Review of Psychology,
2004, Vol. 11, No. 1-2, 35-44

UDC 159.9

Dynamics of two psychomotor activities: Chaotic properties

ANA PROROKOVIĆ and LJILJANA GREGOV

The aim of this study was to find out whether observed time series of complex reaction time (CRT) and tapping intervals (TI) reflect nonlinear, chaotic dynamics of underlying cognitive and motor processes. Seven female subjects, aged between 19-25 participated in 2x2 within groups factor experiment (two levels of task difficulty and two types of performance; single and dual). Seven time series were observed: Four CRT series (single and dual performance of two primary tasks) and three TI time series (single performance and dual performance with two primary tasks. Analysis of CRT series and TI time series included calculation of various nonlinear parameters (correlation dimension, largest Lyapunov exponent), as well as an application of quantitative recurrence analysis (%determinism, divergence) and the surrogate data technique, for testing nonlinearity. The results indicated the existence of chaotic dynamics in human response time, as well as in tapping data. In general, it seems that increased task requirements (difficulty) caused a reduction in the dimensionality of CRT series, i.e. underlying system dynamics. It was also found that one of the nonlinear properties of the CRT series and TI time series was their sensitive dependence on initial conditions. Maximum Lyapunov exponents for CRT and TI time series were greater for more complex tasks, indicating deterministic and chaotic nature of undergoing cognitive and motor processes.

Keywords: nonlinear dynamics, psychomotor activity, dimensionality, predictability

Recent, theoretical and quantitative concepts of chaos theory have been found applicable in the field of psychology and related areas (Guastello, 2000). Chaos theory itself is variously referred to as "deterministic chaos", "the chaos paradigm", "chaotic", "chaos science". Some talk more generally about "dynamic system theory", "nonlinear dynamics" or "theories of complexity" (Ayers, 1997).

Chaos has been formally defined as "stohastic behaviour in a deterministic system", i.e. a system which displays apparent random behaviour, but has an underlying pattern of lawfulness. The most effective contribution of nonlinear analysis resulting from chaos theory is a better understanding of underlying processes in human behavioral dynamics. For example, the phenomenon of chaos (sensitive dependence) itself occurs only in nonlinear interdependent systems. Nonlinearity alone is not enough. The nonlinear revolution, then, is about exploring the nature of nonlinear interdependency, which, in the final analysis, is what all real world systems are (Robertson, 1995).

Although nonlinear methods of data analysis have been in use for more then ten years, there is a rather small number of studies involving dynamical aspects of psychomotor and cognitive behavioral systems. Since reaction time (RT) may provide a good measure of some cognitive dynamics, there has been much more interest in this area recently (Beltz & Kello, 2004; Ding, Chen, & Kelso, 2002; Gilden, Thornton, & Mallon 1995; Gilden 1997; Kelly, Heathote, Heath & Longstaff, 2001, Pressing, 1999). Traditionally, fluctuations of RT are characterized as stochastic (Kelly et al, 2001). From this perspective, these fluctuations are composed of a true score plus random error. Furthermore, common statistical analysis often ignores two sources of information, dynamic structure in the inter-trial RT fluctuations and individual differences in RT variability. Some recent investigations have showed clear evidence for non-random dynamic structure in performance fluctuations: short-term linear dependencies and longer-term rhythms (Gilden et al., 1995, Kelly et al., 2001, Pressing, 1999). These linear de-

Ana Proroković, Department of Psychology, University of Zadar, Krešimirova obala 2, 23000 Zadar, Croatia. E-mail: aprorok@unizd.hr (the address for correspondence).

Ljiljana Gregov, Department of Psychology, University of Zadar, Krešimirova obala 2, 23000 Zadar, Croatia. E-mail: ljgregov@unizd.hr

pendencies between sequential responses have been found in vigilance tasks, serial response task and some perception tasks (Kelly et al, 2001). In general, dynamical analysis (linear and nonlinear) examines data for evidence of dependence between its states at different time intervals and implies that future states are a function of past states. While linear dynamic systems produce relatively simple and regular outputs (time series data), nonlinear dynamic systems usually produce complex output. On the other hand, complex output can be produced by nonlinear interactions between a small number of variables in which case it is considered the low dimensional –chaos. In the same nonlinear context, interaction of large number of variables produces high dimensional or hyper-chaos (Heath, 2000). Existence of low dimensional chaotic structure opens up the possibility that RT fluctuations can be modeled by relatively simple recursive equations with only a few parameters. From this point of view, Cooney & Troyer (1994) proposed a simple nonlinear difference equation to describe the dynamics underlying reaction time on the Steinberg short-term memory scanning task. In this task, subjects are presented with a set of numbers or letters to be remembered. A probe item is given following presentation of the memory set. The task is to make a manual response indicating whether or not the probe item belongs to the memory set. They hypothesized that *trial to trial* variability in speed of access to recently presented information is deterministic in nature and that it reflects important properties of immediate memory. Their model posits that RT is a function of the interaction of two control parameters: susceptibility to interference and episodic activation. Evaluation of the model indicated an exceptionally good fit with the raw empirical observations. Brown and Heathcote (2005) proposed the so-called ballistic (i.e. deterministic within trial) model of RT variability, which is capable to determine relationship between error and correct RT, but can also model other benchmark behavioral phenomena.

Generally, nonlinear analysis of data, necessary includes assessments of various nonlinear parameters and testing nonlinearity of data (Heath, 2000). Most frequently used quantitative indices of nonlinear dynamics are parameters of system dimensionality, Lyapunov exponents and indices resulting from recurrence quantification analysis.

Therefore, the aim of this study was to find out whether series of complex reaction time (CRT) and tapping intervals (TI) contain some kind of nonlinear, chaotic dynamic structure. Several nonlinear statistical techniques have been applied, in attempt to:

- assess the dimensionality of the data;
- assess the possibility that the data are the product of a deterministic system;
- distinguish nonlinear, linear and noise components in time series.

METHOD

Subjects

Seven female subjects (volunteers), university students, aged between 19-25 participated in this study. All subjects were right-handed.

Experimental situations

The study consisted of five comparative experimental situations with duration of twenty minutes each:

1. *CRT task 1* performed as *single* task (independently),
2. *CRT task 2* performed as *single* task (independently),
3. *Tapping task* performed as *single* task (independently),
4. *CRT task 1* performed as *dual* task (simultaneously with tapping task),
5. *CRT task 2* performed as *dual* task (simultaneously with tapping task).

"Dual task" situations included participant's work on two psychomotor tasks simultaneously. The primary task was CRT task, and secondary task was tapping task. It means that subject had to be oriented primarily on CRT task performance, while the tapping task was performed in dependence of allowance of the primary activity.

Those experimental situations produced 7 different types of time series: 3 TI time series (independent performance, simultaneous performance with CRT task 1 and simultaneous performance with CRT task 2) and 4 CRT series (independent and simultaneous performance of CRT task 1 and CRT task 2). The lengths of CRT series vary (interindividually) between 500 and 800 time points and the lengths of TI time series vary between 2500 and 4500 time points.

Before starting the main experiment, subjects exercised single and dual task performance about one hour. The time sequence of experimental situations was rotated by *latin square* principle between subjects.

CRT task description

Tapping task

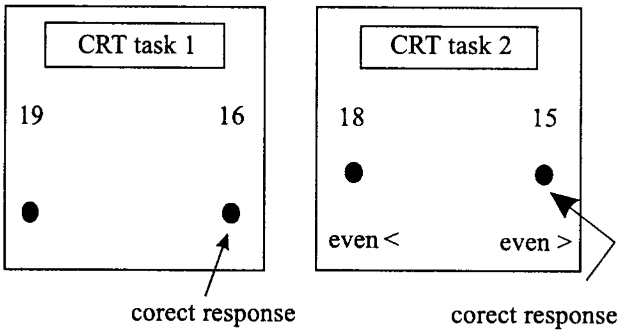
In this simple psychomotor task, subject had to "tap" on one computer key ("space") with his left (non-dominant) hand and by his own rhythm, as regularly, as he could.

CRT task 1

The stimulus (two numbers) was presented on computer screen. One number was randomly chosen odd number, and second number was randomly chosen even number (in the range 1-20). The subject's task was to react on position of even number by pressing the corresponding key with his right (dominant) hand.

CRT task 2

The presentation of stimulus was the same as in the CRT task 1. The subject's task was to react using right key if even number was larger then odd number, or to react using the left key if even number was smaller then odd number.



Program was run under Windows 98 operating system (DirectRT v2004; Jarvis, 2000). For greater precision of the keyboard, stimulus exchange and time registration program uses DirectX drivers.

Nonlinear data analyses

Quantitative nonlinear dynamics parameters and surrogate data were estimated based on the following methods:

1. *Correlation dimension (D_2):* Grassberger & Proccacia (1983) technique;
2. *Largest Lyapunov exponent (LLE):* Wolf, Swift, Swiney, and Vastano (1985) method;
3. *Surrogate data*: Theiler, Eubank, Longtin, Galdrikian and Farmer (1992) adjusted Fourier transform surrogates (AAFT) algorithm;
4. *Recurrence Quantification Analysis RQA*: (Zbilut, Giuliani, & Webber, 1998) determinism (DETER), divergence (DIVERG).

RESULTS

In general, descriptive parameters of CRT series (mean and variability measures) for different experimental situations were sensitive to task difficulty, but could not differentiate single from dual task performance (Table 1). Graphical examples of CRT and TI time series are presented on Figure 1. As could be expected, variability parameters of TI series were greater in situations where tapping task was performed as dual task. Therefore, the main interest of this study was to get insight in this variability, i.e. in dynamic structure of measured time series. Are fluctuations of CRT and TI really random, or maybe deterministic and nonlin-

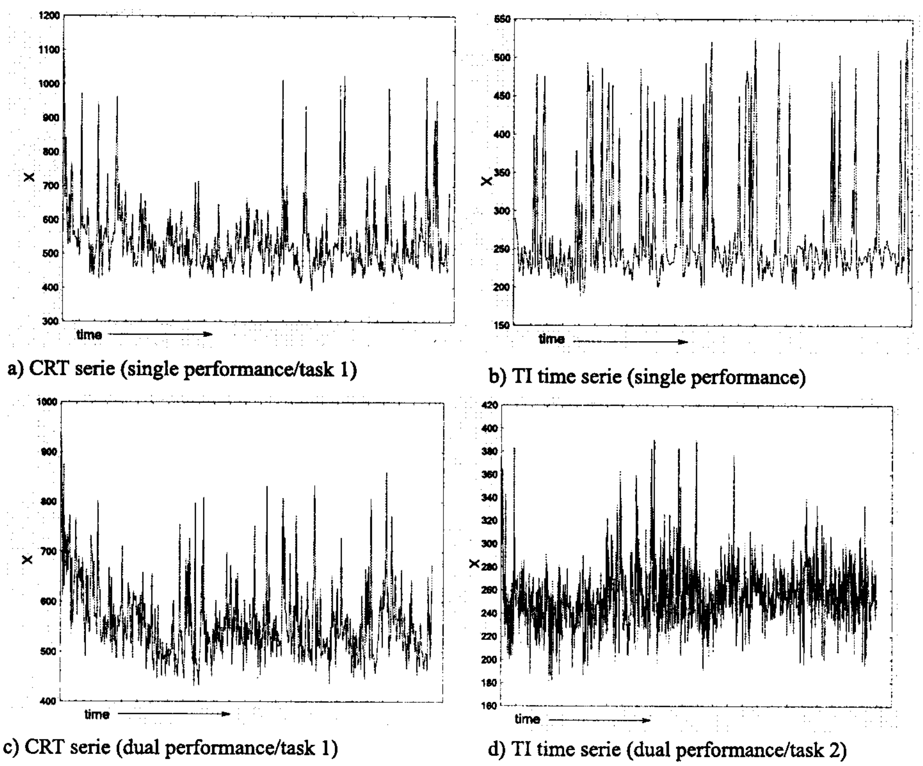


Figure 1. CRT and TI time series (examples)

Table 1

Descriptive statistics for different experimental situations

	M (ms)	*SD* (ms)	*V*	*DM* (ms)	*DSG* (ms)
Tapping task (single)	298.92	70.87	0.24	63.43	67.82
Tapping task (dual with CRT task 1)	313.59	121.43	0.39	112.20	110.27
Tapping task (dual with CRT task 2)	295.06	94.70	0.32	98.00	97.51
CRT task 1 (single)	620.10	121.86	0.20	109.04	104.87
CRT task 1 (dual)	611.09	115.04	0.19	105.79	103.73
CRT task 2 (single)	951.06	226.55	0.24	181.99	180.31
CRT task 2 (dual)	859.59	195.67	0.23	192.04	166.29

Note. V-index of variability (sd/M); DM-index – mean value of absolute differences between successive time intervals; DSG – standard deviation of absolute differences.

ear in nature? To answer this question, different nonlinear analyses were performed.

To distinguish linear components of time series, "white noise" components, and potential chaotic properties, surrogate data technique has been performed. Surrogate time series calculated with algorithms by Theiler et al. (1992) have the same distribution (M, SD) and the same spectra as original time series, but the phases of linear components are randomly shuffled. Thus, comparing the original and surrogate data properties give us an opportunity to discuss about possible nonlinear aspects of the dynamic system (Proroković, 2002).

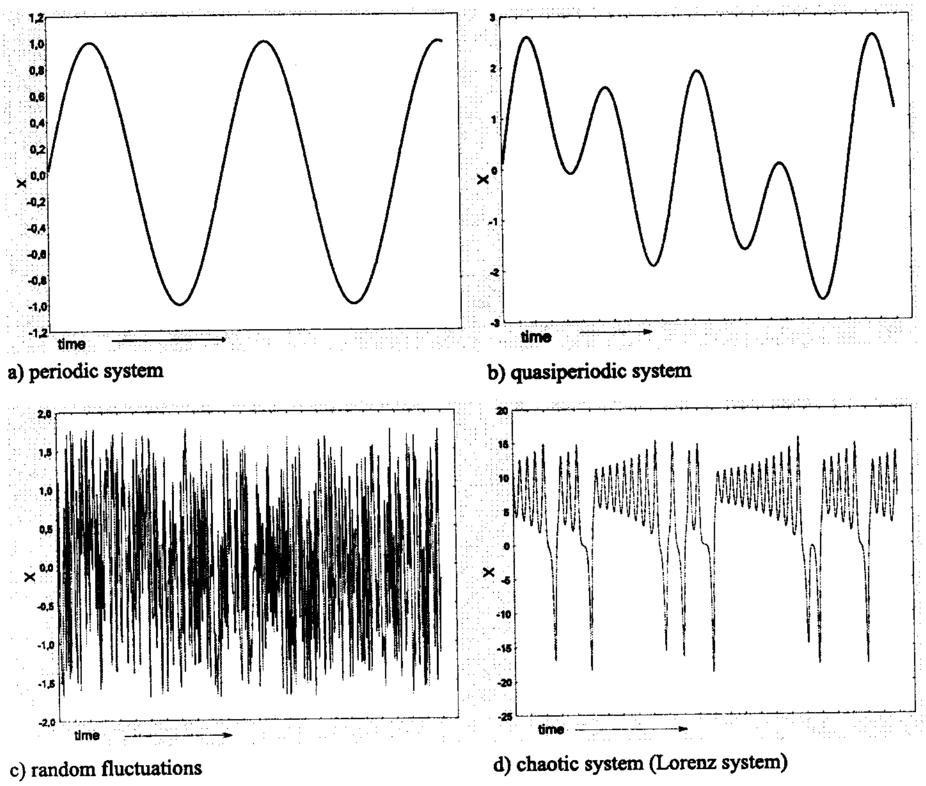


Figure 2. Graphical representations of different dynamical systems

Estimates of dimensionality

The concept of dimension could be used in more than one sense: number of dimensions of Euclidean space, the number of variables in a dynamic system and so-called fractal dimension which often characterizes irregular geometric objects, i.e. dynamic systems presented in phase-space. Fractal dimension is also a term related to fractal objects (for example Cantor set, Koch snowflake, Mandelbrot set, etc.), which are known as self-similar mathematical structures produced by simple repetitive mathematical operations. Some of the most important fractal dimensions for practical applications are: Huseldorff-Besicovitch dimension, correlation dimension, and Hurst exponent.

From a non-linear dynamical perspective, variability is interaction of a limited number of non-independent nonlinear dynamical factors (Frey & Clayton, 1996). Therefore, assessing the fractal properties of an observed time series could be very informative; chaotic system usually has fractal dimension greater than 2, periodic (sine wave) and quasiperiodic systems (interferention of two or more sine waves) between 1 and 2, while random systems (noise) do not have finite dimensionality. Graphical representations of those systems are shown on Figure 2.

Correlation dimension

The most popular attempts to characterize attractors and dimensional complexity have been based on the correlation dimension as proposed by Grassberger and Procaccia (1983). Their algorithm has been performed to calculate correlation dimension for CRT series and TI time series (Figures 3 and 4). Correlation dimension is typically computed as the slope of the correlation integral from a reconstructed state-space.

Correlation dimension could be interpreted as fractal dimensionality, an indicator of the information complexity of a time series, i.e. number of dimensions required to predict behaviour of some dynamic system (Heath, 2000).

For the TI time series correlation dimension varied between 7 and 10, while for the CRT series it varied between 6 and 8. The results are similar to those of Frey and Clayton (1996) and indicate existence of 6-8 non-independent nonlinear dynamical factors, which are determining variability of CRT data. Surrogate data had not finite dimensionality, which could mean that Gaussian noise fits the data much better than linear sinusoidal waves. Although there was a tendency of greater dimensionality in tapping intervals during simultaneous performance (dual task), there was no significant difference between correlation dimensions of TI series in different experimental situations ($F(2,12) = 0.79, p>.05$).

On the contrary, dimensionality of CRT series was significantly lower for more difficult task than for the easier one ($F(1,6) = 6.65, p<.05$), while there was no significant difference in dimensionality of CRT during independent and simultaneous performance ($F(1,6) = 0.13, p>.05$). In general, it seems that increased task requirements caused a reduction in the dimensionality of CRT underlying system dynamic. Comparing these results with descriptive variability parameters (*SD*, *DM*, *V*, *DSG*), we could say that greater variability of CRT in more difficult task did not reflect influence of more random factors and/or linear factors, but could reflect complex dynamics of some nonlinear system. As Kelly et al. (2001) concluded, there is a clear evidence

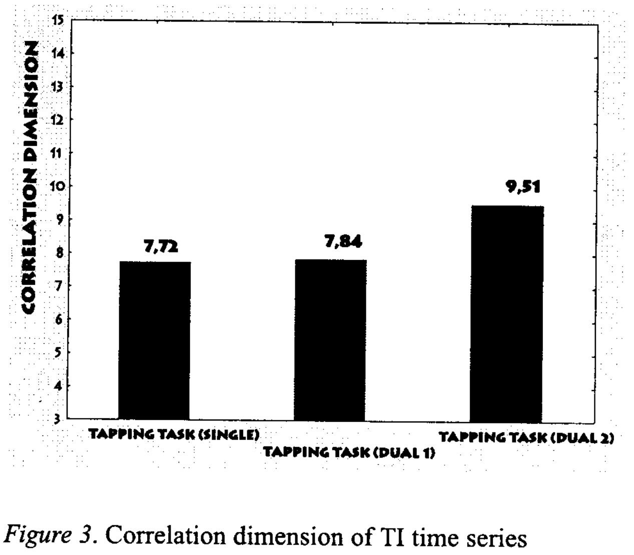


Figure 3. Correlation dimension of TI time series

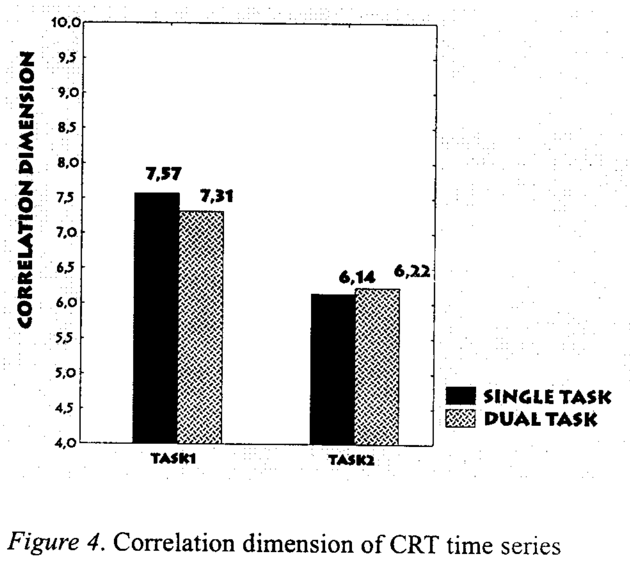


Figure 4. Correlation dimension of CRT time series

for chaotic dynamics in human response time data, but it is difficult to detect low dimensional chaos in experimental data filled with noise. That is because the noise component has significant effect on increased dimensionality of time series. In spite of those facts, it could be speculated that psychomotor dynamics in tapping task, as well as in CRT task, may be of finite dimensionality, which possibly originates from simultaneous psychophysiological processes spanning in range from 6 to 10.

Estimates of predictability (instability)

The correlation dimension is not necessarily correlated with predictability and could not be the only indicator of possible chaotic dynamics. For example, noise is "very complex" but there is no predictability at all. On the other hand, a waveform, which is linearly built up by very many sine waves, may be understood as a complex signal, but it remains very predictable. One of the most important chaotic properties of some dynamic system is sensitivity to the initial conditions (SIC). That is, two nearby points in the phase-space diverge as the orbits (trajectories) of attractor progress. The points are known to diverge exponentially, and this divergence could be examined in terms of Lyapunov exponents spectrum. The Lyapunov exponent and chaos occurs when at least one exponent in the spectrum is positive. A value of zero indicates a periodic, totally predictable system; positive exponents indicate chaotic system behaviour, whereas negative exponents suggest that system does not show chaotic behaviour (noise). Therefore, to gain more information about possible chaotic features of the system dynamics, it should be sufficient to compute the largest Lyapunov exponent. Positive LLE suggests that time series exhibit sensitive dependence on initial condition, which is characteristic of chaotic systems. From this point of view, the less predictable the time series, the greater is LLE.

Two more quantitative parameters related to system predictability are divergence and determinism. These indices are result of quantitative recurrence analysis, another very popular nonlinear approach concerned with graphical representation of system dynamics (Zbilut et al., 1998). Compared with other nonlinear indices, those parameters could also be very informative.

Largest Lyapunov exponent

As we can see, complexity of some dynamic system depends not only of the number of variables defining the system, but on the amount of noise in the data as well. In practical applications, a time series generated by a deterministic, possibly chaotic, nonlinear system will have superimposed measurement noise. This noise will contaminate the computation of quantitative indices such as correlation dimension and LLE (Heath, 2000). So, the next step in analysis was to calculate LLE, that is, a measure of predictability and possible chaotic dynamics (Figures 5 and 6).

The LLE was positive for both time series (TI and CRT), but not in all experimental situations ($F(2,12) = 11.29$, $p<.05$). When task performance was simultaneous, TI time series exhibit sensitive dependence on initial condition, and could be chaotic. On the contrary, surrogate data exhibit no sensitivity on initial condition, i.e. Lyapunov exponent was approximately zero ($F_{TI}(1,6) = 11.5$, $p<.05$, $F_{CRT}(1,6) = 20.92$, $p<.05$). In CRT series LLE was greater for more difficult task, while it was very low for easier task ($F(1,6) =$

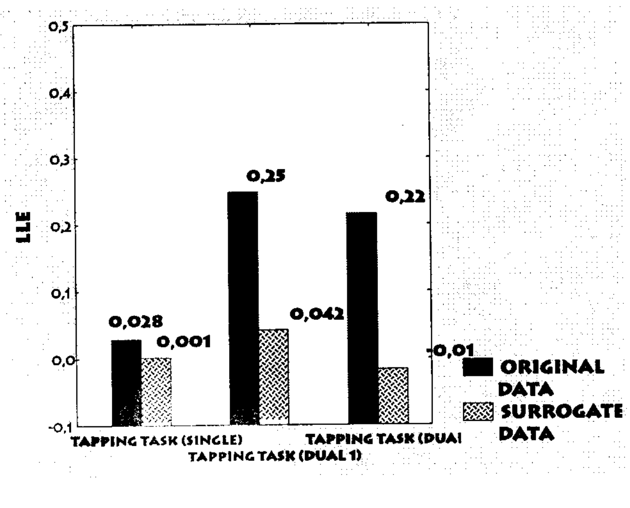


Figure 5. Largest Lyapunov exponent (LLE) of TI time series

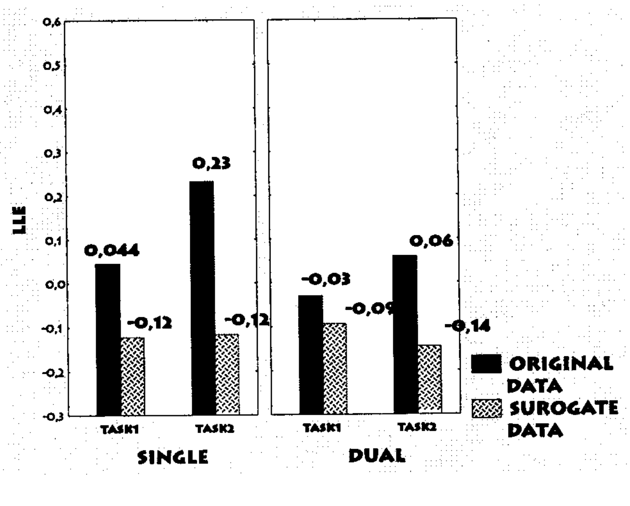


Figure 6. Largest Lyapunov exponent (LLE) of CRT series

3.84, $p<.05$). These results strongly suggest the possibility that CRT series and TI time series are the product of some deterministic, chaotic, underlying dynamic system. Heath (2000) computed Lyapunov spectrum for the reaction time series and obtained LLE of 0.215. Because the sum of Lyapunov exponents was positive and data exhibited high finite dimensionality, he suggested that the reaction time series is nonlinear, but quite noisy. There were not significantly differences in LLE between single and dual performance ($F(1,6) = 1.11$, $p>.05$).

Since application of Wolf's algorithm to short and noisy time series is very spurious, the data of this study could be treated with considerable reserve (Elbert, Ray, Kowalik, Skinner, Graf, & Birbaumer, 1994). Therefore, to gain more information about the non-linearity and predictability of the TI and CRT series, Recurrence quantification analysis (RQA) was applied. Since RQA methodology is independent of limiting constraints such as data set size, data stationarity and statistical distributions (Zbilut et al, 1998), two additionally nonlinear parameters, which are related to predictability of time series, were calculated (divergence and determinism).

Divergence

Divergence of time series trajectories is defined by the reciprocal of the longest line segment in the recurrence plot, plotted in the phase space (rescaled by multiplying by 1000). Examples of recurrence plots for different dynamical systems are shown on Figure 7.

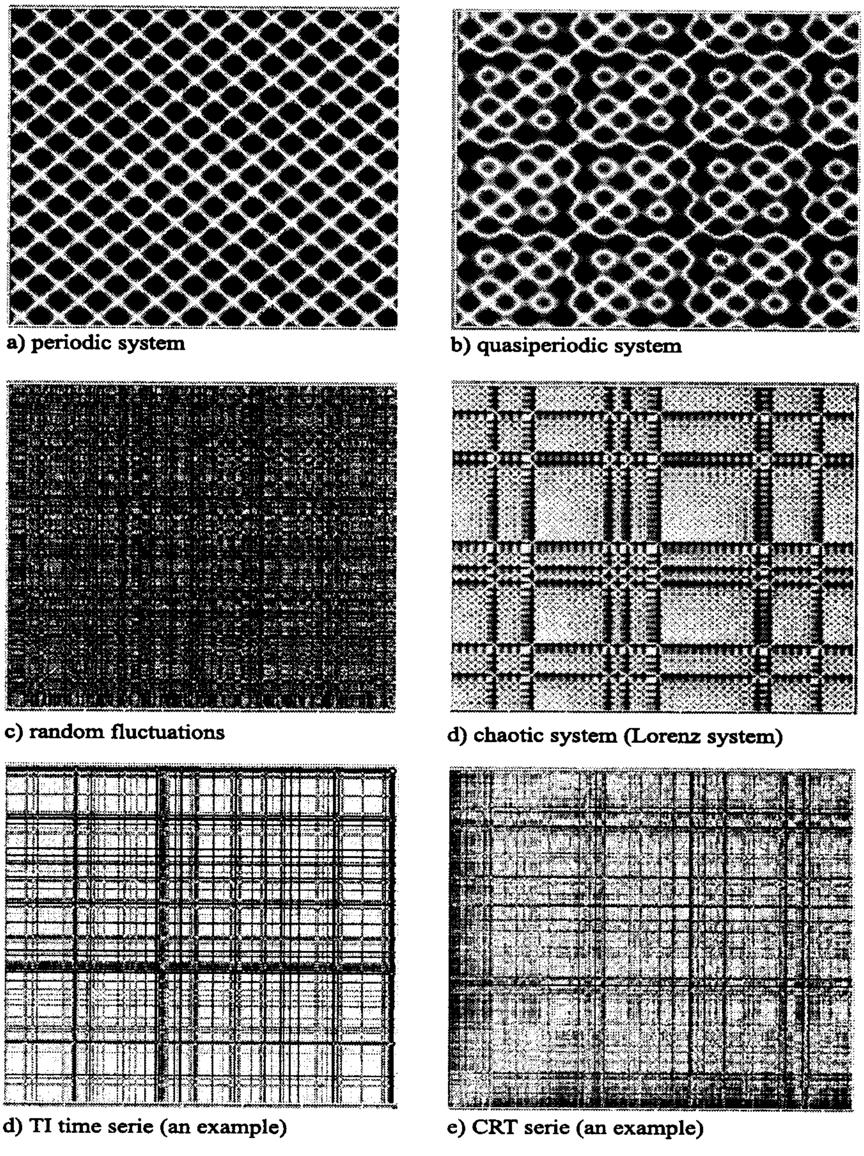


Figure 7. Examples of recurrence plots for different dynamical systems

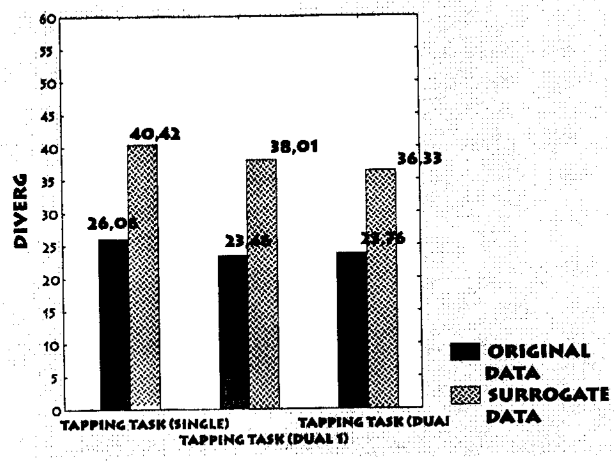


Figure 8. Index of divergence of TI time series

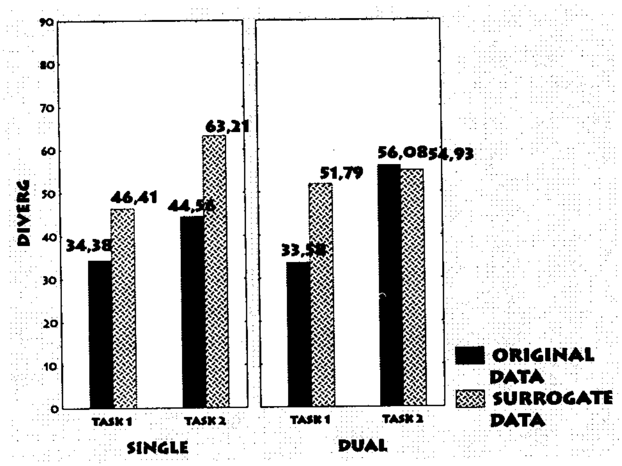


Figure 9. Index of divergence of CRT series

The less predictable the time series, the greater is divergence. Divergence is usually smallest for structured cyclic sample, greatest for the noise sample, while it is intermediate for the chaotic sample (Heath, 2000).

In general, divergence (Figures 8 and 9) was greater for surrogate data than for the original data ($F_{TI}(1,6) = 22.38$, $p<.05$, $F_{CRT}(1,6) = 5.65$, $p<.05$). Similar results were obtained by Heath (2000) on interkey tapping time series, and handwriting velocity data. He suggested the presence of determinism in those time series, but could not differentiate effects of noise component on divergence parameter. According to same author, divergence is related to LLE; the larger the divergence, the greater is evidence for chaos or perhaps noise.

Divergence computed on CRT and TI time series could also be related to correlation dimensions, because surrogate data are primarily saturated with noise. There was also significant task difficulty effect on divergence in CRT series ($F(1,6) = 15.12$, $p<.05$), but not in TI time series ($F(2,12) = 0.32$, $p>.05$). It was greater for more difficult task then for easier task. In comparison to LLE, it could be presented that CRT in more complex task is more chaotic then in easier task. There were no significant differences in divergence of CRT series during independent and simultaneous task performance ($F(1,6) = 0.4$, p>.05).

Determinism

This parameter relates to the predictability of the time series. Determinism (%DETER) is defined by the ratio of the number of recurrent points forming upward diagonal lines and the total number of recurrent points (Heath, 2000). The percentage recurrence (%RECUR) was computed by dividing the number of recurrent points in the upper triangular region of the recurrence plot by the area in that region. Index of determinism is defined by the ratio of %DETER and %RECUR. It is usually greatest for the cyclic

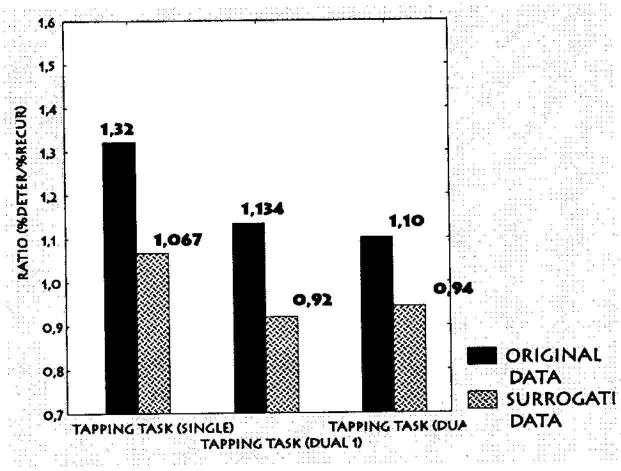


Figure 10. Index of determinism (ratio of %deter/%recur) of TI time series

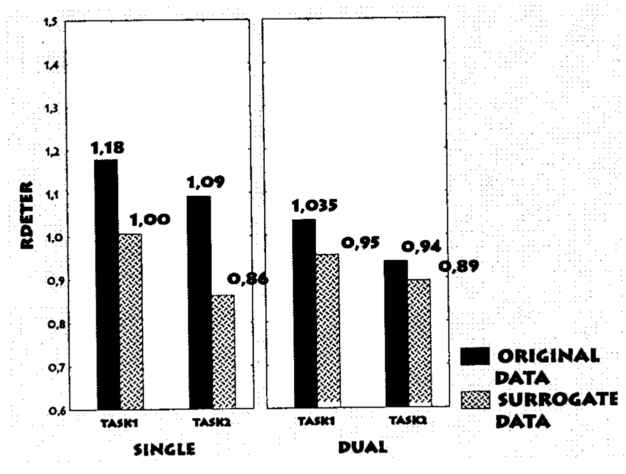


Figure 11. Index of determinism (ratio of %deter/%recur) of CRT series

time series and the lowest for white noise. Chaotic time series has an intermediate level of determinism.

As could be expected, results indicate that level of determinism is significantly lower in surrogate series ($F_{TI}(1,6) = 40.65, p<.05$, $F_{CRT}(1,6) = 16.29, p<.05$; Figures 10 and 11). There were also significant differences in TI time series and CRT series depending on a task difficulty ($F(2,12) = 5.05, p<.05$; $F(1,6) = 6.09, p<.05$). It seems that independent performance of tapping task, as well as CRT series in less complex task, could be primarily determined by linear components. On the contrary, TI series during simultaneous performing and CRT series in more complex task, could be primarily determined by non-linear components. There were no significant differences in determinism of CRT series during independent and simultaneous task performance ($F(1,6) = 2.6, p>.05$).

DISCUSSION AND CONCLUSIONS

Variability of complex reaction times and tapping time intervals could not be interpreted as stochastic fluctuations and could not be treated as error of measurement. Dimensionality of both psychomotor activities (simple tapping activity, response time dynamic) is probably finite and possibly originates from simultaneous psychomotor processes spanning in range from 6 to 10.

Dynamics of both psychomotor activities could be deterministic in nature, that is, produced by non-linear and linear interactions of different variables. The clear evidence of non-linear determinism was found in tapping activity during simultaneous performance, and in the response time dynamics during more difficult task performance. Linear determinism was found in the less complex activities (simple tapping task independently performed and response time dynamics produced by performance of easier CRT task).

Greater variability of output variables produced by more complex task performance does not necessarily mean more complex dynamical structure of underlying system as well. On the contrary, greater variability in output variables could be determined by non-linear interactions between a smaller number of factors, indicating the existence of less complex dynamical structure of underlying processes (low dimensional chaos). Mental load related to easier CRT task is lower than in more complex task, which implies greater influence of more uncontrollable factors (i.e. attention fluctuations, motivation, environmental factors etc.) on psychomotor dynamics. Those uncontrollable factors could reflect on correlation dimension of the dynamic system by making it greater. In contrast, factors which are related directly to task requirements are dominant in the more complex CRT task, mental load is greater, uncontrollable factors' influence on psychomotor activity are probably suppressed and nonlinear interaction of underlying processes are more evident.

Simultaneous performance of two psychomotor activities has significant effect on dynamical properties of secondary activity. Those properties could not be explained only by descriptive variability parameters, but with the changes in whole dynamical structure of underlying processes.

Based on summarised arguments (all computed indices), it could be concluded that complex reaction time and simple tapping activity are most probably product of nonlinear, chaotic dynamics of some (6-10) cognitive and motoric simultaneous and interacting processes.

REFERENCES

Ayers, S. (1997). The application of chaos theory to psychology. *Theory & Psychology*, 7, 377-398.

Beltz, B.C., & Kello, C.T. (2004). The effect of cue predictability on long-range dependencies in response times versus response durations. *Proceedings of the 26th Annual Meeting of the Cognitive Science Society.* Chicago, IL.

Brown, S., & Heathcote, A. (2005). A ballistic model of choice response time. *Psychological Review, 112,* 117-128.

Cooney, J.B., & Troyer, R. (1994). A dynamic model of reaction time in a short-term memory task. *Journal of Experimental Child Psychology, 58,* 200-226

Ding, M., Chen, Y., & Kelso, J.A.S. (2002). Statistical analysis of timing errors. *Brain & Cognition, 48,* 98-106.

Elbert, T., Ray, W.J., Kowalik, Z.J., Skinner, J.E., Graf, K.E., & Birbaumer, N. (1994). Chaos and physiology: deterministic chaos in excitable cell assemblies. *Psychological Reviews, 74,* 1-47.

Frey, B., & Clayton, K. (1996). Examining sequence effects using nonlinear dynamics. *Abstracts for the Sixth Annual International Conference of the SCTPLS.* University of Carolina, Berkeley.

Gilden, D.L. (1997). Fluctuations in the time required for elementary decisions. *Psychological Science, 8,* 296-301.

Gilden, D.L., Thornton, T., & Mallon, M.W. (1995). 1/f noise in human cognition? *Science, 267,* 1837-1839.

Grassberger, P., & Procaccia, I. (1983). Measuring the strangeness of strange attractors. *Physica D, 9,* 189-208.

Guastello, S.J. (2000). Nonlinear dynamics in psychology. *Discrete Dynamics in Nature and Society, 6,* 11-29.

Heath, R. A. (2000). *Nonlinear dynamics: Techniques and applications in psychology*. Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Kelly, A., Heathote, A., Heath, R., & Longstaff, M. (2001). Response time dynamics: Evidence for linear and low-dimensional nonlinear structure in human choice sequences. *Quartetly Journal of Experimental Psychology, 54*, 805-840.

Pressing, J. (1999). Sources for 1/f noise effects in human cognition and performance. Dynamical cognitive science: *Proceedings of the forth Australian Cognitive Science Conference* (CDROM), Newcastle, Australia.

Proroković, A. (2002). Testiranje kaotične dinamike metodom kontrolnih podataka [Testing of chaotic dynamics by the surrogate data method]. *Medica Jadertina, 32*, 107-113.

Robertson, R. (1995). Chaos Theory and the relationship between psychology and science. In R. Robertson & A. Combs (Eds), *Chaos theory in psychology and the life sciences*. Mahwah, New Jersey: Lawrence Erlbaum Associates, 3-17.

Theiler, J., Eubank, S., Longtin, A., Galdrikian, B., & Farmer, J. D. (1992). Testing for nonlinearity in time series: the method of surrogate data. *Physica D, 58*, 77-94.

Zbilut, J.P., Giuliani, A., & Webber, C.L., Jr. (1998). Recurrence quantification analysis and principle components in the detection of short complex signals. *Physics Letters A, 237*; 131-135.

Wolf, A. Swift, H. J.B., Swiney, H.L., & Vastano, A. (1985). Determining Lyapunov exponents from a time series. *Physica, 16*, 285-317.

Jarvis, B. (2000). *Direct RT research software user's guide and reference*. New York: Empirisoft Corporation.