R(S_2)$	$I(S_2)$	i_2
0.2	+0.001251	—0.000001	0.0_51564	+0.000884	—0.000001	0.0_6782
0.4	+0.009733	—0.000066	0.0_49474	+0.006864	—0.000047	0.0_44712
0.6	+0.031213	—0.000726	0.0_39748	+0.021936	—0.000513	0.0_34814
0.8	+0.068230	—0.003799	0.004670	+0.047686	—0.002680	0.002281
1.0	+0.11783	—0.012918	0.014051	+0.081589	—0.009071	0.006739
1.2	+0.16922	—0.032283	0.029678	+0.11528	—0.022448	0.013794
1.4	+0.20354	—0.062675	0.045358	+0.13436	—0.042614	0.019868
1.6	+0.19851	—0.096302	0.048679	+0.12198	—0.061978	0.018721
1.8	+0.13811	—0.11459	0.032204	+0.067453	—0.063025	0.008522
2.0	+0.024463	—0.092653	0.009183	—0.021572	—0.027556	0.001225
2.2	—0.11449	—0.013418	0.013288	—0.11076	+0.072068	0.017462
2.4	—0.23518	+0.11387	0.068275	—0.15196	+0.19791	0.062259
2.6	—0.30273	+0.25345	0.15588	—0.12046	+0.29904	0.10393
2.8	—0.30392	+0.36462	0.22531	—0.031137	+0.31727	0.10163
3.0	—0.23939	+0.41676	0.23099	+0.068343	+0.21874	0.052518
3.2	—0.11699	+0.33255	0.12428	+0.11106	—0.062561	0.016249
3.4	+0.045744	+0.26622	0.072964	+0.050840	—0.21763	0.049947
3.6	+0.21861	+0.048151	0.050111	—0.097098	—0.38646	0.15878
3.8	+0.35644	—0.25576	0.19246	—0.26348	—0.42330	0.24860
4.0	+0.39106	—0.61016	0.52522	—0.35869	—0.31741	0.22941
4.2	+0.24040	—0.91986	0.89922	—0.32902	—0.12462	0.12379
4.4	—0.055608	—1.03875	1.08209	—0.12935	+0.060757	0.020422
4.6	—0.43461	—0.85014	0.91163	—0.12687	+0.17208	0.045706
4.8	—0.70961	—0.32620	0.60995	+0.34838	+0.19321	0.15870
5.0	—0.68922	+0.41560	0.64774	+0.52485	+0.14265	0.29581
5.2	—0.28990	+1.09433	1.28159	+0.55840	+0.06390	0.31590
5.4	+0.34726	+1.43290	2.1738	+0.47589	+0.000000	0.22647
5.6	+0.96926	+1.30864	2.6520	+0.31191	—0.040505	0.098929
5.8	+1.30877	+0.75784	2.2872	+0.084901	—0.070908	0.012236
6.0	+1.17126	+0.012717	1.37202	—0.20000	—0.078693	0.046192
6.2	+0.61800	—0.60077	0.74284	—0.50244	—0.025368	0.25309
6.4	—0.096545	—0.92617	0.86711	—0.74440	+0.10832	0.56586
6.6	—0.73656	—0.97184	1.48700	—0.86606	+0.29441	0.83673
6.8	—1.23482	—0.84288	2.2352	—0.69985	+0.51100	0.75091
7.0	—1.58477	—0.59581	2.8665	—0.31426	+0.58766	0.44410
8.0	+1.49546	+1.37446	4.1255	+0.40800	—1.37032	2.0443
9.0	—1.19101	—2.1139	5.8871	—0.14014	+1.52298	2.3391

TABLE V

Intensity Functions from Mie Theory

$m = 1.25$ $\gamma = 90^0$

α	$R(S_1)$	$I(S_1)$	i_1	$R(S_2)$	$I(S_2)$	i_2
0.2	+0.001259	—0.000001	$0.0_{5}1585$	—0.000000	+0.000000	0.000000
0.4	+0.009975	—0.000066	$0.0_{4}9950$	—0.000036	+0.000000	$0.0_{8}13$
0.6	+0.032986	—0.000727	0.001089	—0.000272	+0.000001	$0.0_{7}740$
0.8	+0.075417	—0.003822	0.005702	—0.001168	+0.000014	$0.0_{5}1364$
1.0	+0.13869	—0.013114	0.019406	—0.003692	+0.000127	$0.0_{4}1365$
1.2	+0.21811	—0.033420	0.048688	—0.009564	+0.000761	$0.0_{4}9205$
1.4	+0.30194	—0.067604	0.095741	—0.021512	+0.003431	$0.0_{3}4746$
1.6	+0.37413	—0.11339	0.15283	—0.042758	+0.012381	0.001982
1.8	+0.41927	—0.16361	0.20255	—0.074033	+0.036701	0.006828
2.0	+0.42540	—0.20984	0.22499	—0.10629	+0.089019	0.019222
2.2	+0.38265	—0.24505	0.20647	—0.11481	+0.17312	0.043151
2.4	+0.28139	—0.26169	0.14766	—0.072658	+0.26609	0.076081
2.6	+0.11340	—0.24544	0.073101	+0.019844	+0.32627	0.10684
2.8	—0.11591	—0.17015	0.042385	+0.13186	+0.32800	0.12497
3.0	—0.36875	—0.007369	0.13603	+0.23167	+0.27393	0.12871
3.2	—0.57221	+0.23557	0.38292	+0.30374	+0.10156	0.10257
3.4	—0.67074	+0.49096	0.69094	+0.34721	+0.060238	0.12419
3.6	—0.67610	+0.68357	0.92437	+0.36352	—0.084122	0.13923
3.8	—0.62930	+0.79366	1.02591	+0.34463	—0.25719	0.18492
4.0	—0.54117	+0.84018	0.99876	+0.26406	—0.45430	0.27612
4.2	—0.45509	+0.83324	0.90139	+0.092274	—0.63743	0.41483
4.4	—0.14374	+0.75131	0.58513	—0.16732	—0.73891	0.57399
4.6	+0.19200	+0.54017	0.32865	—0.45398	—0.70660	0.70537
4.8	+0.54414	+0.14186	0.31621	—0.71019	—0.53821	0.79404
5.0	+0.76461	—0.41634	0.75798	—0.82800	—0.27941	0.76365
5.2	+0.75951	—0.96688	1.51172	—0.84969	+0.012535	0.72214
5.4	+0.60104	—1.34115	2.1599	—0.77857	+0.30660	0.70017
5.6	+0.41961	—1.51180	2.4616	—0.62412	+0.59846	0.74769
5.8	+0.26938	—1.54535	2.4607	—0.36747	+0.87764	0.90529
6.0	+0.11083	—1.50202	2.2683	+0.014053	+1.08886	1.18581
6.2	—0.13929	—1.36441	1.88102	+0.48946	+1.14621	1.55336
6.4	—0.50784	—1.01966	1.29762	+0.96644	+0.99475	1.92354
6.6	—0.87096	—0.35714	0.88612	+1.35560	+0.65320	2.2643
6.8	—0.92269	+0.56377	1.16920	+1.49914	+0.15880	2.2726
7.0	—0.62236	+1.37738	2.2845	+1.56622	—0.33010	2.5620
8.0	+0.84097	+1.85467	4.1470	—0.95377	—1.46277	3.0494
9.0	—0.45624	—1.99780	4.1994	—1.44093	+1.48782	4.2899

TABLE VI

Intensity Functions from Mie Theory

$m = 1.25$ $\gamma = 135^0$

α	$R(S_1)$	$I(S_1)$	i_1	$R(S_2)$	$I(S_2)$	i_2
0.2	+0.001267	—0.000001	$0.0_5 1606$	—0.000896	+0.000001	$0.0_6 803$
0.4	+0.010217	—0.000066	$0.0_3 1044$	—0.007242	+0.000047	$0.0_4 5245$
0.6	+0.034831	—0.000728	0.001214	—0.024766	+0.000515	$0.0_3 6136$
0.8	+0.083102	—0.003845	0.006921	—0.059370	+0.002726	0.003532
1.0	+0.16184	—0.013310	0.026369	—0.11641	+0.009475	0.013640
1.2	+0.27492	—0.034565	0.076778	—0.19963	+0.024822	0.040467
1.4	+0.42270	—0.072594	0.18394	—0.31098	+0.053059	0.099525
1.6	+0.60408	—0.13085	0.38203	—0.45199	+0.098787	0.21405
1.8	+0.81828	—0.21435	0.71552	—0.62376	+0.17027	0.41806
2.0	+1.05995	—0.33384	1.23494	—0.82026	+0.28201	0.75236
2.2	+1.31011	—0.50013	1.96652	—1.01924	+0.44532	1.23716
2.4	+1.53948	—0.70845	2.8719	—1.19239	+0.65011	1.84444
2.6	+1.72964	—0.93735	3.8703	—1.33126	+0.87101	2.5309
2.8	+1.87908	—1.16841	4.8961	—1.44802	+1.10079	3.3085
3.0	+1.98959	—1.39928	5.9165	—1.53910	+1.35827	4.2137
3.2	+2.0499	—1.58068	6.7006	—1.56586	+1.57208	4.9234
3.4	+2.0308	—1.87937	7.6563	—1.49024	+1.94303	5.9962
3.6	+1.89775	—2.1018	8.0192	—1.31863	+2.1857	6.5162
3.8	+1.64101	—2.2495	7.7532	—1.08051	+2.3651	6.7610
4.0	+1.30274	—2.2780	6.8864	—0.78116	+2.4933	6.8267
4.2	+0.92390	—2.2086	5.7314	—0.38241	+2.5618	6.7090
4.4	+0.54008	—2.1012	4.7067	+0.062957	+2.5318	6.4141
4.6	+0.031073	—1.94304	3.7764	+0.57448	+2.3804	5.9963
4.8	—0.59374	—1.63016	3.0099	+1.08223	+2.1135	5.6383
5.0	—1.20297	—1.10992	2.6790	+1.63486	+1.73564	5.6852
5.2	—1.69391	—0.49073	3.1101	+2.1554	+1.23836	6.1795
5.4	—2.1308	+0.11273	4.5531	+2.6265	+0.62365	7.2872
5.6	—2.6066	+0.75467	7.3637	+3.0139	—0.087099	9.0911
5.8	—3.0477	+1.54410	11.6725	+3.2949	—0.86581	11.6063
6.0	—3.2907	+2.4239	16.7040	+3.4531	—1.68229	14.7539
6.2	—3.3323	+3.2181	21.460	+3.4819	—2.5057	18.4020
6.4	—3.2940	+3.8897	25.980	+3.3768	—3.3186	22.416
6.6	—3.1817	+4.5287	30.632	+3.0906	—4.1159	26.493
6.8	—2.9373	+5.1238	34.882	+2.6630	—4.8113	30.240
7.0	—2.3705	+5.5974	36.950	+2.0328	—5.3540	32.798
8.0	+0.96310	+4.6875	22.900	—2.2590	—5.2125	32.273
9.0	+3.7284	—0.96053	14.8234	—5.4519	+0.16158	29.749

TABLE VII

Intensity Functions from Mie Theory

$m = 1.20$ $\gamma = 90^0$

α	$R(S_1)$	$I(S_1)$	i_1	$R(S_2)$	$I(S_2)$	i_2
7.2	—0.77731	+1.00408	1.61238	+1.03734	—0.023278	1.07662
7.4	—0.58754	+1.52009	2.6559	+0.94363	—0.37011	1.02741
7.6	—0.32477	+1.81973	3.4169	+0.73436	—0.68204	1.00447
7.8	—0.066026	+1.91766	3.6818	+0.42848	—0.93532	1.05842
8.0	+0.16813	+1.84742	3.4412	+0.038062	—1.10634	1.22545
8.2	+0.39566	+1.63365	2.8254	—0.41800	—1.15064	1.49870
8.4	+0.64463	+1.25498	1.99051	—0.88067	—1.02841	1.83321
8.6	+0.88174	+0.65192	1.20246	—1.26878	—0.73101	2.1442
8.8	+0.96754	—0.17680	0.96739	—1.50593	—0.29368	2.3541
9.0	+0.76973	—1.05710	1.70995	—1.53666	+0.20767	2.4044
9.2	+0.34433	—1.74983	3.1805	—1.36592	+0.69224	2.3449

TABLE VIII

Specific Turbidities $(\tau/c)_0 \times 10^{-2}$ cm^{-1}. *in Dependence of Relative Diameter* α *from Mie Theory*

$\lambda_0 = 546.1$ mμ; $\varrho_1 = \varrho_2$

α \ m	1.199	1.210	1.214
0.2	0.393	0.436	0.452
0.4	3.08	3.42	3.55
0.6	9.96	11.07	11.47
0.8	22.0	24.5	25.4
1.0	38.9	43.3	44.9
1.2	58.4	65.3	67.9
1.4	78.8	87.9	91.2
1.6	97.7	108.8	113.1
1.8	116.6	130.0	135.1
2.0	137.4	153.6	160.0
2.2	162.1	181.6	188.6
2.4	187.7	210	218
2.6	210	235	244
2.8	229	256	266
3.0	248	276	286
3.2	267	298	309
3.4	286	319	331
3.6	305	339	352
3.8	321	355	368
4.0	337	373	385
4.2	352	389	403
4.4	380	418	432
4.6	380	418	432
4.8	392	430	445
5.0	404	443	457
5.2	415	455	469
5.4	424	463	478
5.6	432	470	484
5.8	440	476	491
6.0	448	485	498
6.2	455	490	502
6.4	460	494	506
6.6	463	498	508
6.8	468	499	509
7.0	470	500	510
8.0	472	494	501
9.0	455	463	467
10.0	425	423	421
11.0	381	373	369
12.0	333	318	312
13.0	283	264	257
14.0	234	217	211
15.0	192	178	173

TABLE IX

Specific Intensities $(I_{90}/I_0c)_0$ cm^{-1}. *in Dependence of Relative Diameter* α *from Mie Theory*

$\lambda_0 = 546.1$ mμ; $\varrho_1 = \varrho_2$

m α	1.199	1.210	1.214
0.2	2.35	2.60	2.69
0.4	18.40	20.4	21.2
0.6	59.0	65.8	68.2
0.8	129.9	145.0	150.0
1.0	223	249	259
1.2	324	362	376
1.4	406	453	471
1.6	442	491	510
1.8	427	473	490
2.0	363	404	419
2.2	274	306	316
2.4	180.0	202	209
2.6	103.1	118.4	124.2
2.8	60.5	73.3	78.3
3.0	68.6	87.1	94.2
3.2	122.9	149.6	159.7
3.4	195.0	231	245
3.6	245	281	294
3.8	256	287	299
4.0	233	259	270
4.2	191.5	215	224
4.4	144.6	162.4	169.5
4.6	101.5	116.5	122.9
4.8	76.7	92.5	99.1
5.0	83.9	106.0	114.6
5.2	124.3	153.2	164.1
5.4	174.0	206	217
5.6	206	234	244
5.8	209	232	240
6.0	189.0	210	218
6.2	157.1	176.4	184.2
6.4	126.6	144.6	151.5
6.6	98.4	113.9	120.4
6.8	83.9	101.2	108.5
7.0	96.2	121.2	132.2
8.0	164.5	182.5	189.4
9.0	101.2	121.7	129.9

TABLE X

Logarithms of Dissymmetries $\log(1/z)_0$ *in Dependence of Relative Diameter* α *from Mie Theory*

m α	1.199	1.210	1.214
0.2	0.989-1	0.988-1	0.988-1
0.4	0.957-1	0.957-1	0.957-1
0.6	0.903-1	0.903-1	0.903-1
0.8	0.828-1	0.827-1	0.827-1
1.0	0.721-1	0.720-1	0.720-1
1.2	0.580-1	0.579-1	0.578-1
1.4	0.386-1	0.380-1	0.379-1
1.6	0.107-1	0.096-1	0.091-1
1.8	0.670-2	0.650-2	0.640-2
2.0	0.910-3	0.860-3	0.844-3
2.2	0.620-3	0.702-3	0.730-3
2.4	0.260-2	0.307-2	0.322-2
2.6	0.520-2	0.550-2	0.559-2
2.8	0.591-2	0.600-2	0.600-2
3.0	0.511-2	0.50-2	0.498-2
3.2	0.288-2	0.250-2	0.237-2
3.4	0.922-3	0.918-3	0.918-3
3.6	0.763-3	0.830-3	0.860-3
3.8	0.146-2	0.232-2	0.262-2
4.0	0.512-2	0.572-2	0.593-2
4.2	0.767-2	0.804-2	0.819-2
4.4	0.915-2	0.939-2	0.947-2
4.6	0.967-2	0.973-2	0.977-2
4.8	0.908-2	0.910-2	0.911-2
5.0	0.837-2	0.860-2	0.870-2
5.2	0.008-1	0.061-1	0.080-1
5.4	0.230-1	0.260-1	0.270-1
5.6	0.285-1	0.280-1	0.278-1
5.8	0.180-1	0.149-1	0.135-1
6.0	0.935-2	0.880-2	0.860-2
6.2	0.559-2	0.508-2	0.490-2
6.4	0.240-2	0.246-2	0.255-2
6.6	0.320-2	0.389-2	0.410-2
6.8	0.510-2	0.563-2	0.580-2
7.0	0.609-2	0.632-2	0.640-2

TABLE XI

Logarithms of Polarization Ratios $\log(\varrho_{90})_0$ *in Dependence of Relative Diameter* α *from Mie Theory*

$\alpha \backslash m$	1.199	1.210	1.214
0.2	—	—	—
0.4	0.861-6	0.917-6	0.937-6
0.6	0.620-5	0.670-5	0.689-5
0.8	0.168-4	0.219-4	0.235-4
1.0	0.626-4	0.678-4	0.696-4
1.2	0.047-3	0.100-3	0.120-3
1.4	0.451-3	0.510-3	0.530-3
1.6	0.847-3	0.910-3	0.930-3
1.8	0.239-2	0.308-2	0.330-2
2.0	0.623-2	0.696-2	0.720-2
2.2	0.007-1	0.080-1	0.106-1
2.4	0.395-1	0.470-1	0.498-1
2.6	0.849-1	0.923-1	0.950-1
2.8	0.388	0.430	0.442
3.0	0.105	0.074	0.063
3.2	0.537-1	0.520-1	0.512-1
3.4	0.179-1	0.196-1	0.200-1
3.6	0.000-1	0.040-1	0.054-1
3.8	0.988-2	0.047-1	0.070-1
4.0	0.119-1	0.191-1	0.220-1
4.2	0.340-1	0.416-1	0.440-1
4.4	0.632-1	0.719-1	0.750-1
4.6	0.010	0.089	0.115
4.8	0.370	0.399	0.406
5.0	0.190	0.151	0.140
5.2	0.760-1	0.740-1	0.736-1
5.4	0.456-1	0.456-1	0.458-1
5.6	0.300-1	0.330-1	0.342-1
5.8	0.280-1	0.334-1	0.354-1
6.0	0.380-1	0.455-1	0.480-1
6.2	0.571-1	0.653-1	0.681-1
6.4	0.820-1	0.902-1	0.932-1
6.6	0.135	0.210	0.231
6.8	0.380	0.397	0.399
7.0	0.174	0.146	0.136
8.0	0.561-1	0.461-1	0.430-1
9.0	0.077	0.103	0.115

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IZVOD

Neke dodatne funkcije rasipanja svijetlosti za polistirenske latekse

Gj. Deželić

Za obradu eksperimentalnih rezultata dobivenih mjerenjem polistirenskih lateksa izračunane su neke dodatne funkcije rasipanja svjetlosti. Podaci za i_1 te i_2, kao i za realne i imaginarne dijelove kompleksnih amplitudnih funkcija $R(S_1)$, $I(S_1)$, $R(S_2)$ i (S_2) za $m = 1{,}15$ i $1{,}25$ i $\gamma = 45^0$, 90^0 i 135^0 prikazani su u tablicama. Iz tih rezultata izračunani su teoretski specifični intenziteti svjetlosti rasute pod kutom od 90^0, dissimetrije i polarizacijski omjeri. Funkcije za $m = 1{,}199$, $1{,}210$ i $1{,}214$ dobivene su grafičkom interpolacijom. Raspravljen je problem određivanja položaja maksima i minima intenziteta kutno rasute svjetlosti.

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