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ON THE SENSITIVITY OF RESIDUAL STRESSES DETERMINED USING THE HOLE-DRILLING METHOD

OSJETLJIVOST MJERENJA ZAOSTALIH NAPREZANJA ZABUŠIVANJEM

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Abstract

The objective of this paper is to examine the effects of measurement error on residual stresses evaluated with the Hole drilling and Integral Method calculation. A detailed analysis of causes and magnitudes of strain measurement error is beyond the scope of this paper. The indicators needed for the error estimation and depth increment distributions with respect to systematic and random errors are discussed. It is shown that the maximum attention must be devoted to random errors. According to presented result it was concluded that practically no more than three points of residual stress distribution can be calculated.

Key words: residual stress measurement, hole drilling technique, sensitivity

Sažetak

Tema je ovoga rada utvrditi utjecaj pogreške mjerenja na zaostala naprezanja primjenom metode zabušivanja i integralne metode proračuna. Ovdje nije dana detaljna analiza iznosa i uzroka pogreške u izmjerenim deformacijama. Opisani su parametri koji su potrebni za procjenu pogreške i rastavljeni na sustavni i slučajni udio. Dokazano je da se prilikom mjerenja veća pozornost mora posvetiti slučajnim pogreškama. Prema prikazanim rezultatima analize zaključena je mogućnost izračunavanja najviše tri točke raspodjele zaostalih naprezanja po dubini.

Ključne riječi: mjerenje zaostalih naprezanja, metoda zabušivanja, osjetljivost

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1. Introduction
Uvod

Residual stresses exist in the object without application of any operational or other external loads. The manufacturing process may introduce them or they may occur or be altered during the life of the structure. Residual stresses can have impact on behavior of the component both during fabrication and in service. It is known that compressive residual stresses can enhance the life or performance of the product and so they are usually considered to be a desired result of the manufacturing process. Conversely, the tensile residual stresses may have detrimental effects, especially in the corrosive environment. Clearly, residual stress state is an important inherent factor as the usual mechanical properties of the material in the consideration of the fatigue-strength and other analysis of design members functionality. Therefore, the integrity of engineering decisions is dependent on the residual stress determination feasibility and understanding of limitations on methods used.

Residual stresses can be measured in nondestructive and in destructive manner. Nondestructive methods are based on the relationship between the physical or crystallographic parameters.

Hole drilling method, ring core method, bending deflection method and sectioning method are the destructive methods of residual stress measurements. Considering the small hole size with respect to the component size, the hole drilling method can be specified as nondestructive, meaning the component remains functional after the measurement, and therefore usually is referred to as 'semi-destructive' method. The hole drilling strain gage method requires only a small fraction of the time, effort and cost associated with other destructive methods of residual stress measurements, and is commonly used in practical field applications. Destruction of the state of equilibrium of residual stresses in mechanical component creates the relaxation of residual stresses. That process implies a change in strains, which can be measured and mathematically related with relaxed stresses. The first used, and the simplest approach is by analytical solution for the strain distribution nearby the circular hole. That approach is valid for the hole drilled completely through a thin plate in which the residual stress is uniformly distributed through the thickness [1]. Machine parts are rarely thin enough and shallow hole must be used. It was empirically deduced when the depth of the hole is bigger than the hole diameter and the relieved stresses are uniform, the measured relaxed strains are approximately equal to those obtained by analytical solution [2,3,4]. The calculations quoted are called Equivalent Uniform Stress Method, which give the weighted average of the relaxed stresses.

In many cases residual stress fields are significantly non-uniform, particularly in areas close to the surface, so their averaging can result in an incorrect estimate. It is difficult to predict the distribution of residual stresses, and stress uniformity assumption cannot be implemented a priori.

Therefore, it is recommended to apply non-uniform instead of one-step residual stress measurements [2,3]. Non uniform residual stress measurements use relieved strain data taken after several successive small increments of hole depth. There are recommendations of ASTM [2] to estimate the validity of using the Equivalent Uniform Stress Method.

That standard specifies checking measured strain relaxation data to confirm that the residual stresses are sufficiently uniform. This validation is to be performed by comparison of the measured relaxed strain distribution shapes with standard curves. Validation test could yield two conclusions and quantitative results respectively:

case 1: When the shape of the measured strains are similar to the strain distribution obtained with constant stresses, it is confirmed that the relieved stresses are constant with depth, so the calculated stress value is accurate enough.

case 2: The difference between the measured distribution and the distribution obtained with constant stress requires the rejection of measurement results and leads to two possible conclusions: (1) that the measured residual stresses significantly vary with depth and due to averaging error the analysis results can not be used, (2) that the measurement procedure is not accurate enough.

Comment on possible conclusions: knowing the nature of generation of residual stresses, the case (1) can be expected with high frequency. In this case the dilemma is not resolved whether the distribution has a variation with depth, or the measurement include a significant error. A possible resolution of this dilemma may be attained by analysis presented in this paper.

There are some efforts to resolve stress fluctuations through the depth by using strains measured after successive increments of hole depth [4,5,6,7,8,9]. It is possible to yield a common conclusion that the ability of the method to measure sub-surface stresses is limited by the high sensitivity of the hole drilling method to the measurement error. In order to overcome this drawback the maximum depth of analysis and the number of calculation points must be decreased. In other words, the poor selection of calculation parameters could adversely affect, or even invalidate, the result.

The theoretically acceptable calculation methods [5,6,8,9] can distinguish between the influence of error due to gradient and error sensitivity of the method. Schajer [5] was the first to assess that the method allows line approximation of the stress distribution. Vangi [10] established, using the numerical estimate of error, the possibility of getting 4 to 5 points. Zuccarelo [11] established possibility of getting up to 8 points.

The analysis of sensitivity of the Integral Method calculations accomplished by means of numerical analysis will be shown. In the next chapters, the various types of error in strain measurement and its impact on final results are investigated. In this way, it is possible to characterize the procedure of optimal depth increments determination, and also to identify some fundamental limitations.

2. Background
Osnove postupka mjerenja

This section comments on data analysis techniques. For conceptual simplicity theoretical aspects will not be discussed in details.

Successive relieving of stresses is a nonlinear process because the stresses in formerly relieved layers have an impact on changes in strain [4]. It was shown that calibration constants obtained by finite elements closely correlate with experimental results and that the theoretically correct representation of the stress relieving process can only be obtained by using numerical models.

Numerical calibration is to be performed by application of the opposite stresses to the hole boundary to account for the presence of the hole, figure 1. Applying the stresses over the increment thickness ΔH solves the problem of nonlinearity, i.e. separation of influence of stresses at certain depth H when the depth of the hole is h.

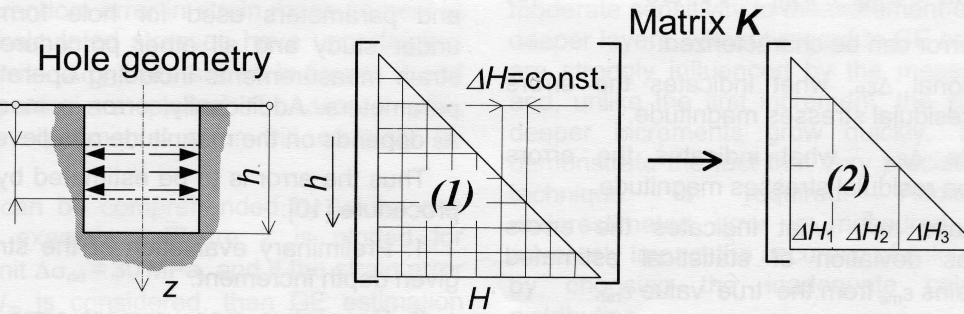

Figure 1. Simplified schematic representation of the stress relaxation model and calibration constant matrix *K*.

Slika 1. Shematski prikaz modela otpuštanja naprezanja i matrice konstanti baždarenja K.

The relation between the strain relief measured by each element of strain-gage rosette and residual stresses, according to the Integral Method [6,8,9], is given by equation *(1)*. The vector ε represents a set of strains measured at the particular hole depth, and σ represents a set of equivalent uniform residual stresses within corresponding hole depth increment ΔH.

$$\varepsilon = \mathbf{K} \cdot \sigma \tag{1}$$

Matrix *K* represents calibration constants and depends on the hole geometry, the strain-gage geometry, the elastic properties of the material under study and the distribution of depth increments. It was shown [4] that the specifications regarding hole depth, size of the specimen and the hole radius should be given in terms of mean gage radius, *R*.

The other theoretically acceptable approach, also based on finite element calculated calibration data, is Power Series Method [4]. Superposing calibration functions in a least-squares analysis of the measured strain relief's derives the residual stress field. Calibration functions are obtained by finite element calculations applying power series stress profiles with depth.

Theoretically, the Integral Method and the Power Series Method, in the limit case of reducing increment thickness will yield correct results. The calibration matrix is the base of the Integral Method, and using this matrix, the Power Series Method calculations can be made, but not inversely. Since all other calculation methods can be shown as modification of the Integral Method [8], matrix representation of the calibration constants is justified. Attached are the calibration constants of the rosette HBM RY61, developed and used for calculations in this paper.

St. Venant's principle indicates that the surface strain response becomes increasingly insensitive to stress in deeper increments. Consequently, stress calculation becomes highly sensitive to strain measurement error as the depth from the surface *H* increases, and hole depth increment ΔH decreases. In order to reduce sensitivity of stress calculation, hole depth increments must be taken progressively larger [9,10]. The original calibration matrix is schematically shown on the figure 1, indicated as *(1)*. Resized matrix is indicated as *(2)*.

3. Indicator needed for the error estimation

Parametri potrebni za procjenu pogreške

The term error signifies a deviation of the result from some 'true' value and distinction must be drawn between an estimate of errors in measured strains $\Delta\varepsilon$ and errors in calculated stresses $\Delta\sigma$ (the quantity that to be measured). These stresses are hereafter referred as the measurement error and the $\Delta\varepsilon$ is referred as the strain measurement error.

The sensitivity of the hole drilling method is defined in the matrix *K* and depth increments used in calculation can be determined only based on estimated strain measurement error. Thus, the variation of the calculation steps in order to obtain stress uncertainty values within preset limits will result in minimum possible increment thickness. It must be noted that the residual stresses calculated by Integral Method are equivalent uniform residual stresses within depth increments and the increasing size of the depth increments rises the averaging error.

In case of absolute accurate measurements of the residual stress σ_{ref}, according to equation *(1)*, strains ε_{ref} should be acquired. In practice the relation between the 'true' value of stresses σ_{ref} and the measured strains ε_{me} is:

$$\varepsilon_{me} = \mathbf{K} \cdot (\sigma_{ref} + \Delta\sigma) \tag{2}$$

where $\Delta\sigma$ is the resulting error in calculated stresses.

The strain measurement error $\Delta\varepsilon$ is a discrepancy between the measured strains ε_{me} and the calculated strains ε_{ref}.

$$\Delta\varepsilon = \varepsilon_{me} - \varepsilon_{ref} = \mathbf{K} \cdot \Delta\sigma \quad (3)$$

Generally error can be characterized:

- as proportional $\Delta\varepsilon_P$, what indicates the errors depenent on residual stresses magnitude.
- as absolute $\Delta\varepsilon_A$, what indicates the errors independent on residual stresses magnitude.
- as systematic $\Delta\varepsilon^S$, what indicates the errors determined as deviation of statistical estimated measured strains ε_{me} from the 'true' value ε_{ref}.
- as random $\Delta\varepsilon^R$, what indicates the fluctuations in results which yield results that differ from experimnet to experiment.

Therefore an arbitrary strain measurement error can be decomposed:

$$\begin{aligned}\Delta\varepsilon &= \Delta\varepsilon_P + \Delta\varepsilon_A = \Delta\varepsilon^S + \Delta\varepsilon^R \\ &= \Delta\varepsilon_P^S + \Delta\varepsilon_P^R + \Delta\varepsilon_A^S + \Delta\varepsilon_A^R\end{aligned} \quad (4)$$

The measurement error can be defined as sum of the proportional and absolute fractions.

$$\Delta\sigma = \Delta\sigma_P + \Delta\sigma_A \quad (5)$$

Using the equation *(1)*, absolute and proportional strain measurement errors can be written:

$$\Delta\varepsilon_A = \Delta\varepsilon_A^S + \Delta\varepsilon_A^R = \mathbf{K} \cdot \Delta\sigma_A \quad (6)$$

$$\Delta\varepsilon_P = \Delta\varepsilon_P^S + \Delta\varepsilon_P^R = \mathbf{K} \cdot \Delta\sigma_P \quad (7)$$

Proportional fraction of error $\Delta\varepsilon_P$ is to be determined by a procedure similar to the experimental calibration relating the strain gage readings to imposed stresses and presented in the form of calibration constants ΔK.

$$\Delta\varepsilon_P = \Delta\mathbf{K} \cdot \sigma_{ref} = \Delta\varepsilon_{me} - \mathbf{K} \cdot \Delta\sigma_{ref} \quad (8)$$

In that procedure the absolute strain measurement error $\Delta\varepsilon_A$ must be controlled or compensated. All of the members in equation *(8)* are vectors and matrix ΔK can only be calculated in diagonal form by using Equivalent Uniform Stress Method calculation. If matrix ΔK can be determined in lower triangular form the correction of measurements can be applied. Unfortunately the experimental approach alone is insufficient for the corrections. Suggested experimental analysis must provide data for estimation of the measured strain error.

The depth increment determination is based on criteria that calculated residual stresses must be accurate up to a preset error tolerance σ_{dd}. According to equations *(2)-(1)* (measured - reference), the distribution of layers depends on satisfying the condition that the error obtained is equal or slightly less than agreed.

$$\Delta\varepsilon \approx \mathbf{K} \cdot \Delta\sigma_{dd} \quad (9)$$

Unknown quantity $\Delta\varepsilon$ is intrinsic to the procedure and parameters used for hole forming, material under study and all other procedures involved in strain measurements including operator dependent parameters. Additionally, error in measured strains $\Delta\varepsilon$ depends on the magnitude of relieved stresses.

Thus the error is to be estimated by the following procedure [10]:

1. Preliminary evaluation of the stresses for the given depth increment.

2. Respective measurement error estimation $\Delta\varepsilon$ by using the calculated stresses and error indicators.

3. If the estimated error is higher than preset error the size of analyzed layer must be increased, otherwise the size of the increment is to be decreased.

Calculation steps from 1 to 3 must be repeated until the convergence is reached which yields the increment size and respective average stresses. After the particular increment is calculated the procedure is to be repeated with a new subincrement. If the bottom depth of the analyzed increment is higher than the theoretical maximum [9] (approximately *h/r* = 0.6) and the estimated error does not fulfil the preset error criteria the calculation is finished. If the stresses for the remaining depth increment are calculated they must be indicated with a respective estimated error which is higher than the preset error.

In this paper the impact of strain measurement error ($\Delta\varepsilon$) on error in calculated stresses ($\Delta\sigma$) is evaluated assuming limiting error estimation (LE) and Gaussian error propagation (GE). The LE approach yields a highest possible error estimation, and statistically has a very low probability. Assuming the error can be estimated as the first term of Gaussian formula for error distribution, where n is the number of depth increments.

$$\Delta\sigma(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n) = \sqrt{\sum_{i=1}^{n} \left| \frac{\partial\sigma(\varepsilon)}{\partial\varepsilon_i} \Delta\varepsilon_i \right|^2} \quad (10)$$

This estimation is valid when systematic errors can be neglected. Thus, the basic assumption of error estimation must be assured by quantifying the random and systematic errors for measurement procedure used.

4. Numerical results
Rezultati numeričke analize

In the following analysis these two approaches of error estimation will be described and their relation will be discussed.

To characterize the essential nature of the sensitivity problem, equation *(9)* was employed and the strain measurement error ($\Delta\varepsilon$) was taken as independent of stress state. First the effect of random error will be considered. Depth increments for the given random error in strain measurement in cases when calculated stresses have uncertainties below fixed limits ($\Delta\sigma_{dd}$), are shown in figures 2 and 3. The diagrams are plotted for $\Delta\varepsilon > \pm 2\ \mu^{m}/_{m}$ because the experimental error will certainly be higher.

Diagrams can be comprehended by taking one point as an example. Figure 2 is plotted for uncertainty limit $\Delta\sigma_{dd} = 30$ MPa, and if the strain error of $\Delta\varepsilon = \pm 5\ \mu^{m}/_{m}$ is considered, than GE estimation yields first increment up to 0.23 mm and second up to 0.62 mm. For the same error level the LE estimation yields the 0.27 / 1.29 mm distribution of depth increments. Stresses calculated for the increment size from the last calculated to the depth limit of analysis (1.5 mm) have a stress error higher than a given value.

The second depth increment in figure 2 can be reached for $\Delta\varepsilon < \pm 8\mu\varepsilon$. In figure 3, the uncertainties limit $\Delta\sigma_{dd}$ is increased. That figure illustrates that one point but, considering the values that can be obtained in practice, unlikely two additional points of residual stress distribution can be calculated. Those stresses are associated with double uncertainties and the contribution to overall estimation of residual stress distribution is disputable. The preliminary conclusion is that only the first increment has a moderate sensitivity to measurement error. All of the deeper layers, without regard to GE or LE approach, are strongly influenced by the measurement error and, unlike the first increment, the slopes of all of deeper increments grow quickly. Those figures demonstrate the fact that very precise experimental technique is required. Additionally, the underestimated error in measured strains could adversely impact the accuracy of calculated stresses by choosing the inadequate calculation step distribution.

A second example, figures 4 and 5, shows influence of relaxed strain hypothetically obtained with systematic and random error. Due to simplicity the distributions for third or deeper calculation layer were not plotted.

Distributions shown on the figures 2, 4 and 5 have the same maximum error level. The GE distributions on figures 2 and 5, according to equation *(10)*, are the same as displayed on figure 2. It can be seen that the LE estimate yields a better distribution of increments when the systematic error rises. Thus, the basic assumption of GE error propagation must be confirmed by determining the fractions of systematic and random errors. The

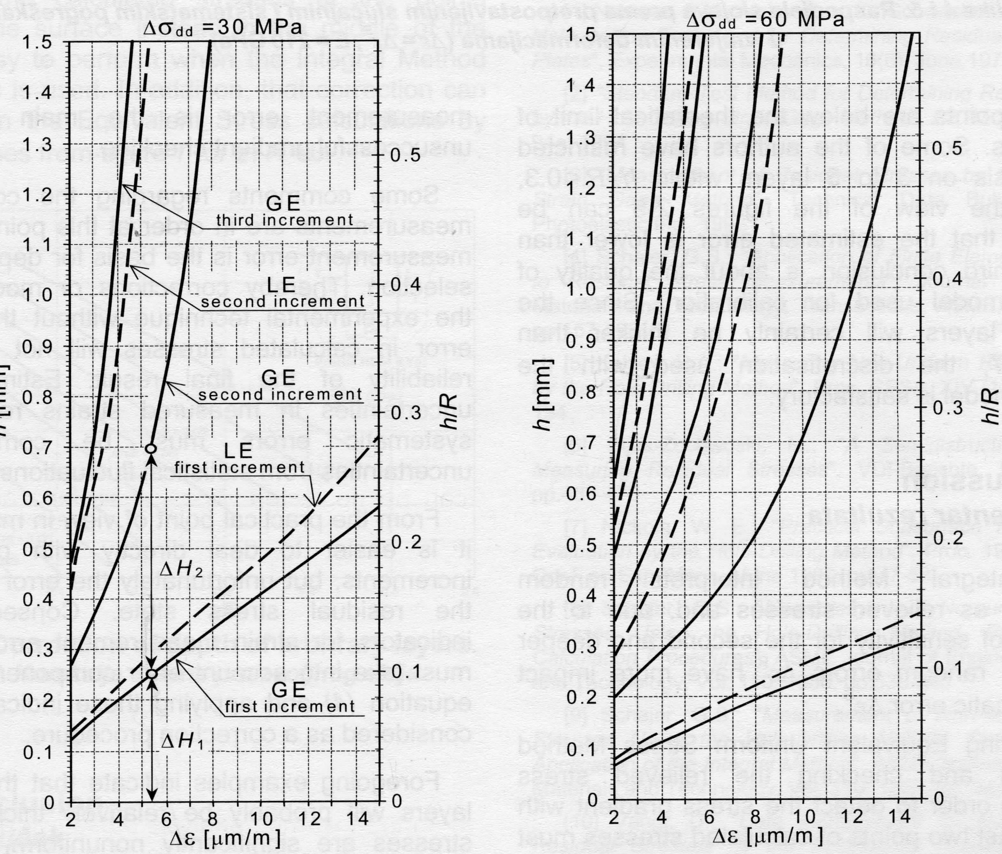

Figure 2-3. Distribution of the depth increments assuming random error in measured strains ($\Delta\varepsilon = \Delta\varepsilon^{R}$, $E = 210$ GPa).

Slike 2 i 3. Raspodjele slojeva uz pretpostavljenu samo slučajnu pogrešku u izmjerenim deformacijama ($\Delta\varepsilon = \Delta\varepsilon^{R}$, $E = 210$ GPa).

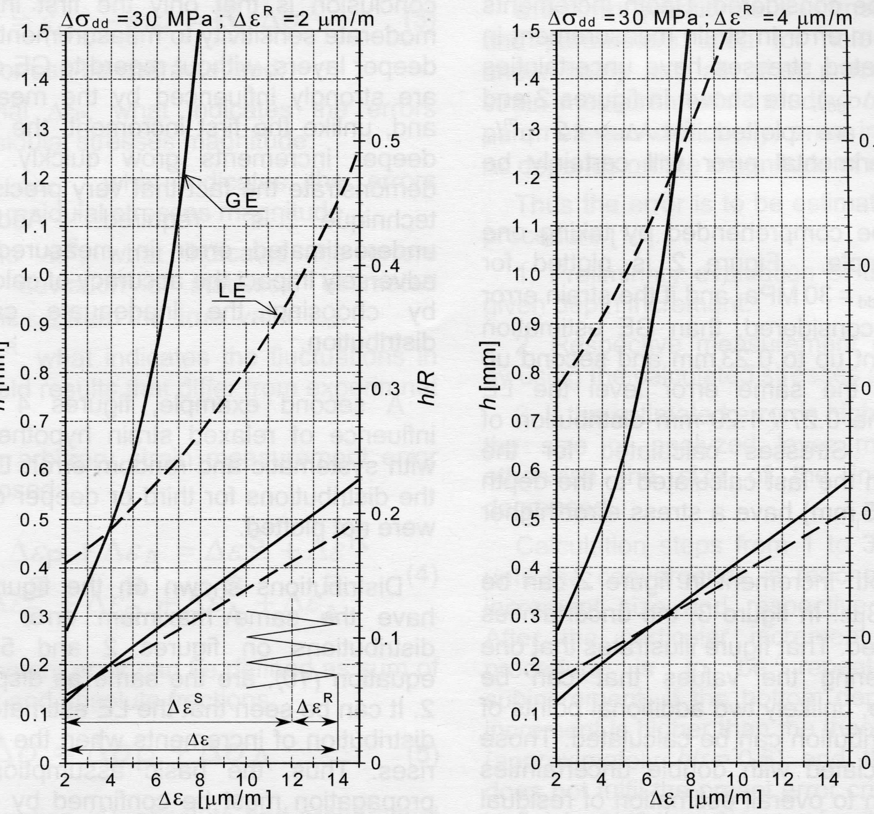

Figure 4-5. Distribution of the depth increments assuming random and systematic error in measured strains ($\Delta\varepsilon = \Delta\varepsilon^S \pm \Delta\varepsilon^R$).

Slike 4 i 5. Raspodjele slojeva prema pretpostavljenim slučajnim i sistematskim pogreškama u izmjerenim deformacijama ($\Delta\varepsilon = \Delta\varepsilon^R$, E = 210 GPa).

calculated points are below the theoretical limit of the analysis. Some of the authors have restricted their analysis on 3 to 5 layers within $h/R < 0.3$, which in the view of the figures 2-5 can be interpreted that the estimated error is lower than $\pm 5\,\mu^m/_m$. Third conclusion is about the quality of numerical model used for calibration. Since the calculated layers will certainly be thicker than $\Delta H/R = 0.07$, the discretisation used with the numerical model is satisfactory.

5. Discussion
Komentar rezultata

The Integral Method interprets random fluctuations as relieved stresses and, due to the rapid drop of sensitivity for the second and deeper increments, random errors $\Delta\varepsilon^R$ have more impact than systematic error $\Delta\varepsilon^S$.

Considering Equivalent Uniform Stress Method calculations and checking the relieved stress gradient. In order to detect the stress gradient with depth at least two points of calculated stresses must be found in figures 2-4. Error ranges that fit that criterion are below $\pm 10\,\mu^m/_m$. Accordingly to [2] precision of strain measurements alone of $\pm 2\,\mu^m/_m$ is required. Consequently, considering a number of possible sources of error involved in hole forming, especially when the applications are in-field, the measurement error is the main reason for unsuccessful gradient checking.

Some comments regarding the corrections of measurements are in order at this point. The strain measurement error is the basis for depth increment selection. Thereby corrections or modifications of the experimental technique without the estimated error in calculated stresses will not improve the reliability of the final result. Estimate of the uncertainties in measured strains resulting from systematic errors must be combined with uncertainties from statistical fluctuations.

From the practical point of view in many respects it is easier to deal directly with preset depth increments, but unfortunately the error depends on the residual stress state. Consequently the indicators for strain measurement error estimation must take into account error components written in equation *(4)* and applying those indicators can be considered as a correction procedure.

Foregoing examples indicate that the calculated layers will probably be relatively thick. If relaxed stresses are significantly nonuniform, due to the assumption that the stresses are constant within the calculation step, the averaging error will be in the direction of underestimating the maximum stress, figure 6. Power Series Method by using the set of functions to reconstruct the stress distribution has inherent property of depth correction.

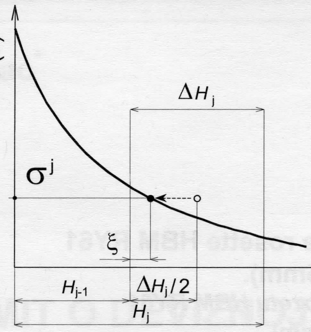

Figure 6. Schematic representation of the depth correction.

Slika 6. Shematski prikaz korekcije dubine.

Vangi [10] describes the procedure to set a position of calculated stresses within the increment, ξ. Theoretically this position varies with variation of the stress state, the increment size and depth. When the linear approximation of the stresses is used ξ becomes independent of the stress variation. Numerical trials on the various stress distributions carried out in this paper show that the procedure satisfactorily approximates stress distributions. A correction procedure was applied to variable-size calculation steps and the resulting ξ values are shown in figure 7.

Setting the calculated stresses at the points closer to the surface instead in the middle of the layer is easy to perform when the Integral Method calculations is used. In addition, that correction can be useful in the Equivalent Stress calculations by using ξ values from figure 7 for $z/R = 0$.

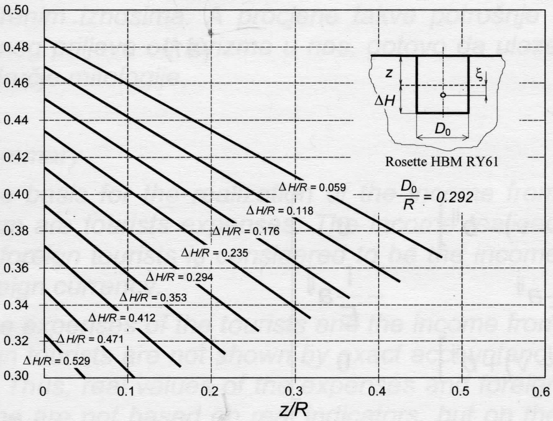

Figure 7. Correction of depth of arbitrary layers.

Slika 7. Korekcija dubine proizvoljnog sloja.

6. Conclusion
Zaključak

An investigation on error influence on Integral Method calculations has been made.

Schajer [9] was reported that, when using Integral Method calculations, three or four depth increments are a practical maximum. This investigation shows that three or unlikely four points of residual stress distribution can be calculated.

The analysis presented suggests that in case of straightforward measurements and stress calculations applying Equivalent Uniform Stress Method the increment of size $h/R = 0.10 \div 0.15$ should be applied. Than the averaging error will be decreased and by setting the calculated stresses closer to the surface, the results will be correctly interpreted.

This investigation highlights that maximum attention must be devoted to random errors. In case of hole drilling measurements, random errors cannot be statistically decreased by repeaing measurements and regression analysis of acquired relaxed strains is the only way to average out random errors.

Selecting inadequate calculation step size could increase error in calculated stresses. Therefore, in order to achieve reliable results indicators for optimal step selection must be developed and measurement procedure with all parameters involved must be tested.

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APPENDIX
PRILOG

The data needed for calculations, see equation (1).

Table a1. Calibration constants for strain gage rosette HBM RY61 (D_0=1.5mm, ΔH=0.15mm, R=2.55mm).

Tablica a1. Tablice konstanti baždarenja za rozetu HBM RY61 (D_0=1.5mm, ΔH=0.15mm, R=2.55mm).

a^{ij}

i \ j	1	2	3	4	5	6	7	8	9	10
1	0.01369									
2	0.01876	0.01397								
3	0.02174	0.01814	0.01251							
4	0.02368	0.02025	0.01599	0.01020						
5	0.02490	0.02158	0.01756	0.01303	0.00767					
6	0.02569	0.02238	0.01849	0.01420	0.00988	0.00530				
7	0.02618	0.02289	0.01903	0.01486	0.01074	0.00697	0.00330			
8	0.02651	0.02320	0.01935	0.01523	0.01121	0.00759	0.00451	0.00171		
9	0.02671	0.02340	0.01955	0.01544	0.01146	0.00791	0.00494	0.00255	0.00053	
10	0.02685	0.02353	0.01968	0.01557	0.01160	0.00808	0.00516	0.00283	0.00107	0.00032

b^{ij}

i \ j	1	2	3	4	5	6	7	8	9	10
1	0.02751									
2	0.03624	0.03015								
3	0.04182	0.03814	0.02942							
4	0.04570	0.04256	0.03652	0.02668						
5	0.04827	0.04548	0.04001	0.03276	0.02297					
6	0.04994	0.04735	0.04219	0.03549	0.02800	0.01903				
7	0.05100	0.04853	0.04354	0.03713	0.03011	0.02307	0.01531			
8	0.05166	0.04927	0.04437	0.03810	0.03133	0.02468	0.01847	0.01202		
9	0.05207	0.04973	0.04488	0.03868	0.03203	0.02558	0.01968	0.01443	0.00923	
10	0.05232	0.05001	0.04519	0.03903	0.03244	0.02608	0.02033	0.01533	0.01104	0.00694

$$\begin{bmatrix} \boldsymbol{\varepsilon}^1 \\ \cdots \\ \begin{bmatrix} \varepsilon_a^i \\ \varepsilon_b^i \\ \varepsilon_c^i \end{bmatrix} \\ \cdots \\ \boldsymbol{\varepsilon}^n \end{bmatrix} = \begin{bmatrix} \boldsymbol{K}^{11} & & 0 \\ \cdots & \boldsymbol{K}^{ij} & \cdots \\ \cdots & \cdots & \cdots \\ \boldsymbol{K}^{1n} & \cdots & \boldsymbol{K}^{nn} \end{bmatrix} \begin{bmatrix} \boldsymbol{\sigma}^1 \\ \cdots \\ \begin{bmatrix} \sigma_x^j \\ \sigma_x^j \\ \tau_{xy}^j \end{bmatrix} \\ \cdots \\ \boldsymbol{\sigma}^n \end{bmatrix} \qquad (a1)$$

$$\boldsymbol{K}^{ij} = \begin{bmatrix} -\frac{1}{2E}\left[a^{ij}(1+\nu)+b^{ij}\right] & -\frac{1}{2E}\left[a^{ij}(1+\nu)-b^{ij}\right] & 0 \\ -\frac{1}{2E}a^{ij} & \frac{1}{2E}a^{ij} & -\frac{1}{E}a^{ij} \\ -\frac{1}{2E}\left[a^{ij}(1+\nu)-b^{ij}\right] & -\frac{1}{2E}\left[a^{ij}(1+\nu)+b^{ij}\right] & 0 \end{bmatrix}$$

σ^j_x, σ^j_y = normal stress components in directions *a* and *b* within the calculation increment j, (*a*, *b* and *c* are used by manufacturer to denote the gages, a^{ij} and b^{ij} are the dimensionless calibration constants)

τ^j_{xy} = shear stress normal to directions *a* and *b* within the calculation increment j,

$\varepsilon^i_a, \varepsilon^i_b, \varepsilon^i_c$ = strain relaxation measured at gages *a*, *b* and *c* at the hole depth i,

n = number of increments,

E = Young's modulus, ν = Poisson's ratio.

Index i indicates hole depth (h = i ΔH), and index j indicates particular increment (H = i ΔH).