# Ligand Field Splitting of $d$-Orbitals in Eight Coordinated Complexes of Square Antiprism Structure 

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#### Abstract

The splitting of $d$-orbitals in the ligand field of a square antiprism structure is given. Fivefold degenerate level splits into $A_{1}+E_{2}+E_{3}$. The energy order of $d$ orbitals is obtained from known order of $d$ orbitals in a cube considering the effect of distortion of a cube into an antiprism on each of $d$ orbitals. It follows that $\mathrm{d}_{\mathrm{z}^{2}}$ orbital is substantially more stable than others.


The UV spectra and magnetic properties of transition metal complexes may be interpreted by the splitting of the $d$ orbitals in the electrostatic field of the ligands. Bethe ${ }^{1}$ has described the splitting of different degenerate orbitals under the influence of the outer electrostatic fields in crystals. Following this work Ilse and Hartmann ${ }^{2}$ successfully explained the observed absorption spectra of a few metal ions, assuming that the absorption is due to a promotion of an electron from a lower to a higher level of the originally degenerate set of levels. Several authors have later studied a large number of complexes of different symmetry and structure and most of the important cases have now been examined. However, the ligand field splittings for complexes of the Archimedean square antiprism geometry have not yet been reported. This problem is of interest since the antiprism as a coordination polyhedron is found ${ }^{3}$ in the methyl acetylacetonates of $\mathrm{Zr}, \mathrm{Ce}, \mathrm{Th}$ and U .

The splitting of the degenerate levels depends on the symmetry properties of the perturbation potential field only. This is determined by the symmetry of the coordination polyhedron and can be obtained by the methods of group theory. The square antiprism belongs to the symmetry point group $\mathrm{D}_{4 \mathrm{~d}}$ and for this group the fivefold degenerate $d$ orbitals split into one single and two double degenerate levels (see Appendix). The order of these levels depends generally on the geometry of the structure considered and can be calculated by perturbation theory (for an example see ref. 4). The calculation of the perturbation matrix elements is very lengthy since the corresponding integrals involve the factor $1 / \mathrm{r}_{12}$. However, in the case of the antiprism, the order of $d$ orbitals can be obtained by a comparison of the known order of the splitting of $d$ orbitals in a tetrahedron and a cube.

A cube and a tetrahedron are simply related: alternate corners of a cube make a tetrahedron. Therefore the splitting for a cube is twice as large as

## APPENDIX

Characters for $(21+1)$ dimensional representation can be obtained using

$$
\begin{array}{ll}
x(\mathrm{I}) & =21+1 \\
x\left(\mathrm{C}_{2}\right) & =(-)^{1} \\
x\left(\mathrm{C}_{4}\right) & =(-)^{1 / 2} \\
x(\sigma) & =1 \\
x\left(\mathrm{~S}_{4}\right) & =\sum_{-1}^{+1}(-)^{1+|m|} \mathrm{e}^{\mathrm{im} \frac{\pi}{2}}
\end{array}
$$

These relations are derived from representation matrices of symmetry elements; and are described at some length for example by Kelen ${ }^{6}$, where the explanation of the symbols used and the origin of the formulae can be found.

The characters of the $n$ dimensional representations of the square antiprism are given in Table I. This table can be compared with the character table for the point group $\mathrm{D}_{4 \mathrm{~d}}$ (see for example Herzberg7).

TABLE I
Characters of the n dimensional representations of the rotation group restricted by the symmetry properties of square antiprism.

| 1 | I | $\mathrm{S}_{8}$ | $\mathrm{C}_{4}$ | $\mathrm{~S}_{8}{ }^{8}$ | $\mathrm{C}_{2}{ }^{\prime \prime}$ | $\mathrm{C}_{2}$ | $\sigma_{\mathrm{d}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 3 | $1+\sqrt{2}$ | 1 | $1-\sqrt{2}$ | -1 | 1 | 1 |
| 2 | 5 | $1+\sqrt{2}$ | -1 | $1-\sqrt{2}$ | -1 | 1 | 1 |
| 3 | 7 | 1 | -1 | 1 | -1 | 1 | 1 |
|  | TABLE II |  |  |  |  |  |  |

Symmetry types and characters for the point group $\mathrm{D}_{4 \mathrm{~d}}$

| $\mathrm{D}_{4 \mathrm{~d}}$ | I | $\mathrm{S}_{8}$ | $\mathrm{C}_{4}$ | $\mathrm{~S}_{8}{ }^{3}$ | $\mathrm{C}_{2}{ }^{\prime \prime}$ | $\mathrm{C}_{2}$ | $\sigma_{d}$ |
| :--- | ---: | ---: | ---: | ---: | :---: | ---: | ---: |
| $\mathrm{~A}_{1}$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\mathrm{~A}_{2}$ | 1 | 1 | 1 | 1 | 1 | -1 | -1 |
| $\mathrm{~B}_{1}$ | 1 | -1 | 1 | -1 | 1 | 1 | -1 |
| $\mathrm{~B}_{2}$ | 1 | -1 | 1 | -1 | 1 | -1 | 1 |
| $\mathrm{E}_{1}$ | 2 | $\sqrt{2}$ | 0 | $-\sqrt{2}$ | -2 | 0 | 0 |
| $\mathrm{E}_{2}$ | 2 | 0 | -2 | 0 | 2 | 0 | 0 |
| $\mathrm{E}_{\mathbf{2}}$ | 2 | $-\sqrt{2}$ | 0 | $\sqrt{2}$ | -2 | 0 | 0 |

The point group $D_{4 d}$ has four one-dimensional and three two-dimensional irreducible representations Table II. Therefore every term of free atom of a higher degree of degeneracy than two will split in the crystal field of an anti-prism. By a comparison of the two tables of characters one obtains:

$$
\begin{array}{lll}
\mathrm{l}=0 & s & \mathrm{~A}_{1} \\
\mathrm{l}=1 & p & \mathrm{~A}_{1}+\mathrm{E}_{3} \\
\mathrm{l}=2 & d & \mathrm{~A}_{1}+\mathrm{E}_{2}+\mathrm{E}_{3} \\
\mathrm{l}=3 & \mathrm{f} & \mathrm{~A}_{1}+\mathrm{E}_{1}+\mathrm{E}_{2}+\mathrm{E}_{3}
\end{array}
$$

i. e., Table I can be obtained from Table II by performing the above indicated additions of the corresponding rows. We see that the three $p$ orbitals and the five $\dot{d}$ orbitals are no longer equivalent. The five $d$ orbitals are split into three terms $A_{1}, E_{2}$ and $E_{3}$. That is, the orbitals are not all equally efficient in avoiding negative charges situated at the corners of the antiprism. Knowing the forms of these orbitals, these results are to be expected.

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## IZVOD

## Cijepanje $d$ orbitala kompleksa koordinacije osam u ligandnom polju strukture kvadratične antiprizme

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Opisano je cijepanje degeneriranoga terma $d$ elektrona $u$ elektrostatskom polju kompleksa geometrije kvadratične antiprizme. Peterostruko degenerirani term se cijepa $u$ tri nivoa: $A_{1}+E_{2}+E_{3}$. Energetski redoslijed novih nivoa dobiven je iz poznatoga redoslijeda $d$ orbitala u kocki razmatrajući efekt deformacije kocke u antiprizmu na pojedine $d$ orbitale. Rezultat je znatna stabilizacija $d_{i 2}$ orbitale u odnosu na ostale $d$ orbitale.

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