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CALCULATION OF COMPONENTS OF THE RHUMB LINE INTERSECTION WITH THE EQUATOR

IZRAČUN ELEMENATA PRESJECIŠTA LOKSODROME S EKVATOROM

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Summary

The paper presents a mathematical method ensuring a greater accuracy in calculating the co-ordinates of rhumb line intersection with the equator as well as the rhumb line distance from the departure point to the point of rhumb line intersection with the equator. The basic condition requested for the proposed method is to approximate the shape of Earth to a selected reference biaxial spheroid.

Key words : rhumb line, equator, intersection

Sažetak

U radu je izložen matematički postupak pomoću kojeg se mogu preciznije izračunati koordinate presjecišta loksodrome s ekvatorom i loksodromska udaljenost od pozicije polaska do pozicije presjecišta loksodrome s ekvatorom.Osnovni uvjet za predloženi postupak je da se oblik Zemlje aproksimira odabranim referentnim dvoosnim rotacijskim elipsoidom.

Ključne riječi: loksodroma, ekvator, presjecište

1. Introduction
Uvod

In maritime surface navigation the most common and simplest method of sailing is the rhumb line sailing. In the case when the departure point (P_1) is in one hemisphere, and the destination point (P_2) in the other the rhumb line crossing the two points will intersect the earth equator at the point S_L , Fig. 1 and 2. To perform these calculations the shape of the Earth should be approximated to a selected reference biaxial spheroid. The objective of the paper is to work out a mathematical method that, if compared with the traditional methods so far employed, will ensure a more accurate determination of the rhumb line co-ordinates with the equator as well as the rhumb line distance from the departure point (P_1) to the point of rhumb line intersection with the equator (S_L).

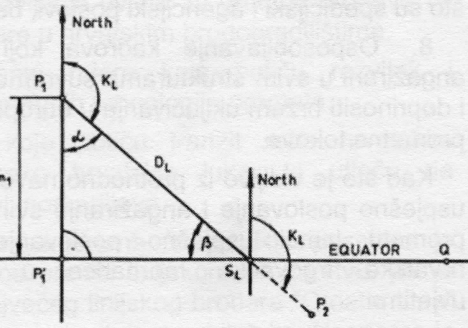

Figure 1. Rhumb line triangle on a selected reference biaxial spheroid

Slika 1. Loksodromski trokut na odabranom referentnom dvoosnom elipsoidu

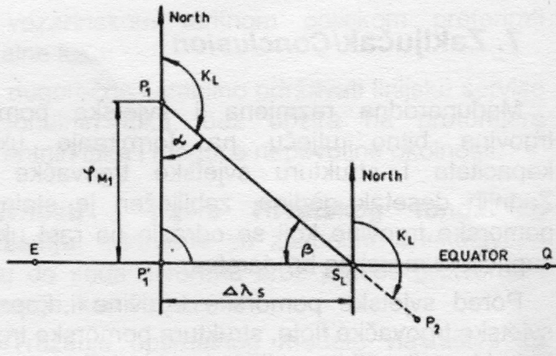

Figure 2. Rhumb line triangle on the Mercator projection chart

Slika 2. Loksodromski trokut na navigacijskoj karti Mercatorove projekcije

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2. Calculating the co-ordinates of the rhumb line intersection with the Equator
Izračunavanje koordinata presjecišta loksodrome s ekvatorom

The geometric relationships among relative values in the rhumb line triangle of the Mercator projection chart are shown in Fig. 2, where:

The leg $P_1P'_1 = \Delta\varphi_M = \varphi_{M1} - 0 = \varphi_{M1}$ ... of the Mercator latitude of the departure point is expressed in minutes of arc and can be calculated by the following equation :

$$\varphi_{M1} = 7915{,}70446 \log \left[tg\left(45^\circ + \frac{\varphi_1}{2}\right)\left(\frac{1 - e\sin\varphi_1}{1 + e\sin\varphi_1}\right)^{\frac{e}{2}} \right] \quad (1)$$

The equation (1) may be applied to any of the selected reference biaxial spheroid on condition that:

$e = \frac{\sqrt{a^2 - b^2}}{a}$ ... is the first numerical excentricity of the selected spheroid

a – equator semi-axis of the spheroid

b – meridian semi-axis of the spheroid

K_L – general rhumb line course between the points P_1 i P_2

β - inclination of rhumb line, i.e. the angle formed by the rhumb line on intersecting the earth equator EQ

α - angle between the legs $P_1P'_1$ and the hypotenuse P_1S_L

Between the angle α and angle K_L there is the following functional interdependency :

Case I : $\varphi_1 > \varphi_2$ (Point P_1 in the north, point P_2 in the south hemisphere)

a) **II. Navigational quadrant**

$\alpha = 180^\circ - K_L$

$90^\circ < K_L < 180^\circ \Rightarrow$ navigating EASTWARD (sign $\Delta\lambda_S$ +)

b) **III: Navigational quadrant**

$\alpha = K_L - 180^\circ$

$180^\circ < K_L < 270^\circ \Rightarrow$ navigating WESTWARD (sign $\Delta\lambda_S$ -)

Case II): $\varphi_1 < \varphi_2$ (Point P_1 in the south, point P_2 in the north hemisphere)

c) **I. Navigational quadrant**

$\alpha = K_L$

$0^\circ < K_L < 90^\circ \Rightarrow$ navigating EASTWARD (sign $\Delta\lambda_S$ +)

d) **IV. Navigational quadrant**

$\alpha = 360^\circ - K_L$

$270^\circ < K_L < 360^\circ \Rightarrow$ navigating WESTWARD (sign $\Delta\lambda_S$ -)

$\Delta\lambda_S$ – relative co-ordinate, or difference of longitude between the meridian point of rhumb line intersection with the equator S_L and the meridian of departure point P_1

From the rhumb line triangle shown on the Mercator projection chart (Fig. 2) $\Delta\lambda_S$ can be worked out as follows:

$$\Delta\lambda_S = \varphi_{M1}\ \mathrm{tg}\alpha \quad (2)$$

The co-ordinates of point S_L are : ($\varphi = 00^\circ\ 00'\ 00''$, $\lambda = \lambda_1 + \Delta\lambda_S$).

3. Calculating the rhumb line distance from point P_1 to point S_L
Izračunavanje loksodromske udaljenosti od pozicije P_1 do pozicije S_L

The geometric relationships of relative values in the rhumb line triangle on the reference biaxial spheroid are shown in Fig. 1, where :

the leg $P_1P'_1 = \Delta\varphi_m = \varphi_{m1} - 0 = \varphi_{m1}$ ... is the meridian distance from the equator to the depature point P_1 expressed in nautical miles and can be worked out by the following equations :

$$\varphi_{m1} = \frac{a(1-e^2)}{1852}\left(\frac{A}{\delta}\varphi_1^\circ - \frac{B}{2}\sin 2\varphi_1 + \frac{C}{4}\sin 4\varphi_1 - \frac{D}{6}\sin 6\varphi_1 + \dots\right) \quad (3)$$

a – equator semi -axis of the selected biaxial spheroid expressed in metres

$$\delta = \frac{180}{\pi}, \quad e = \frac{\sqrt{a^2 - b^2}}{a}$$

φ_1° - latitude of departure point P_1 expressed in degrees

$$A = 1 + \frac{3}{4}e^2 + \frac{45}{64}e^4 + \frac{350}{512}e^6 + ...$$

$$B = \frac{3}{4}e^2 + \frac{60}{64}e^4 + \frac{525}{512}e^6 + ...$$

$$C = \frac{15}{64}e^4 + \frac{210}{512}e^6 + ... \quad (4)$$

$$D = \frac{35}{512}e^6 + ...$$

β - inclination of rhumb line

α - angle between the leg $P_1P'_1$ and hypotenuse P_1S_L

hypotenuse $P_1S_L = D_L$ – rhumb line distance in nautical miles from the departure point P_1 to the point of rhumb line intersection with the equator S_L

From the rhumb line triangle shown in Fig. 1, D_L can be worked out as follows :

$$D_L = \frac{\varphi_{m1}}{\cos\alpha} \quad (5)$$

4. Illustrative example
Demonstracijski primjer

Departure point P_1 (φ_1 = 35°26′ N , λ_1 = 139°36′ E). General rhumb line course

K_L = 109°25′. Work out the co-ordinates of point S_L and the value D_L for WGS-84. Use the traditional method of calculation of the same data considering the shape of the Earth as an approximate sphere.

4.1. WGS-84

a) Co-ordinates of point S_L

α = 70°35′ , φ_{M1} = 2262,7594′

$\Delta\lambda_S$ = + 106°59′30″

S_L (φ = 00°00′00″ , λ = 113°24′30″ W)

b) Rhumb line distance from P_1 to S_L

φ_{m1} = 2118,0718 M

D_L = 6371,3768 M

4.2. The Earth as a sphere
Zemlja kao kugla

a) Co-ordinates of point S_L

φ_{M1} = 2276,1116′

$\Delta\lambda_S$ = + 107°37′22″

S_L (φ = 00°00′00″ , λ = 112°46′38″ W)

b) Rhumb line distance from P_1 to S_L

φ_1 = 2126 M

D_L = 6395,2257 M

5. Conclusion/*Zaključak*

The rhumb line is a spiral curve of the unchanged navigational course asymptotically approaching the pole of the hemisfere the ship is sailing in. On the Mercator projection the curve is shown by the direction enabling calculation of the rhumb line sailing course. When the departure point and destination point are in different hemispheres, the rhumb line intersects the earth equator at point S_L. By means of the rhumb line triangle shown on the Mercator projection chart it is possible to calculate the co-ordinates of the rhumb line intersection with the equator or the general rhumb line (K_L) and the rhumb line distance from the departure point P_1 to point S_L can be calculated by means of the rhumb line triangle formed on the selected reference biaxial spheroid. If, however, the leg of the rhumb line triangle lying on the meridian of departure point is used to calculate the meridian distance from the earth equator to the departure point (φ_{m1}) , and not from the latitude of departure point (φ_1) , a more accurate result can be obtained. The whole procedure of the method is dealt with in detail in the paper.

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